Quantum Assisted Secure Multiparty Computation

Manuel Batalha dos Santos

Thesis defence

18 September 2024

TÉCNICO LISBOA

Outline

Outline

- Motivation and outcomes

Outline

- Motivation and outcomes
- Quantum and classical oblivious transfer

Outline

- Motivation and outcomes
- Quantum and classical oblivious transfer
- Private phylogenetic trees

Outline

- Motivation and outcomes
- Quantum and classical oblivious transfer
- Private phylogenetic trees
- Quantum oblivious linear evaluation

Motivation

SMC

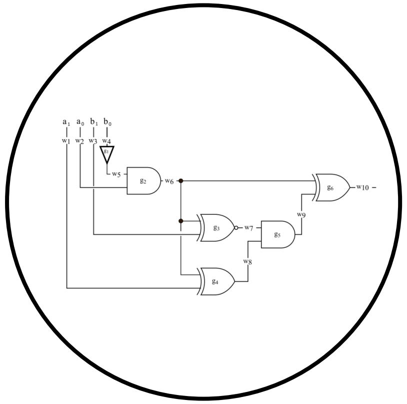
Motivation

SMC

Motivation

SMC

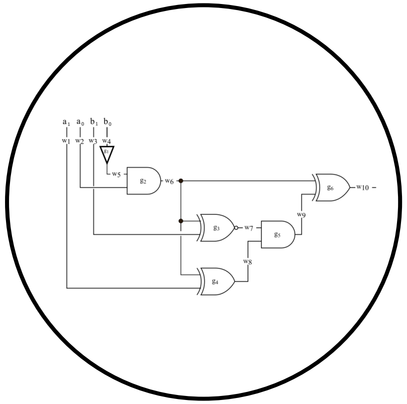
Boolean



Motivation

SMC

Boolean



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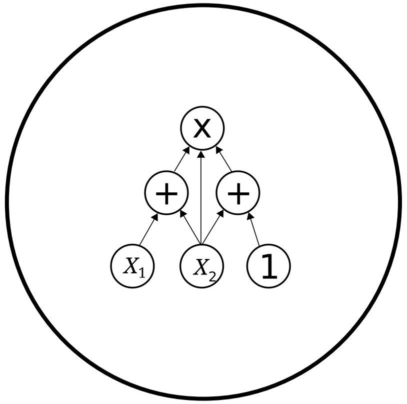
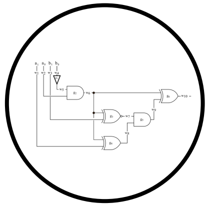
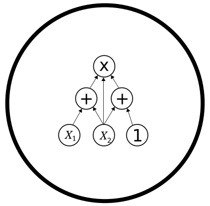


Motivation

SMC

Circuit



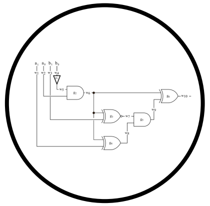


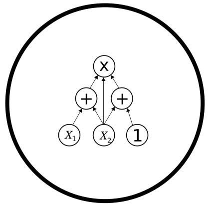
Motivation

SMC

Primitive

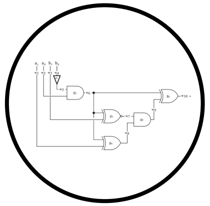
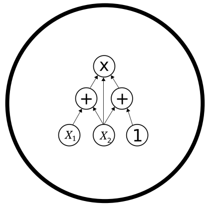
Circuit





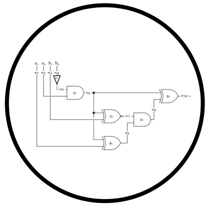
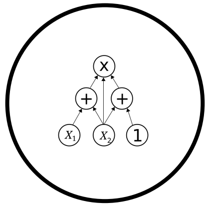
Motivation

SMC

Primitive	Oblivious Transfer	
Circuit		

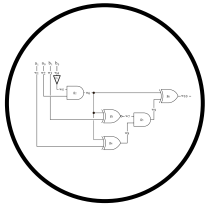
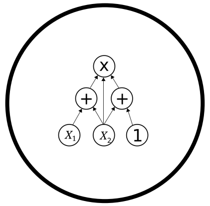
Motivation

SMC

Primitive	Oblivious Transfer	Oblivious Linear Evaluation
Circuit		

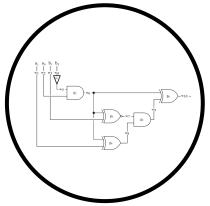
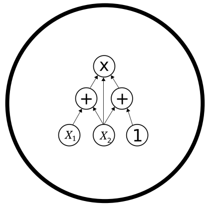
Motivation

SMC

	Classic	Classic
Primitive	Oblivious Transfer	Oblivious Linear Evaluation
Circuit		

Motivation

SMC

	Quantum	Classic	Quantum	Classic
Primitive	Oblivious Transfer		Oblivious Linear Evaluation	
Circuit				

Motivation

SMC

Quantum

Classic

Quantum

Classic

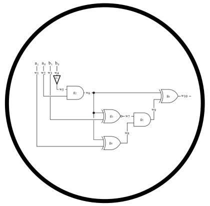
[BBCS'91]

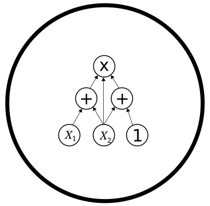
Primitive

Oblivious Transfer

Oblivious Linear Evaluation

Circuit





Motivation

SMC

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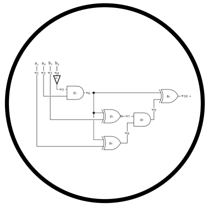
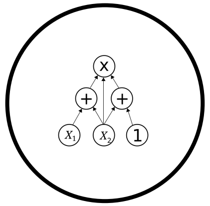
Circuit				

Motivation

SMC

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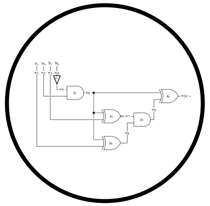
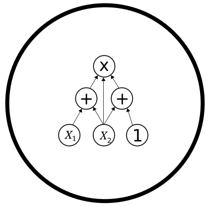
Circuit						

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SMC

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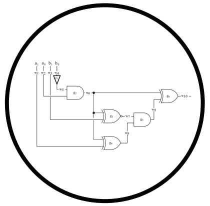
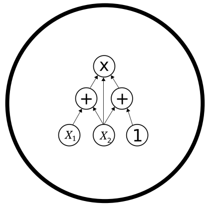
Circuit						

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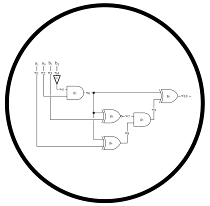
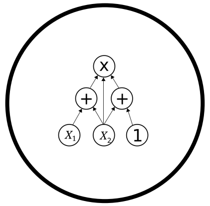
Circuit						

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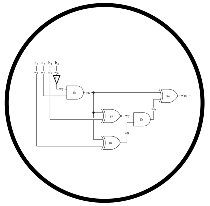
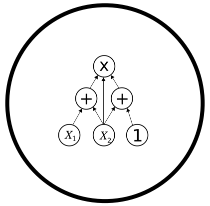
Circuit						

Motivation

SMC

Quantum | Classic

Quantum | Classic

Primitive

[BBCS'91]
[FS'09]
[DFS+05]
[WST'08] …

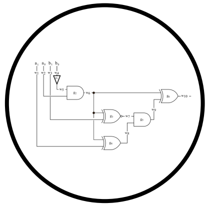
Oblivious Transfer

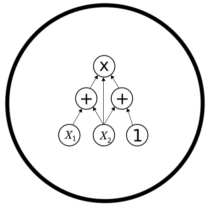
Review

[EGL'85]
SimpleOT
…
libscapi
[YAV'22]

Oblivious Linear Evaluation

Circuit





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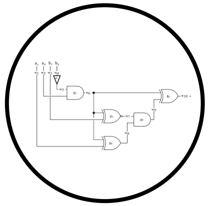
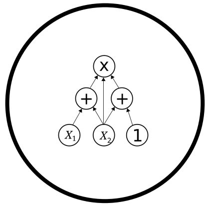
Circuit						

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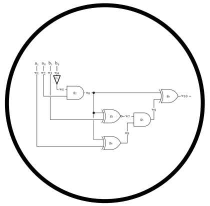
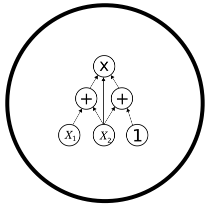
Circuit				
Application		Secure auctions Secure voting		

Oblivious Transfer

Review

Oblivious Linear Evaluation

Motivation

SMC

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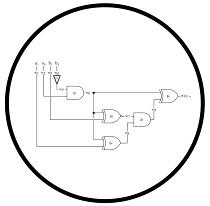
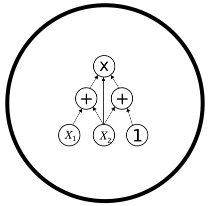
Circuit				
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Motivation

SMC

Quantum | Classic

Quantum | Classic

Primitive

[BBCS'91]
[FS'09]
[DFS+05]
[WST'08] …

Oblivious Transfer

Review

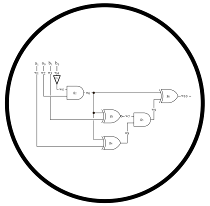
[EGL'85]
SimpleOT
…

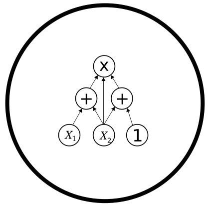
libscapi
[YAV'22]

Oblivious Linear Evaluation

[NP'99]
TinyOLE
…

Circuit





Application

Secure auctions
Secure voting

Motivation

SMC

Quantum | Classic

Primitive

[BBCS'91]
[FS'09]
[DFS+05]
[WST'08] …

Oblivious Transfer

Review

[EGL'85]
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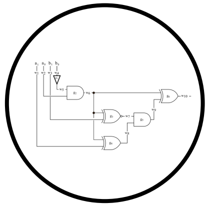
…

libscapi
[YAV'22]

Circuit



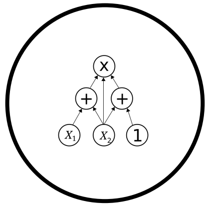
Application

Secure auctions
Secure voting

Quantum | Classic

Oblivious Linear Evaluation

[NP'99]
TinyOLE
…



Motivation

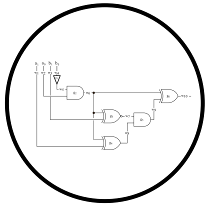
SMC

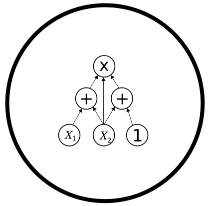
	Quantum	Classic	Quantum	Classic
Primitive	[BBCS'91] [FS'09] [DFS+05] [WST'08] …	[EGL'85] SimpleOT … libscapi [YAV'22]		[NP'99] TinyOLE …
Circuit				MASCOT
Application		Secure auctions Secure voting		

Oblivious Transfer

Review

Oblivious Linear Evaluation





OT reduction into SPDZ

Motivation

SMC

Quantum | Classic | Quantum | Classic

Primitive

[BBCS'91]
[FS'09]
[DFS+05]
[WST'08] …

Oblivious Transfer

Review

[EGL'85]
SimpleOT
…

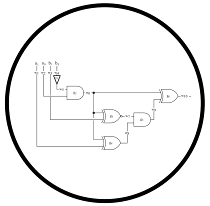
libscapi
[YAV'22]

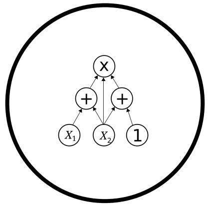
Oblivious Linear Evaluation

[NP'99]
TinyOLE
…

MP-SPDZ

Circuit





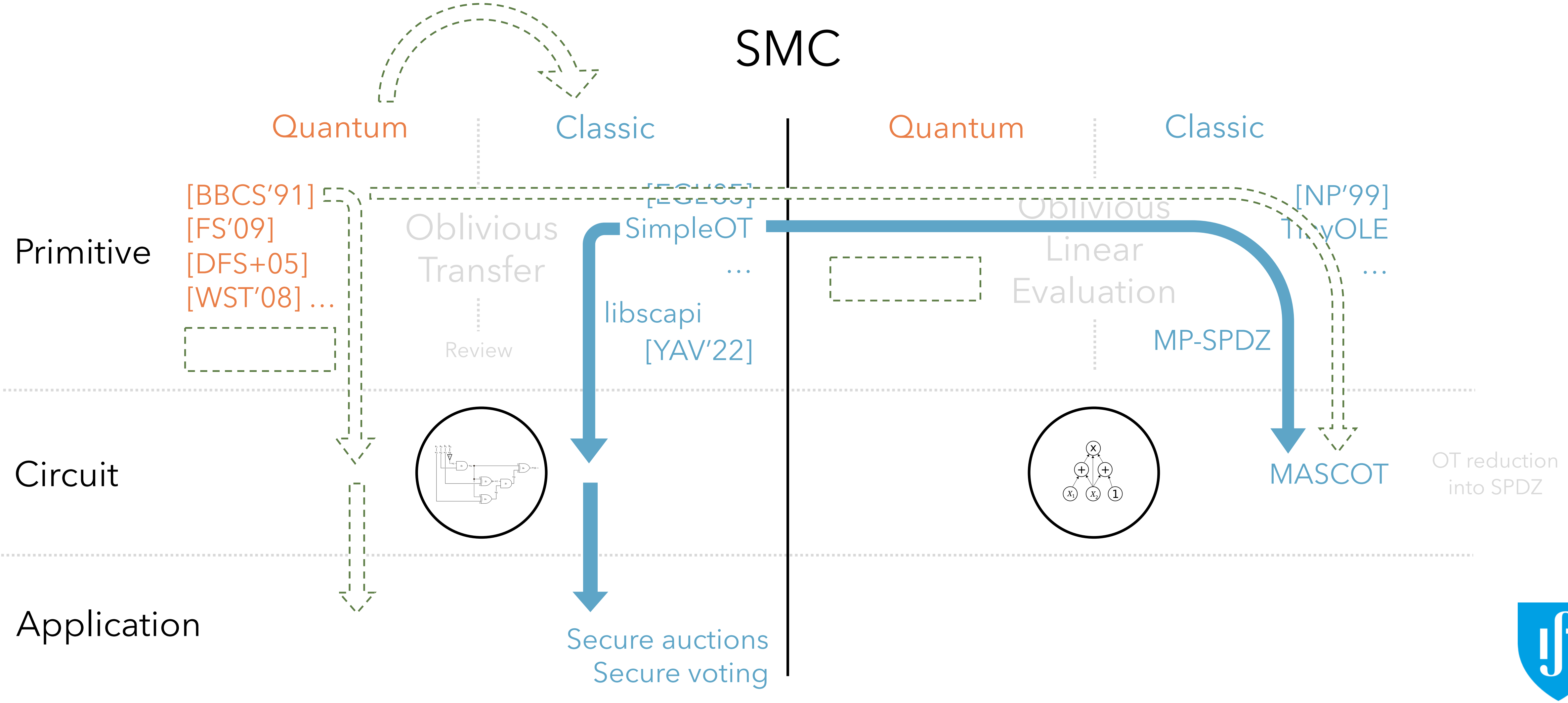
MASCOT

OT reduction into SPDZ

Application

Secure auctions
Secure voting

Motivation


SMC
Quantum
Classic
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[BBCS'91]
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Oblivious Transfer
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…
MP-SPDZ
MASCOT
OT reduction into SPDZ
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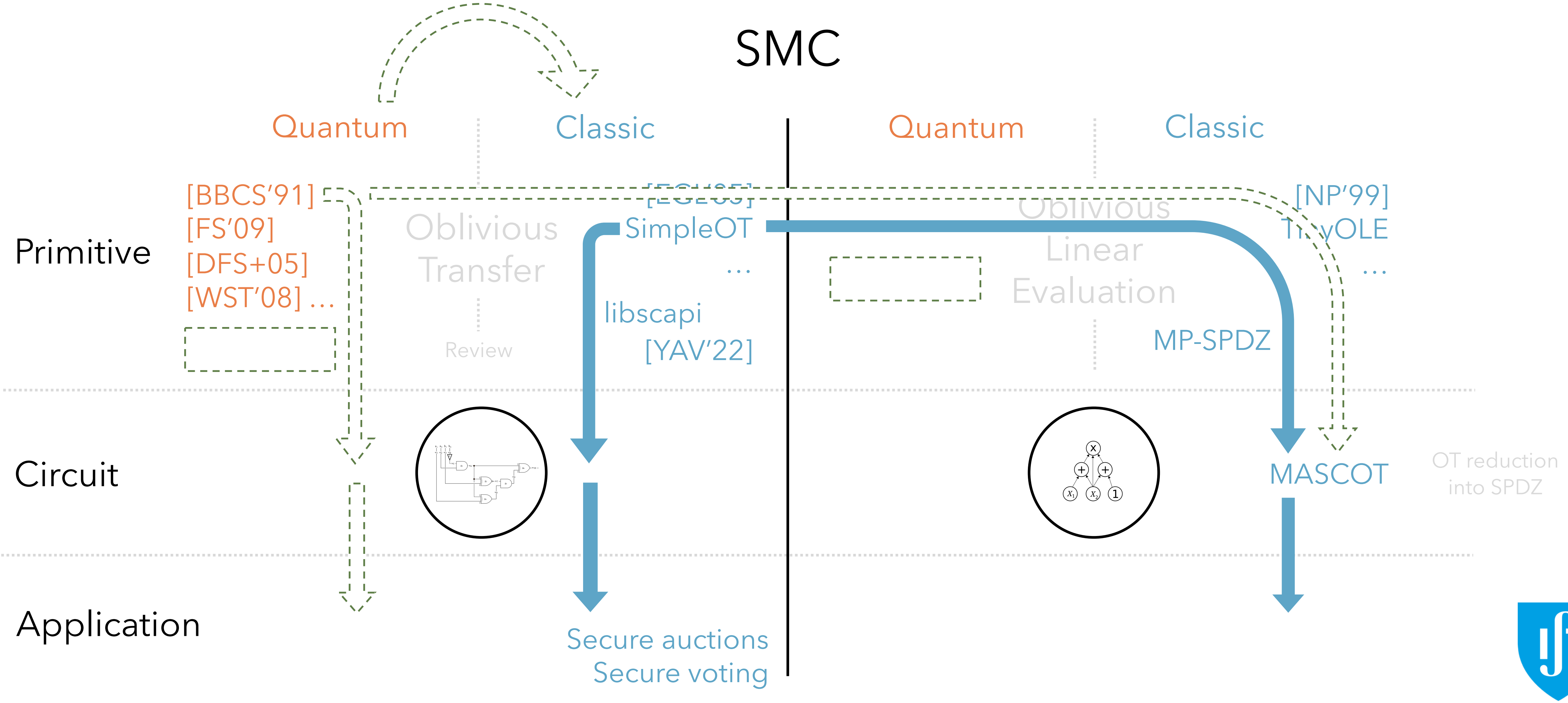
Motivation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Review
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MP-SPDZ
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OT reduction into SPDZ
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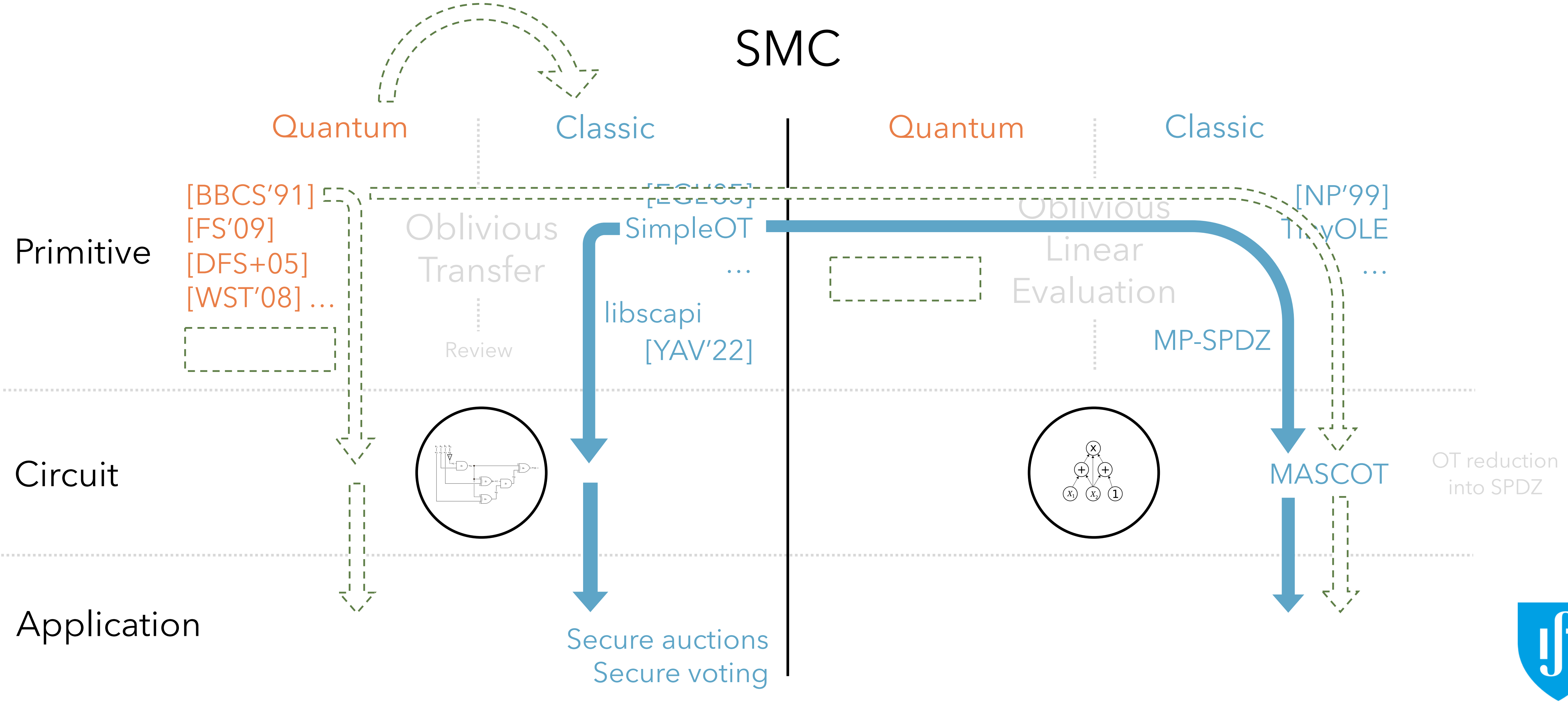
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Secure auctions
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Oblivious
Linear
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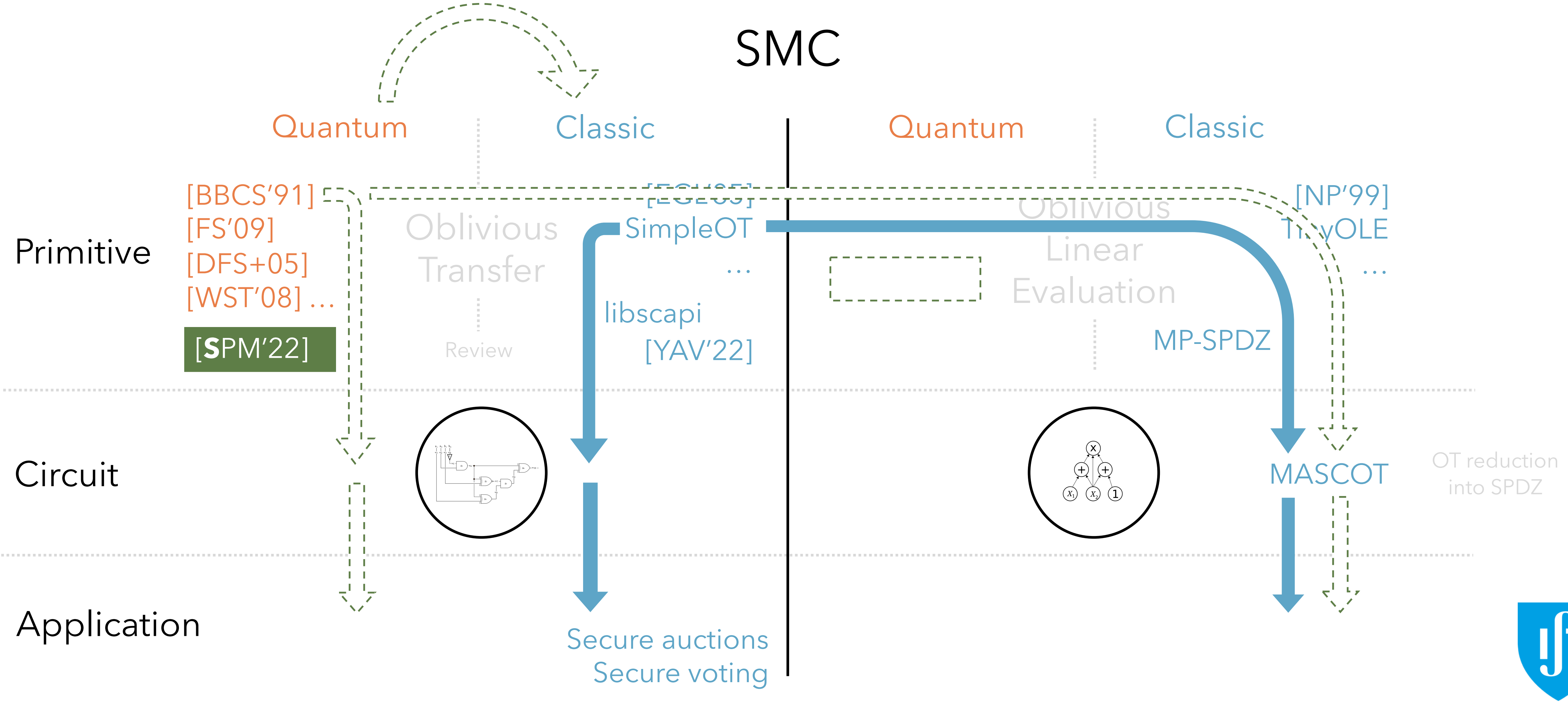
Outcomes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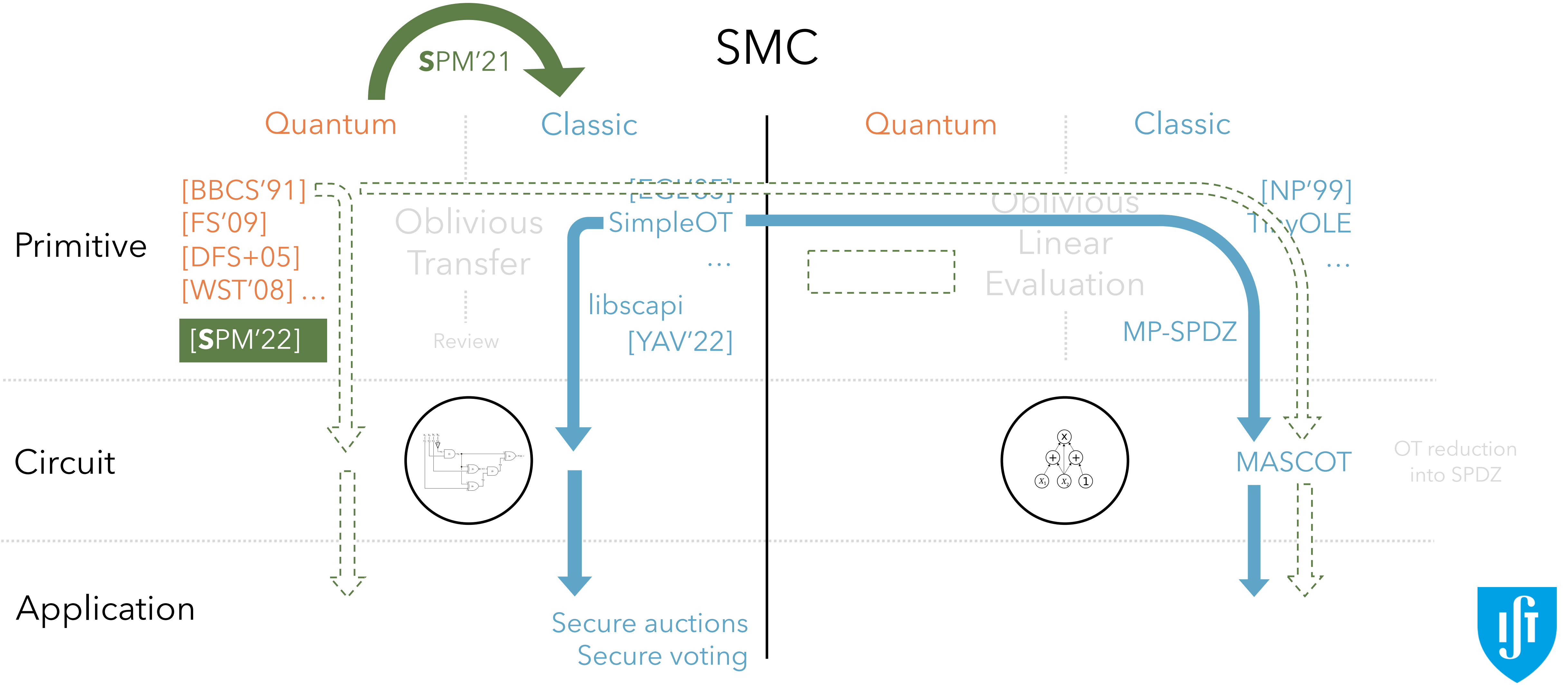

