INTRODUCCIÓN A LA INFERENCIA BAYESIANA CON R

José Manuel Velasco Cabo

mvelascc@ucm.es

http://github.com/manelvcmb

Instalación OpenBugs

www.openbugs.net/w/Downloads

Linux Download

Source package

Should be usable on any x86 (PC) Linux platform. On 64 bit Linux, the necessary 32-bit C development packages are required.

Compilation has been successful on 64-bit Ubuntu using the g++-multilib package,

Download: OpenBUGS-3.2.3.tar.gz

To install this, unpack by typing

```
tar zxvf OpenBUGS-3.2.3.tar.gz
cd OpenBUGS-3.2.3
```

then compile and install by typing

```
./configure
make
sudo make install
```

Instalación JAGS

https://launchpad.net/ubuntu/+source/jags

To install this, unpack by typing

```
tar zxvf OpenBUGS-3.2.3.tar.gz
cd OpenBUGS-3.2.3
```

then compile and install by typing

```
./configure
make
sudo make install
```

Instalación RSTAN

Dentro de Rstudio, en la consola de R:

```
source('http://mc-stan.org/rstan/install.R', echo = TRUE, max.deparse.length = 2000)

install_rstan()
```

Índice

- Teorema de Bayes. Inferencia Bayesiana
- Posterior y Prior Conjugados
- MCMC
  - BUGS
  - JAGS
  - STAN
  - MCMCPACK
- Diagnosis de Convergencia

El Origen

Un matemático visita a Bayes con dos gemelos recién nacidos

El Origen

¿Vienes a bautizar a los gemelos?

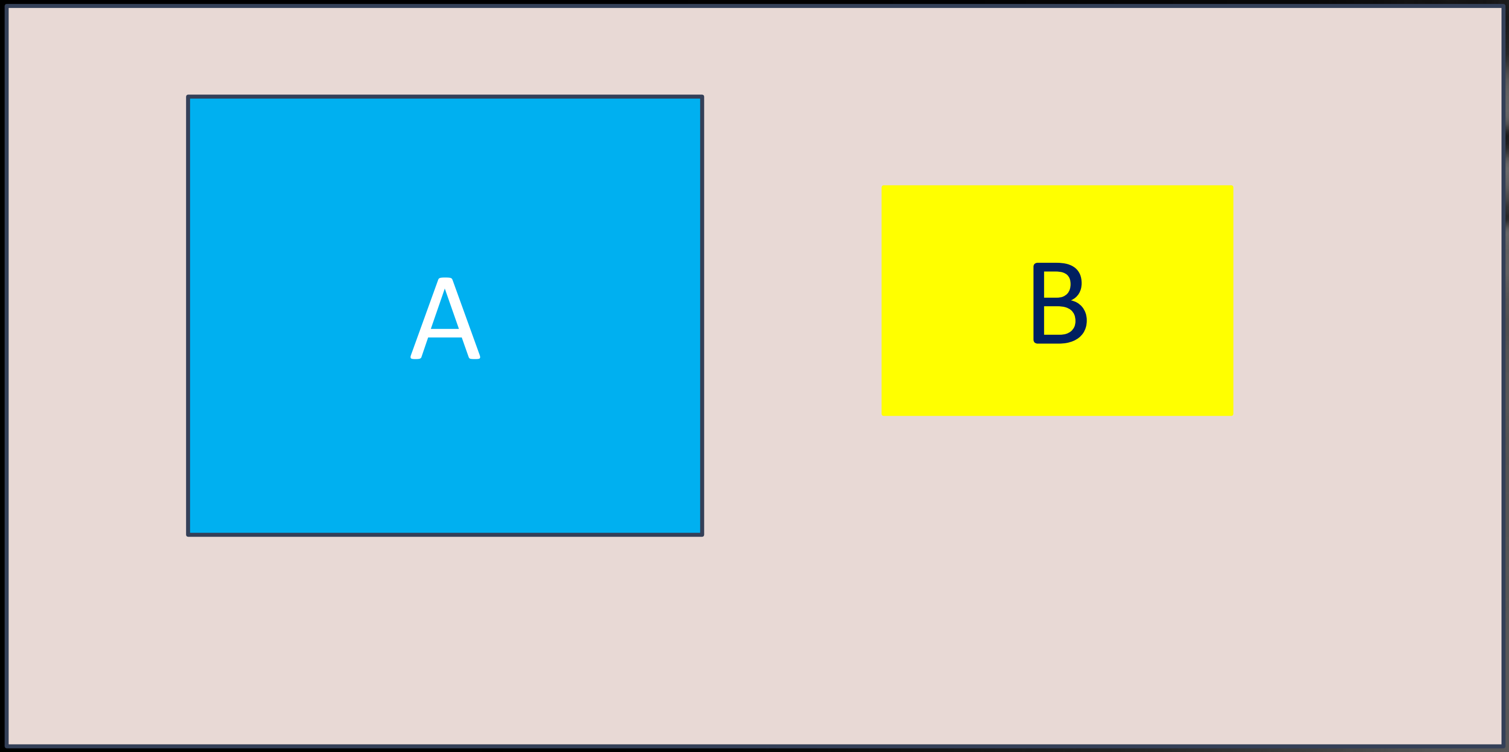
El Origen

No, sólo quiero bautizar a uno.
El otro servirá como grupo de control.
Cuando sean mayores les haré un test,
calcularé el p-valor y …

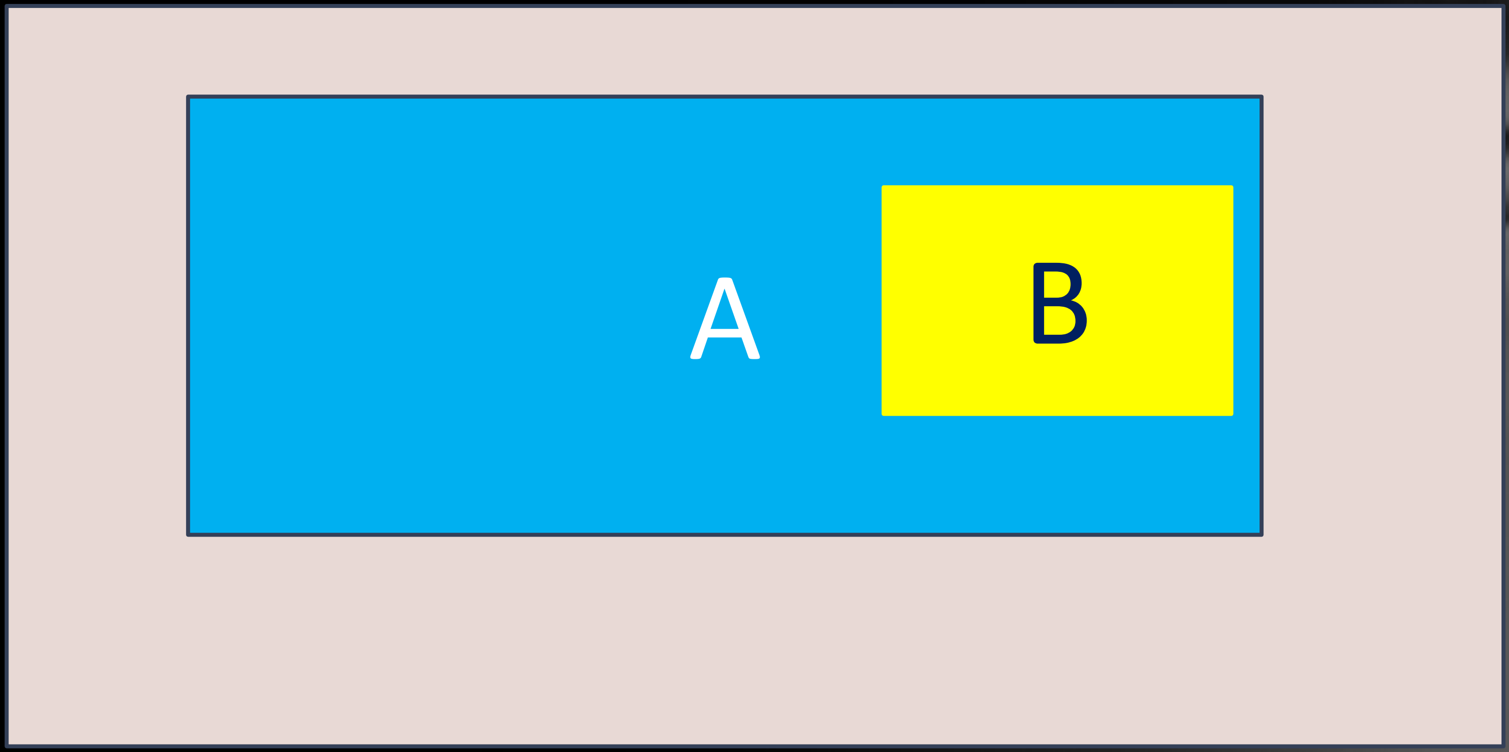
El Origen

Me @#*% en los p valores …

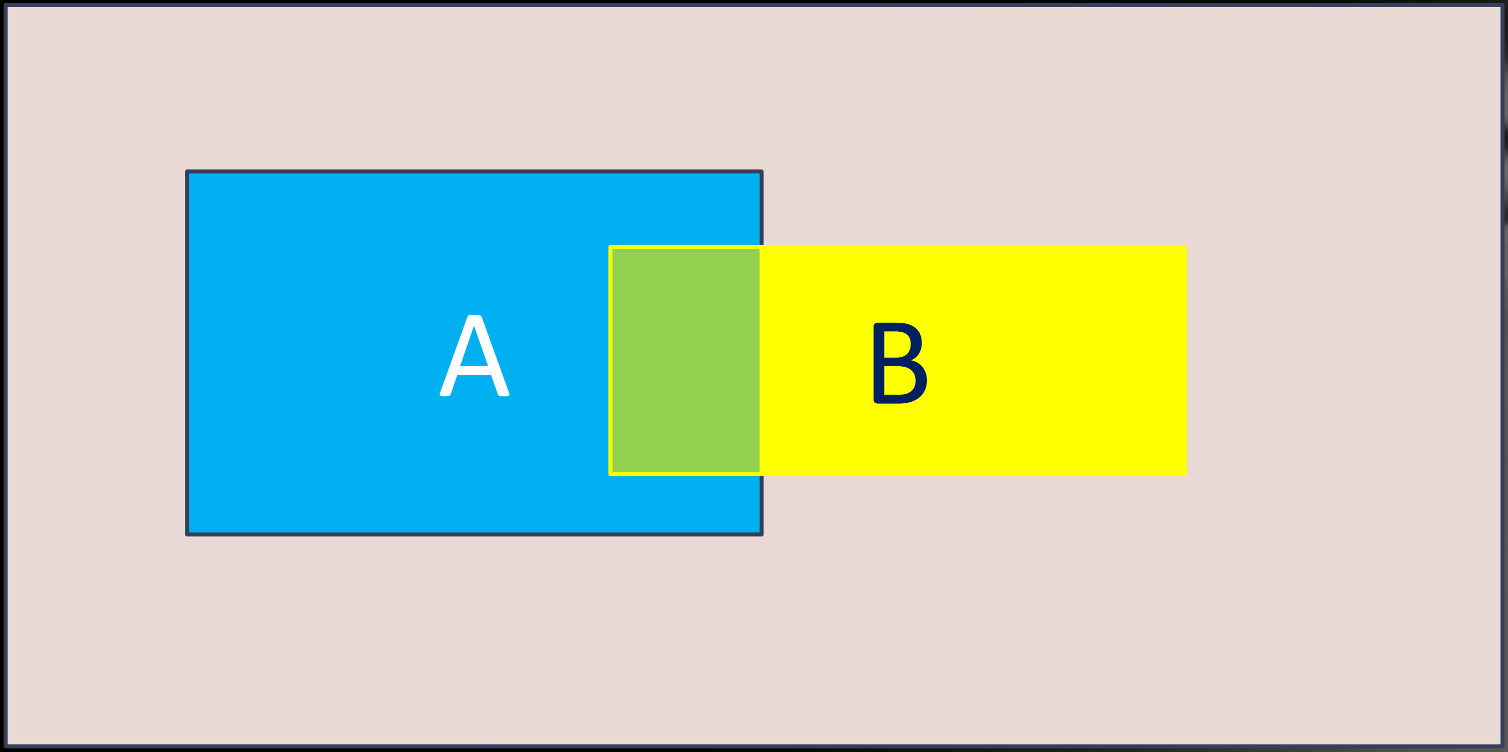
Probabilidades Condicionadas



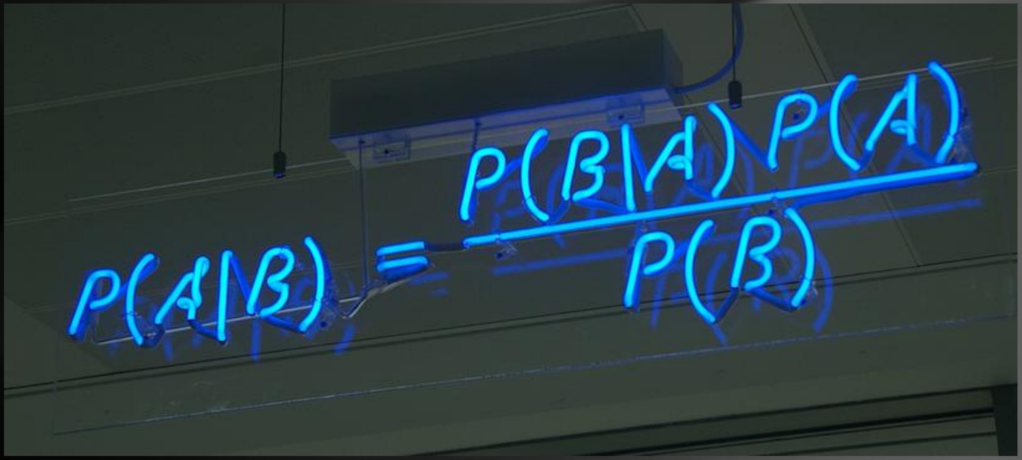
Probabilidades Condicionadas



Probabilidades Condicionadas



Teorema de Bayes



Inferencia Frecuentista vs Bayesiana

P(Datos|Hipótesis) ⇒ Test –> p-valor → Rechazo Hipótesis / → No Rechazo

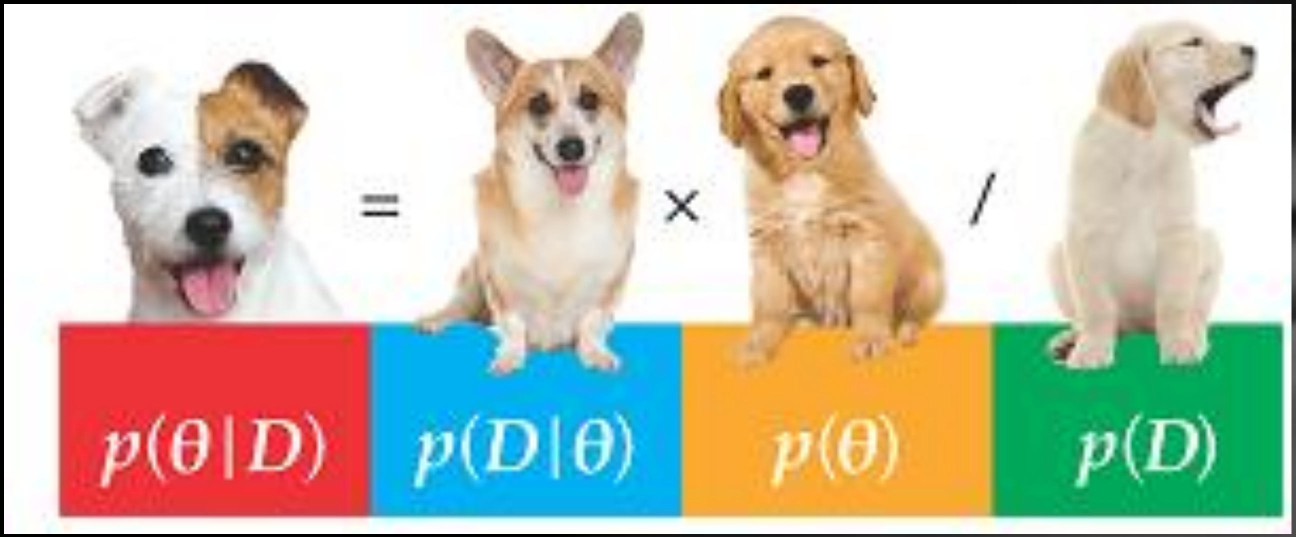
$$P(\text{Hipótesis}|\text{Datos}) = \frac{P(\text{Datos}|\text{Hipótesis}) \times P(\text{Hipótesis})}{P(\text{Datos})}$$

P(Hipótesis|Datos) → P(Datos|Hipótesis) x P(Hipótesis)

Posterior → Verosimilitud x Priori

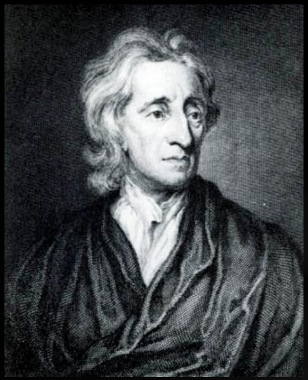
Creencia después Datos → Datos x Creencia antes Datos

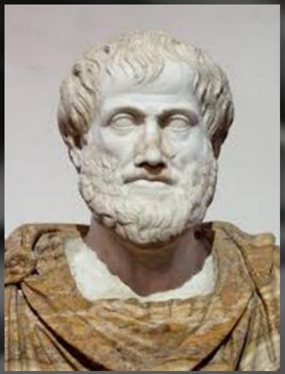
Inferencia Bayesiana



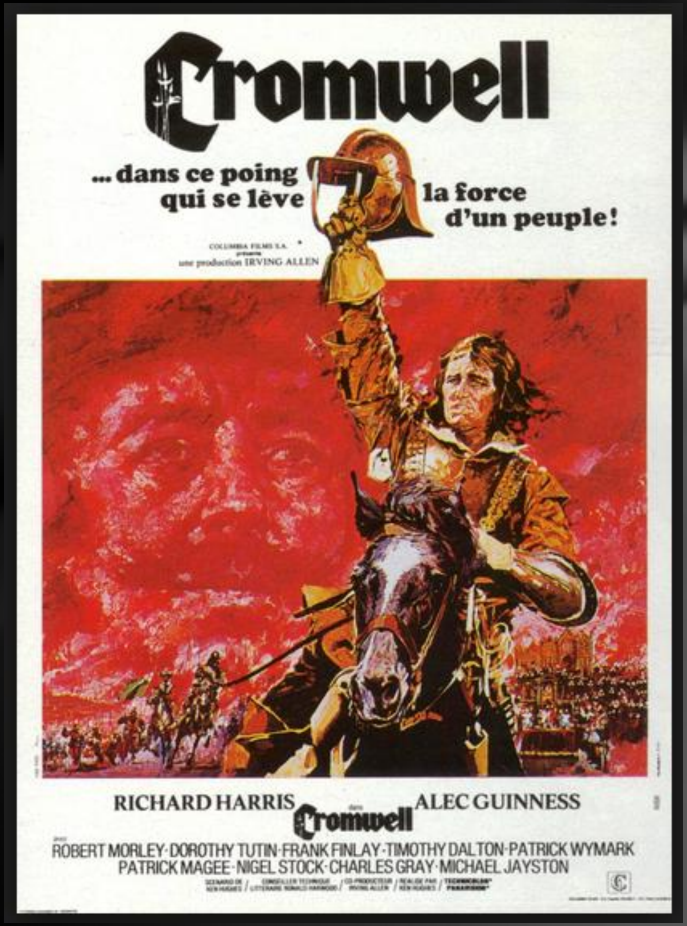
Portada del próximo libro de Kruschke que saldrá en noviembre de 2014