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SPM'21
Quantum
Classic
Quantum
Classic
Primitive
Circuit
Application
[BBCS'91]
[FS'09]
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[WST'08] …
[SPM'22]
Oblivious Transfer
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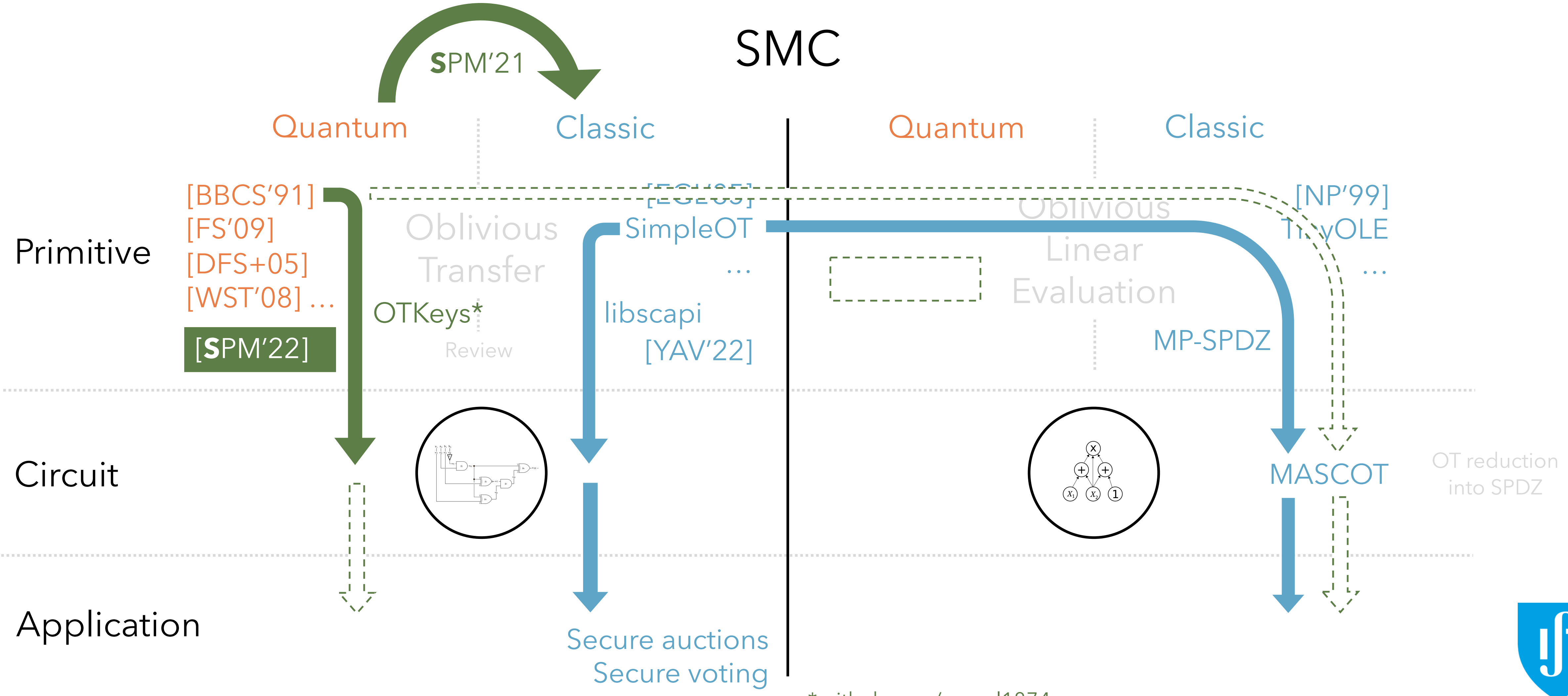
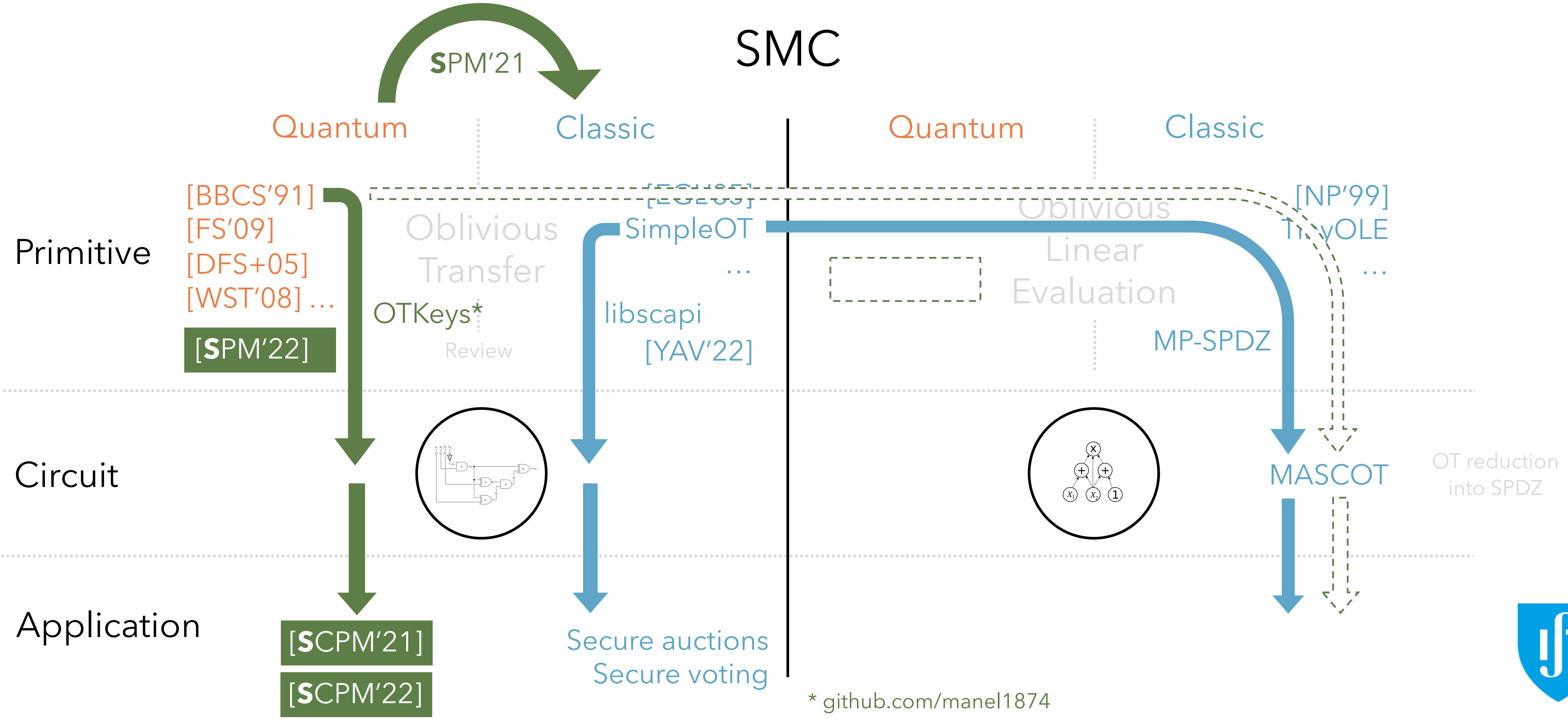
Outcomes



* github.com/manel1874

Outcomes



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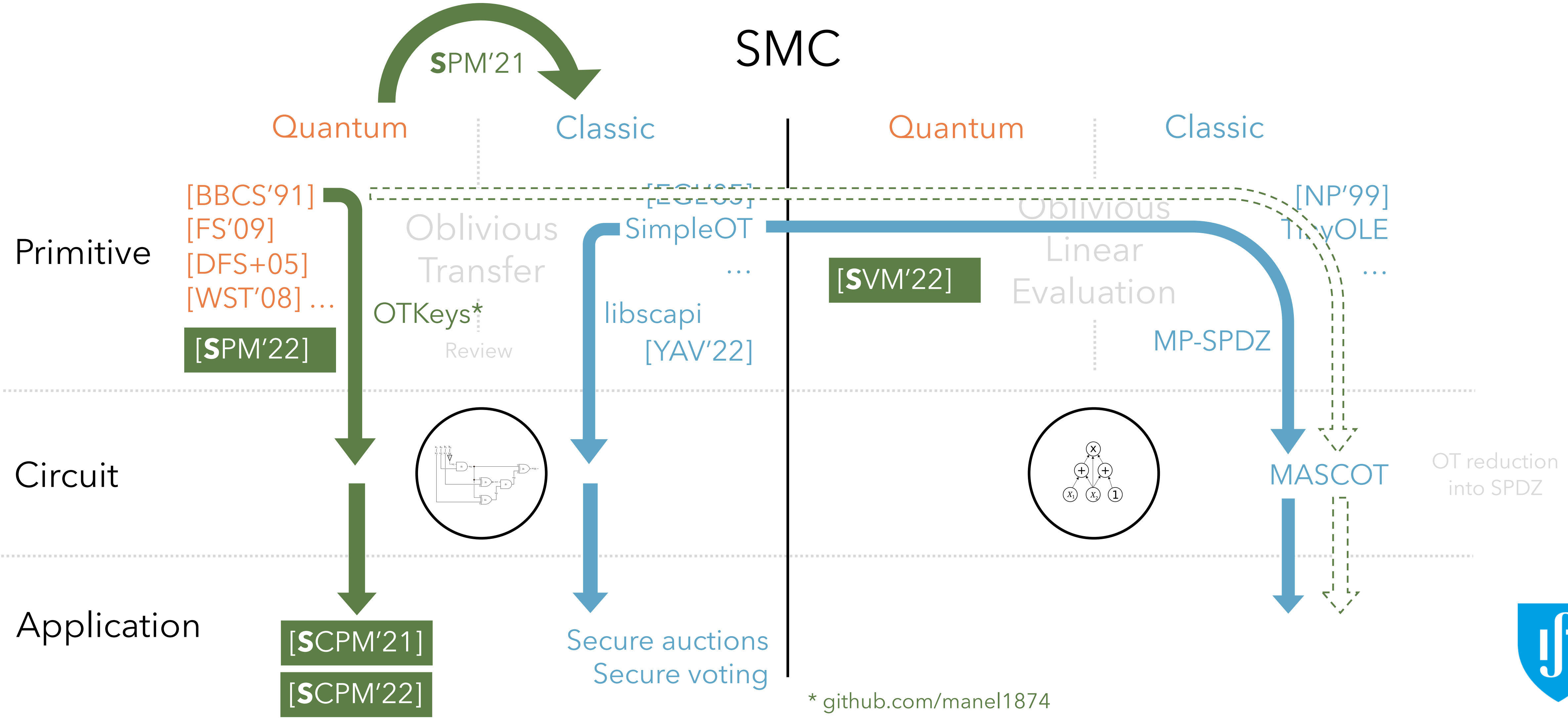
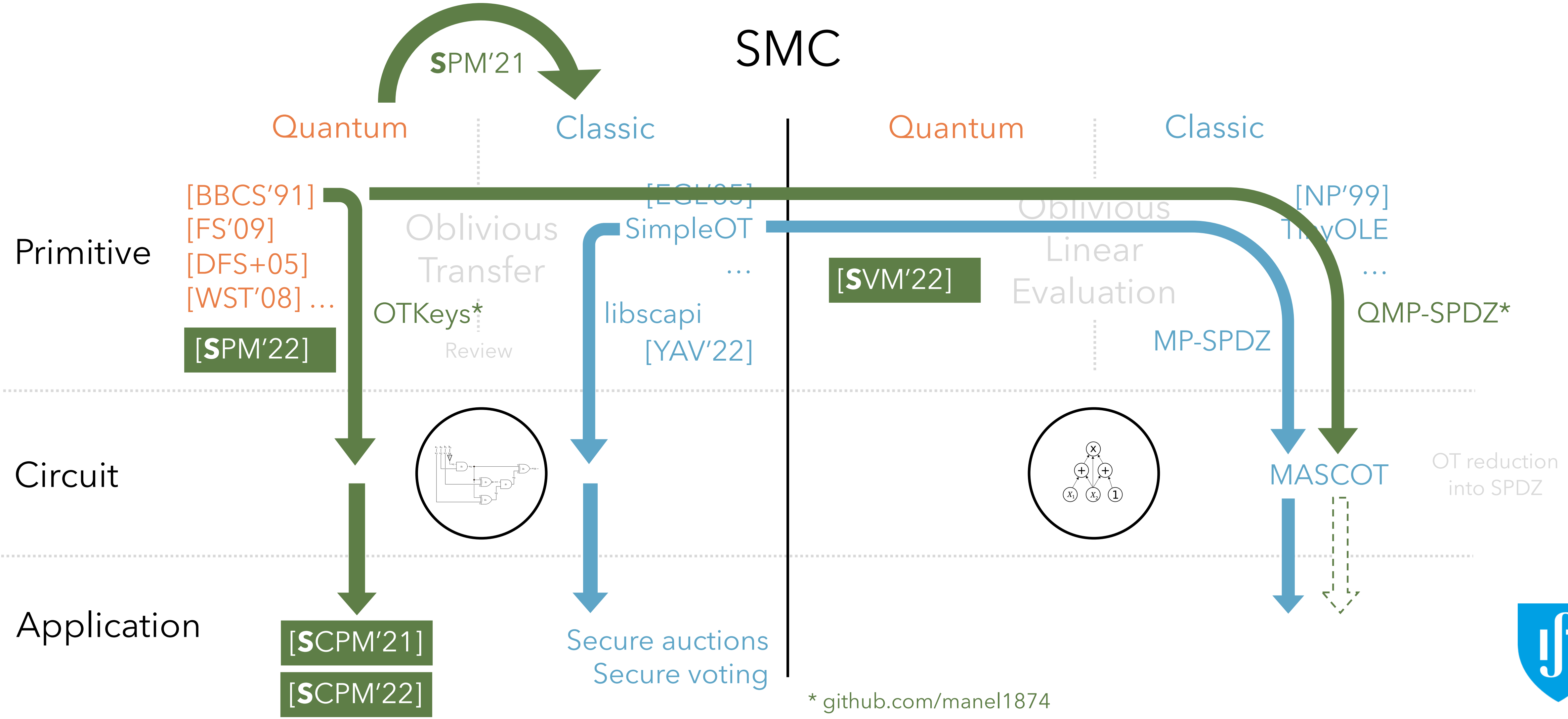
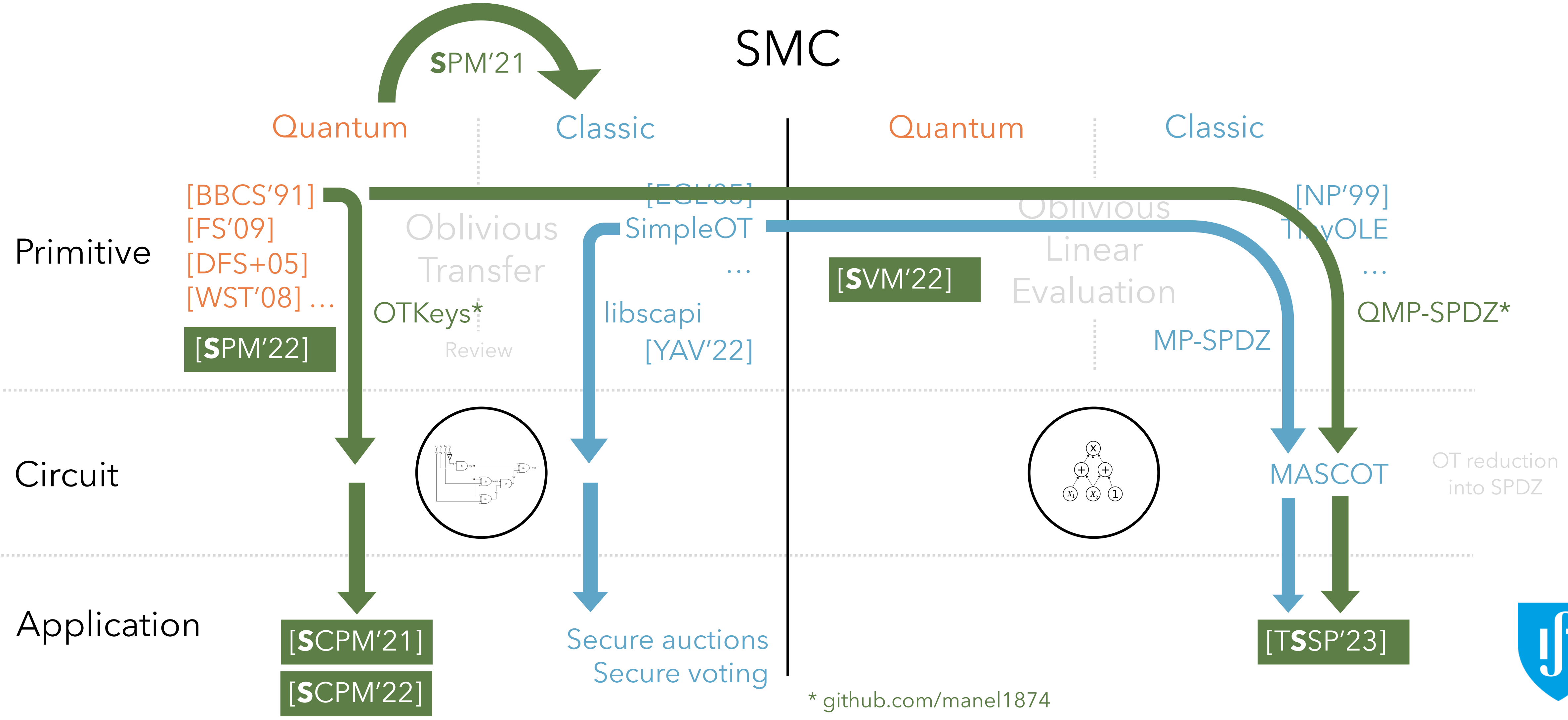

SPM'21
SMC
Quantum
Classic
Quantum
Classic
Primitive
[BBCS'91]
[FS'09]
[DFS+05]
[WST'08] …
[SPM'22]
Oblivious
Transfer
OTKeys*
Review
SimpleOT
…
libscapi
[YAV'22]
[SVM'22]
Oblivious
Linear
Evaluation
MP-SPDZ
[NP'99]
…
Circuit
MASCOT
OT reduction
into SPDZ
Application
[SCPM'21]
[SCPM'22]
Secure auctions
Secure voting
* github.com/manel1874

Outcomes


SMC
SPM'21
Quantum
Classic
Quantum
Classic
Primitive
Circuit
Application
[BBCS'91]
[FS'09]
[DFS+05]
[WST'08] …
[SPM'22]
Oblivious Transfer
OTKeys*
Review
SimpleOT
…
libscapi
[YAV'22]
[SVM'22]
Oblivious Linear Evaluation
[NP'99]
…
QMP-SPDZ*
MP-SPDZ
MASCOT
OT reduction into SPDZ
[SCPM'21]
[SCPM'22]
Secure auctions
Secure voting
* github.com/manel1874

Outcomes



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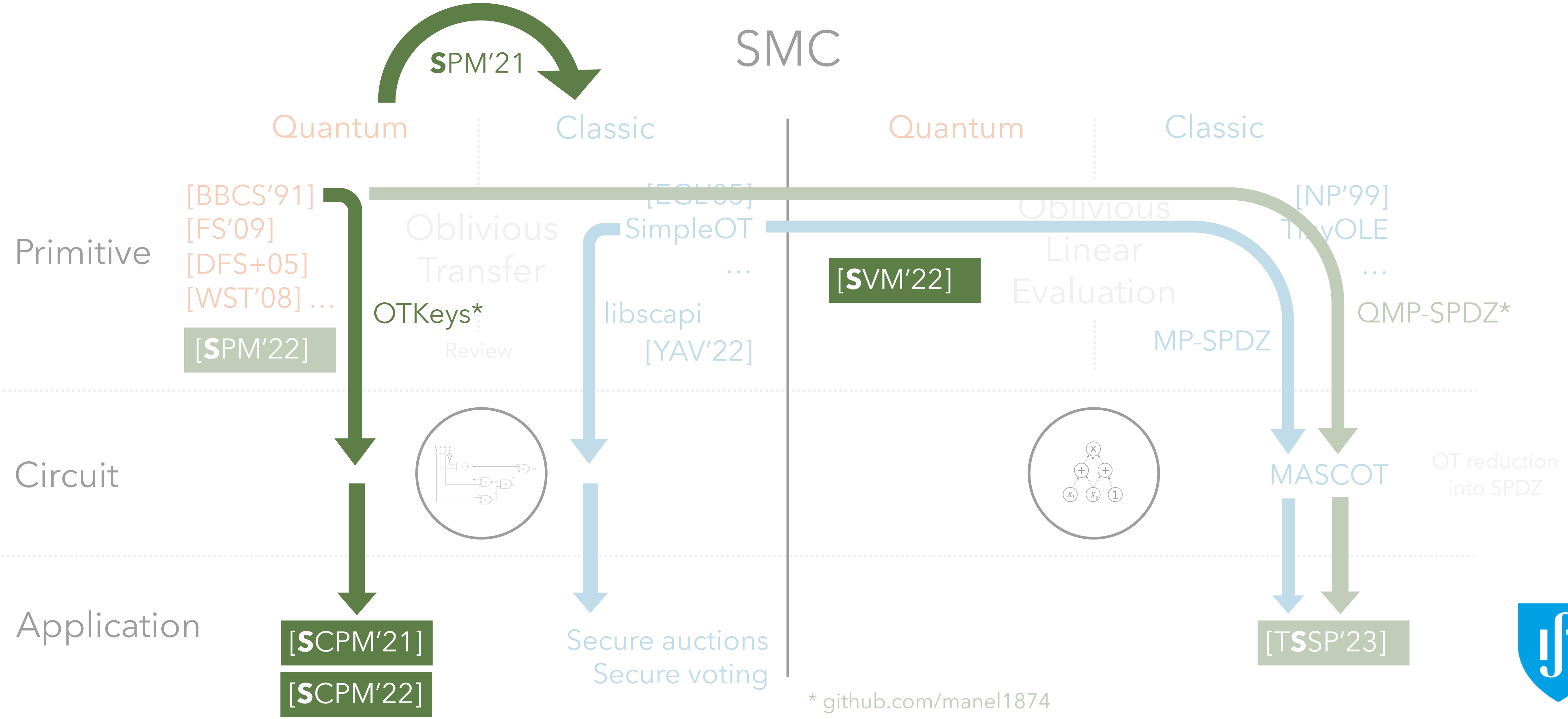
Outcomes