Inducción - Deducción

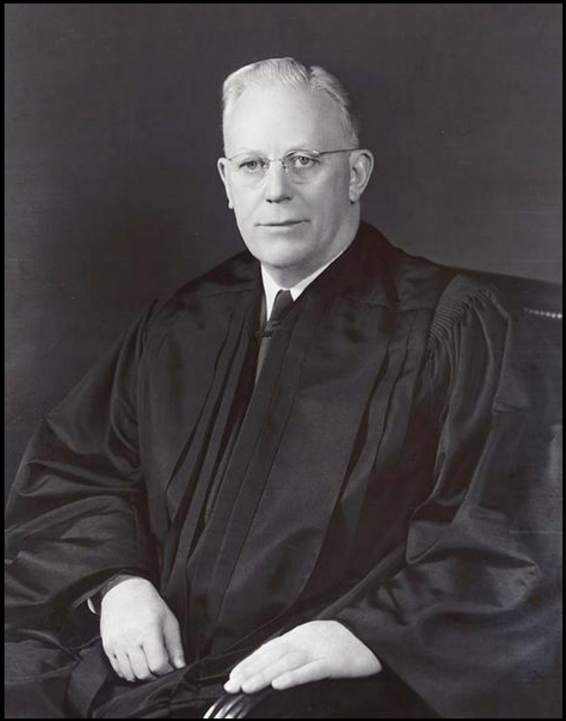





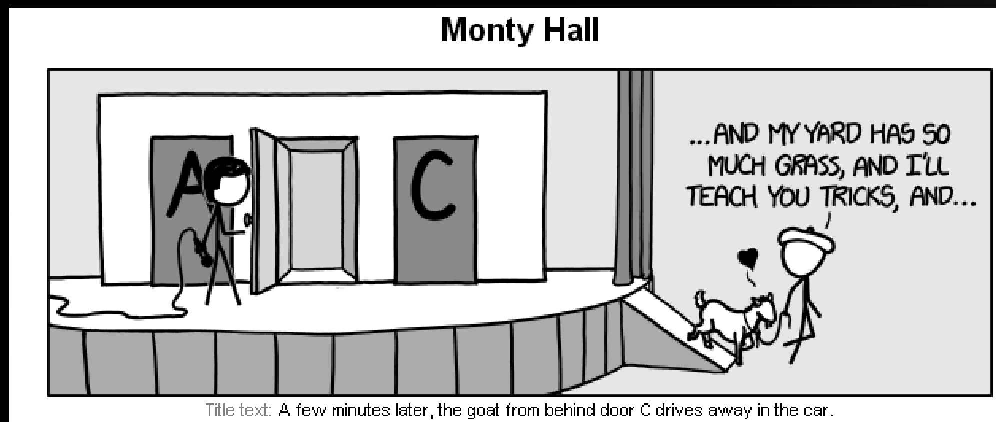
Regla de Cromwell

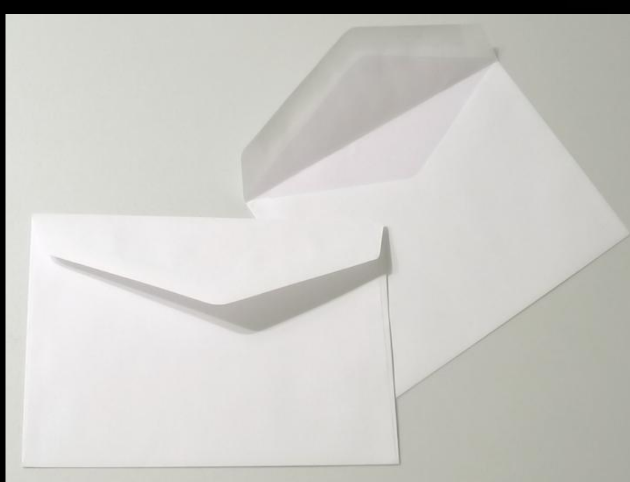


Earl Warren. Gobernador de California

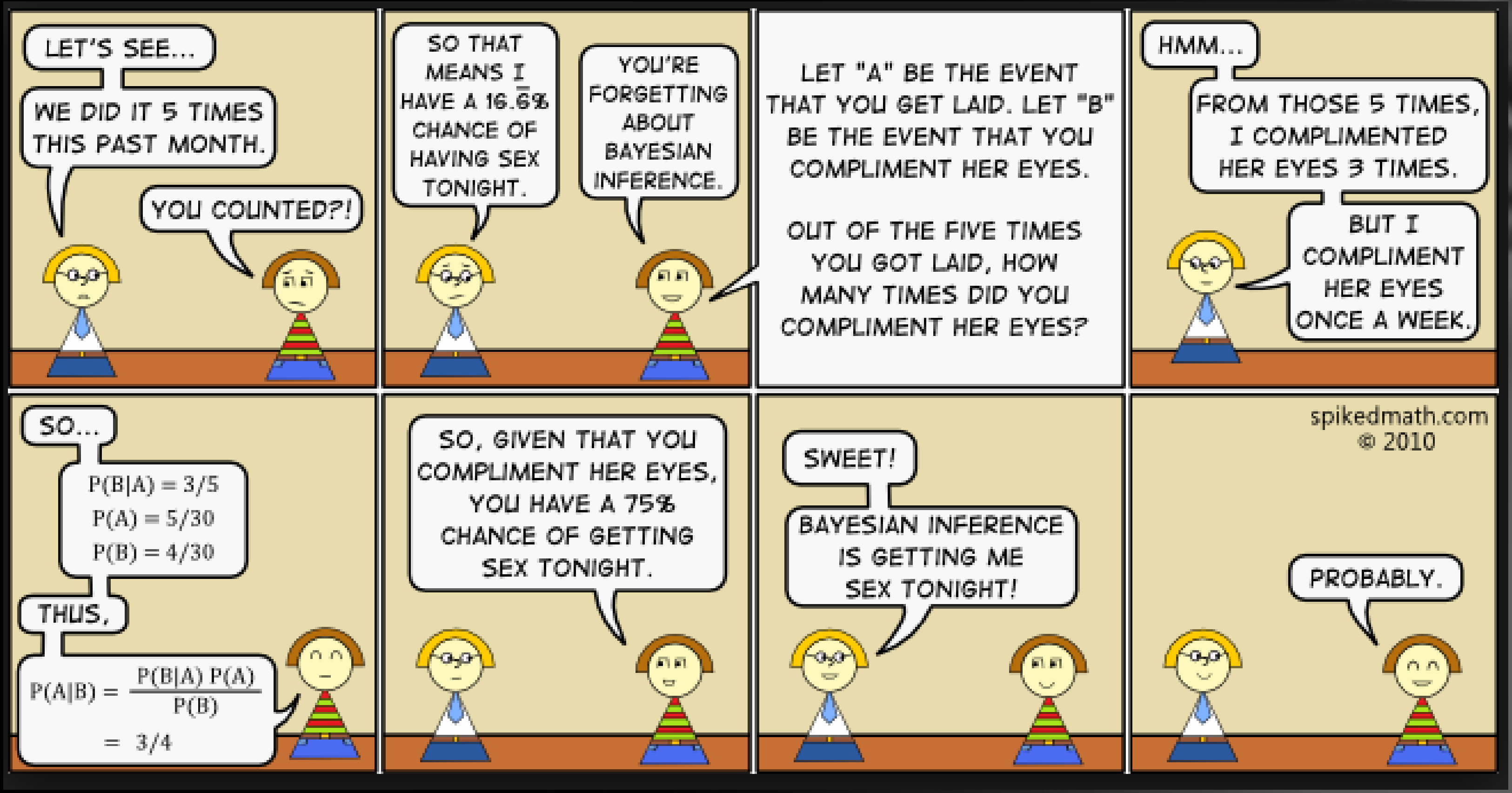


Monty Hall – Paradoja de los dos sobres – Tres cartas

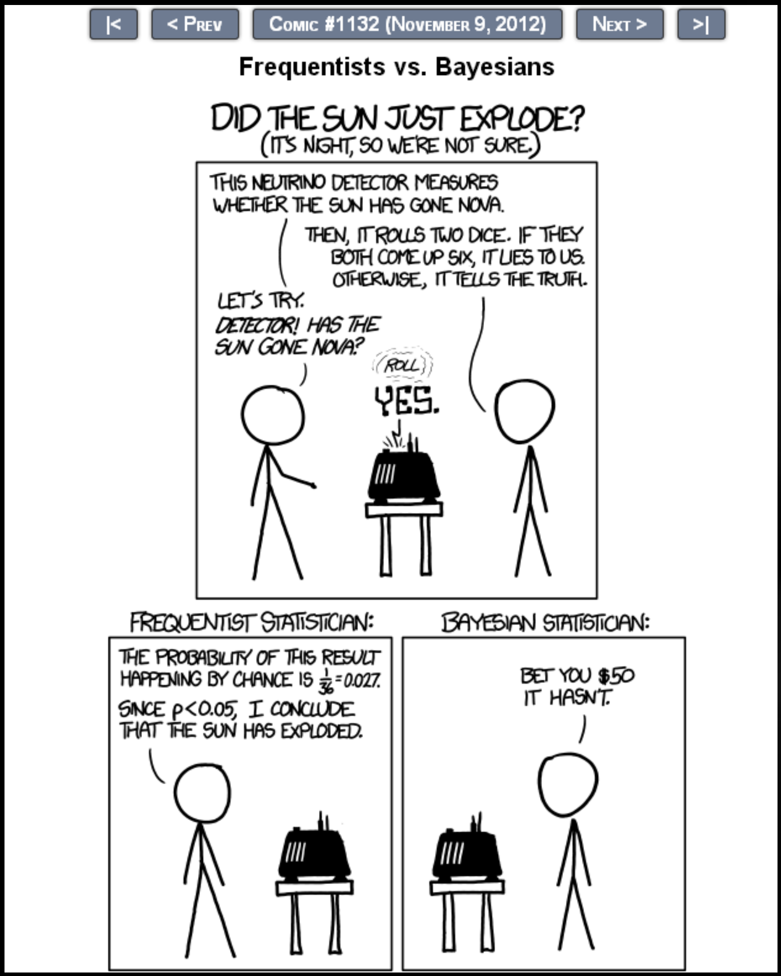




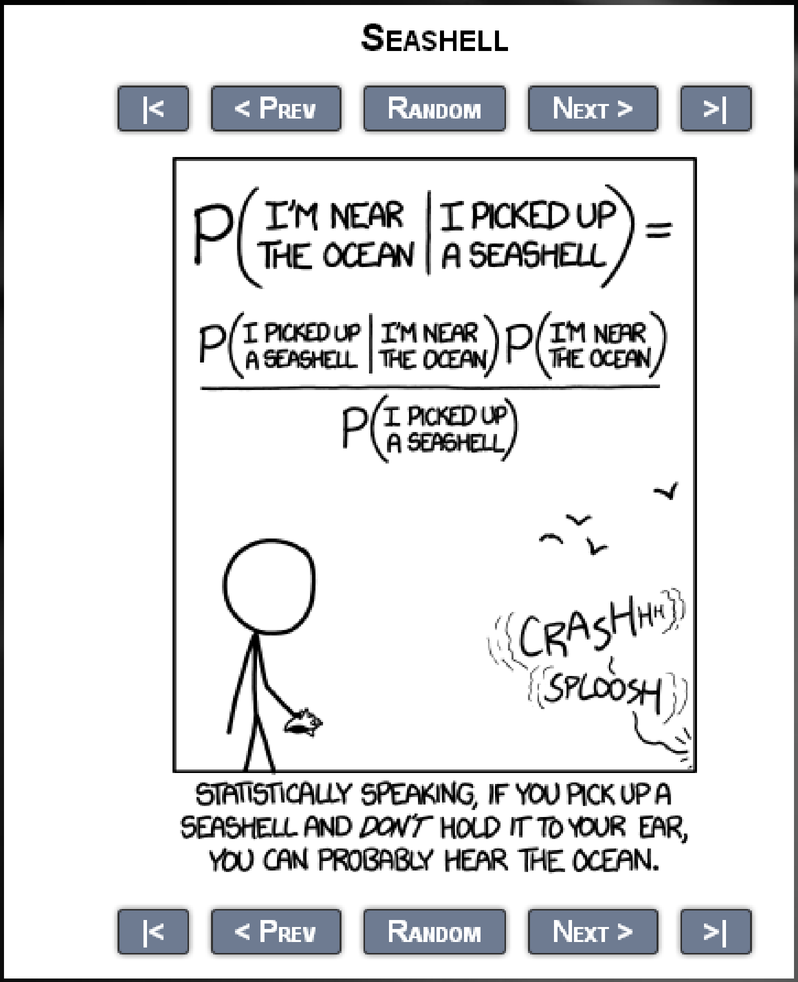
Teorema de Bayes



XKCD



|<
< Prev
Comic #1132 (November 9, 2012)
Next >
>|
Frequentists vs. Bayesians
DID THE SUN JUST EXPLODE?
(IT'S NIGHT, SO WE'RE NOT SURE.)
THIS NEUTRINO DETECTOR MEASURES WHETHER THE SUN HAS GONE NOVA.
THEN, IT ROLLS TWO DICE. IF THEY BOTH COME UP SIX, IT LIES TO US. OTHERWISE, IT TELLS THE TRUTH.
LET'S TRY.
DETECTOR! HAS THE SUN GONE NOVA?
ROLL
YES.
FREQUENTIST STATISTICIAN:
THE PROBABILITY OF THIS RESULT HAPPENING BY CHANCE IS $\frac{1}{36}$=0.027.
SINCE p<0.05, I CONCLUDE THAT THE SUN HAS EXPLODED.
BAYESIAN STATISTICIAN:
BET YOU $50 IT HASN'T.