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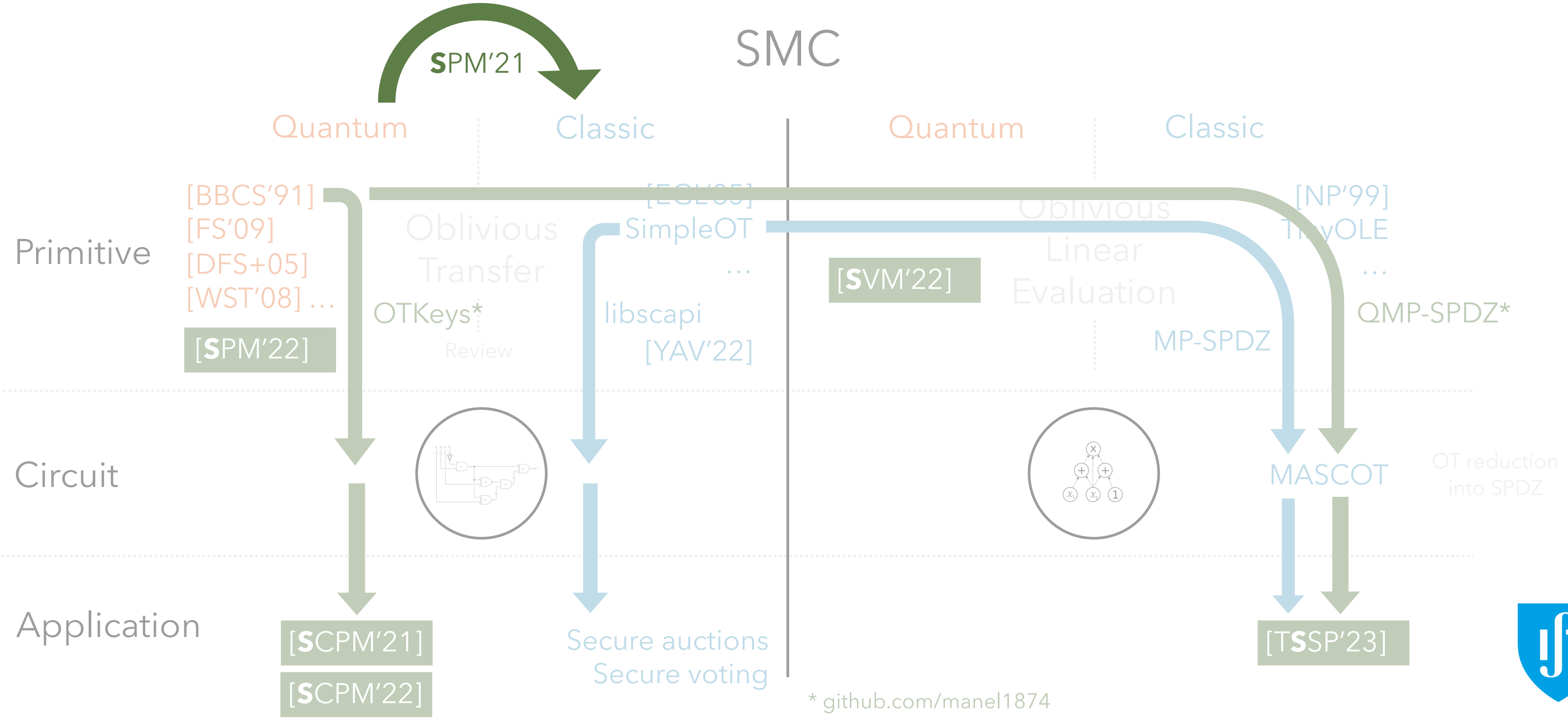
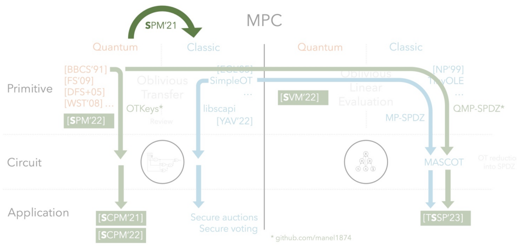
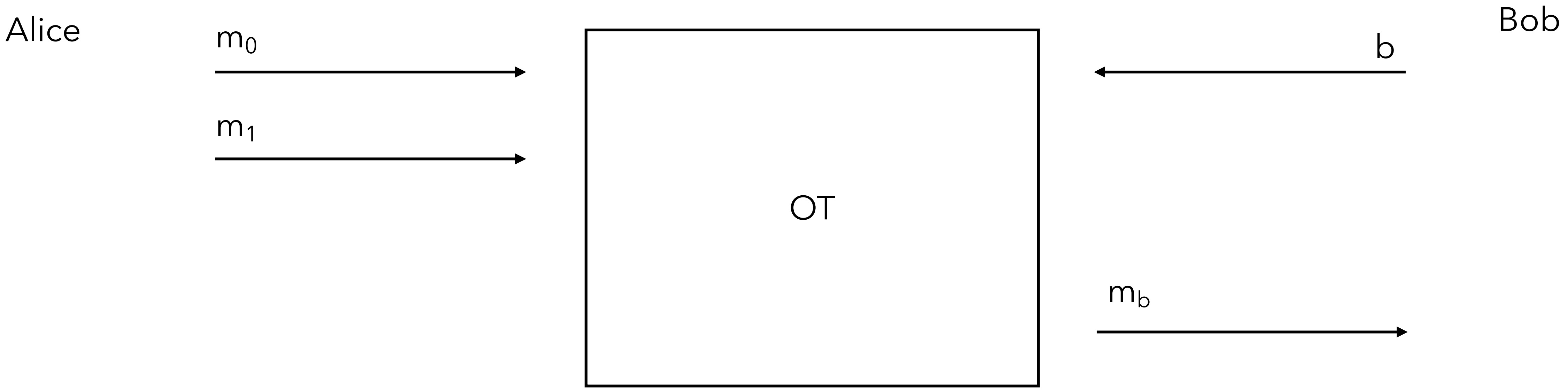
Quantum and classical OT



Oblivious Transfer





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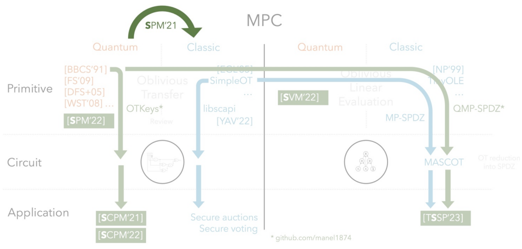


Quantum

[BBCS'91]
[DFS+05]
[WST'08]
[FS'09]
...

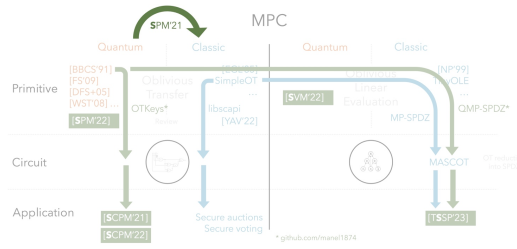
Classic

[EGL'85]
[BM'89]
[NP'01]
SimpleOT
...

No previous work

How can we compare?

Quantum and classical OT



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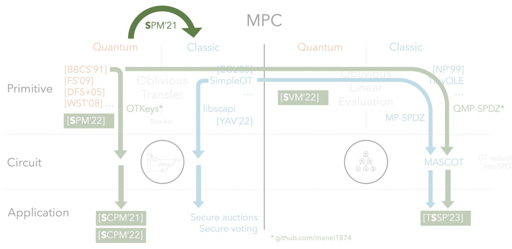
How can we compare?

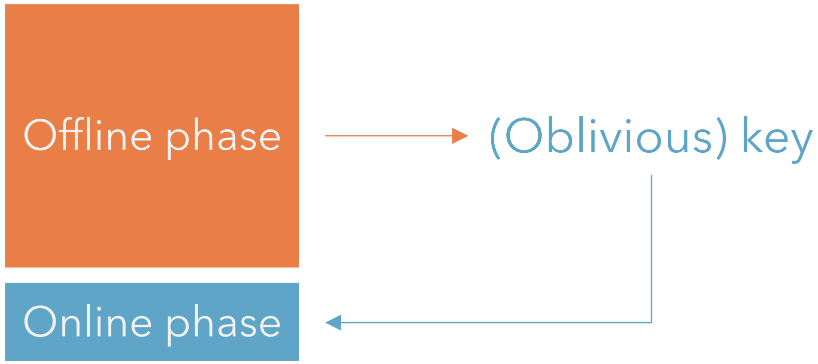
Comparable structure?
Corresponding phases with same technology?
Any practical insight?

Quantum and classical OT



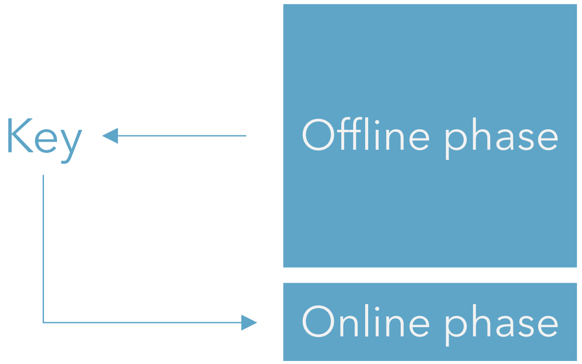
Quantum

[BBCS'91]



Classic

Base OT OT Extension



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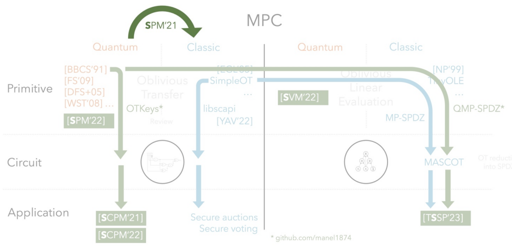

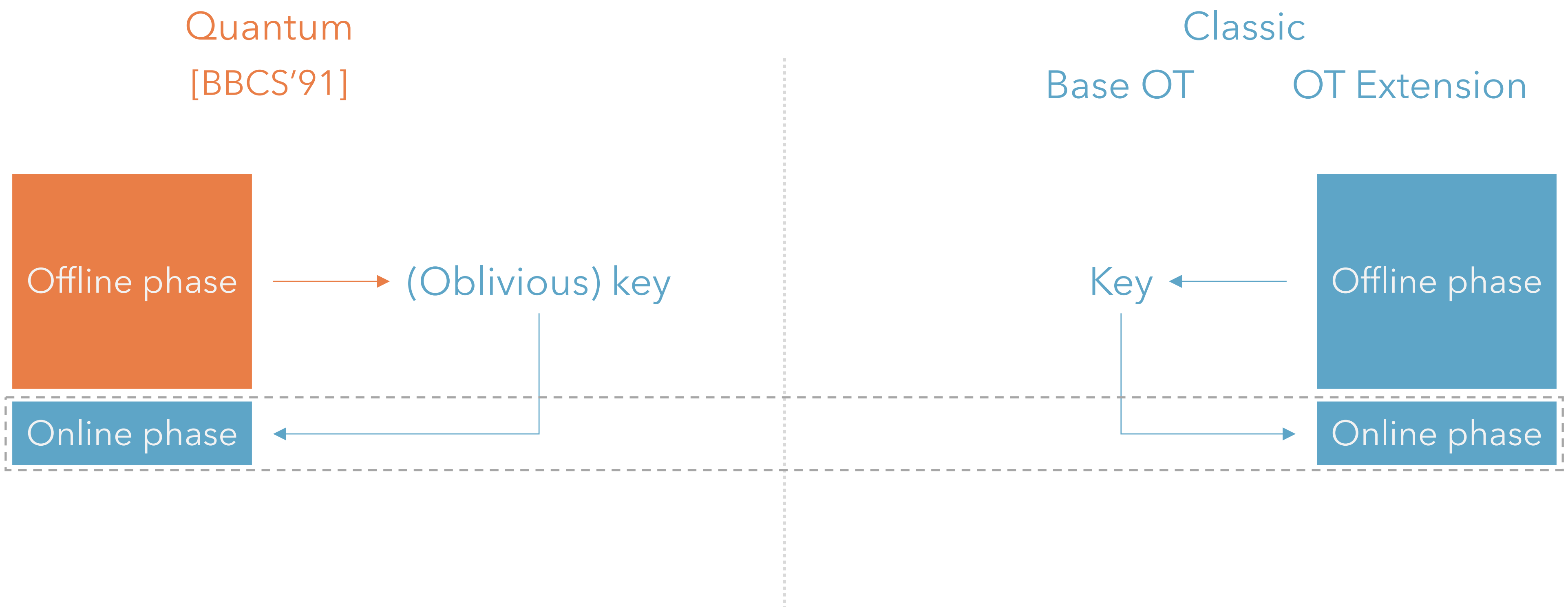

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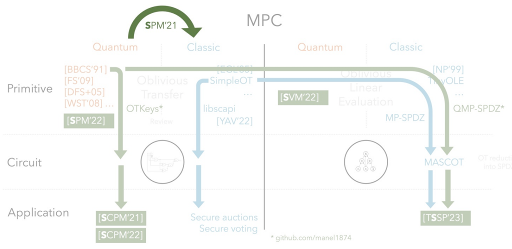

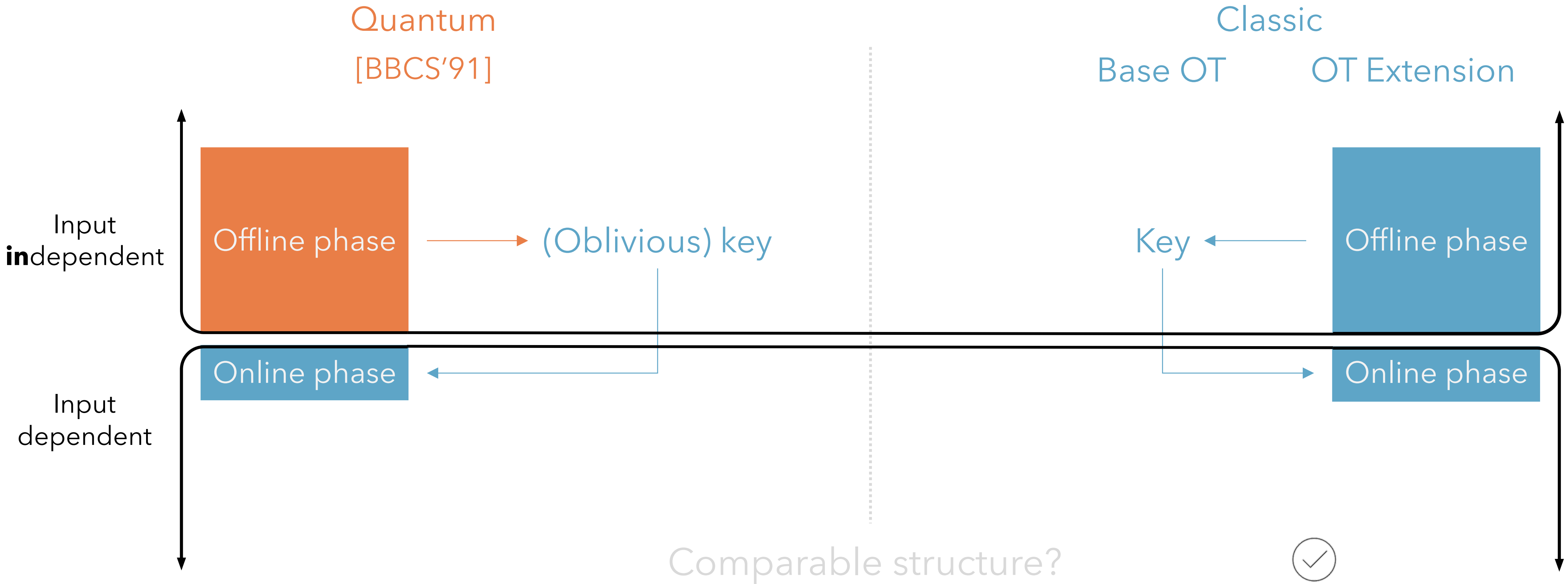

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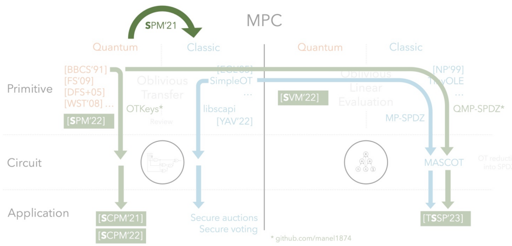

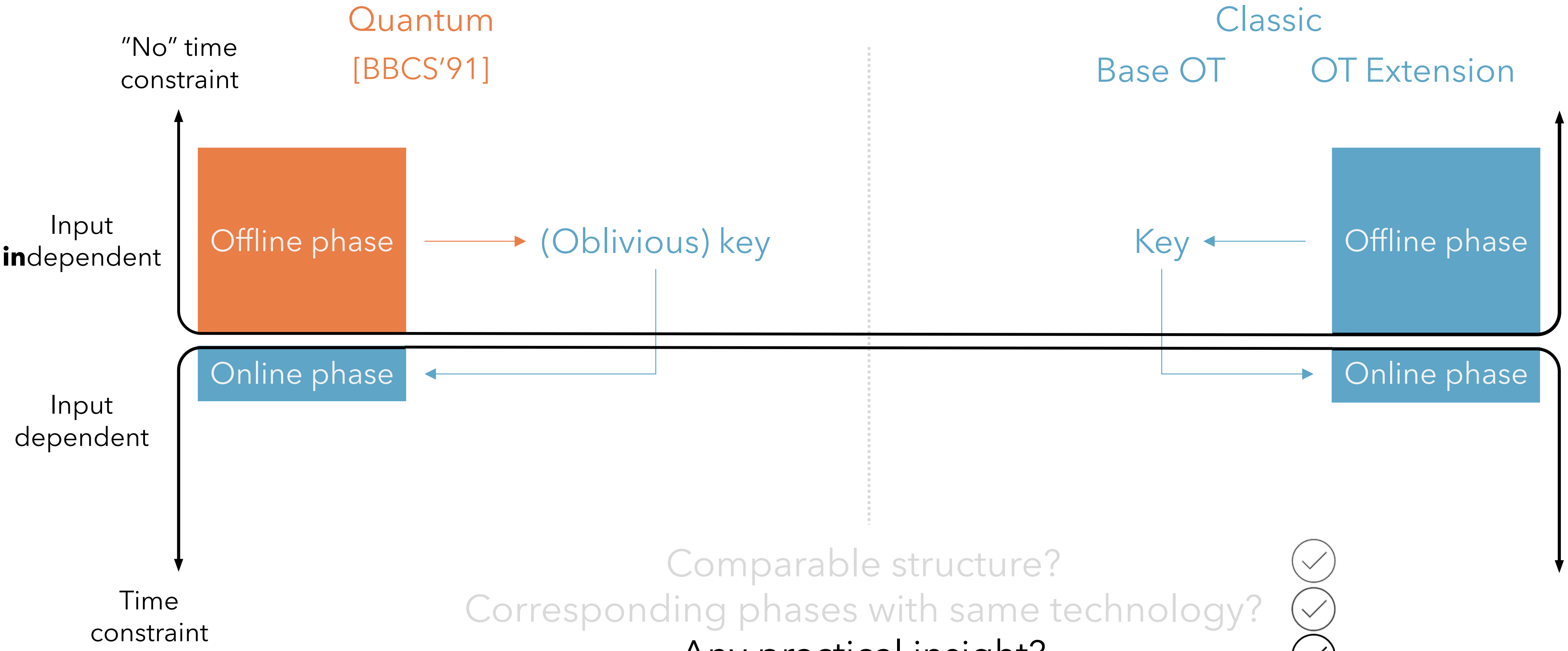

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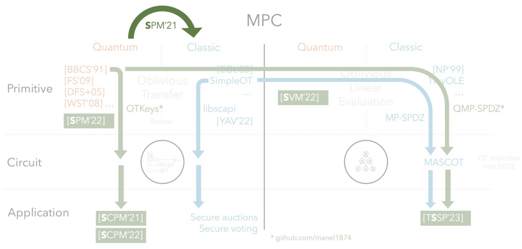


Classic

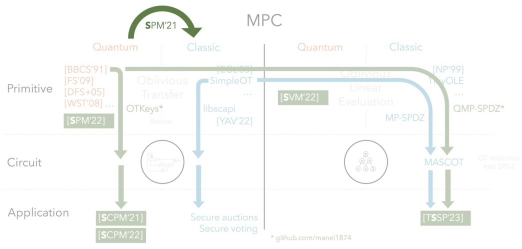
Base OT

OT Extension

Quantum

[BBCS'91]

Quantum and classical OT



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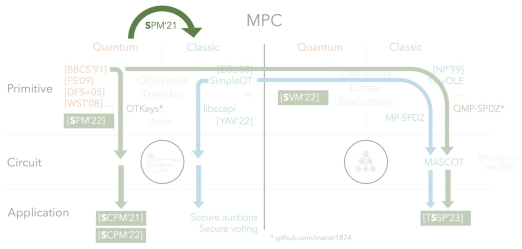
Quantum

[BBCS'91]

Issue: PK operations



Quantum and classical OT

Classic

Base OT

OT Extension

Quantum

[BBCS'91]

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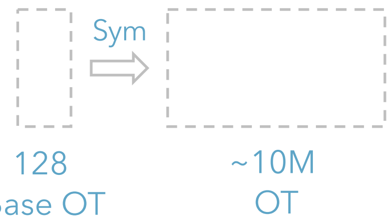
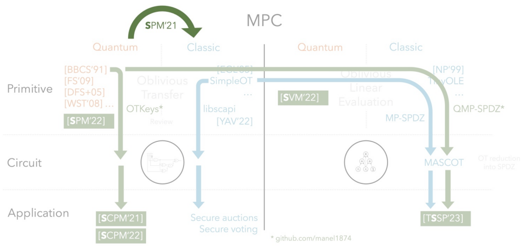


Quantum and classical OT



Classic

Base OT

	OT/s
[NP'01]	56
SimpleOT	1 375
NTRU-OT	728
Kyber-OT	41

<

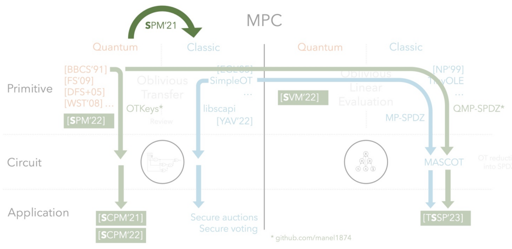
OT Extension

	10M OT
[ALSZ'13]	2.68 s
[KOS'15]	3.35 s

Quantum

[BBCS'91]

Quantum and classical OT



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OT Extension

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Quantum

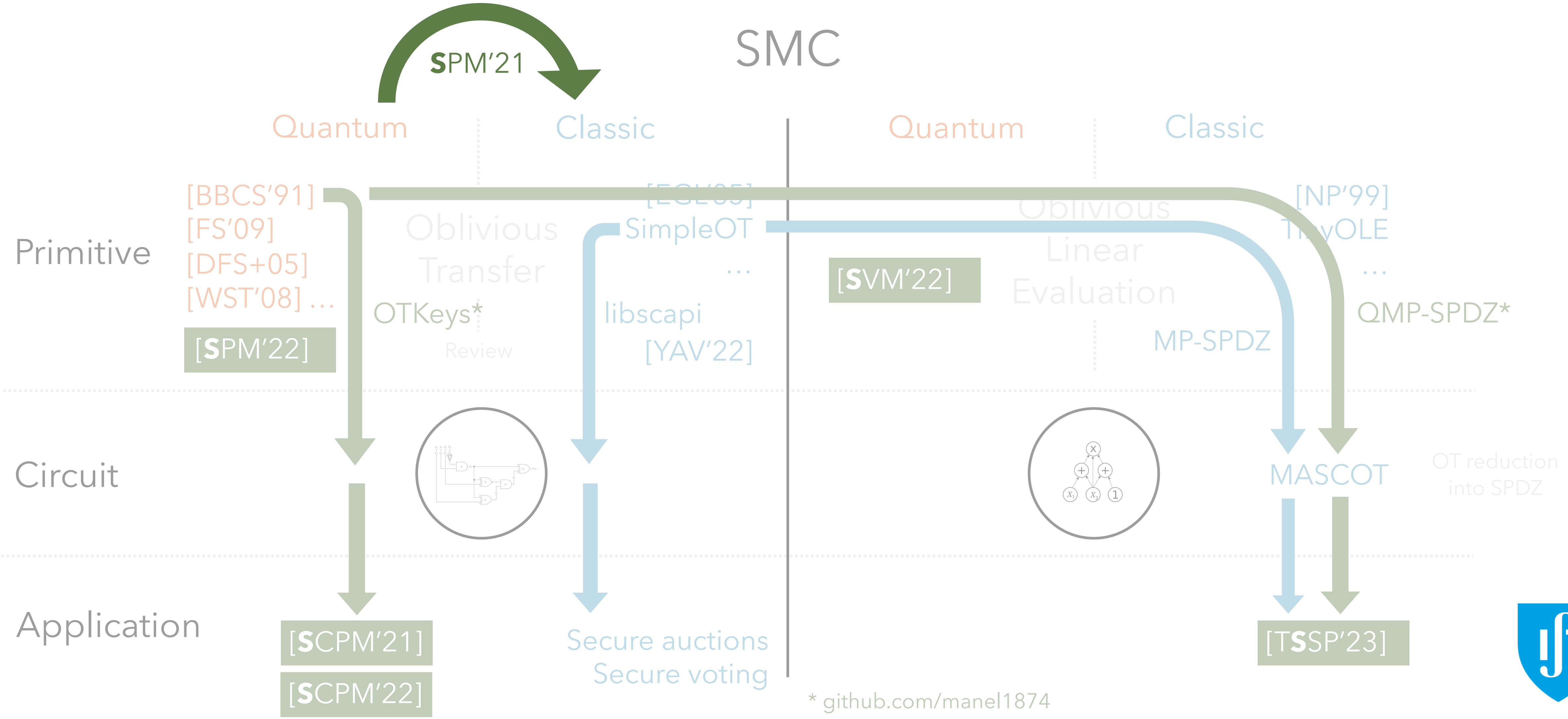
[BBCS'91]

Online phase for m OTs

	Computation	Communication
[ALSZ'13]	$O^{ALSZ} - O^{BBCS} > m \log m$	$C^{ALSZ} - C^{BBCS} = 0$
[KOS'15]	$O^{KOS} - O^{BBCS} > m \log m + 5ml$	$C^{KOS} - C^{BBCS} \gtrsim 0$

BBCS

Quantum and classical OT



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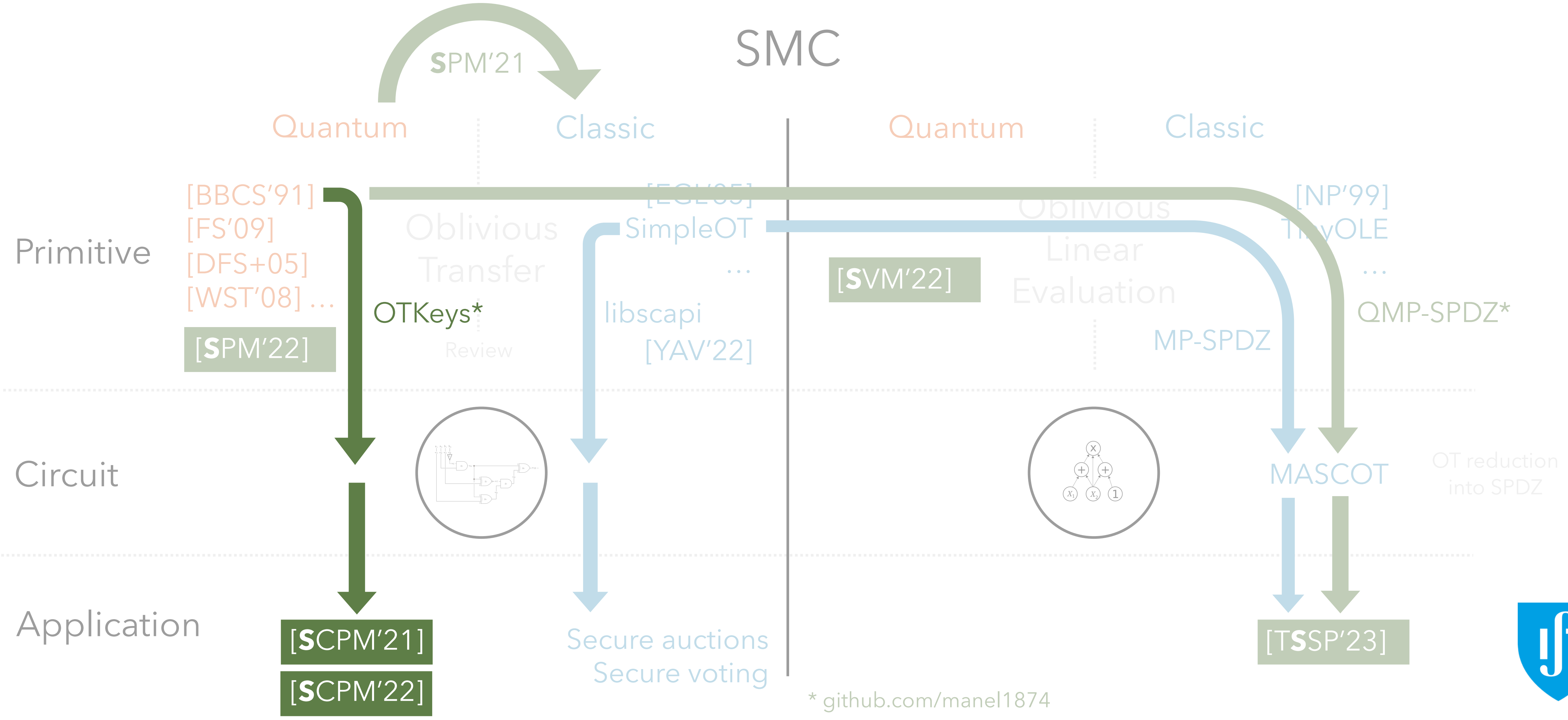
Private phylogenetic trees



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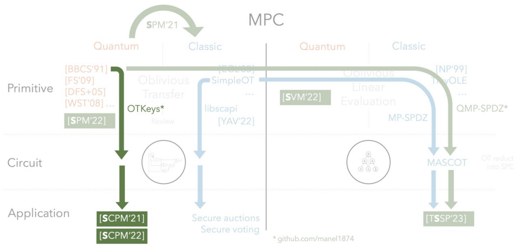
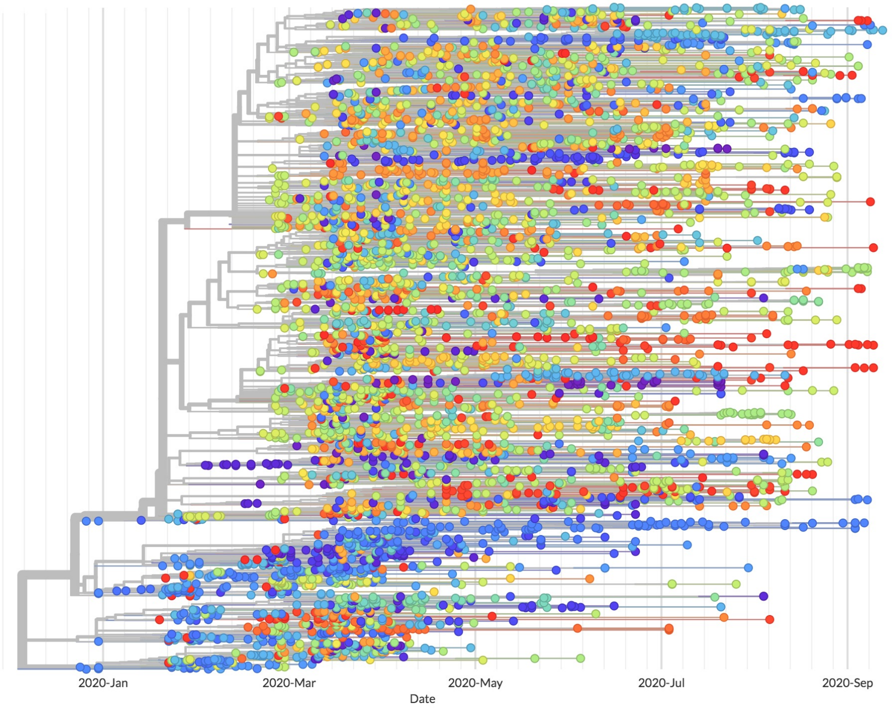


Shows the **evolutionary relationship** between **DNA** sequences in a **tree**.