SEASHELL
|<
< Prev
Random
Next >
>|
P(I'M NEAR THE OCEAN | I PICKED UP A SEASHELL) =
P(I PICKED UP A SEASHELL | I'M NEAR THE OCEAN) P(I'M NEAR THE OCEAN)
P(I PICKED UP A SEASHELL)
CRASHHH
SPLOOSH
STATISTICALLY SPEAKING, IF YOU PICK UP A SEASHELL AND DON'T HOLD IT TO YOUR EAR, YOU CAN PROBABLY HEAR THE OCEAN.
|<
< Prev
Random
Next >
>|

Índice

- Teorema de Bayes. Inferencia Bayesiana
- Posterior y Prior Conjugados
- MCMC
  - BUGS
  - JAGS
  - STAN
  - MCMCPACK
- Diagnosis de Convergencia

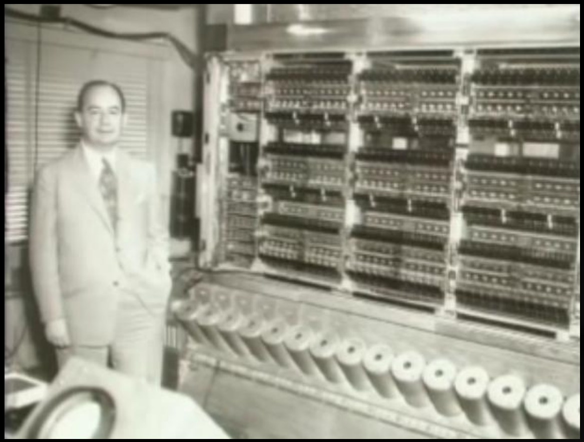
Stanislaw (Stan) Ulam

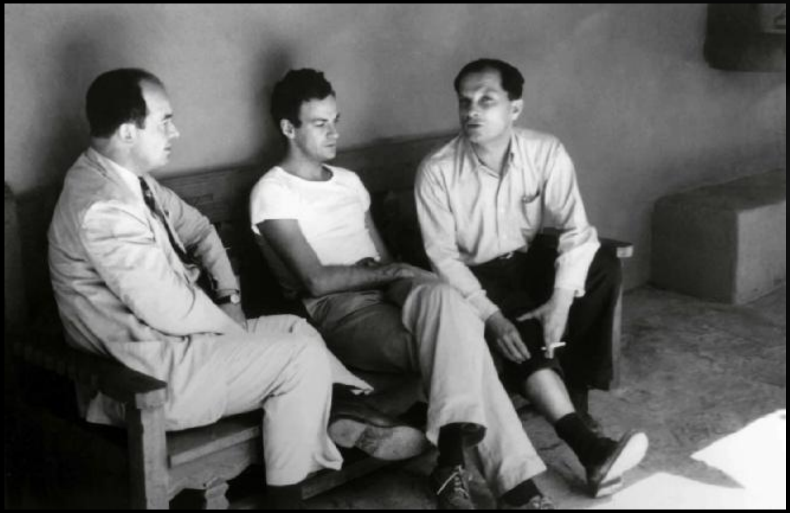
Once in my life I had a mathematical dream which proved correct. I was twenty years old. I thought, my God, this is wonderful, I won't have to work, it will all come in dreams. But it never happened again. . .

All my father does is think, think, think . . .

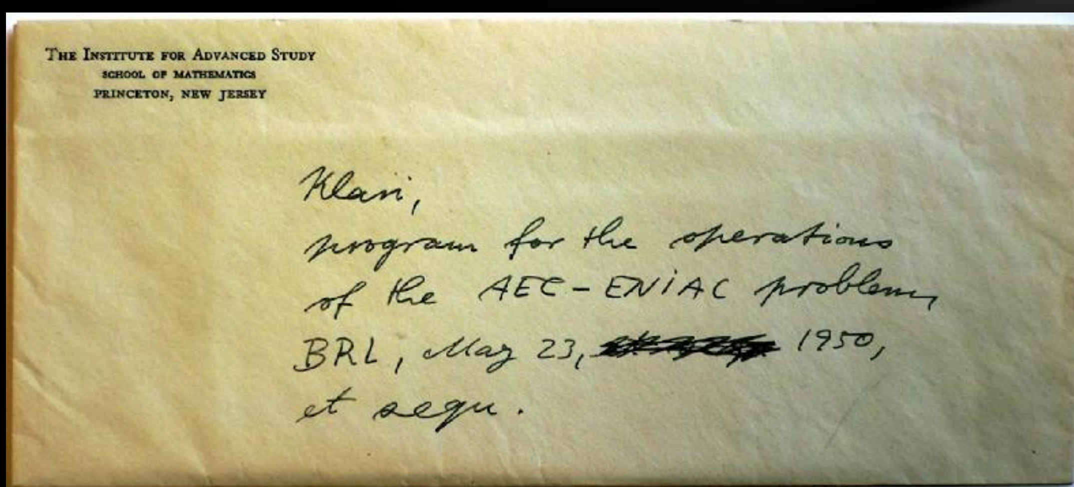
— Claire Ulam

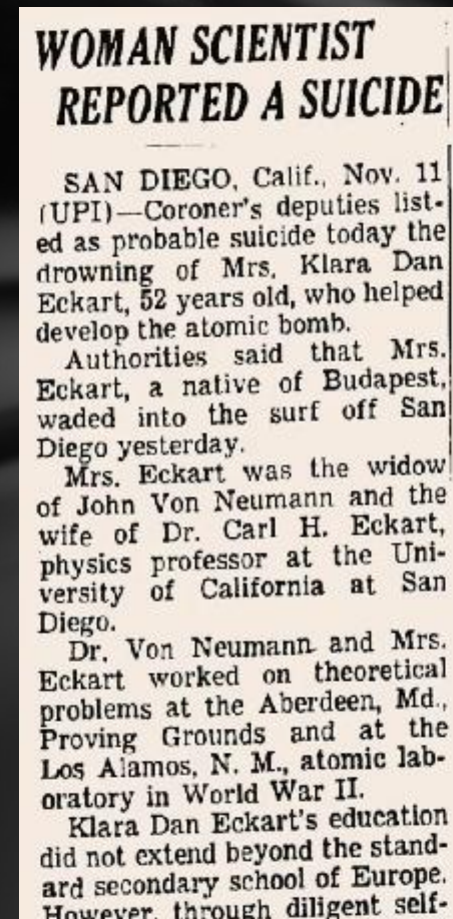
John von Neumann





Klara von Neumann (Dan Eckart)





WOMAN SCIENTIST REPORTED A SUICIDE

SAN DIEGO, Calif., Nov. 11 (UPI)—Coroner's deputies listed as probable suicide today the drowning of Mrs. Klara Dan Eckart, 52 years old, who helped develop the atomic bomb.

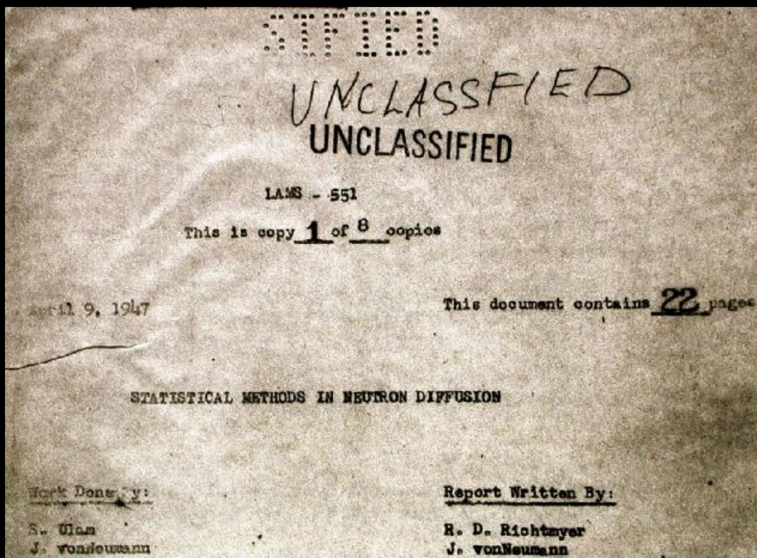
Authorities said that Mrs. Eckart, a native of Budapest, waded into the surf off San Diego yesterday.

Mrs. Eckart was the widow of John Von Neumann and the wife of Dr. Carl H. Eckart, physics professor at the University of California at San Diego.

Dr. Von Neumann and Mrs. Eckart worked on theoretical problems at the Aberdeen, Md., Proving Grounds and at the Los Alamos, N. M., atomic laboratory in World War II.

Klara Dan Eckart's education did not extend beyond the standard secondary school of Europe. However, through diligent self-

Statistical Methods in Neutron Diffusion


UNCLASSIFIED

UNCLASSIFIED

LAMS - 551

This is copy 1 of 8 copies

April 9, 1947

This document contains 22 pages

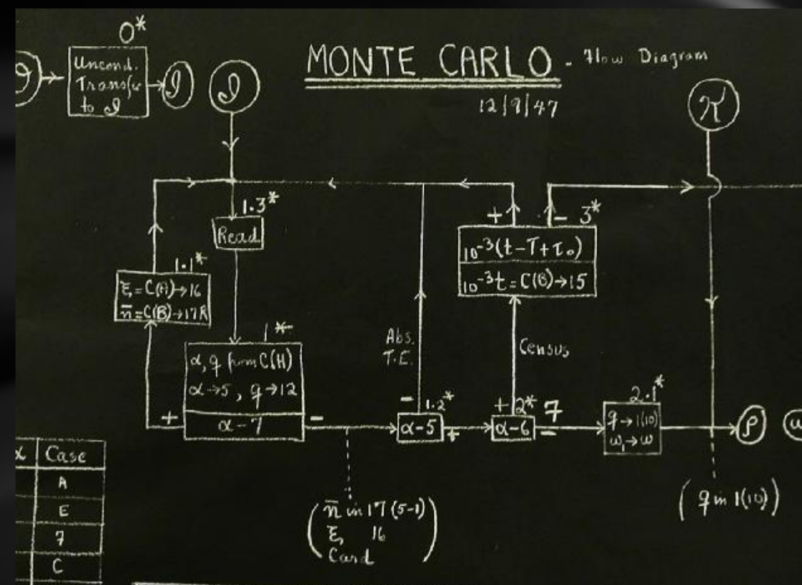
STATISTICAL METHODS IN NEUTRON DIFFUSION

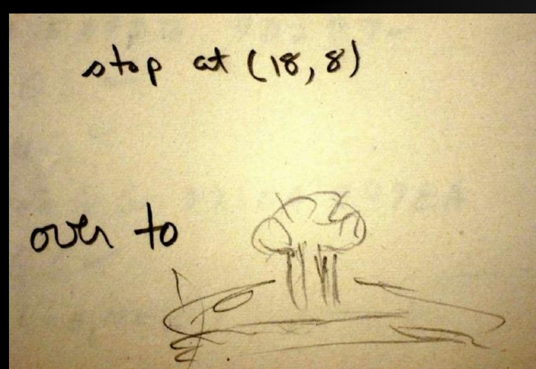
Work Done By:

S. Ulam
J. vonNeumann

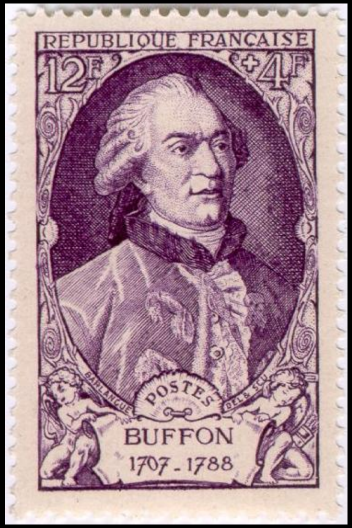
Report Written By:

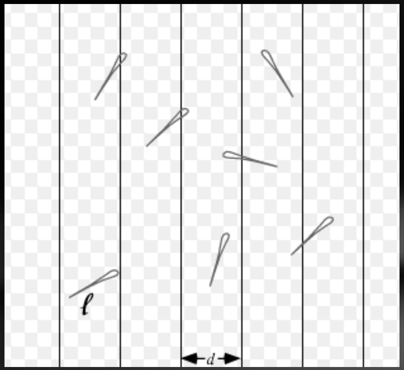
R. D. Richtmyer
J. vonNeumann

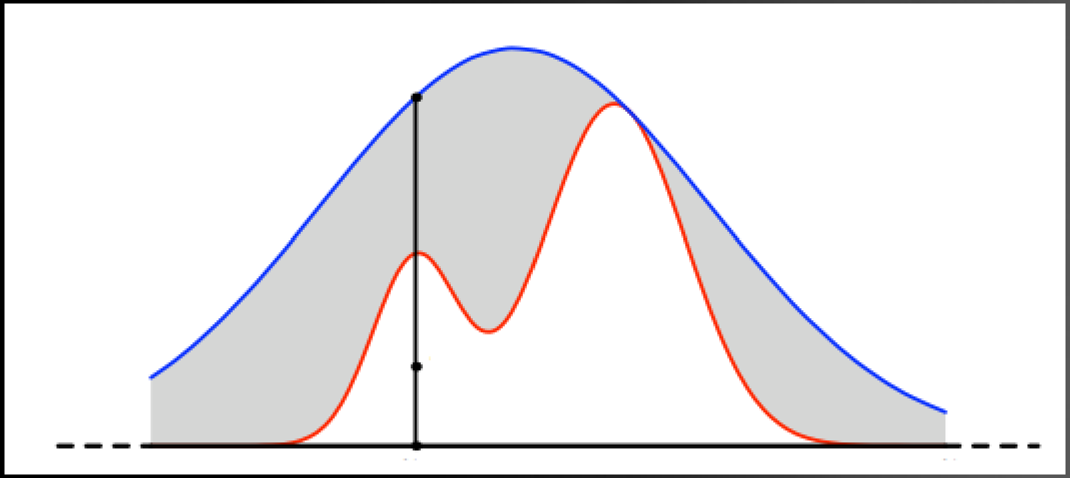




Aguja de Bufón - Método GRID







Metropolis

Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953) Equations of State Calculations by Fast Computing Machines. *J. Chem. Phys.*, **21**, 1087–1092.

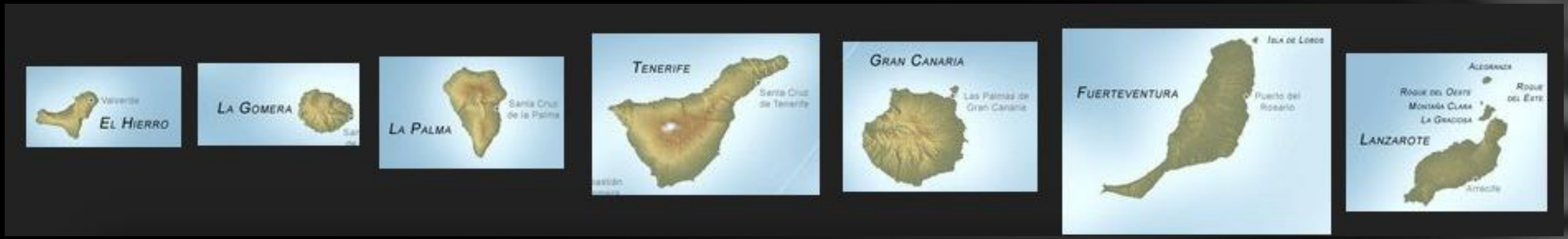
Paseo Aleatorio por las Canarias

- Unos políticos quieren hacer campaña en las islas Canarias.
- Quieren dedicar a cada isla un tiempo proporcional a su población
- Sin embargo, estudiaron estadística en un par de tardes y no les dio tiempo a aprender a normalizar

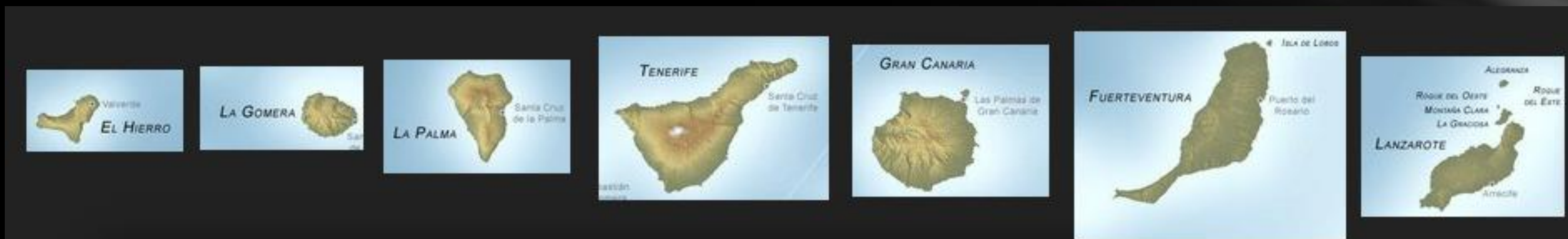
- ¿Qué pueden hacer?

Idea de John Kruschke

Paseo Aleatorio por las Canarias

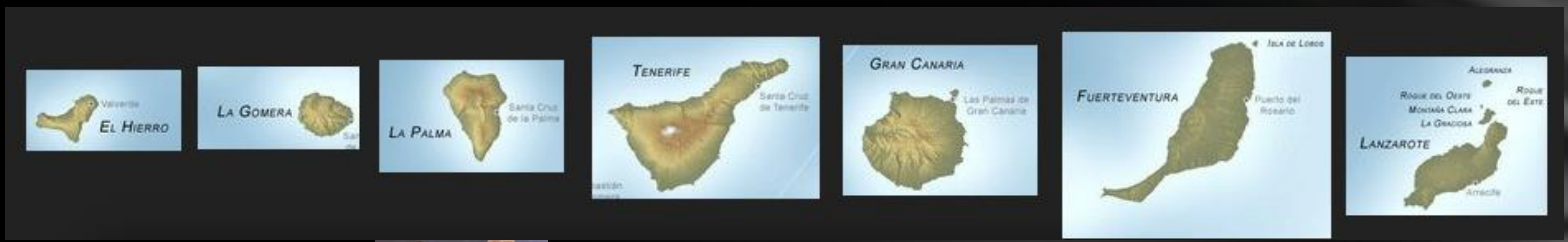


Paseo Aleatorio por las Canarias



Empezamos en una isla cualquiera. Por ejemplo: La Palma.

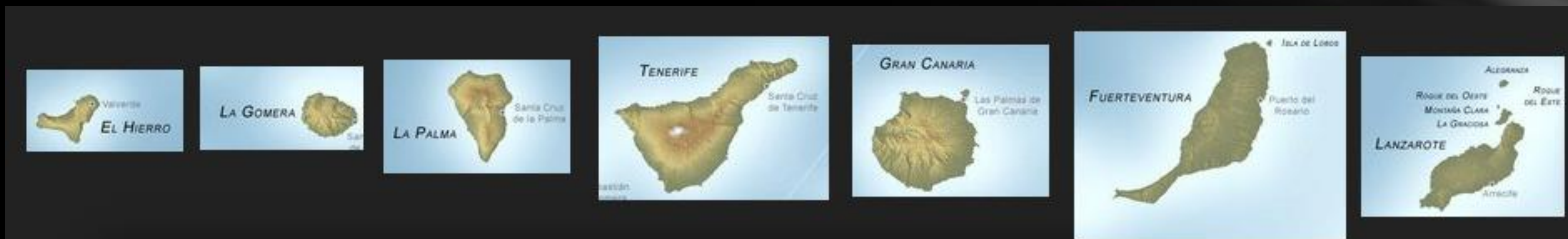
Paseo Aleatorio por las Canarias



Elegimos al azar una de las islas adyacentes. Por ejemplo: Tenerife

La población de Tenerife es mayor que la de La Palma → Nos movemos a Tenerife

Paseo Aleatorio por las Canarias

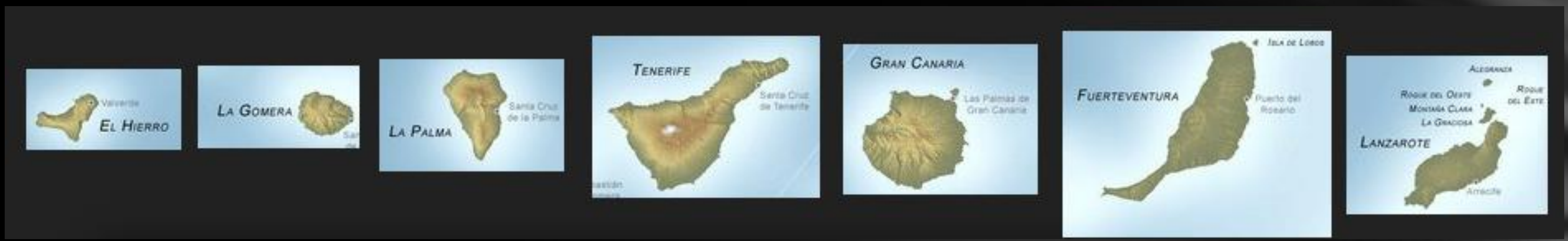


De nuevo, elegimos al azar una de las islas adyacentes.
Por ejemplo: Gran Canaria

La población de Tenerife es mayor que la de Gran Canaria →

$$\frac{\text{Población de Gran Canaria}}{\text{Población de Tenerife}} = 0.9$$

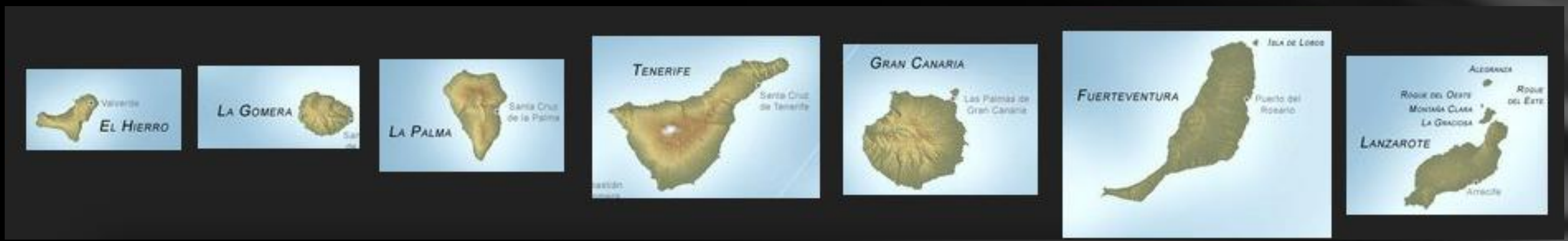
Paseo Aleatorio por las Canarias



Elegimos un número aleatorio entre 0 y 1 → 0.7

$$\frac{\text{Población de Gran Canaria}}{\text{Población de Tenerife}} = 0.9 > 0.7$$ → Nos movemos a Gran Canaria