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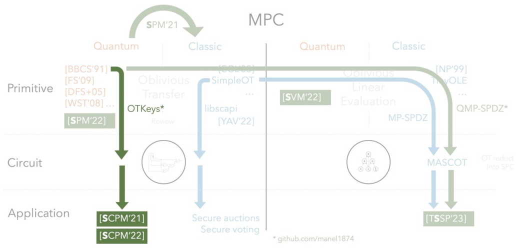
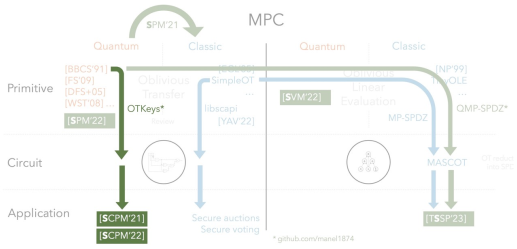


Results summary

- Tailored SMC protocol for phylogenetic trees algorithms

Private phylogenetic trees

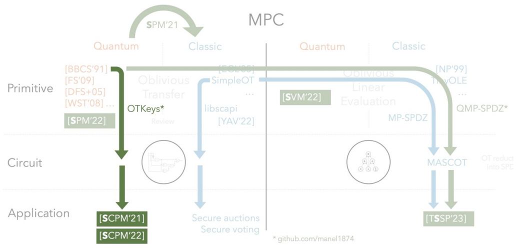


Results summary

- Tailored SMC protocol for phylogenetic trees algorithms

- Classical implementation
    - CBMC-GC: circuit generation
    - MPC-Benchmark: yao protocol based on Libscapi
    - PHYLIP: phylogeny analysis

Private phylogenetic trees



Results summary

- Tailored SMC protocol for phylogenetic trees algorithms
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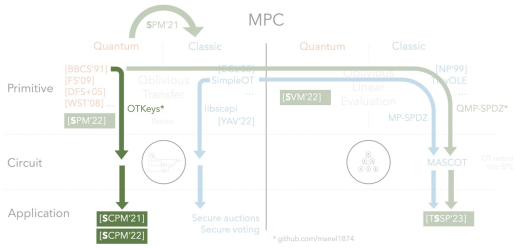
  - CBMC-GC: circuit generation
  - MPC-Benchmark: yao protocol based on Libscapi
  - PHYLIP: phylogeny analysis
- Integrate BBCS based protocol into Libscapi



Private phylogenetic trees

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- Tailored SMC protocol for phylogenetic trees algorithms
- Classical implementation
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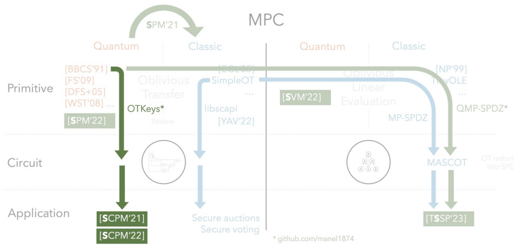
  - PHYLIP: phylogeny analysis
- Integrate BBCS based protocol into Libscapi
- Benchmark classical and quantum approaches

Private phylogenetic trees



Distance based: trees depend on the matrix distance of genes

Computation: simple

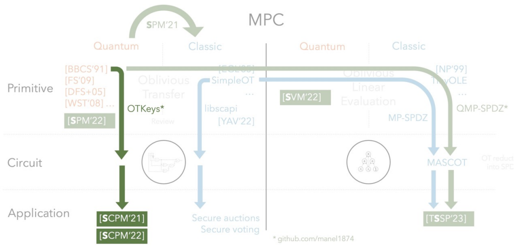
Algorithms:
- UPGMA
- NJ
- FM

Character based: search for the tree that optimizes the evolution the most

Computation: complex

Algorithms:
- Maximum Parsimony
- Maximum Likelihood

Private phylogenetic trees



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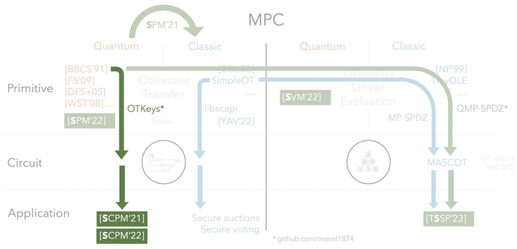
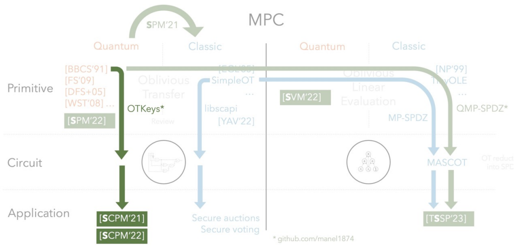


Algorithms:

- UPGMA
- NJ
- FM

Distances:

- JC
- K2P
- F84
- LogDet



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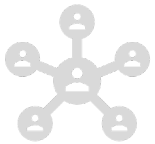
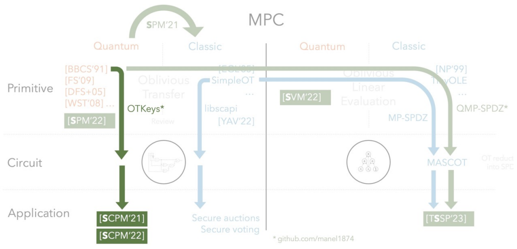


Part 1: Compute the distance matrix

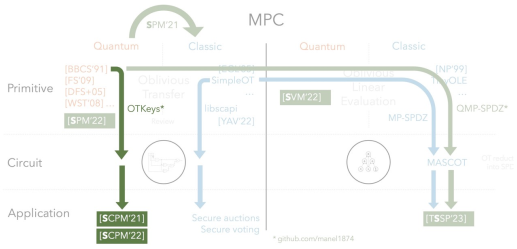
Private phylogenetic trees



Part 1: Compute the distance matrix

Part 2: Iteratively group the genes through some specific method

Private phylogenetic trees



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SMC for distances

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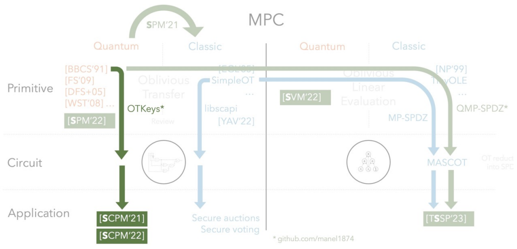


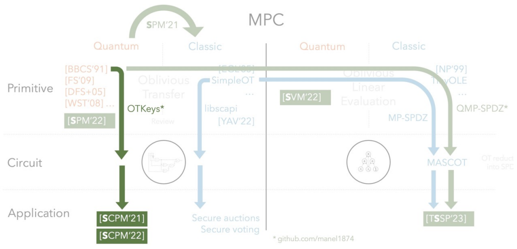
Part 1: Compute the distance matrix

SMC for distances

Part 2: Iteratively group the genes through some specific method

No interaction

Quantum-assisted system

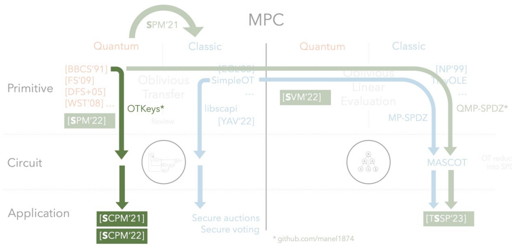


[BBCS'91]

Offline phase

Online phase

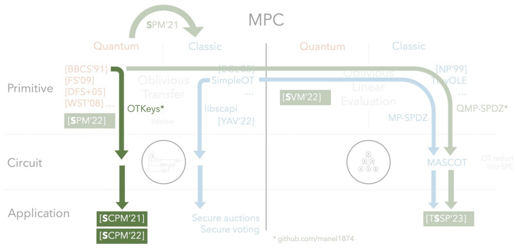
Quantum-assisted system



[BBCS'91]

Offline phase → Oblivious key → Online phase → OT

Quantum-assisted system



[BBCS'91]

Offline phase → Oblivious key → Online phase → OT

Alice

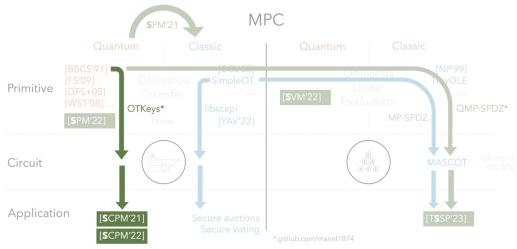
Bob

OK_A= 011001100101

OK_B= 001101100011

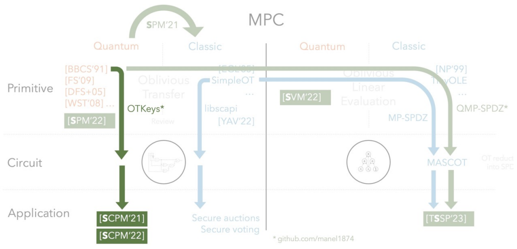
X = 010110010110

Quantum-assisted system



[BBCS'91]

Offline phase → Oblivious key → Online phase → OT → Yao protocol



Performance evaluation

Setup:

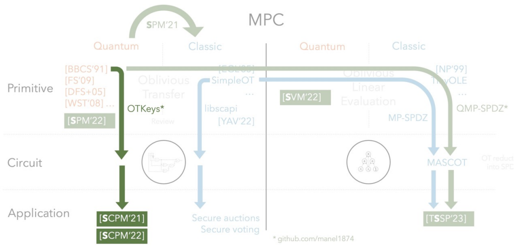
- **3 parties:** VMs running Ubuntu 16.04.3
- **30** SARS-CoV-2 genome **sequences*** with **32 000 length**

Boolean circuit:

- ~3 minutes (CBMC-GC)
- ~2.2 million gates
- 128 000 input wires

*GISAID database

Performance evaluation



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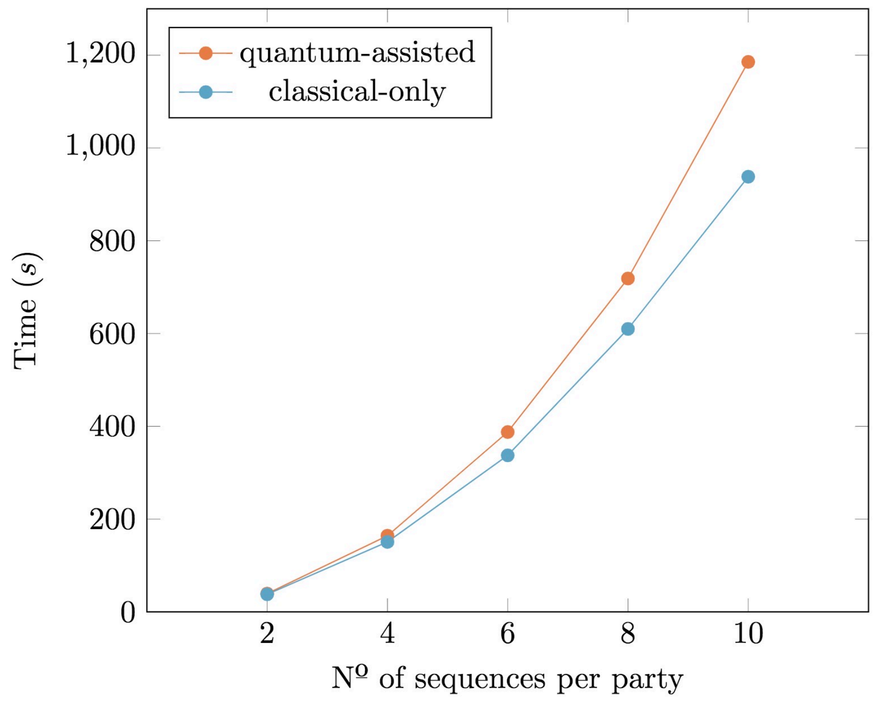
Total running time



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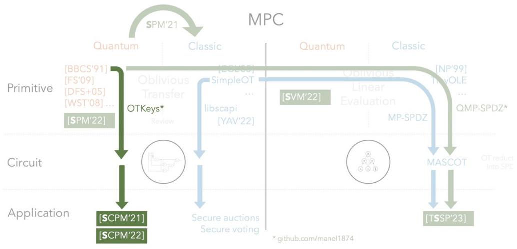

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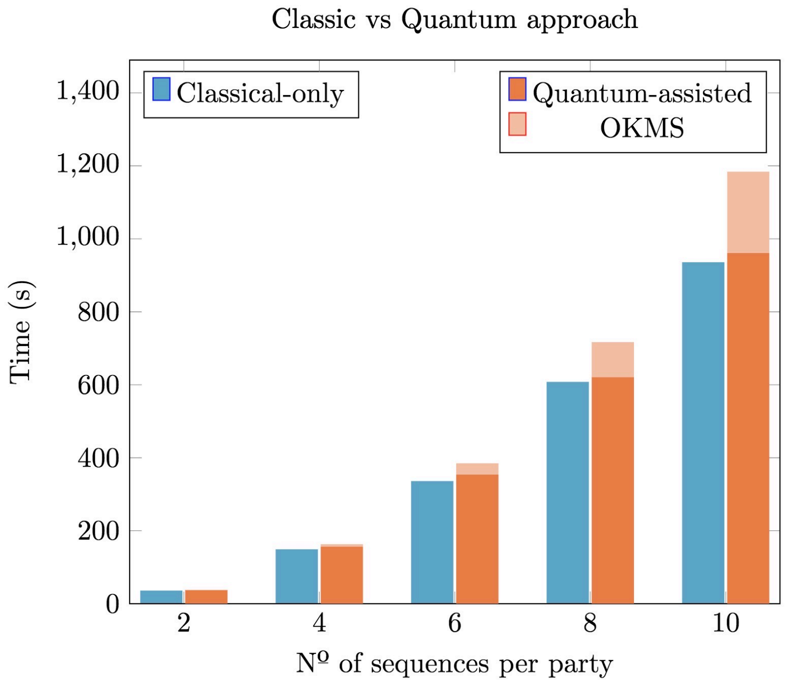

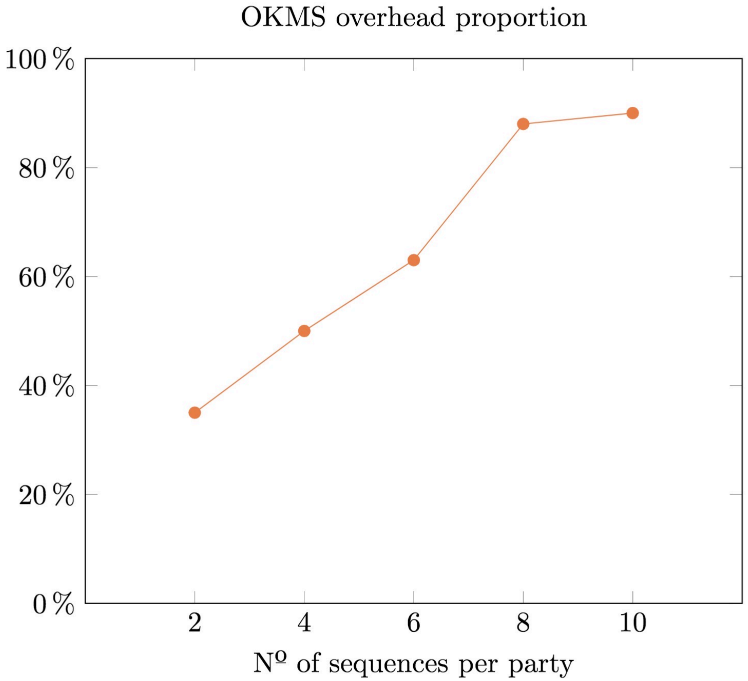

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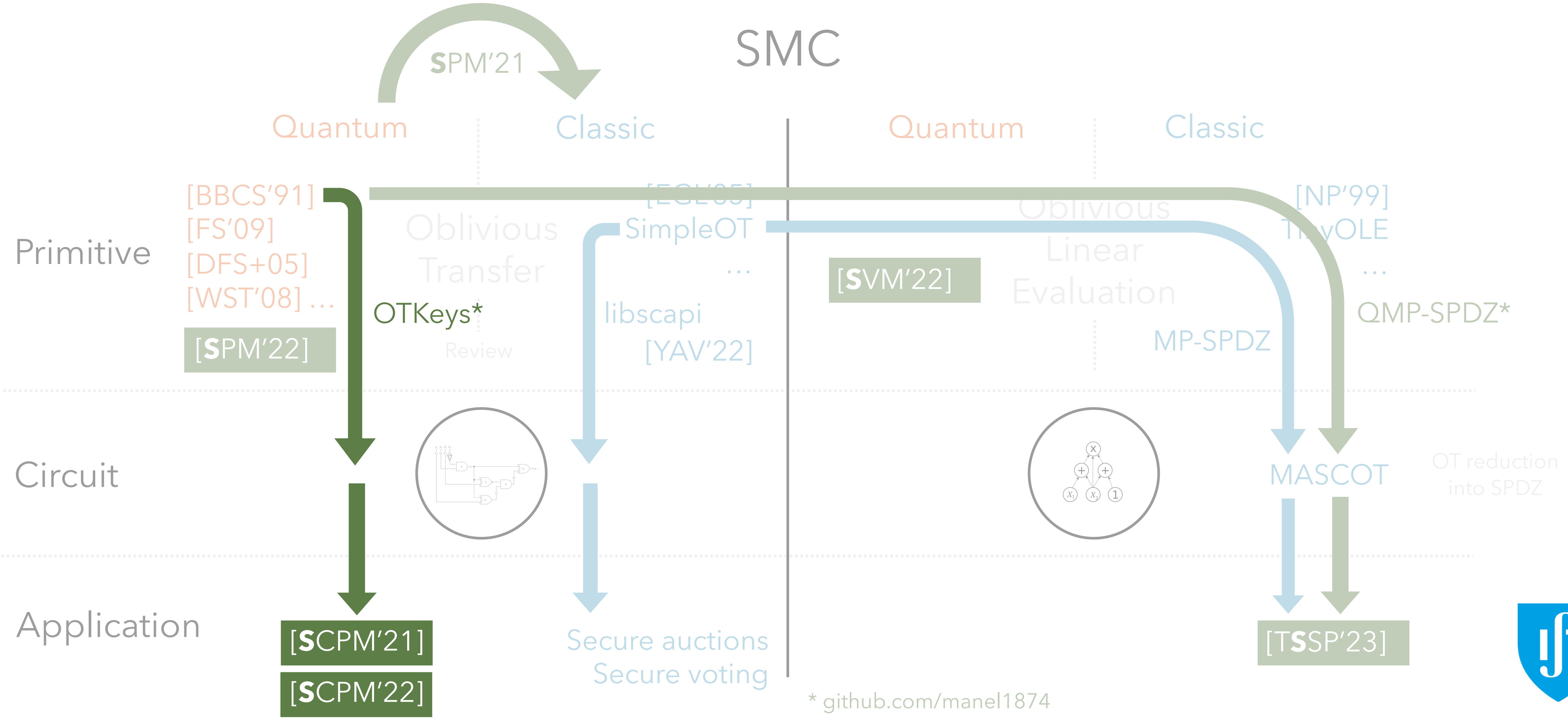
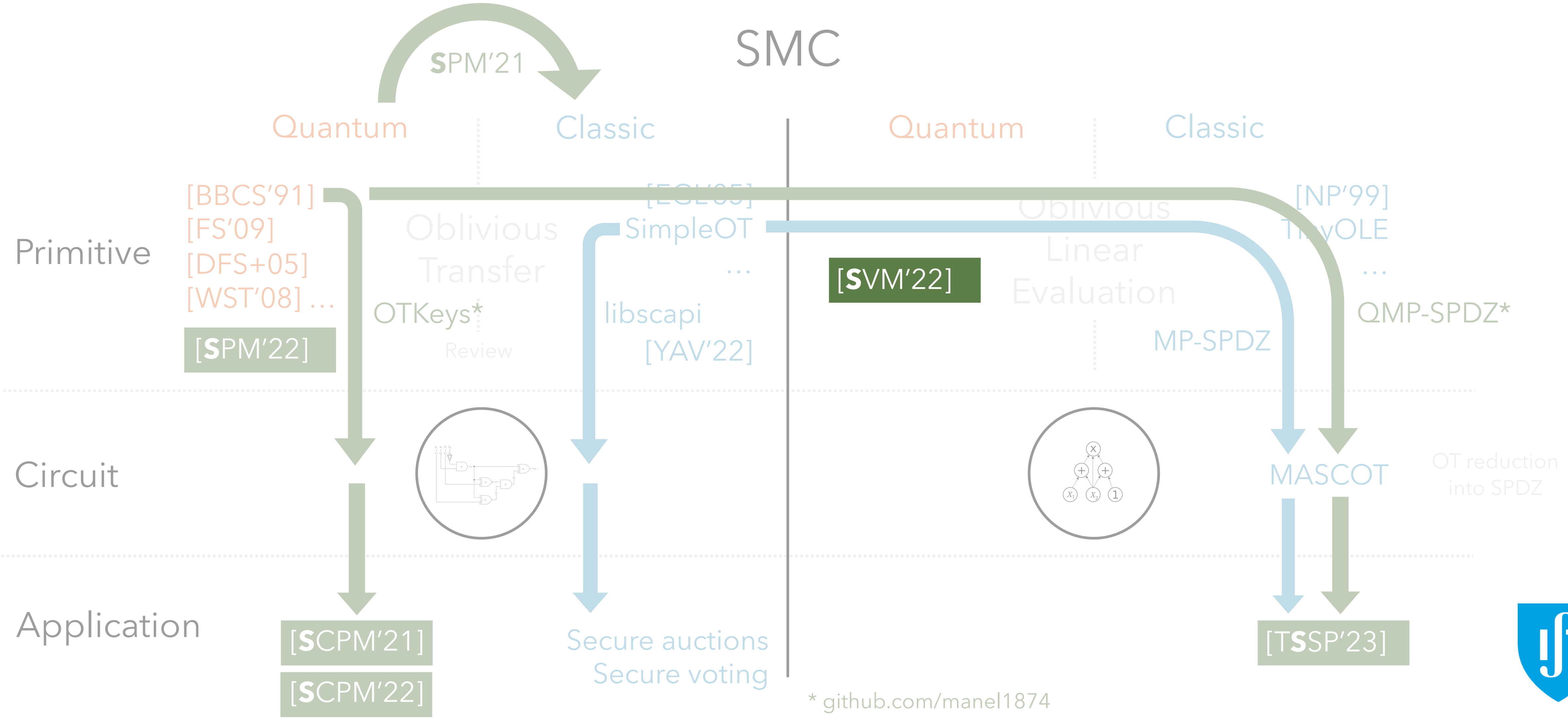


* github.com/manel1874

Quantum OLE



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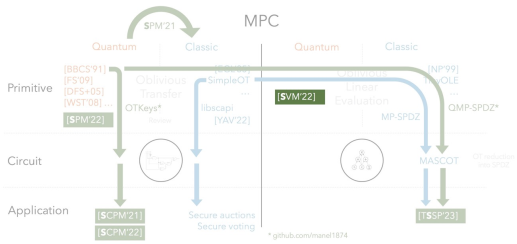
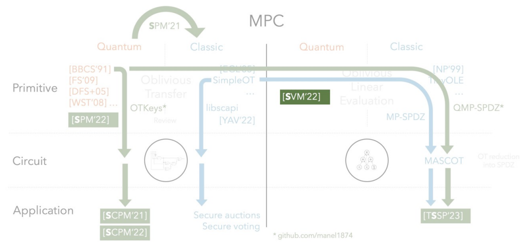