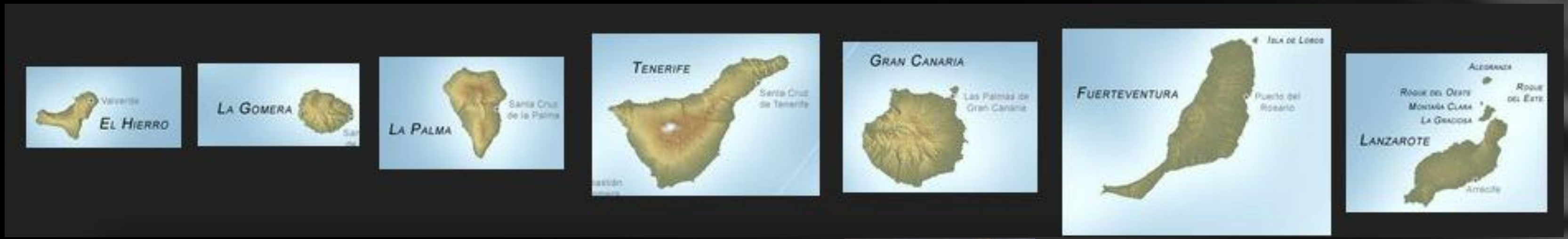
Paseo Aleatorio por las Canarias



Elegimos una isla adyacente al azar → Fuerteventura

De nuevo, la población de Gran Canaria es mayor que la de Fuerteventura →

Paseo Aleatorio por las Canarias

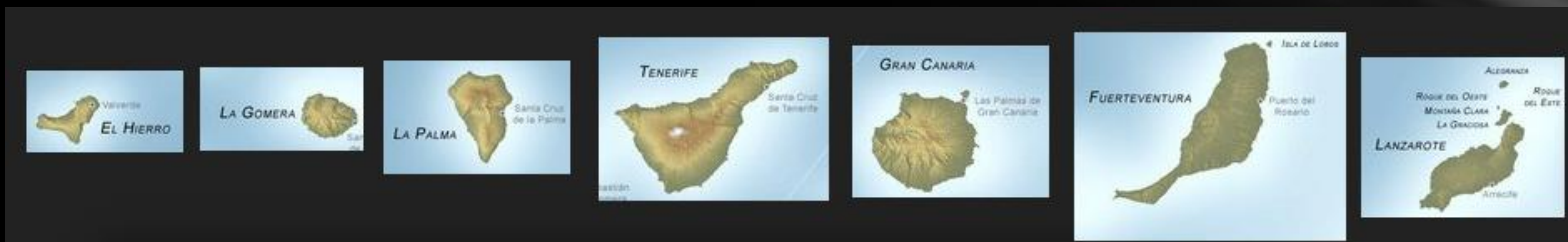


Elegimos un número aleatorio entre 0 y 1 → 0.4

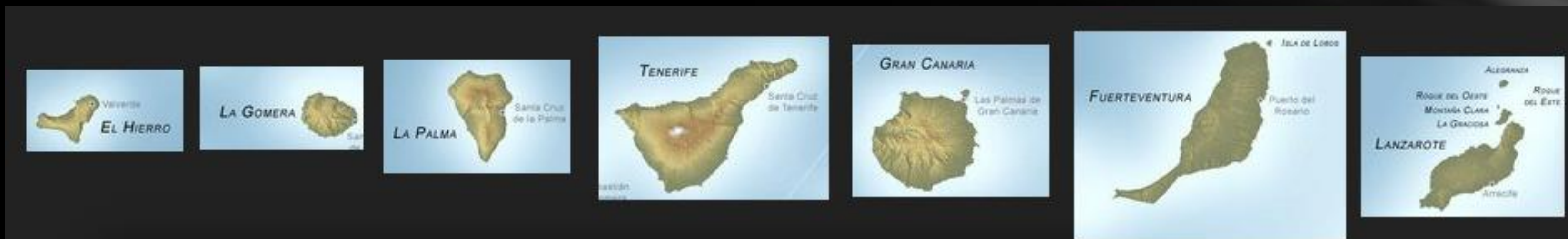
$$\frac{\text{Población de Gran Canaria}}{\text{Población de Tenerife}} = 0.9 > 0.7$$ → Nos quedamos en Gran Canaria

¡Rechazamos el movimiento a Fuerteventura!

Paseo Aleatorio por las Canarias

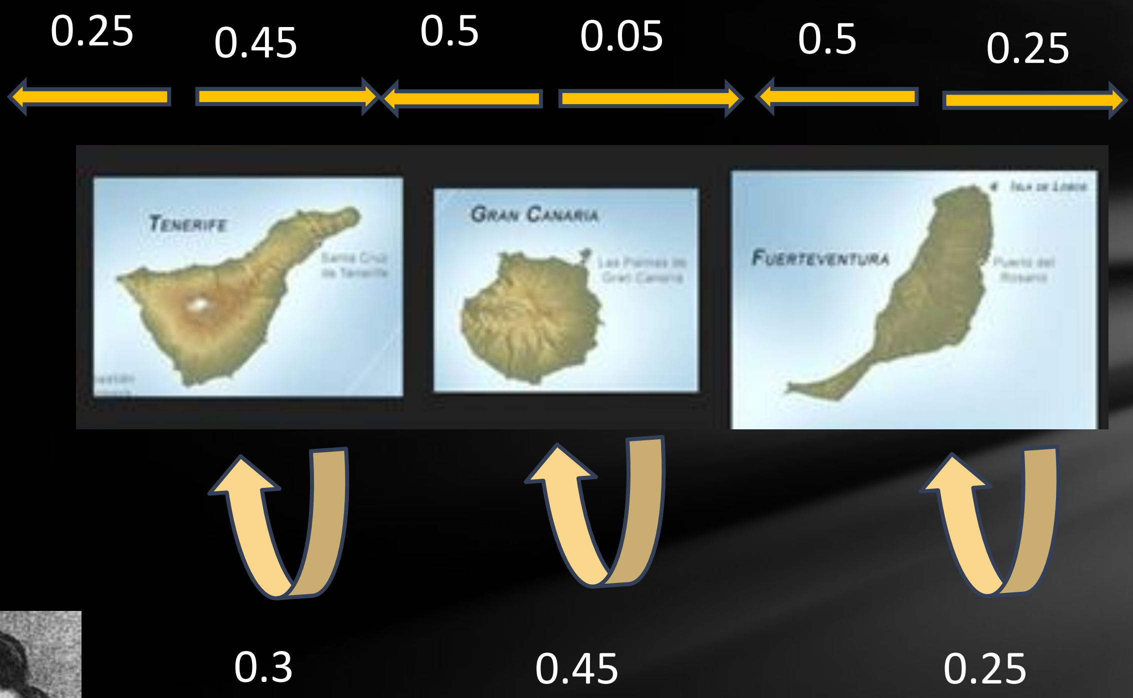


Paseo Aleatorio por las Canarias

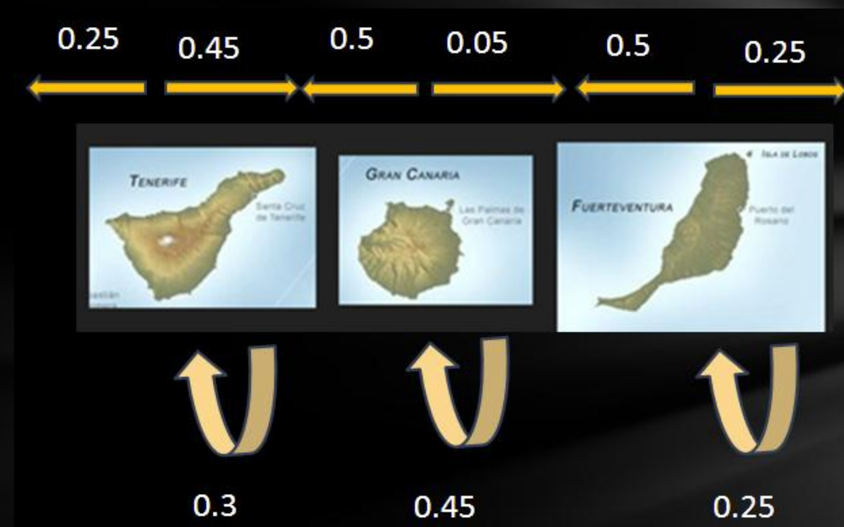


Si el paseo es lo suficientemente largo, al final el tiempo que pasan en cada isla es proporcional a su población relativa

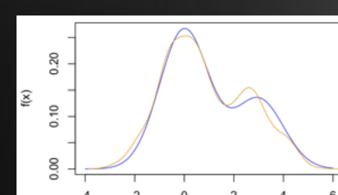
Cadena de Markov



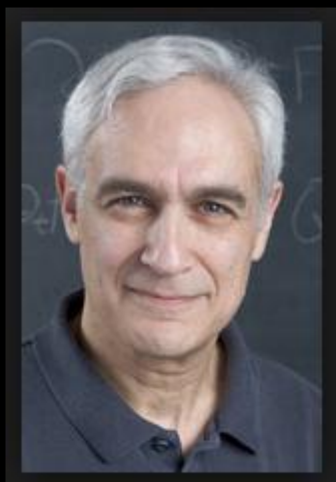
Ergodicidad

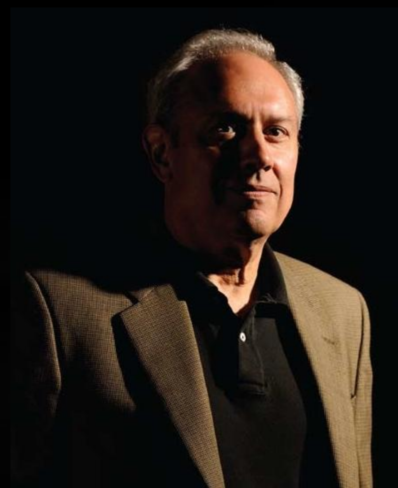


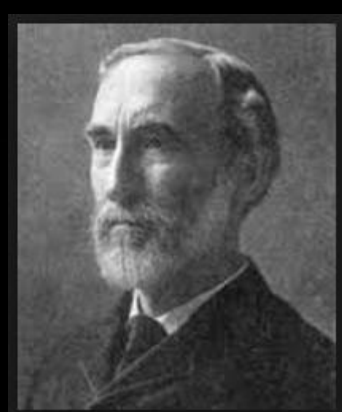
$$\frac{1}{n}\sum_{j=1}^{n} f \circ T^{j}(x) \rightarrow \int f \, dm$$