Results summary

- Oblivious Linear Evaluation (OLE)
- Vector OLE

Quantum OLE

Results summary



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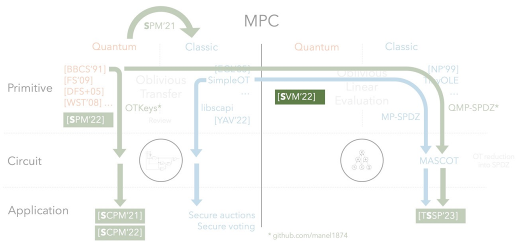

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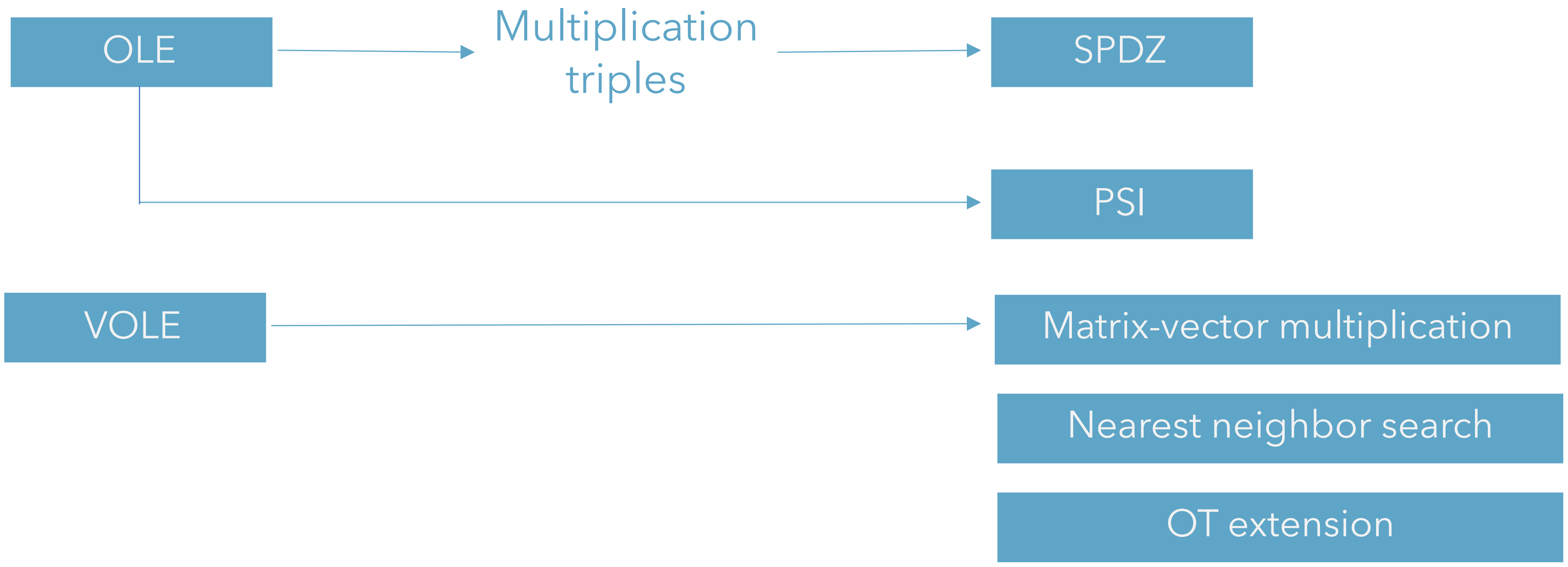

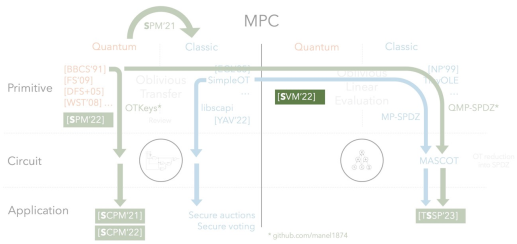

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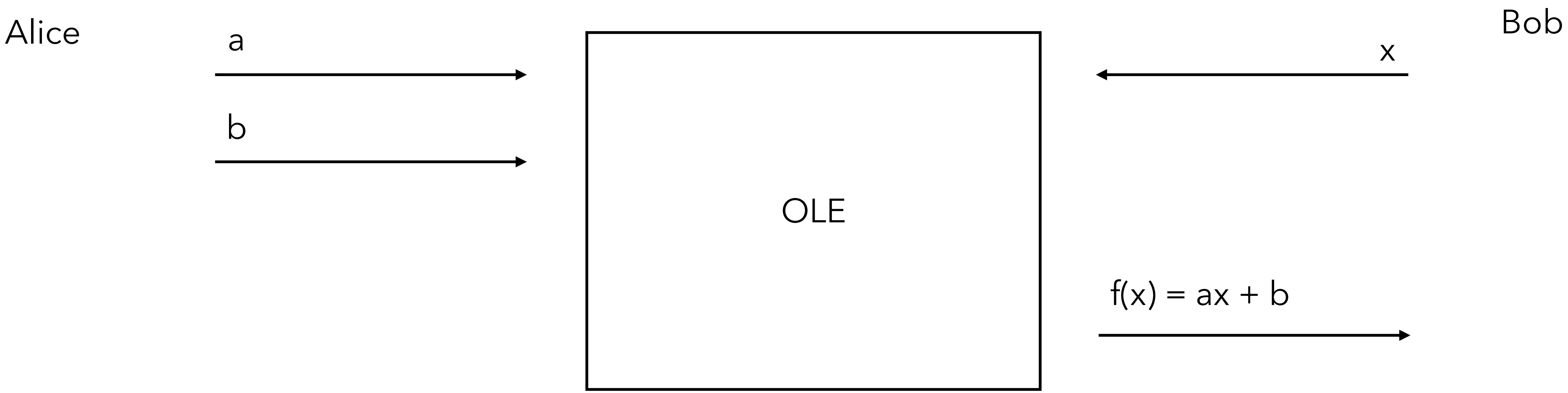
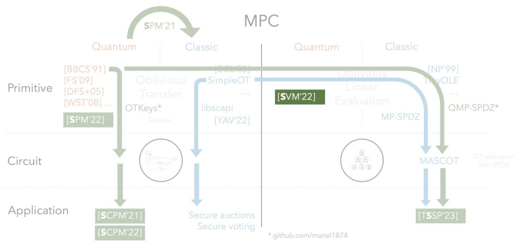
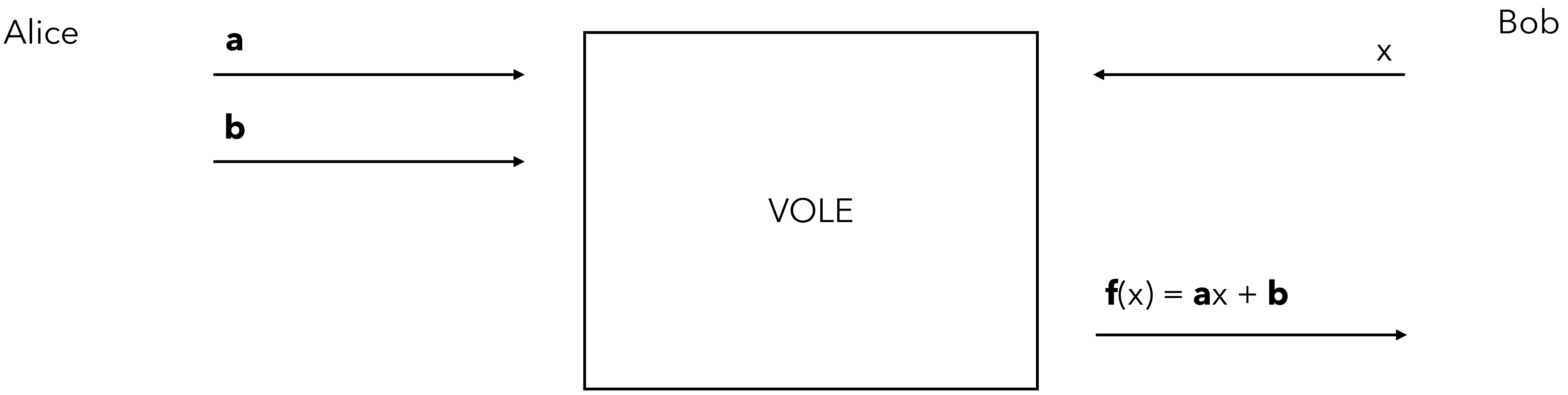


Quantum OLE

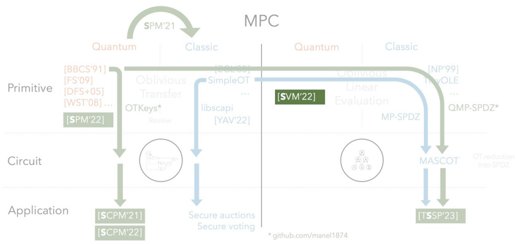


Results summary

- Oblivious Linear Evaluation (OLE)
- Vector OLE

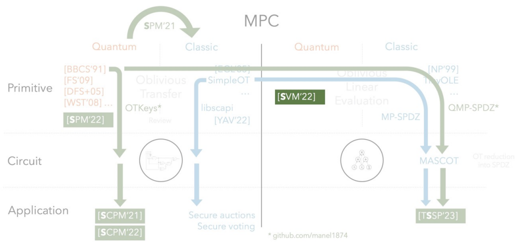


Quantum OLE | Main tool



In an Hilbert space of dimension *d*

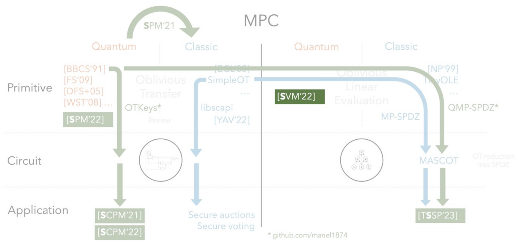
Quantum OLE | Main tool



In an Hilbert space of dimension *d*, there exists a set of MUBs

$$\{|e_r^x\rangle\}_{r\in\mathbb{Z}_d}$$

Quantum OLE | Main tool



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$$\mathcal{B}_1 = \{|\phi_1\rangle, \ldots, |\phi_d\rangle\}$$

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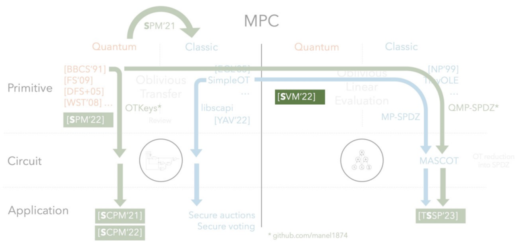
$$|\langle\psi_i|\phi_j\rangle| = \frac{1}{\sqrt{d}}$$

Quantum OLE | Main tool



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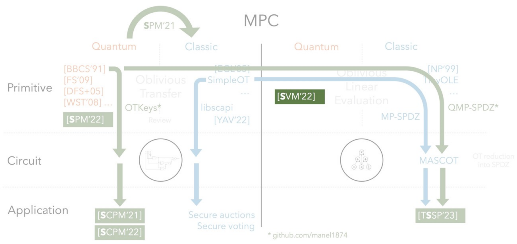
Quantum OLE | Main tool



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which, upon the action of the Heisenberg-Weyl operators, V_a^b

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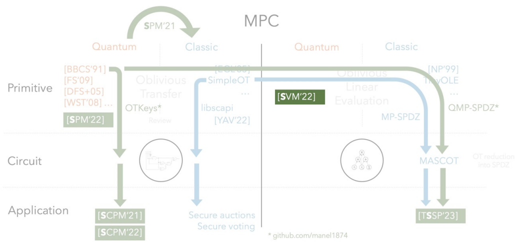
Quantum OLE | Main tool



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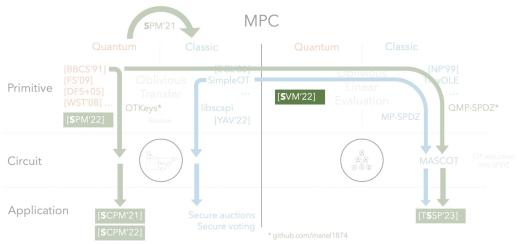
$$V_a^b := V_0^b V_a^0 = \sum_{l=0}^{d-1} \omega^{(l+a)b} |l+a\rangle\langle l|$$

Quantum OLE | Main tool



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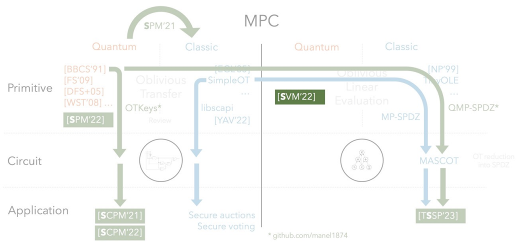

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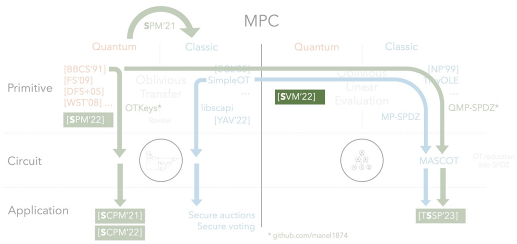
Alice, (a,b)

Bob, x

Quantum OLE | Main tool



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Alice, (a,b)

Bob, x

$|e_r^x\rangle$

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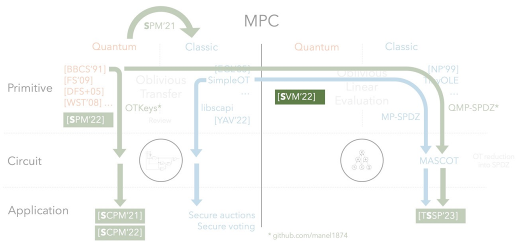

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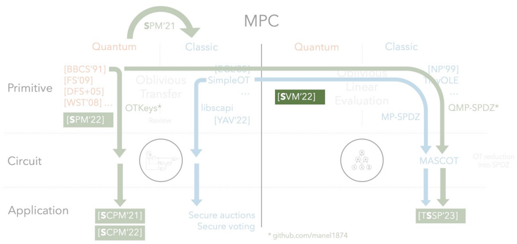
Alice, (a,b) Bob, x

$|e_r^x\rangle \longleftarrow |e_r^x\rangle$

Quantum OLE | Main tool



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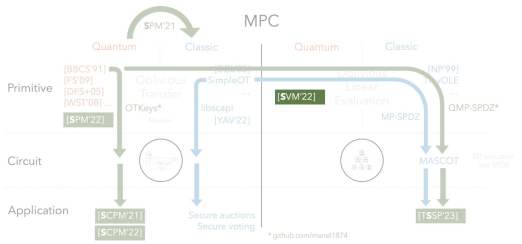
Alice, (a,b) Bob, x

$$|e_r^x\rangle \longleftarrow |e_r^x\rangle$$

$$V_a^b\,|e_r^x\rangle$$

Quantum OLE | Main tool



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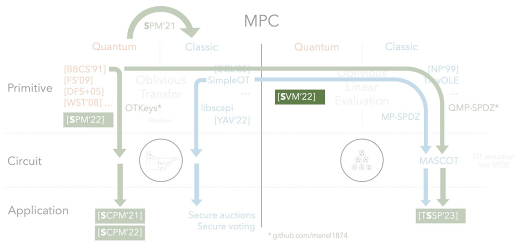
Quantum OLE | Main tool



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Alice, (a,b) — Bob, x

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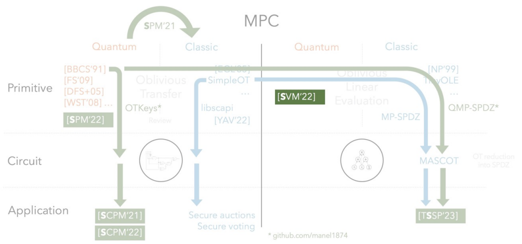
Quantum OLE | Main tool



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Alice, (a,b)

Bob, x

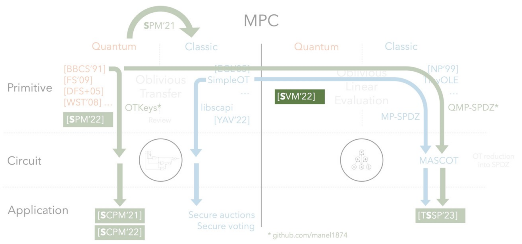
Attack:

$$|B_{a,b}\rangle = (\mathbb{1}\otimes V_a^b)\,|B_{0,0}\rangle$$

$|e_r^x\rangle$ $|e_{ax-b+r}^x\rangle$

$|e_r^x\rangle$ $|e_{ax-b+r}^x\rangle$

Quantum OLE | Main tool



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Alice, (a,b)

Bob, x

Attack:

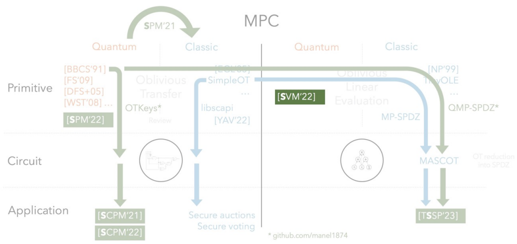
Commit-and-open phase

$$|B_{a,b}\rangle = (1\otimes V_a^b)\,|B_{0,0}\rangle$$

$|e_r^x\rangle \leftarrow$ $|e_r^x\rangle$

$|e_{ax-b+r}^x\rangle \rightarrow$ $|e_{ax-b+r}^x\rangle$

Quantum OLE | Protocol



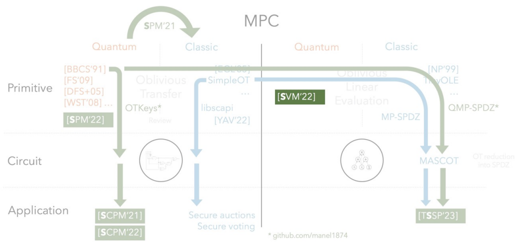
Alice, (a,b)

Bob, x

Quantum phase

Classical phase

Quantum OLE | Protocol



Alice, (a,b)

Bob, x

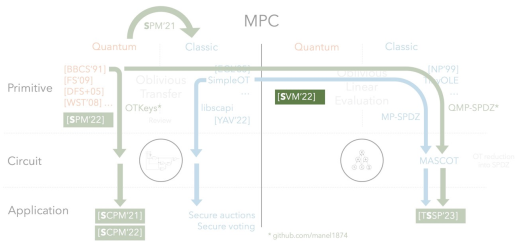
$i \in [m]$

$\left|e_{r_i}^{x_i^0}\right\rangle$

Quantum phase

Classical phase

Quantum OLE | Protocol



Alice, (a,b)

Bob, x

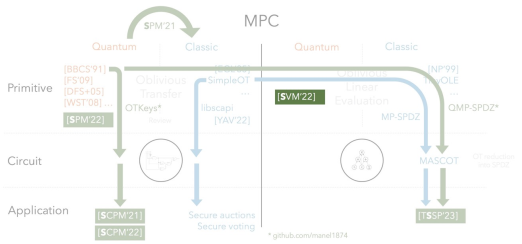
Quantum phase

$i \in [m]$

$\left|e^{x_i^0}_{r_i}\right\rangle \longleftarrow \left|e^{x_i^0}_{r_i}\right\rangle$

$V^{b_i^0}_{a_i^0}\left|e^{x_i^0}_{r_i}\right\rangle$

Classical phase



Quantum OLE | Protocol

Alice, (a,b)

Bob, x

Quantum phase

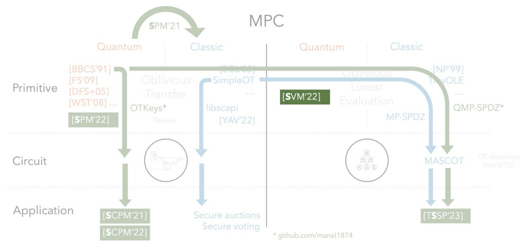
$i \in [m]$

$\left|e_{r_i}^{x_i^0}\right\rangle \longleftarrow \left|e_{r_i}^{x_i^0}\right\rangle$

$V_{a_i^0}^{b_i^0}\left|e_{r_i}^{x_i^0}\right\rangle$

Classical phase

Quantum OLE | Protocol



Alice, (a,b)

Bob, x

$i \in [m]$

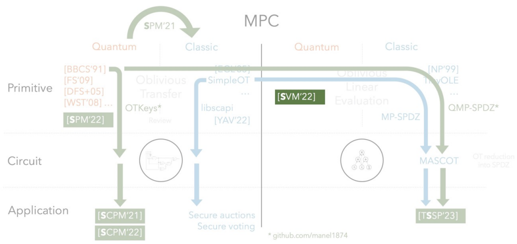
$\left| e^{x_i^0}_{r_i} \right\rangle \longleftarrow \left| e^{x_i^0}_{r_i} \right\rangle$

$\left| e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i} \right\rangle$

Quantum phase

Classical phase

Quantum OLE | Protocol



Quantum phase

Classical phase

Alice, (a,b)

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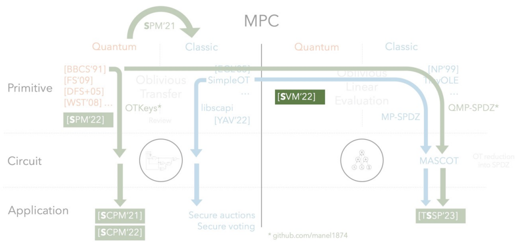
Commit-and-open phase

Quantum OLE | Protocol



Quantum phase

Classical phase

Alice, (a,b)

Bob, x

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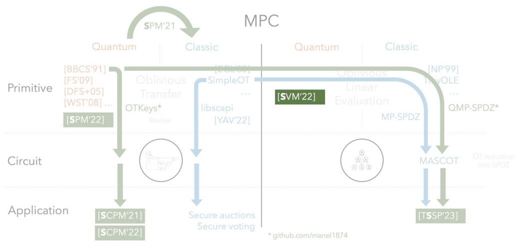
$\longleftarrow$ $\texttt{commit}(i, x_i^0, r_i)_{i\in[m]}$

Commit-and-open phase

Quantum OLE | Protocol



Quantum phase

Classical phase

Alice, (a,b) | Bob, x

$i \in [m]$

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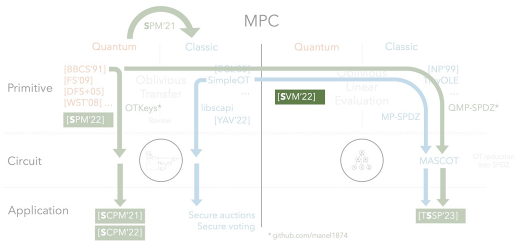
$T \subset [m]$ **Commit-and-open phase** $\mathtt{commit}(i, x_i^0, r_i)_{i\in[m]}$

Quantum OLE | Protocol



Quantum phase

Alice, (a,b) — Bob, x

$$i \in [m]$$

$\left| e^{x_i^0}_{r_i} \right\rangle \longleftarrow \left| e^{x_i^0}_{r_i} \right\rangle$

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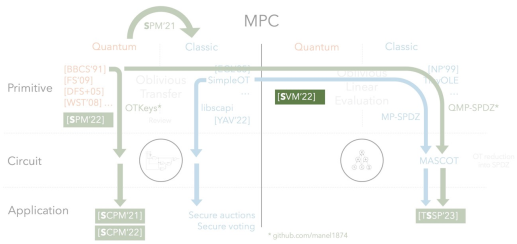
$T \subset [m]$ — **Commit-and-open phase** — $\texttt{commit}(i, x_i^0, r_i)_{i\in[m]}$, $\texttt{open}(i, x_i^0, r_i)_{i\in T}$

Classical phase

Quantum OLE | Protocol



Quantum phase

Alice, (a,b) | Bob, x

$i \in [m]$

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$\left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle$

Commit-and-open phase

$T \subset [m]$

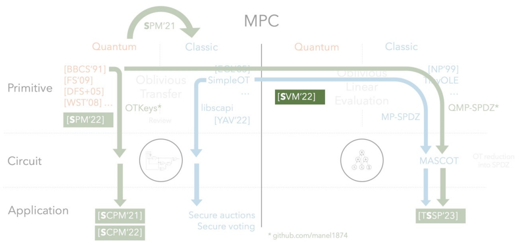
$\texttt{commit}(i, x_i^0, r_i)_{i \in [m]}$

$\texttt{open}(i, x_i^0, r_i)_{i \in T}$

$\left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle \longrightarrow$

Classical phase

Quantum OLE | Protocol



Quantum phase

Classical phase

Alice, (a,b) — Bob, x

$i \in [m]$

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$\left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle$

Commit-and-open phase

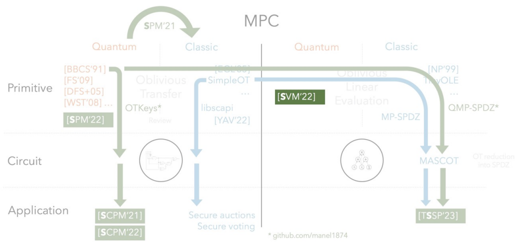
$T \subset [m]$

$\texttt{commit}(i, x_i^0, r_i)_{i\in[m]}$

$\texttt{open}(i, x_i^0, r_i)_{i\in T}$

$\left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle \longrightarrow \left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle$

Quantum OLE | Protocol



Alice, (a,b) — Bob, x

Quantum phase

$i \in [m]$

$\left|e^{x_i^0}_{r_i}\right\rangle \longleftarrow \left|e^{x_i^0}_{r_i}\right\rangle$

$\left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle$

$T \subset [m]$ — **Commit-and-open phase** — $\texttt{commit}(i, x_i^0, r_i)_{i\in[m]}$, $\texttt{open}(i, x_i^0, r_i)_{i\in T}$

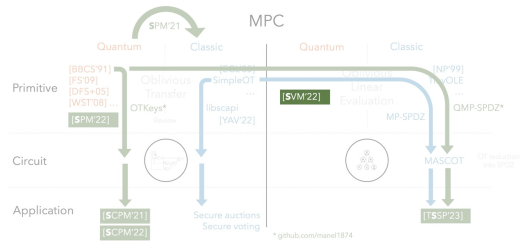
$\left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle \longrightarrow \left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle$

Classical phase

Security:

$$H_{\min}(\mathbf{F}_0|B')_{\sigma_{\mathbf{F}_0 B'}} \geq \frac{n \log d}{2}(1 - h_d(\zeta))$$