Gibbs Sampling

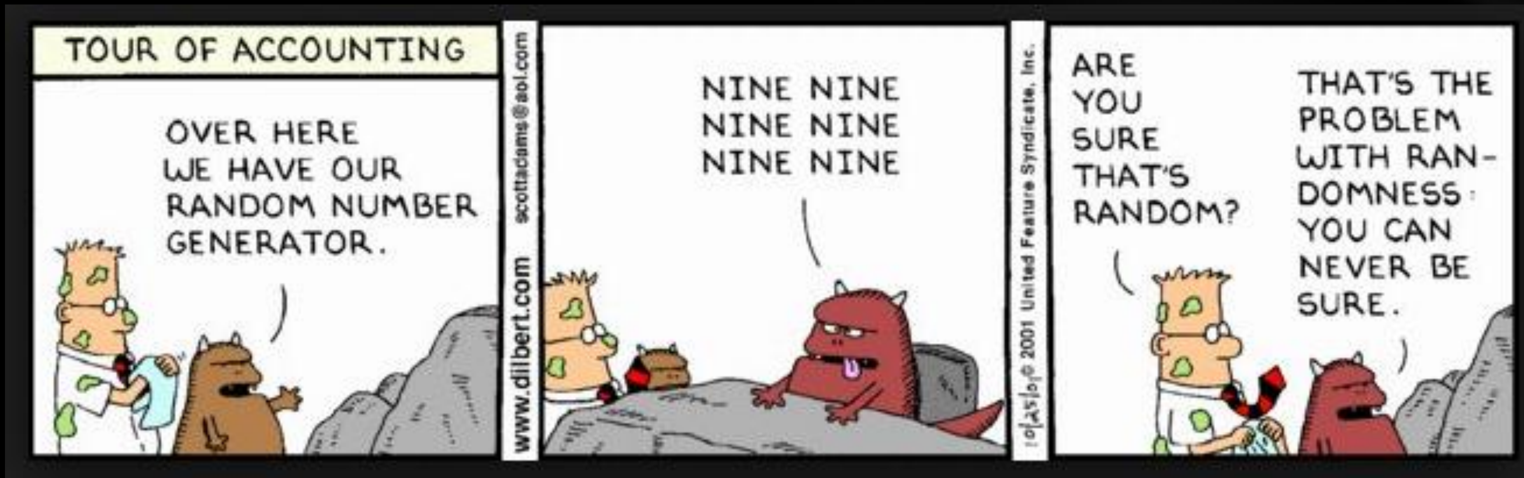


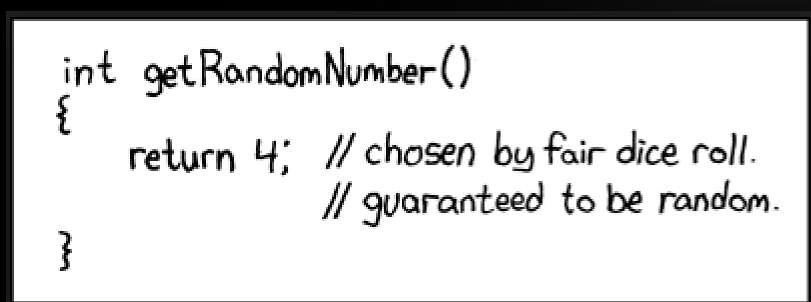


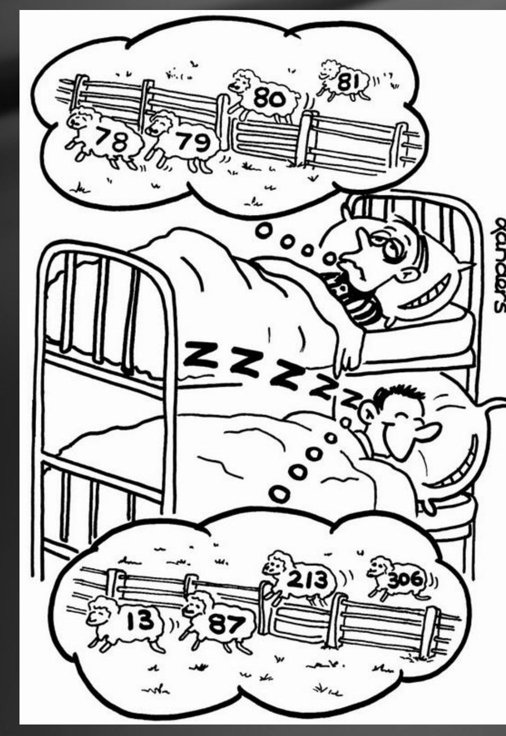


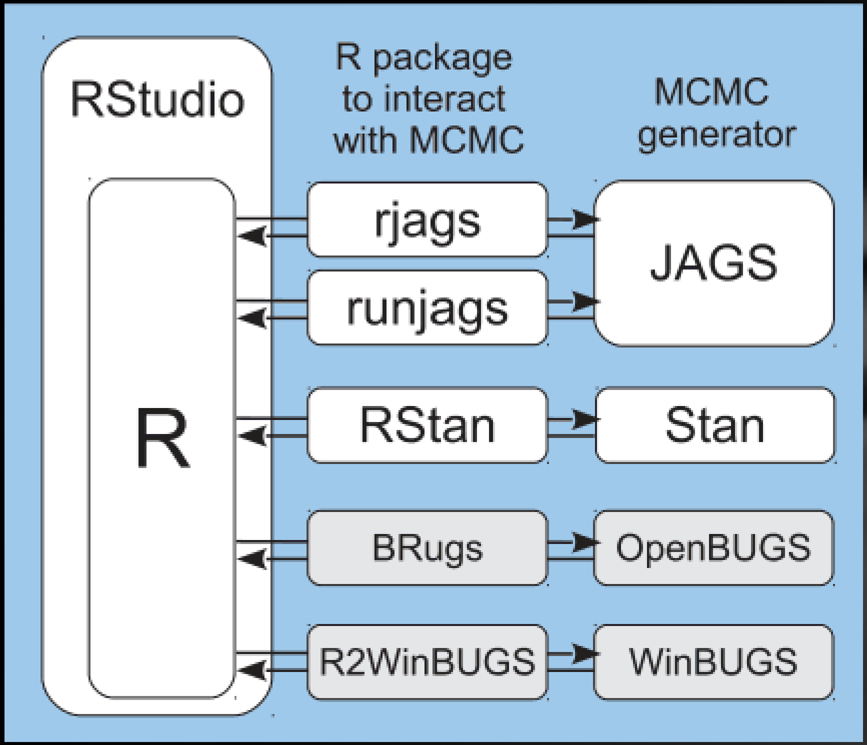
Geman and Geman (1984) Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images. *IEEE Trans. Pattern Anal. Machine Intelligence*, **6**, 721–741.*

$$
\begin{aligned}
\theta_1^{(j)} &\sim \pi\left(\theta_1 \,\middle|\, \theta_2^{(j-1)}, \theta_3^{(j-1)}, \ldots, \theta_k^{(j-1)}\right) \\
\theta_2^{(j)} &\sim \pi\left(\theta_2 \,\middle|\, \theta_1^{(j)}, \theta_3^{(j-1)}, \ldots, \theta_k^{(j-1)}\right) \\
\theta_3^{(j)} &\sim \pi\left(\theta_3 \,\middle|\, \theta_1^{(j)}, \theta_2^{(j)}, \theta_3^{(j-1)}, \ldots, \theta_k^{(j-1)}\right) \\
&\vdots \\
\theta_k^{(j)} &\sim \pi\left(\theta_k \,\middle|\, \theta_1^{(j)}, \theta_2^{(j)}, \ldots, \theta_{k-1}^{(j)}\right)
\end{aligned}
$$


TOUR OF ACCOUNTING
OVER HERE WE HAVE OUR RANDOM NUMBER GENERATOR.
NINE NINE NINE NINE NINE NINE
ARE YOU SURE THAT'S RANDOM?
THAT'S THE PROBLEM WITH RANDOMNESS: YOU CAN NEVER BE SURE.
www.dilbert.com
scottadams@aol.com
10/25/01 © 2001 United Feature Syndicate, Inc.


int getRandomNumber()
{
return 4; // chosen by fair dice roll.
// guaranteed to be random.
}

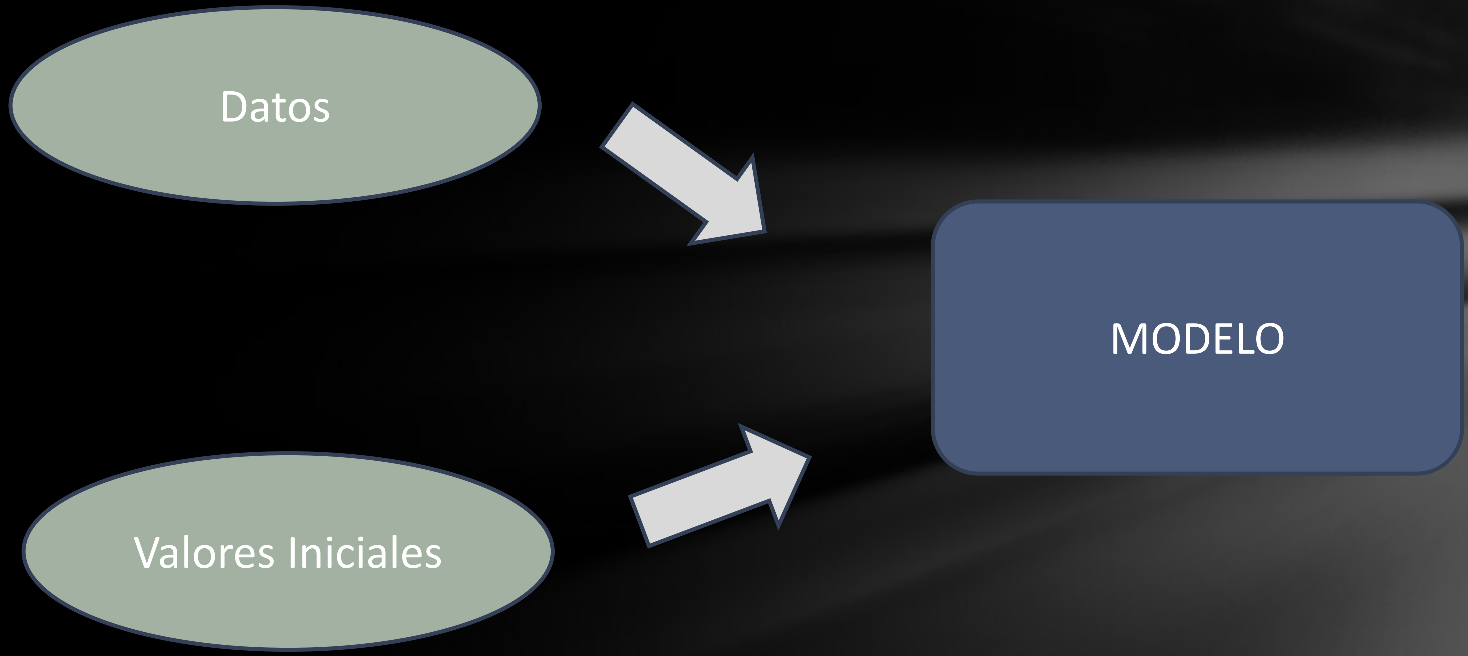

78
79
80
81
ZZZZZ
13
87
213
306


RStudio
R
R package to interact with MCMC
MCMC generator
rjags
runjags
JAGS
RStan
Stan
BRugs
OpenBUGS
R2WinBUGS
WinBUGS

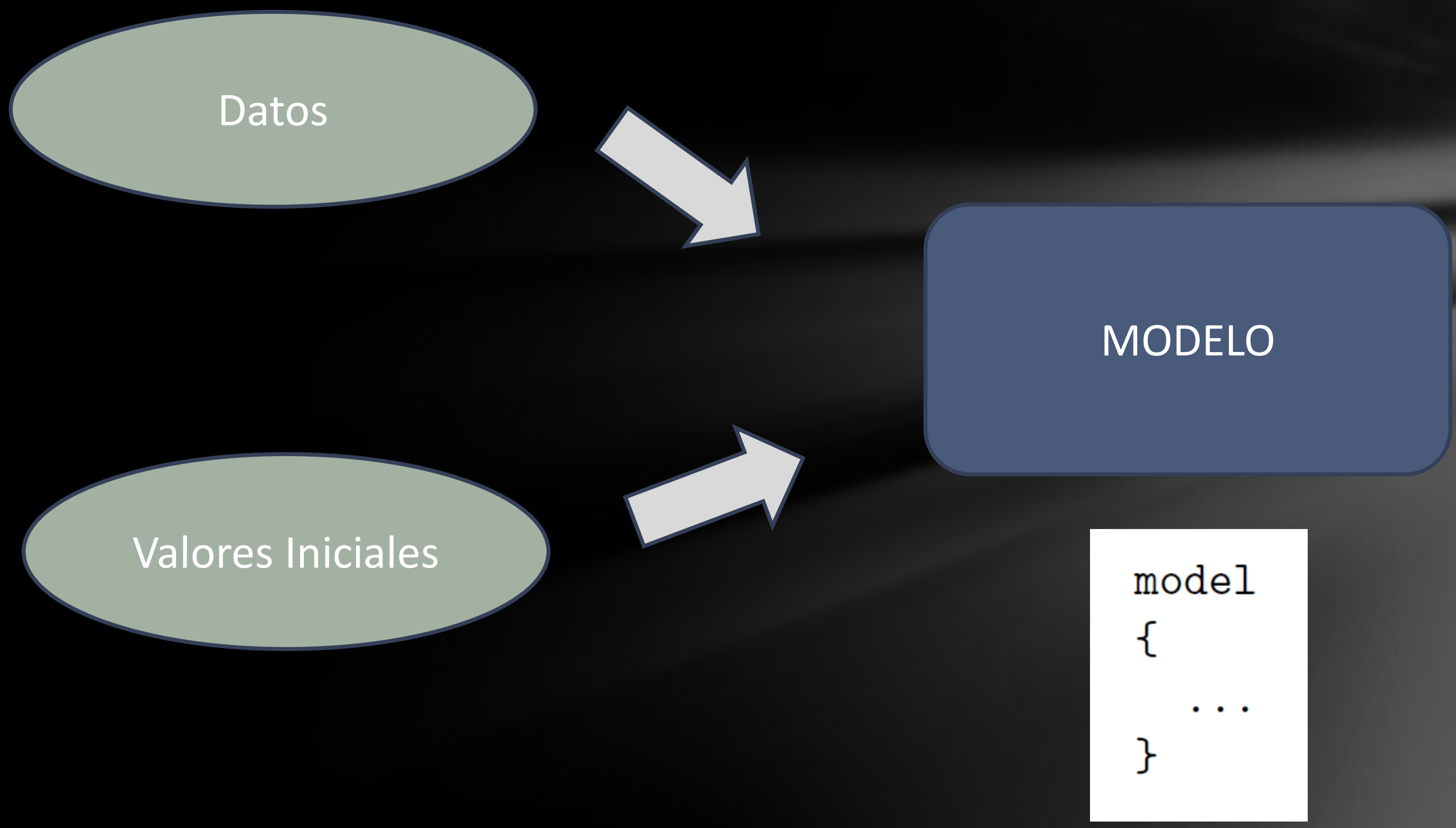
BUGS - JAGS

- BUGS → Bayesian inference Using Gibbs Sampling
- JAGS → Just Another Gibbs Sampler
- Stan → Stan (Ulam) → Hamiltonian Monte Carlo