Quantum OLE | Protocol



Alice, (a,b) | Bob, x

Quantum phase

$i \in [m]$

$|e^{x_i^0}_{r_i}\rangle \longleftarrow |e^{x_i^0}_{r_i}\rangle$

$|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\rangle$

$T \subset [m]$

Commit-and-open phase

$\texttt{commit}(i, x_i^0, r_i)_{i\in[m]}$

$\texttt{open}(i, x_i^0, r_i)_{i\in T}$

$|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\rangle \longrightarrow |e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\rangle$

Security:

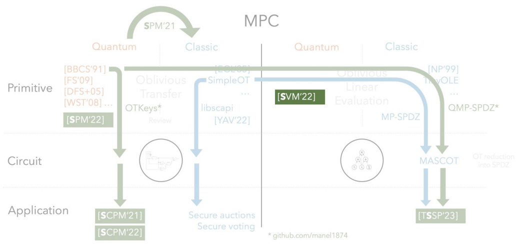
$$H_{\min}(\mathbf{F}_0|B')_{\sigma_{\mathbf{F}_0 B'}} \geq \frac{n \log d}{2}(1 - h_d(\zeta))$$

Classical phase

Derandomization:

n ROLE $\longrightarrow$ n OLE

Quantum OLE | Protocol



Quantum phase

Alice, (a,b)

Bob, x

$i\in[m]$

$\left|e^{x_i^0}_{r_i}\right\rangle \longleftarrow \left|e^{x_i^0}_{r_i}\right\rangle$

$\left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle$

$T \subset [m]$

Commit-and-open phase

$\texttt{commit}(i, x_i^0, r_i)_{i\in[m]}$

$\texttt{open}(i, x_i^0, r_i)_{i\in T}$

$\left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle \longrightarrow \left|e^{x_i^0}_{a_i^0 x_i^0 - b_i^0 + r_i}\right\rangle$

Security:

$$H_{\min}(\mathbf{F}_0|B')_{\sigma_{\mathbf{F}_0 B'}} \geq \frac{n \log d}{2}(1 - h_d(\zeta))$$

Classical phase

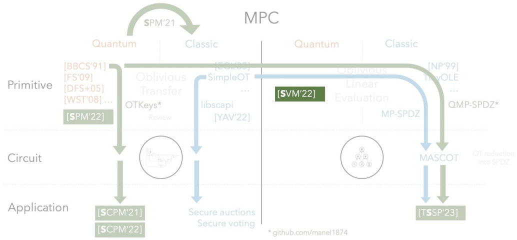
Derandomization:

n ROLE $\longrightarrow$ n OLE

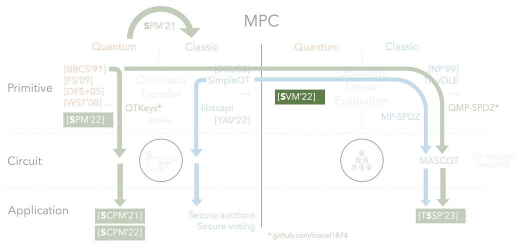
Extraction: Privacy amplification + Combiner

n OLE $\longrightarrow$ 1 OLE

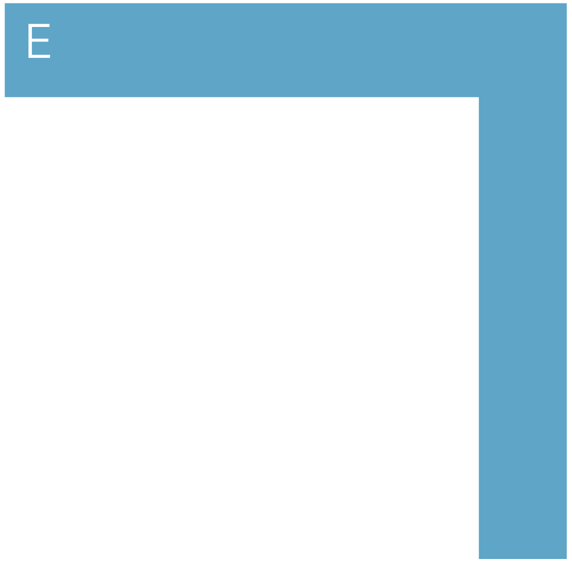
Quantum OLE | UC security



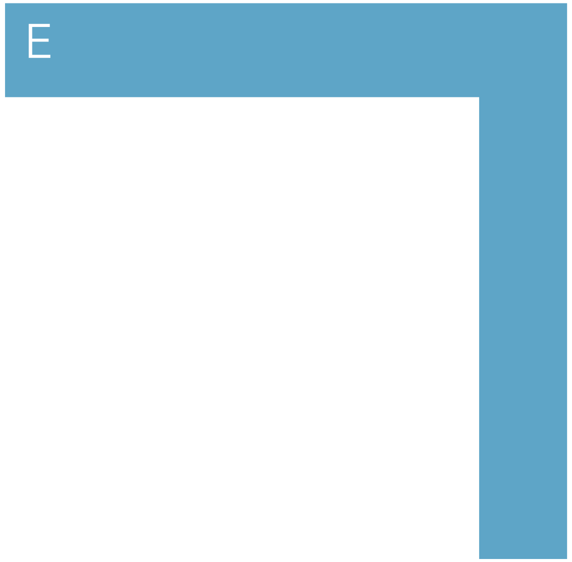
Quantum OLE | UC security



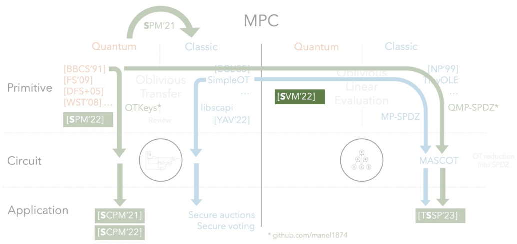
Ideal



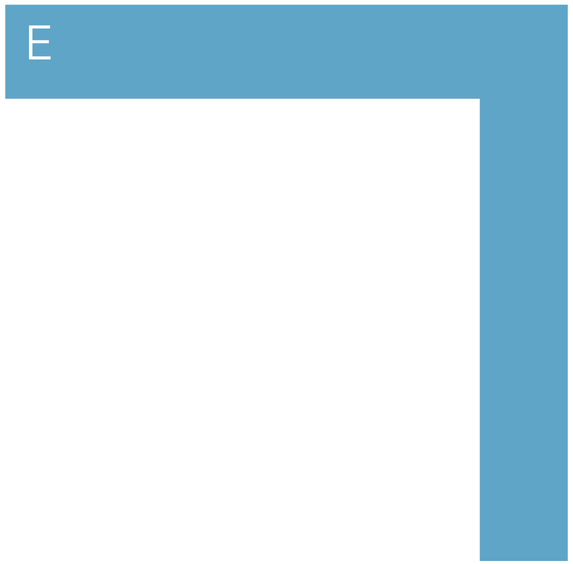
Real



Quantum OLE | UC security



Ideal



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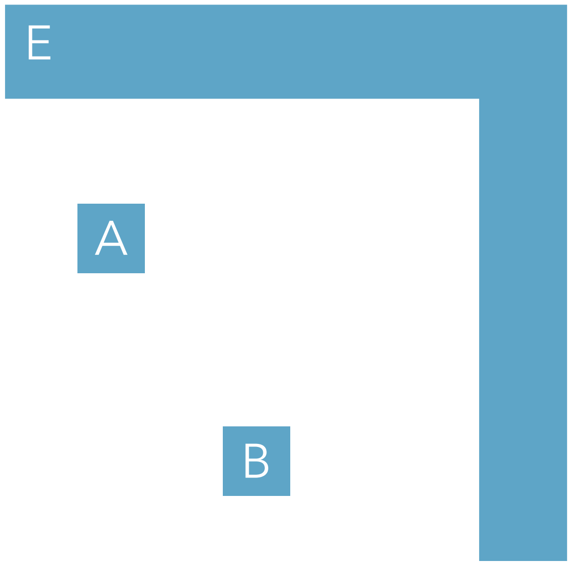
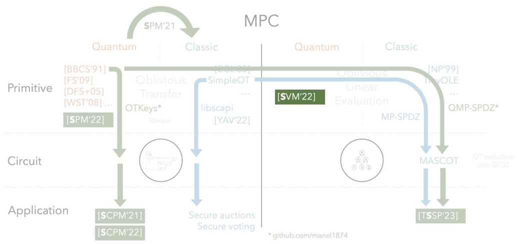
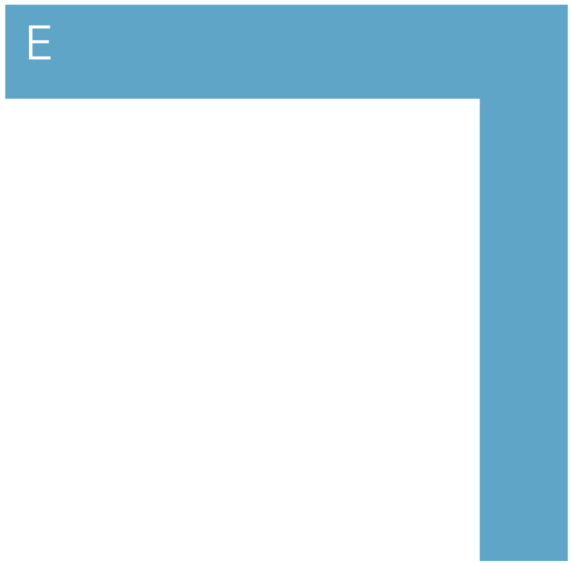


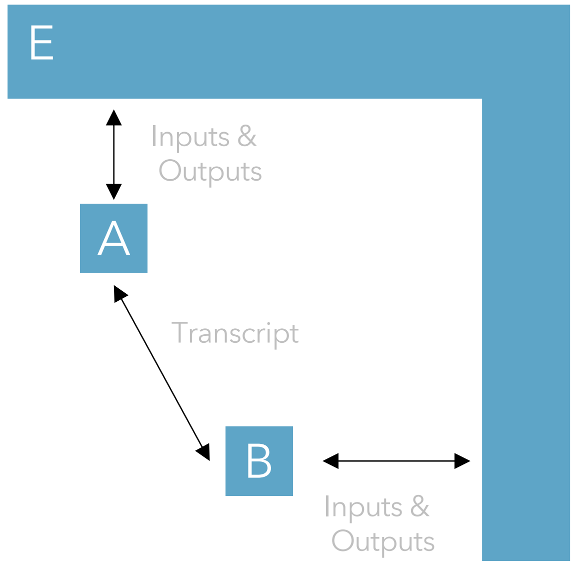
Quantum OLE | UC security



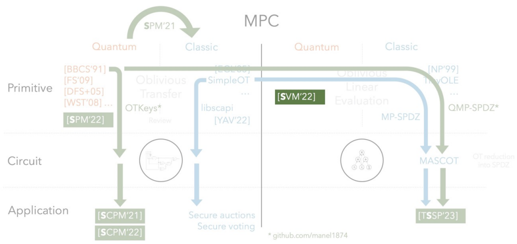
Ideal



Real



Quantum OLE | UC security



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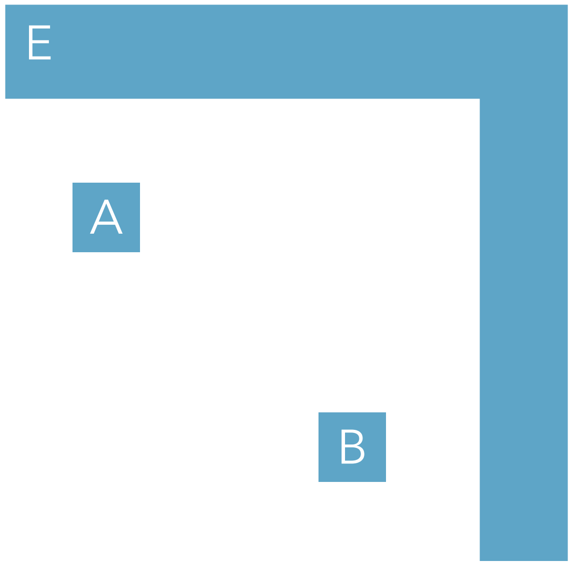

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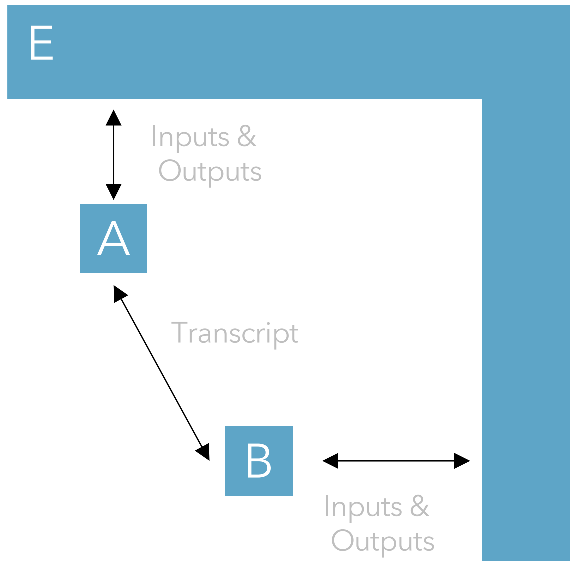
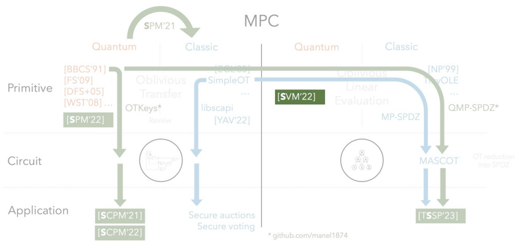


Quantum OLE | UC security



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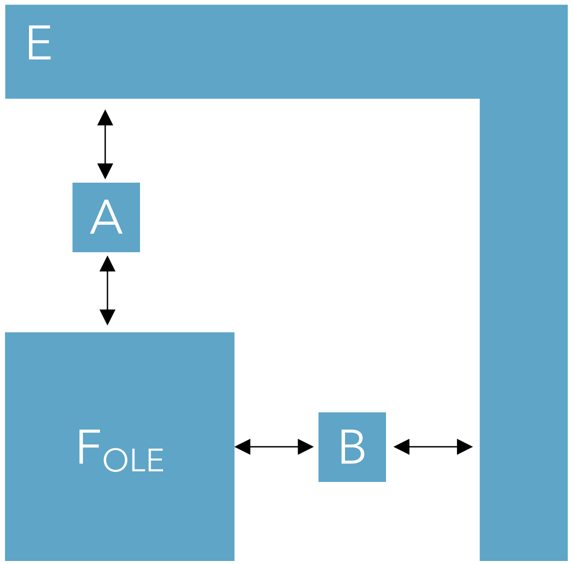

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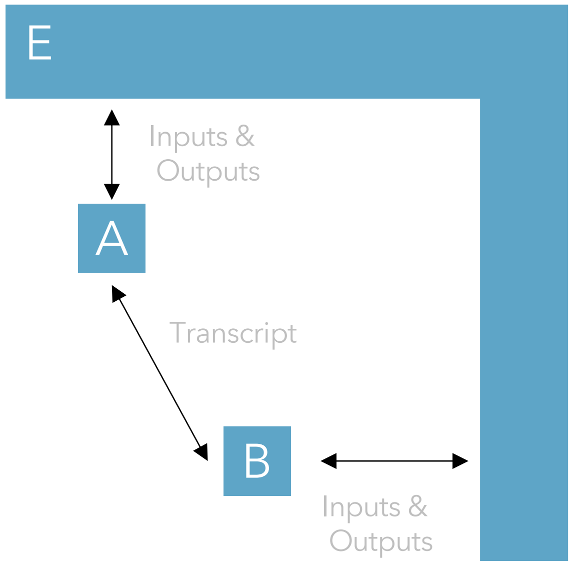
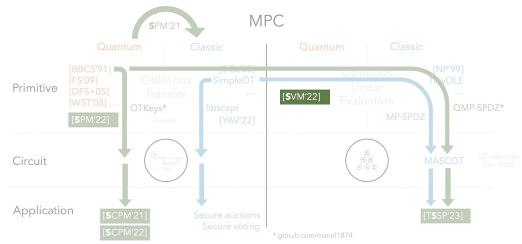
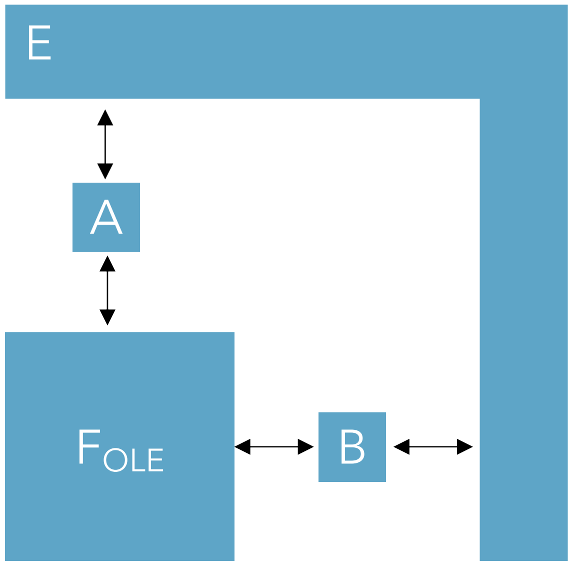


Quantum OLE | UC security

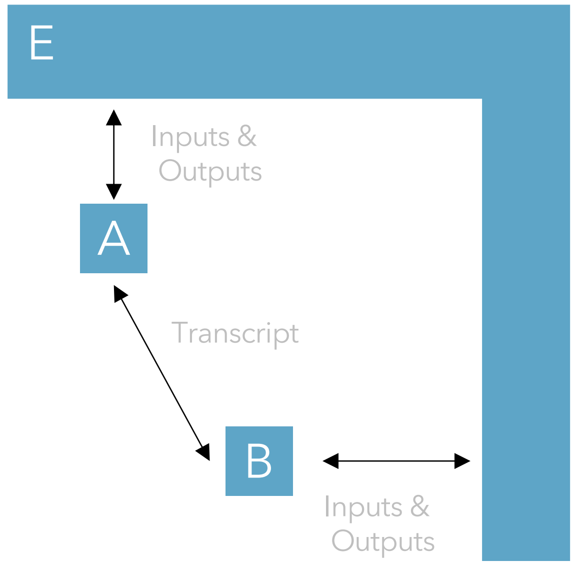


Ideal

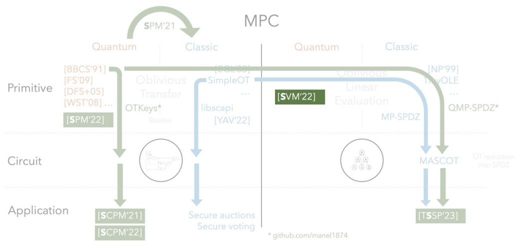


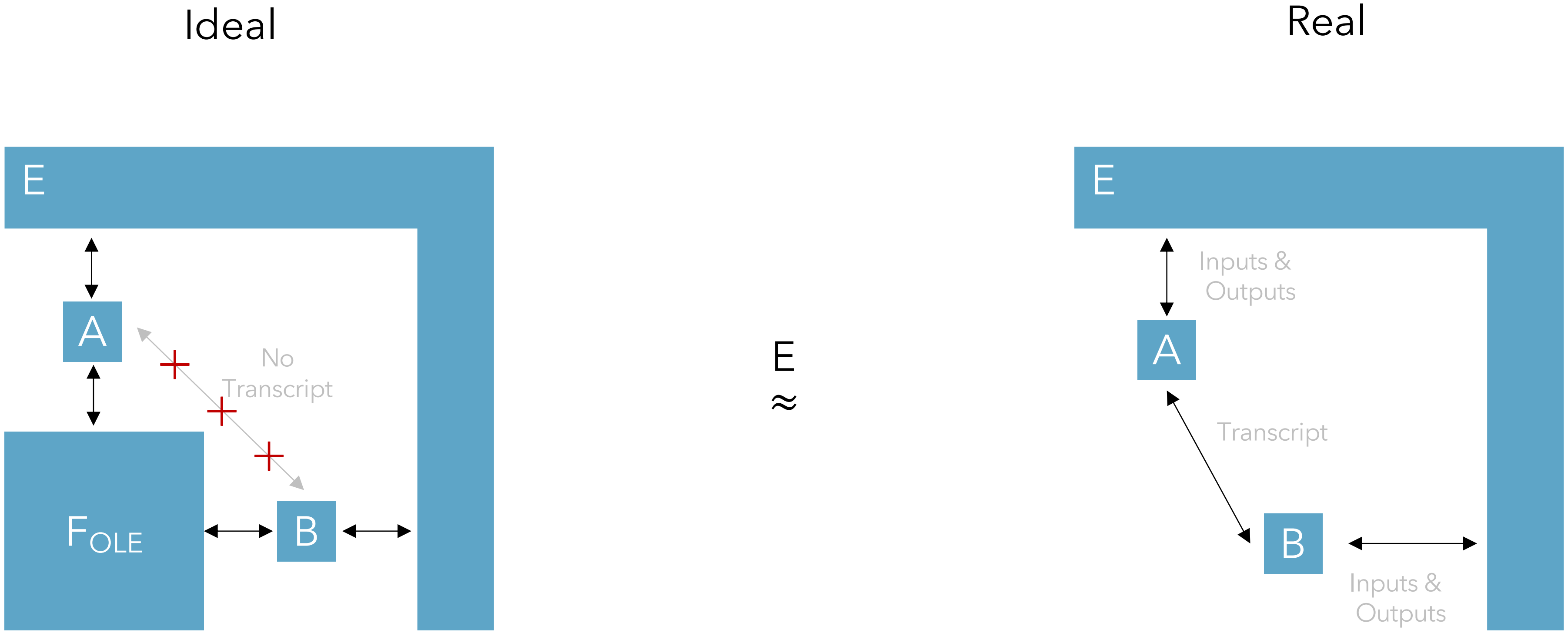
$\overset{E}{\approx}$

Real

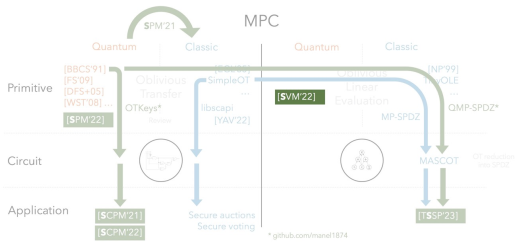


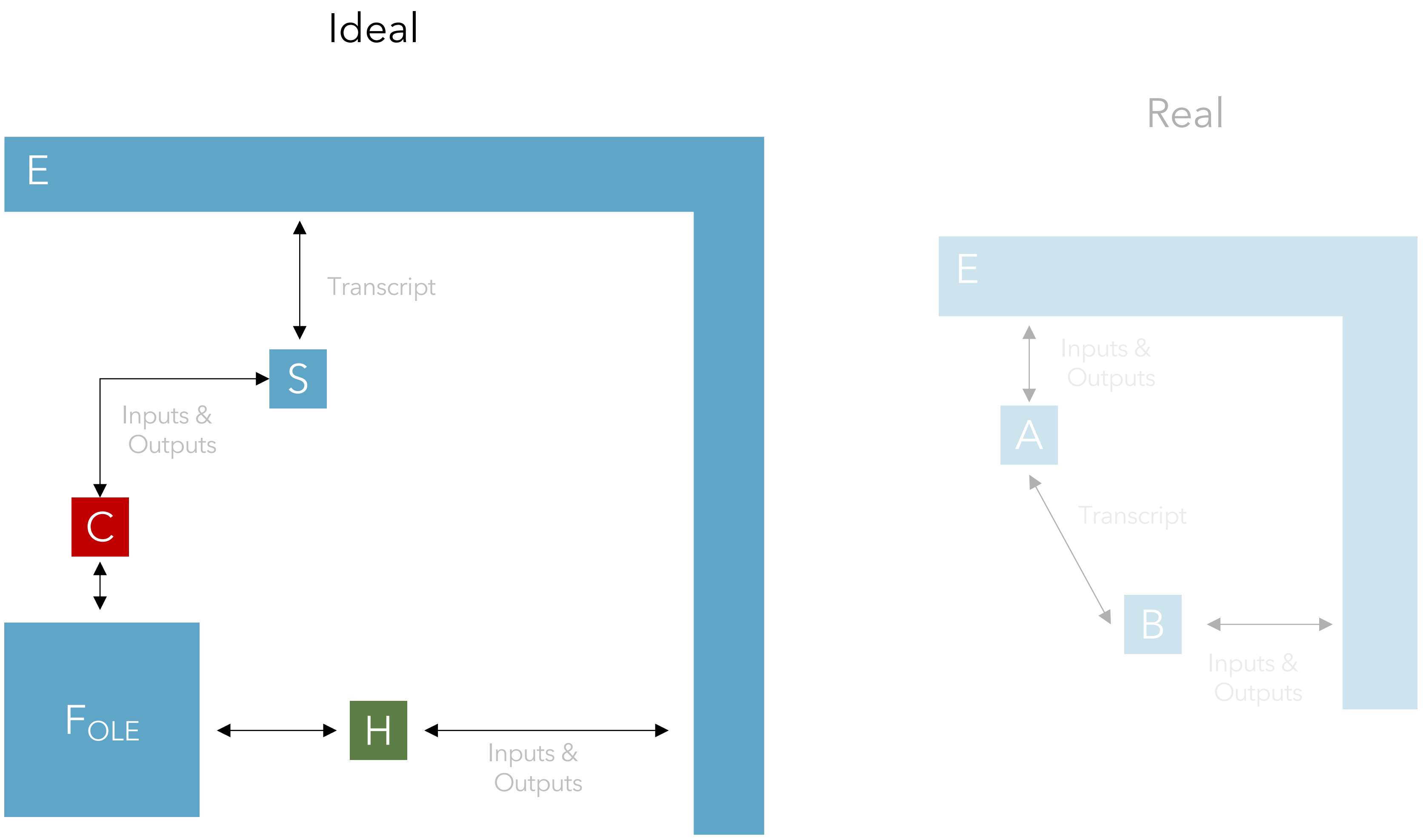
Quantum OLE | UC security



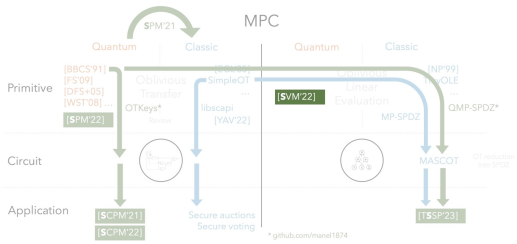


Quantum OLE | UC security



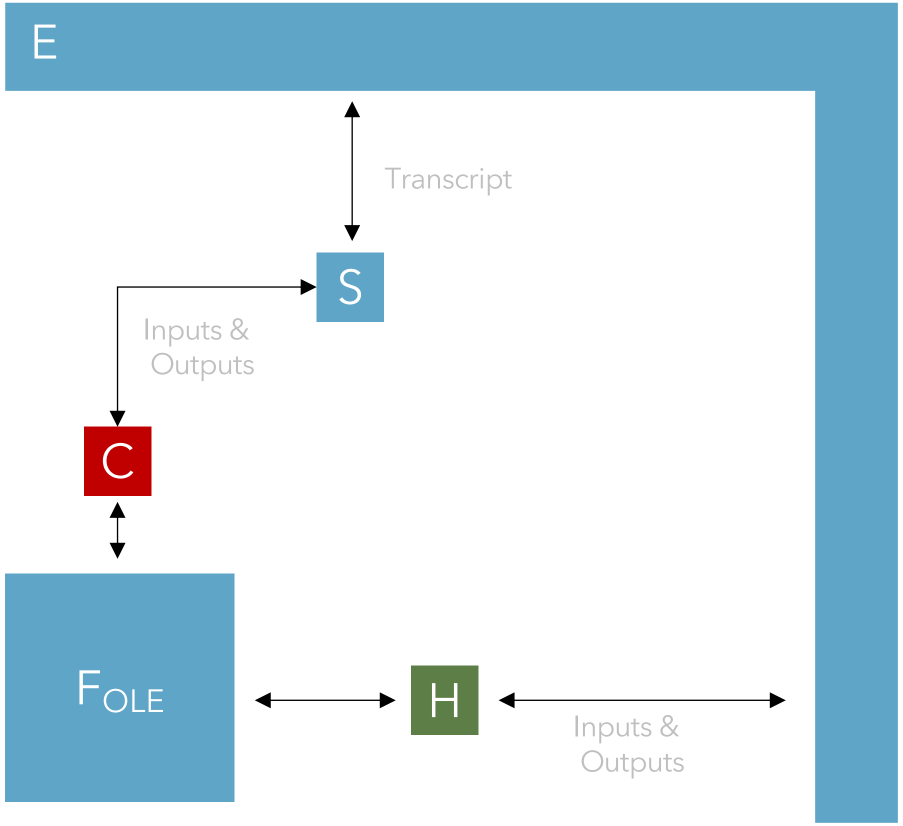


Quantum OLE | UC security

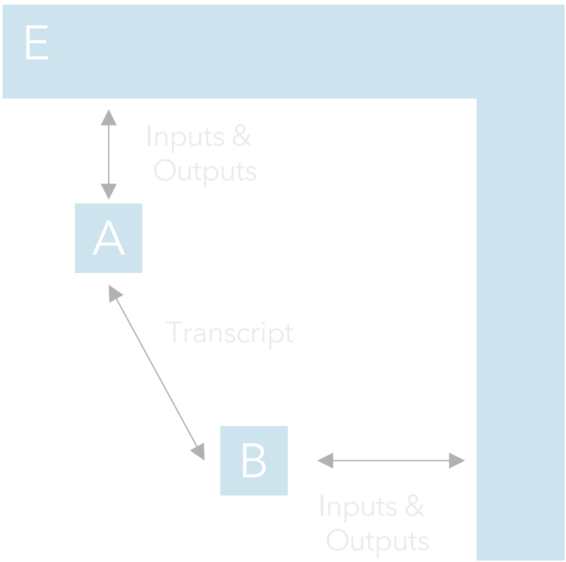


Indistinguishability:

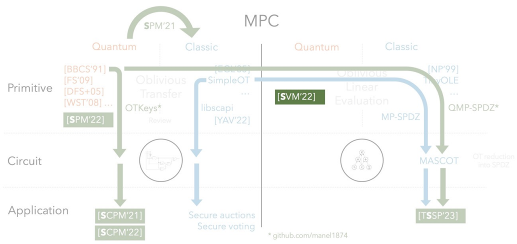
Ideal



Real

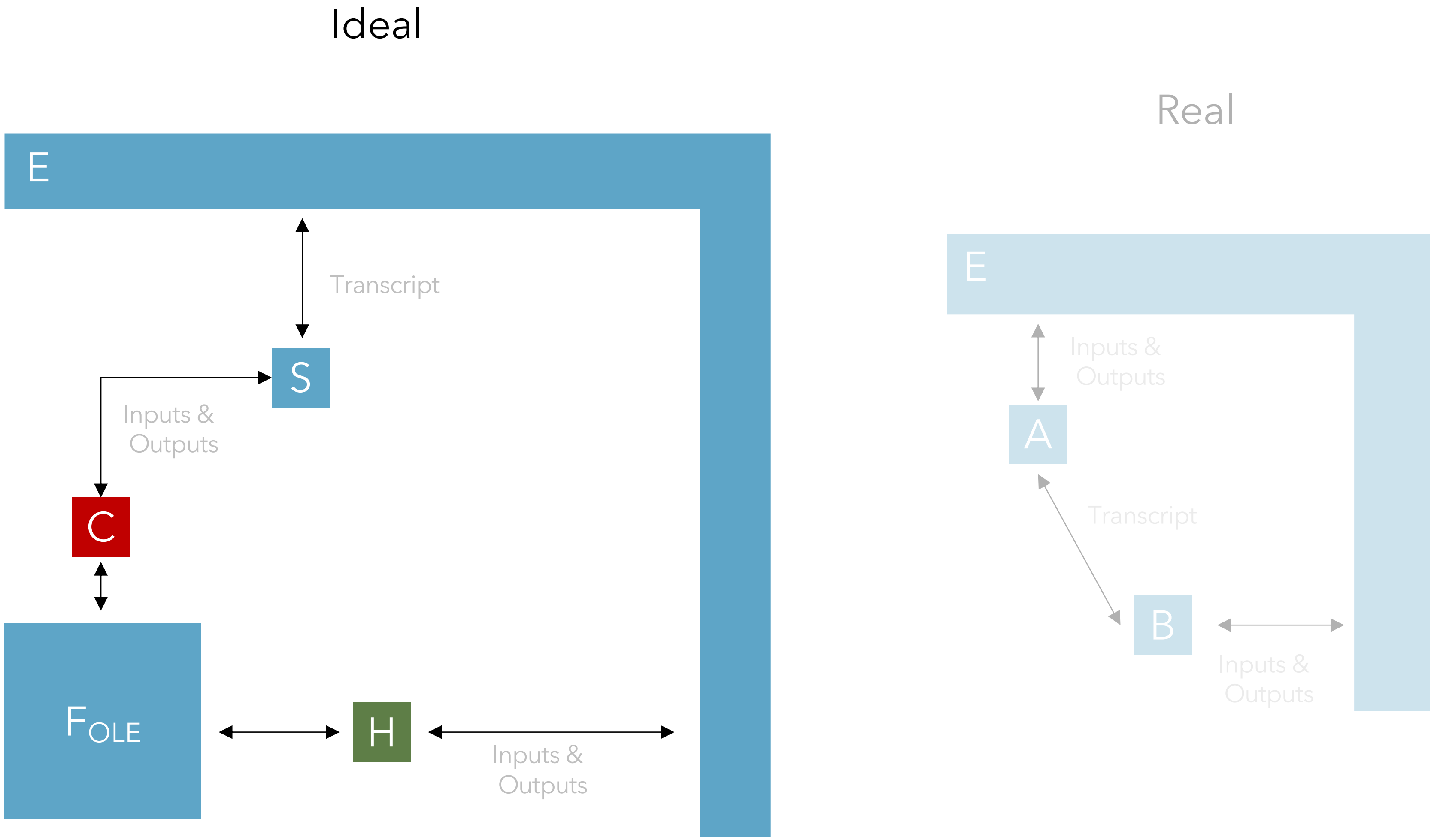


Quantum OLE | UC security

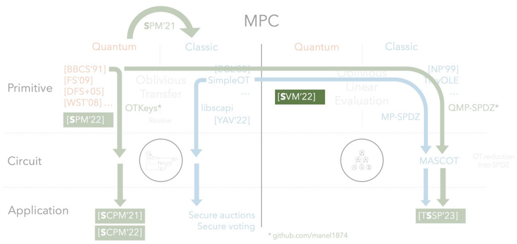


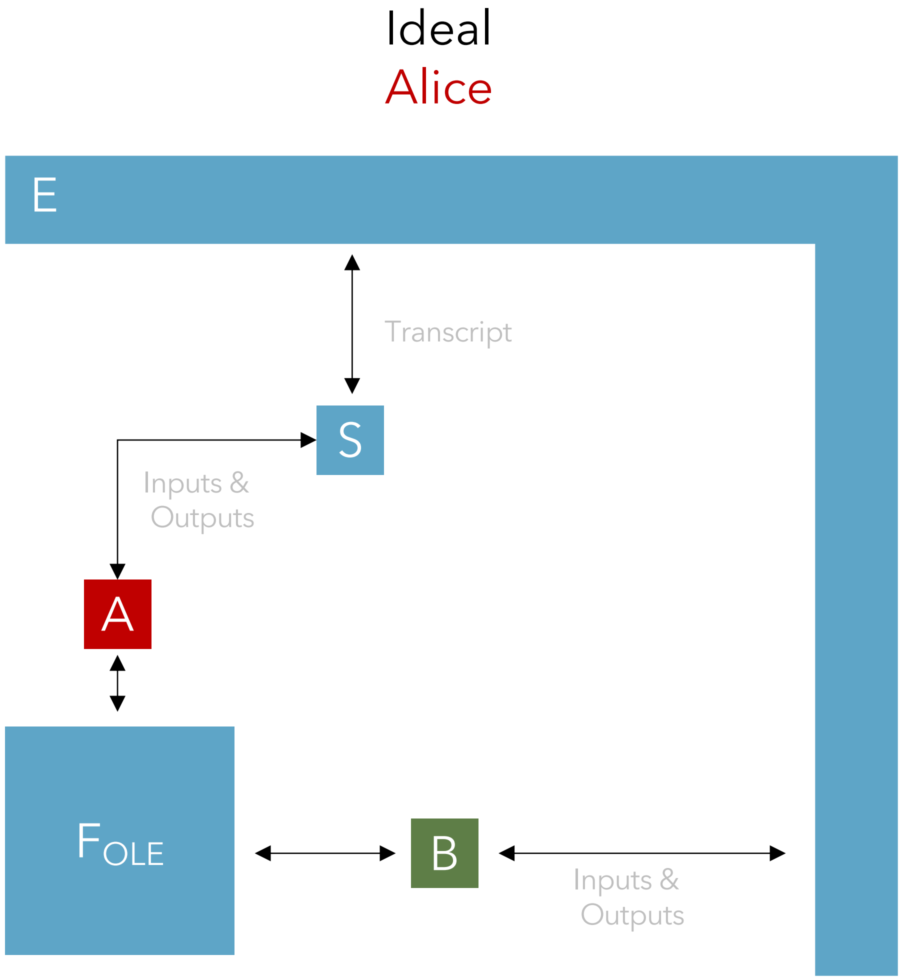
Indistinguishability:

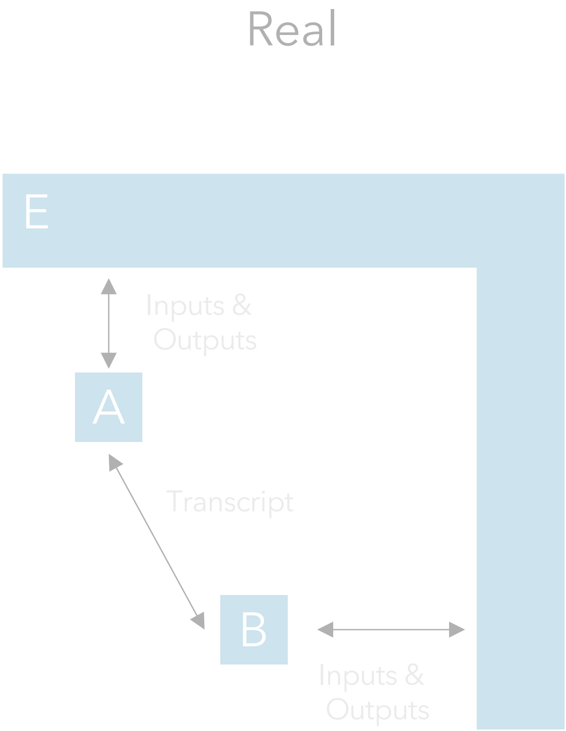
Extractability:



Quantum OLE | UC security



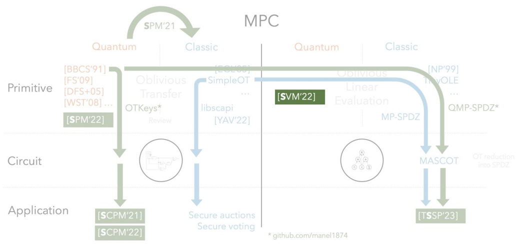




Indistinguishability:

Extractability:

Quantum OLE | UC security



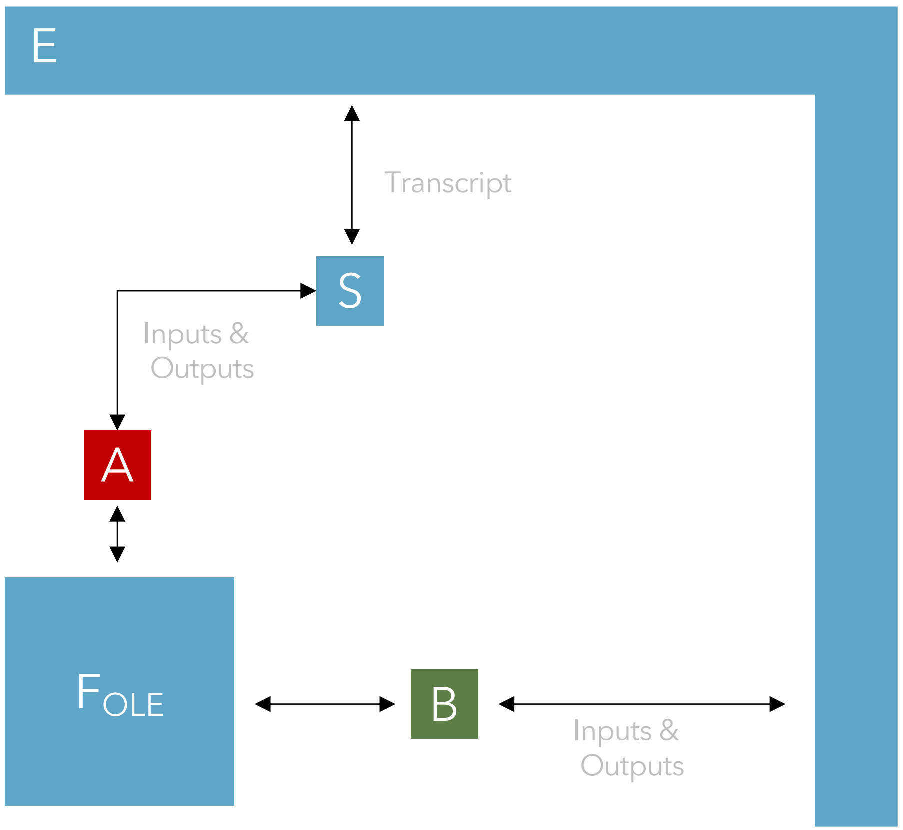
Indistinguishability:

Fake commitments

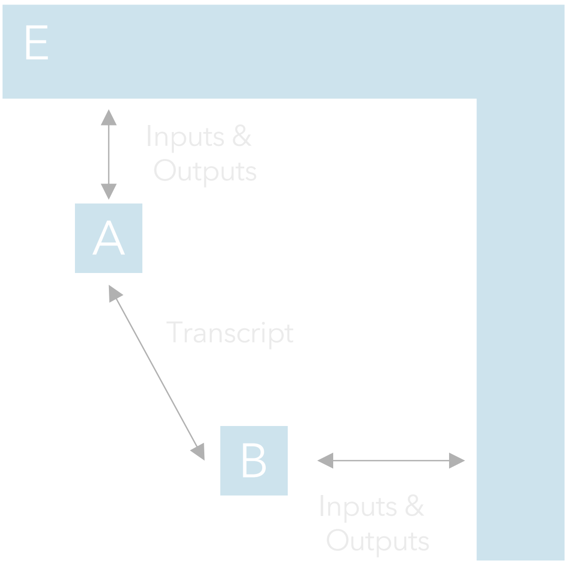
Extractability:

Ideal

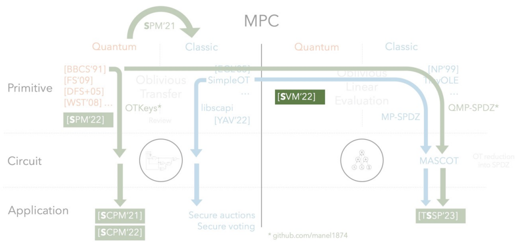
Alice



Real



Quantum OLE | UC security



Indistinguishability:

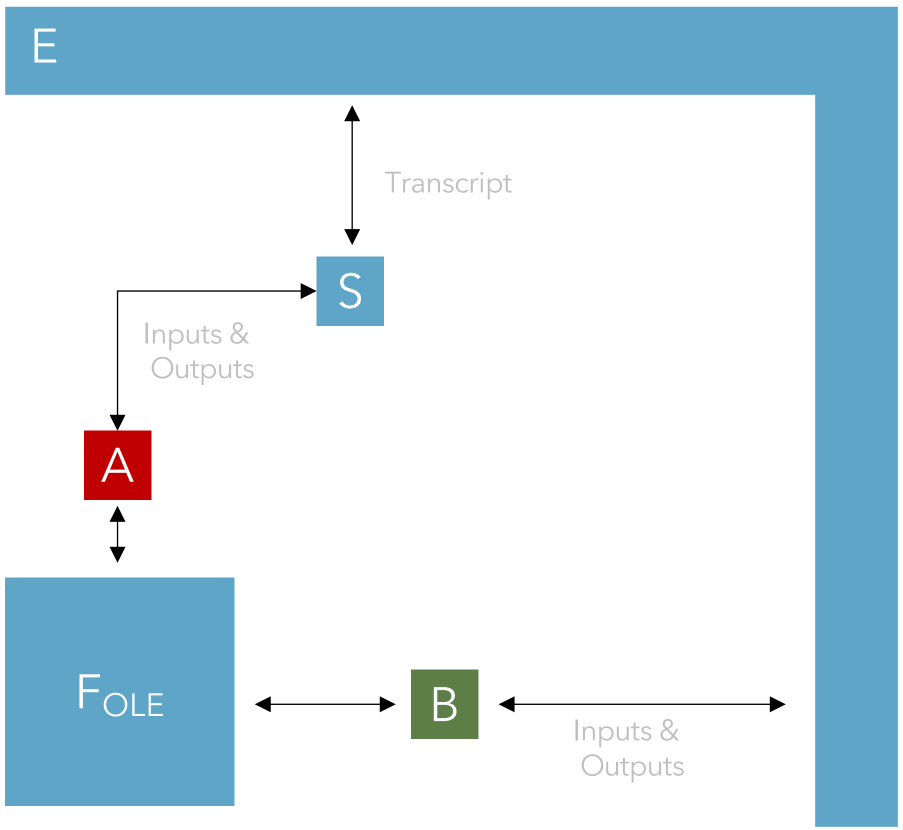
Fake commitments

Extractability:

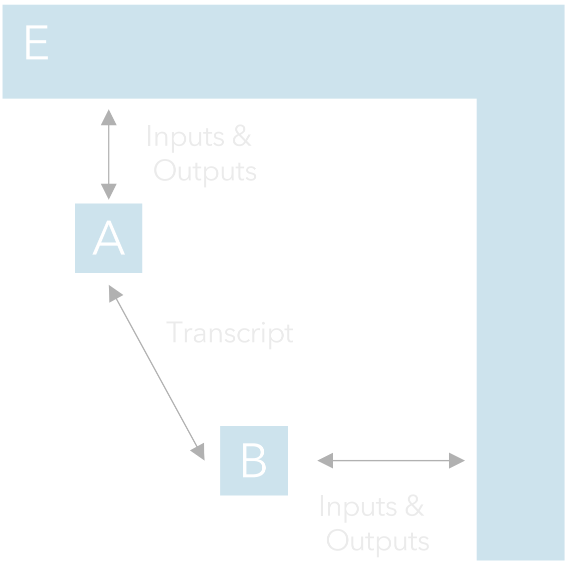
$$|B_{a,b}\rangle \quad = \quad (\mathbb{1} \otimes V_a^b)\,|B_{0,0}\rangle$$

Ideal

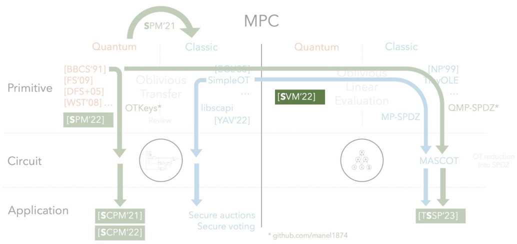
Alice



Real

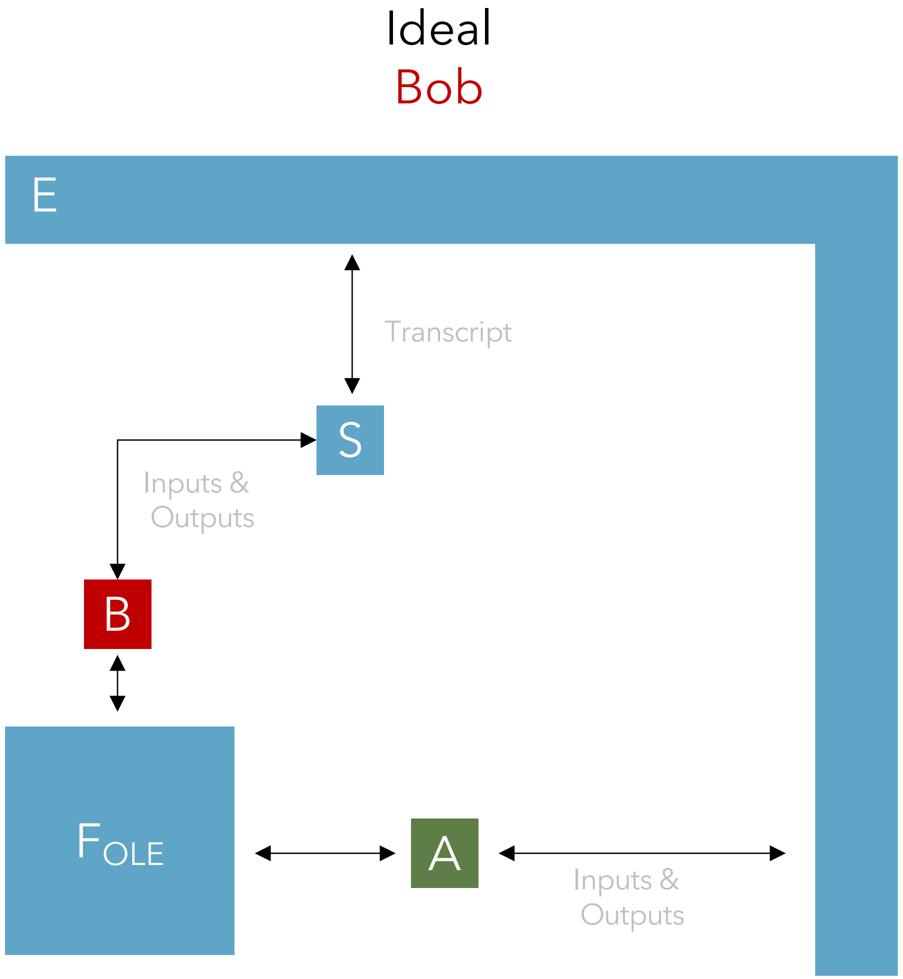


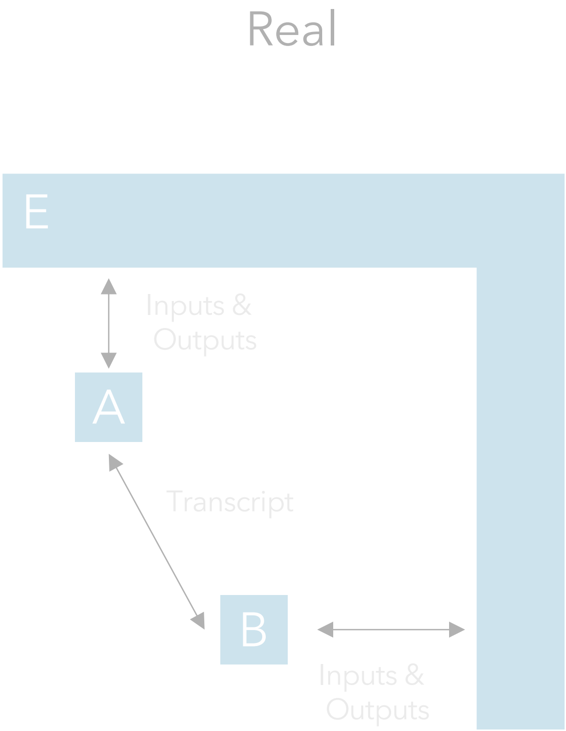
Quantum OLE | UC security



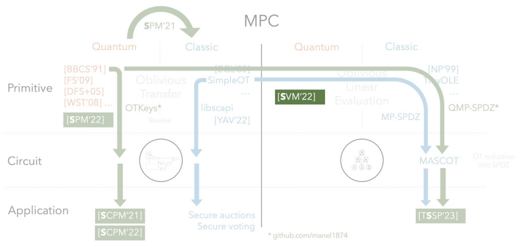
Indistinguishability:

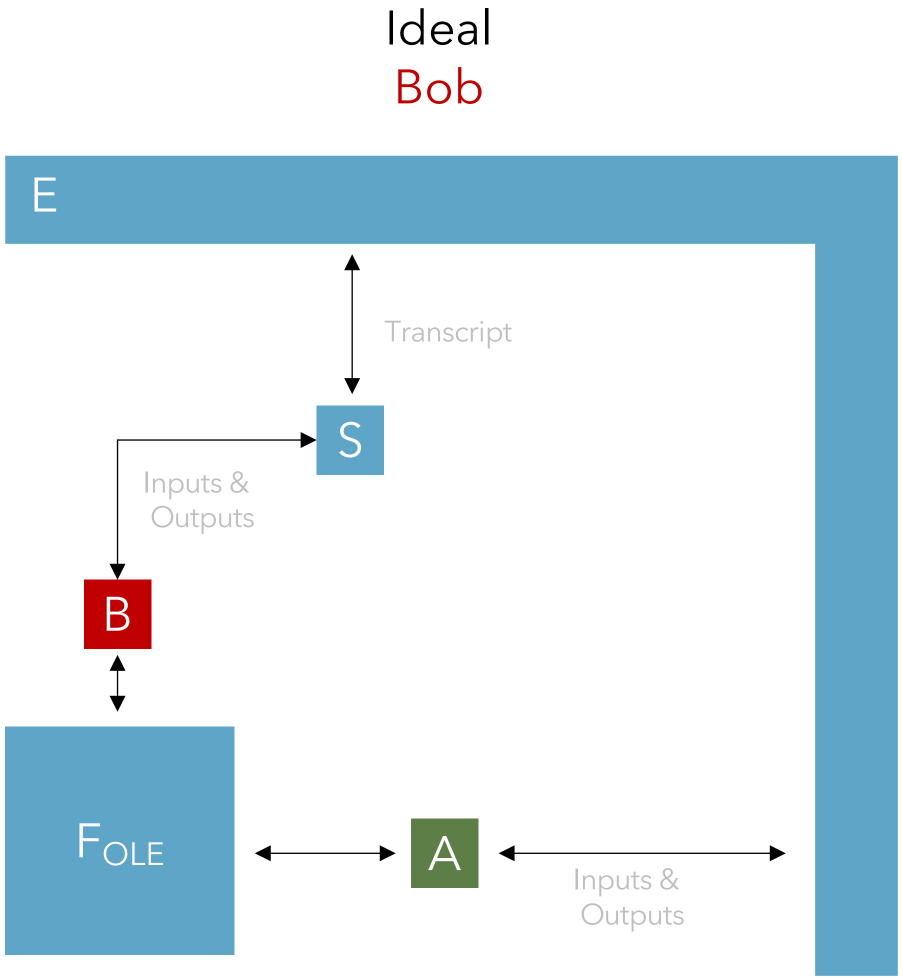
Extractability:

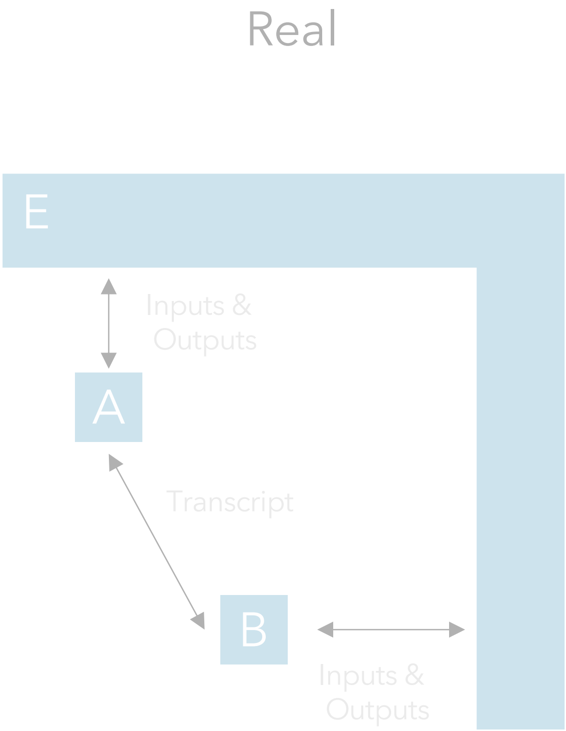




Quantum OLE | UC security





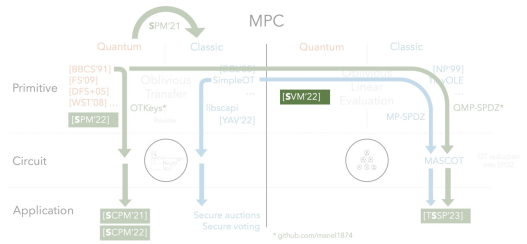


Indistinguishability:

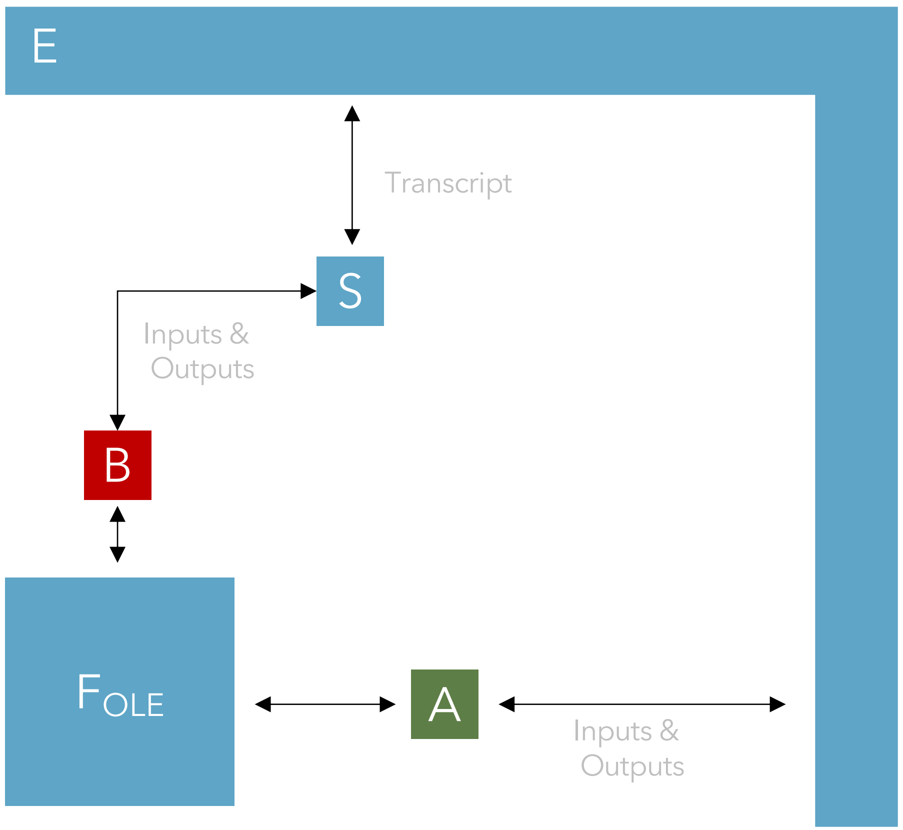
$$H_{\min}(\mathbf{F}_{\boldsymbol{a}} \,|\, \mathbf{Y}E) \geq \frac{n \log d}{2}(1 - h_d(\zeta))$$

Extractability:

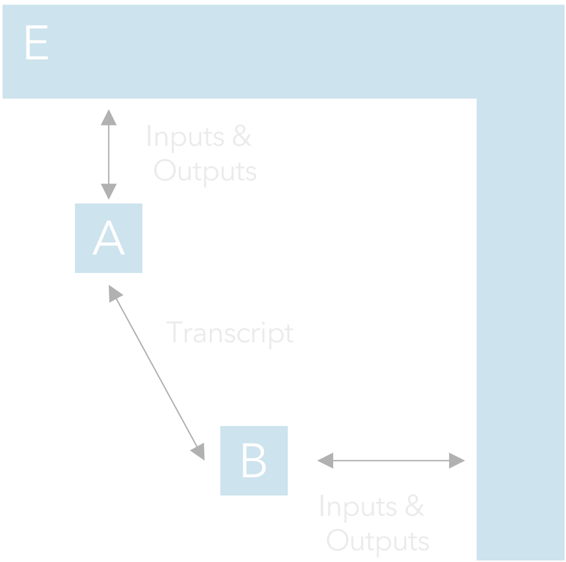
Quantum OLE | UC security



Ideal
Bob



Real



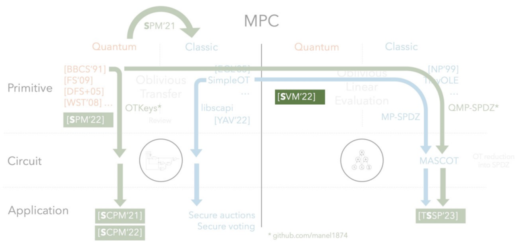
Indistinguishability:

$$H_{\min}(\mathbf{F}_{\boldsymbol{a}} \,|\, \mathbf{Y}E) \geq \frac{n \log d}{2}(1 - h_d(\zeta))$$

$$\mathbf{a} \xrightarrow[\text{Amplification}]{\text{Privacy}} \mathrm{a}$$

Extractability:

Quantum OLE | UC security



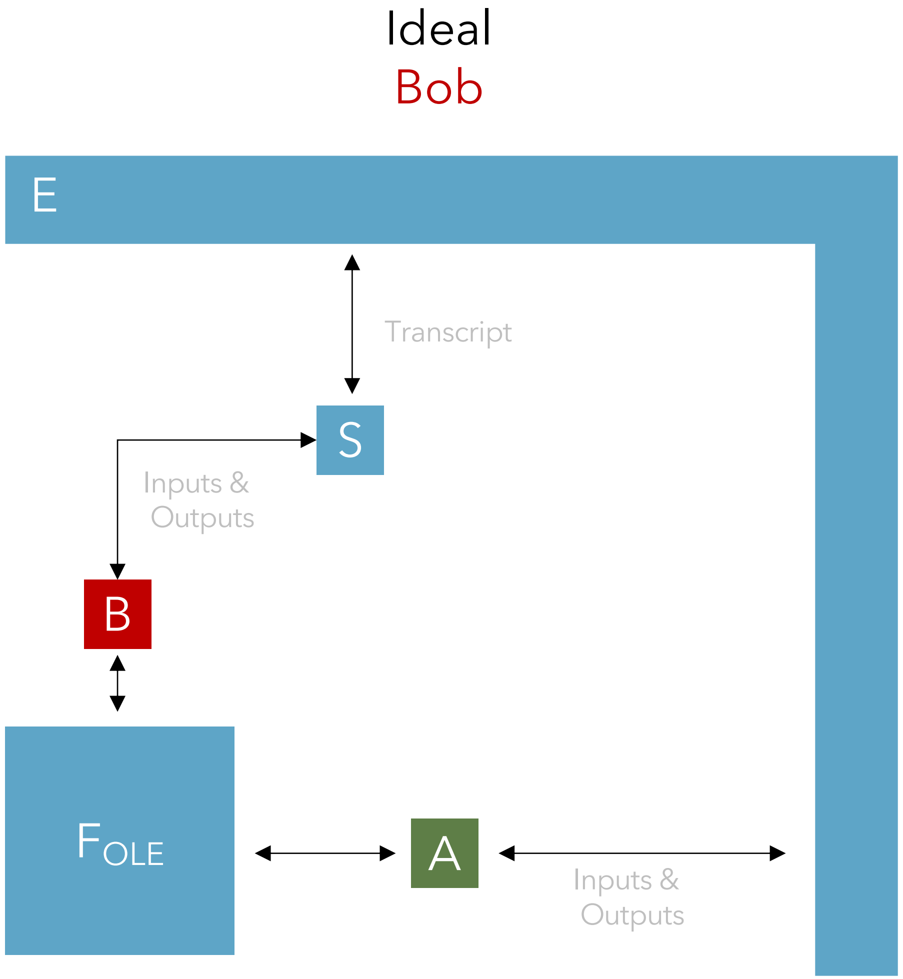
Indistinguishability:

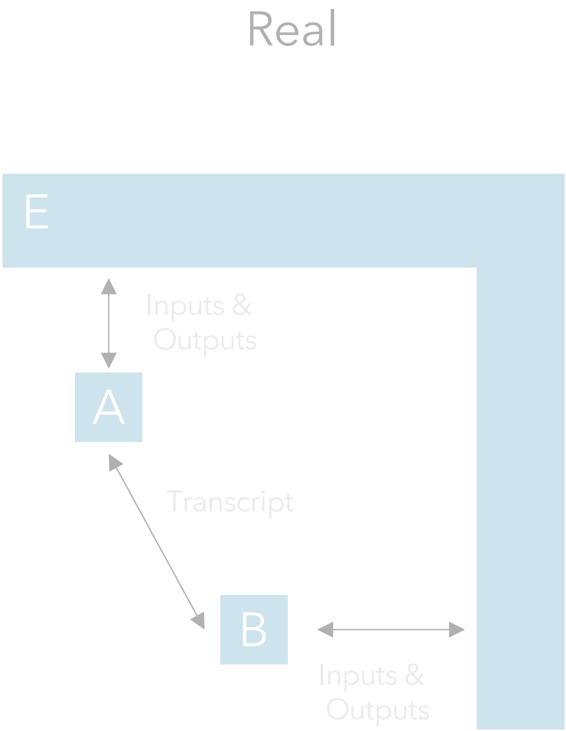
$$H_{\min}(\mathbf{F}_{\boldsymbol{a}} \,|\, \mathbf{Y}E) \geq \frac{n \log d}{2}(1 - h_d(\zeta))$$

$\mathbf{a} \xrightarrow[\text{Amplification}]{\text{Privacy}} \mathrm{a}$

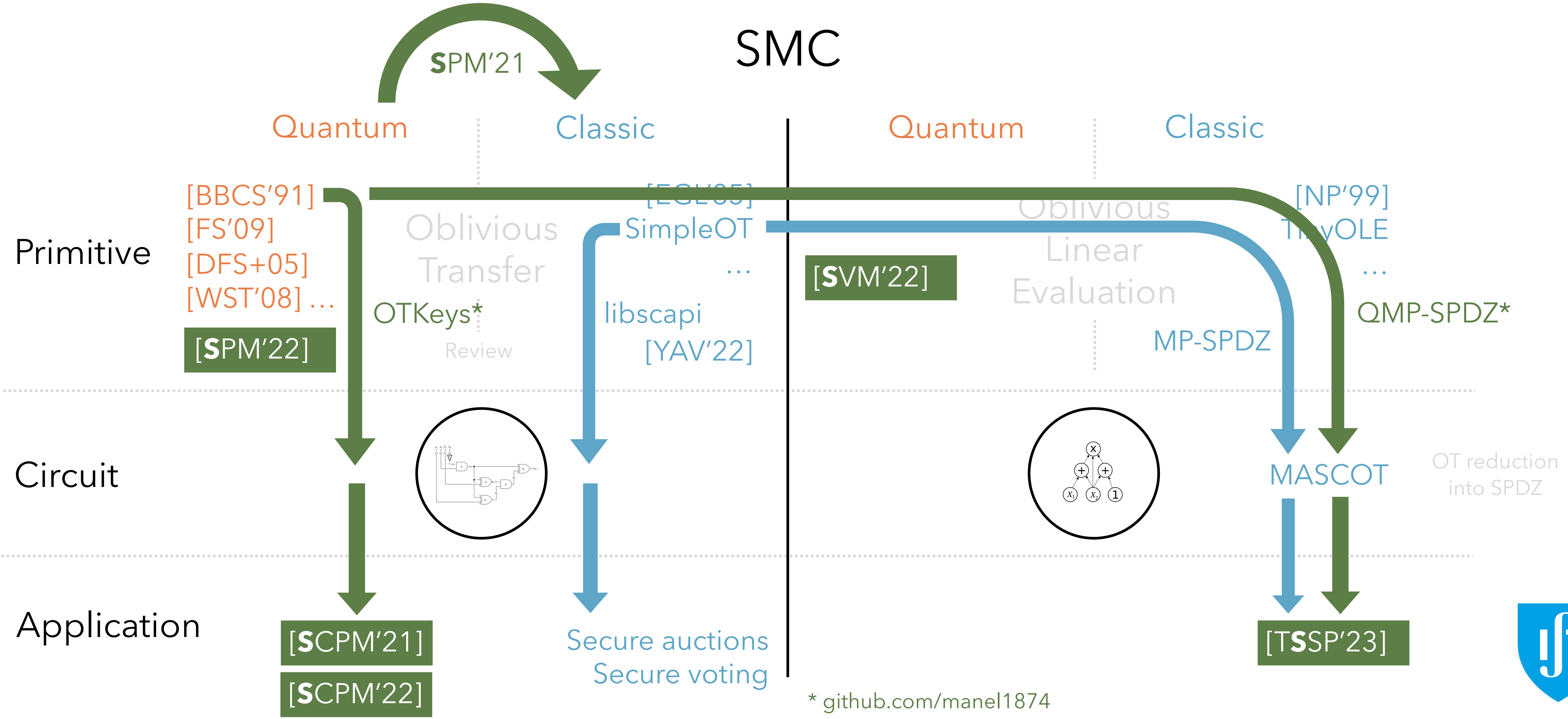
Extractability:

Com. functionality



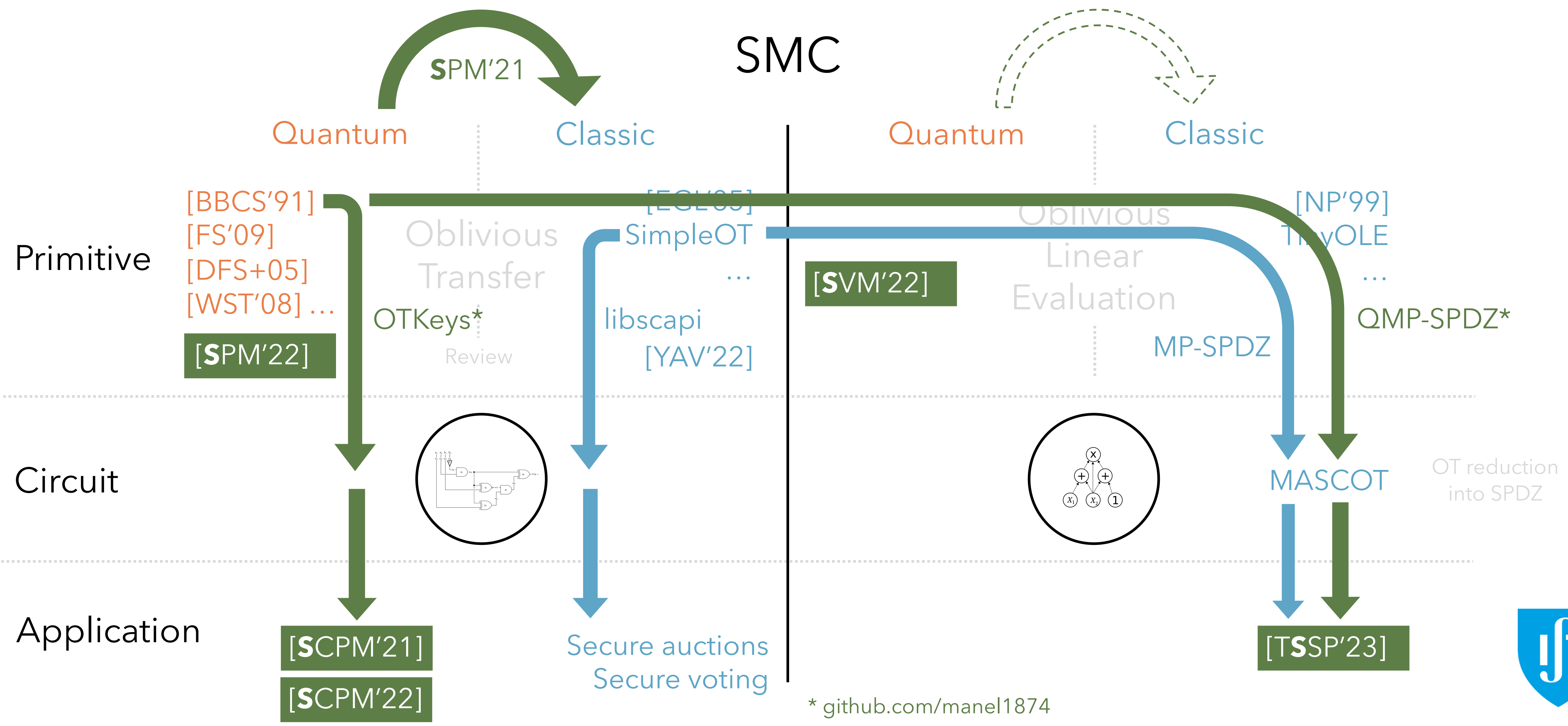


Future work



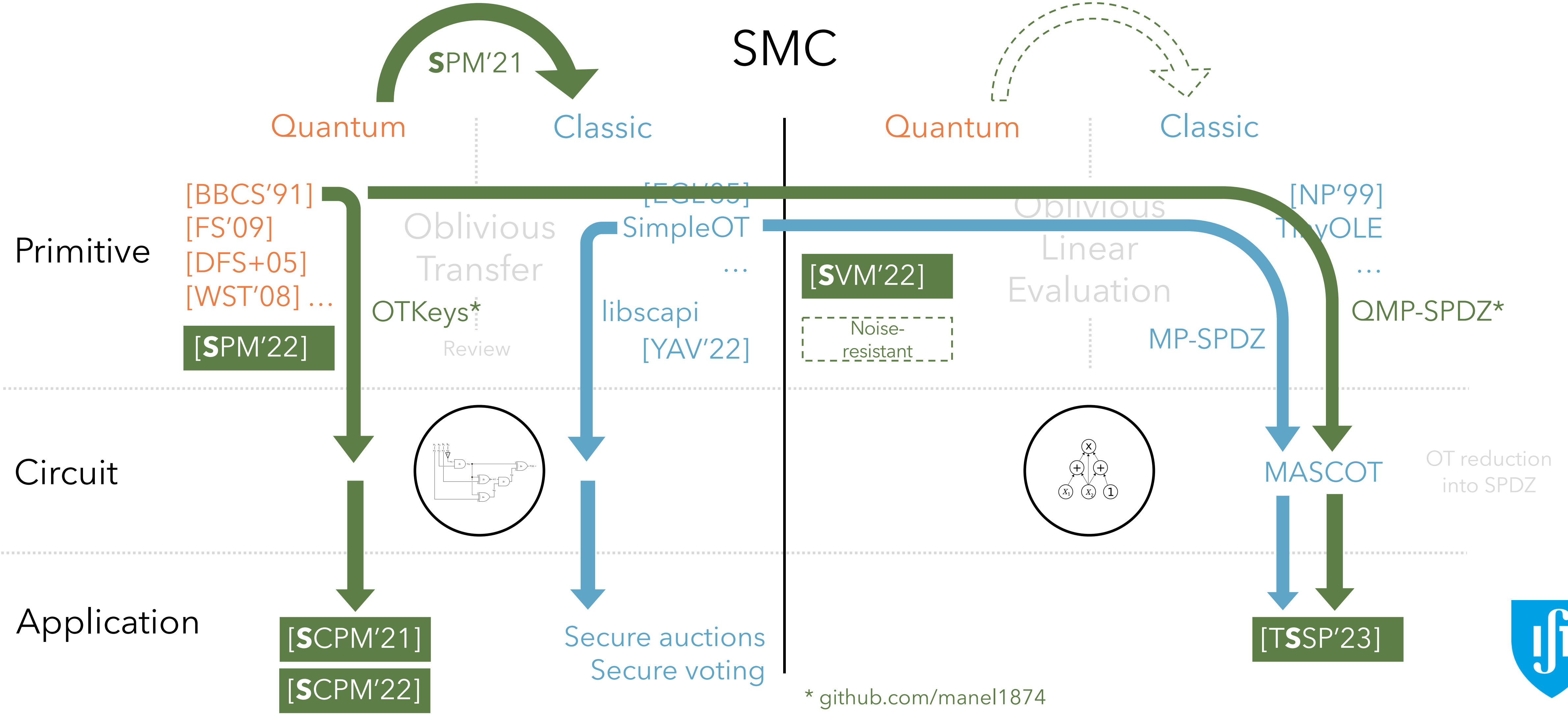
SMC
SPM'21
Quantum
Classic
Quantum
Classic
Primitive
Circuit
Application
[BBCS'91]
[FS'09]
[DFS+05]
[WST'08] …
[SPM'22]
Oblivious Transfer
OTKeys*
Review
SimpleOT
…
libscapi
[YAV'22]
[SVM'22]
Oblivious Linear Evaluation
[NP'99]
…
QMP-SPDZ*
MP-SPDZ
MASCOT
OT reduction into SPDZ
[SCPM'21]
[SCPM'22]
Secure auctions
Secure voting
[TSSP'23]
* github.com/manel1874

Future work



* github.com/manel1874

Future work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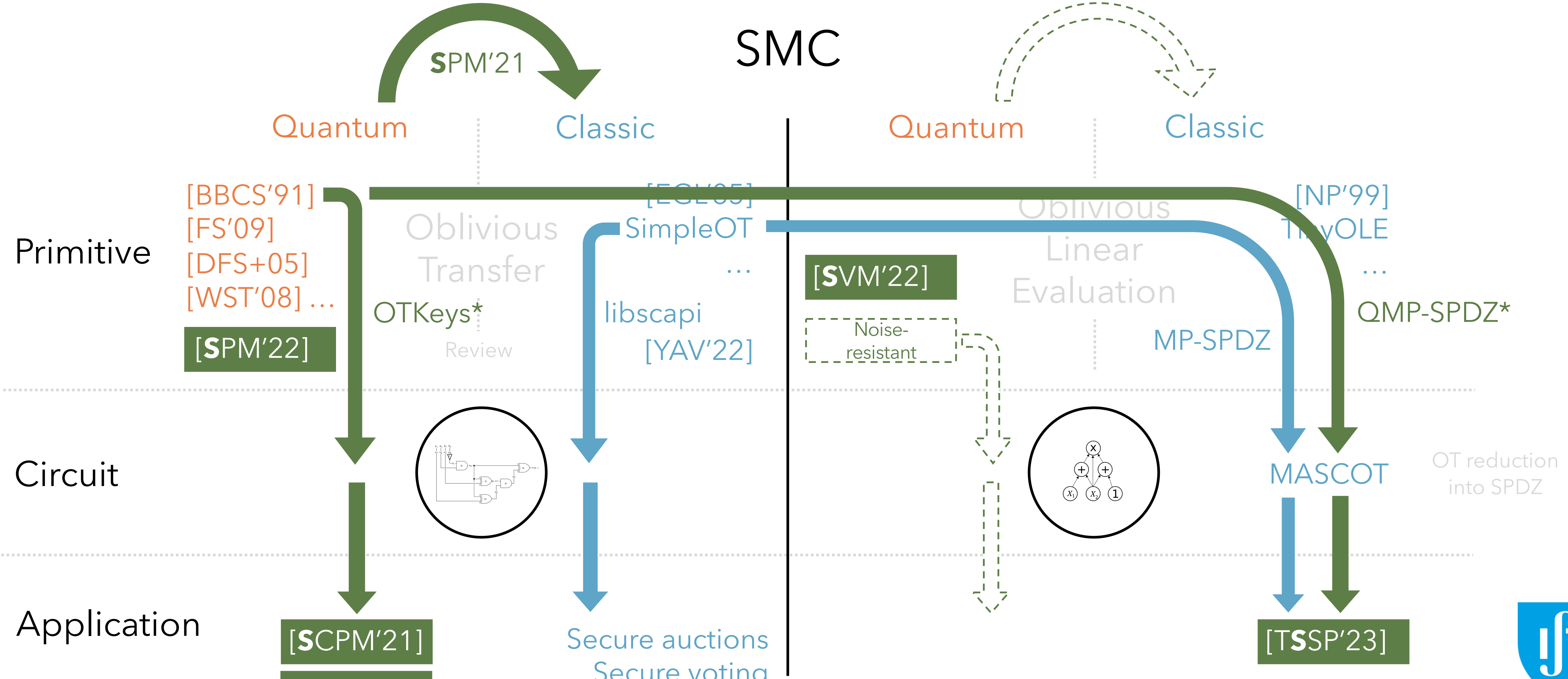
Future work



* github.com/manel1874

Thank you

I acknowledge Fundação para a Ciência e a Tecnologia (FCT, Portugal) for its support through the PhD grant SFRH/BD/ 144806/2019 in the context of the Doctoral Program in the Information Security (IS).