BUGS - JAGS



BUGS - JAGS



BUGS - JAGS

MODELO

```
alpha <- 1
```

Asignación determinística

```
model
{
   ...
}
```

BUGS - JAGS

MODELO

```
alpha <- 1
```

Asignación determinística

```
p ~ dbeta(1, 1)
```

Asignación estocástica

```
model
{
   ...
}
```

BUGS - JAGS

MODELO

```
model
{
   ...
}
```

```
alpha <- 1
```

Asignación determinística

```
p ~ dbeta(1, 1)
```

Asignación estocástica

```
for (i in 1:N)
{
  x[i] ~ dbern(p)
}
```

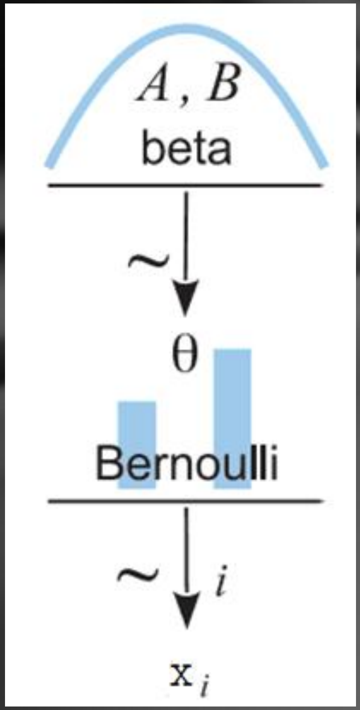
Bucles

BUGS - JAGS

MODELO

```
model
{
   ...
}
```

```
model
{
  alpha <- 1
  beta <- 1

  θ ~ dbeta(alpha, beta)

  for (i in 1:N)
  {
    x[i] ~ dbern(p)
  }
}
```



¡BUGS es un lenguaje descriptivo!

BUGS - JAGS

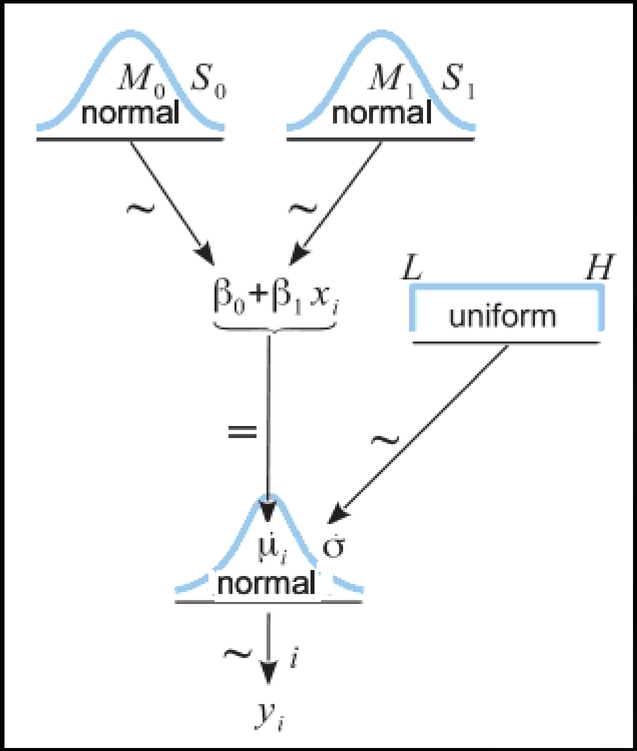
$$y_i = ax_i + b + \epsilon_i$$

$$\epsilon_i \sim \mathcal{N}(0, 1)$$

$$y_i \sim \mathcal{N}(ax_i + b, 1)$$

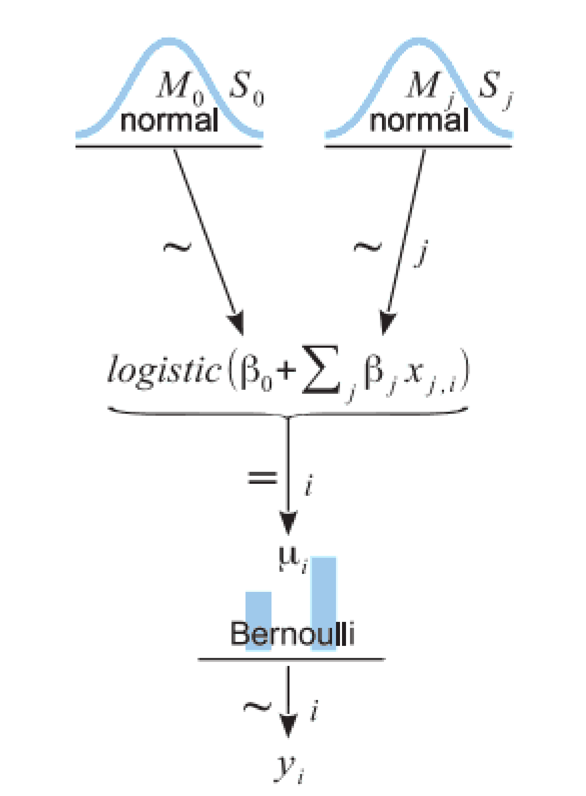
```
y[i] ~ dnorm(a * x[i] + b, 1)
```

JAGS – Regresión Lineal



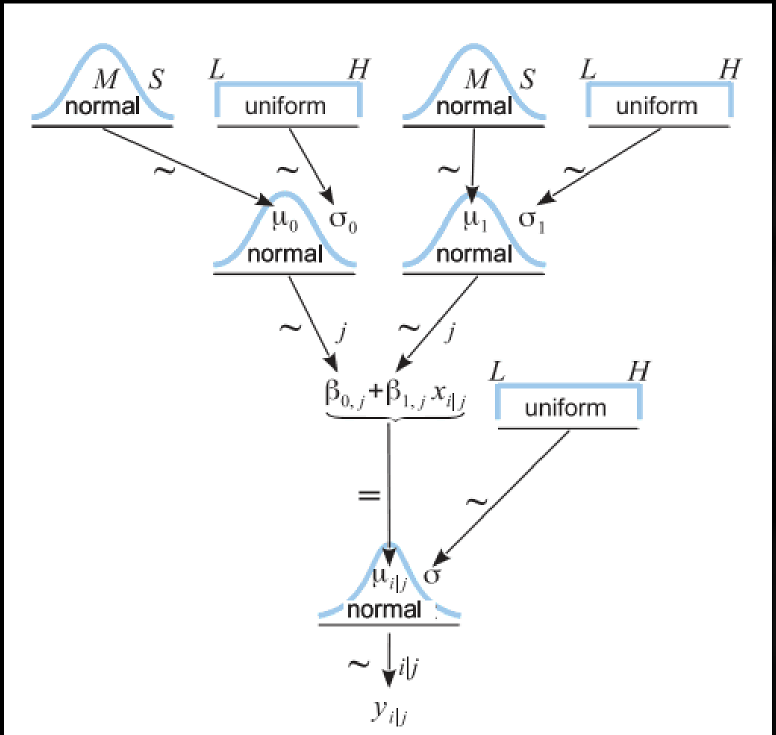

```
model
{
  a ~ dnorm(0, 0.0001)
  b ~ dnorm(0, 0.0001)

  tau <- pow(sigma, -2)
  sigma ~ dunif(0, 100)

  for (i in 1:N)
  {
    mu[i] <- a * x[i] + b
    y[i] ~ dnorm(mu[i], tau)
  }
}
```

JAGS – Regresión Logística



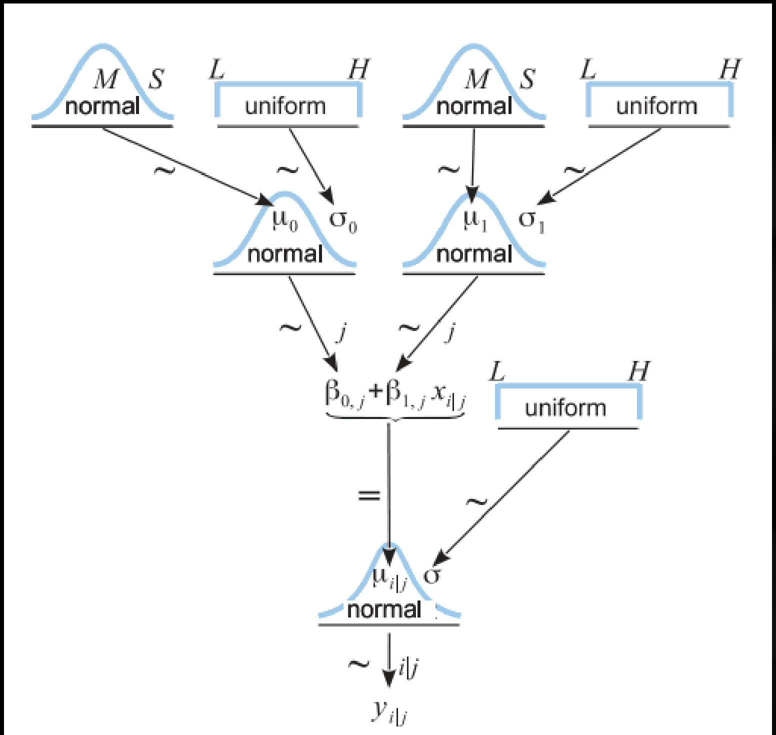

```
model
{
  a ~ dnorm(0, 0.0001)
  b ~ dnorm(0, 0.0001)

  for (i in 1:N)
  {
    y[i] ~ dbern(p[i])
    logit(p[i]) <- a * x[i] + b
  }
}
```

JAGS – Regresión Lineal Jerárquica




```
model
{
  mu.a ~ dnorm(0, 0.0001)
  mu.b ~ dnorm(0, 0.0001)
  ...

  for (j in 1:K)
  {
    a[j] ~ dnorm(mu.a, tau.a)
    b[j] ~ dnorm(mu.b, tau.b)
  }
  for (i in 1:N)
  {
    mu[i] <- a[g[i]] * x[i] + b[g[i]]
    y[i] ~ dnorm(mu[i], tau)
  }
}
```

JAGS – Regresión Lineal Jerárquica

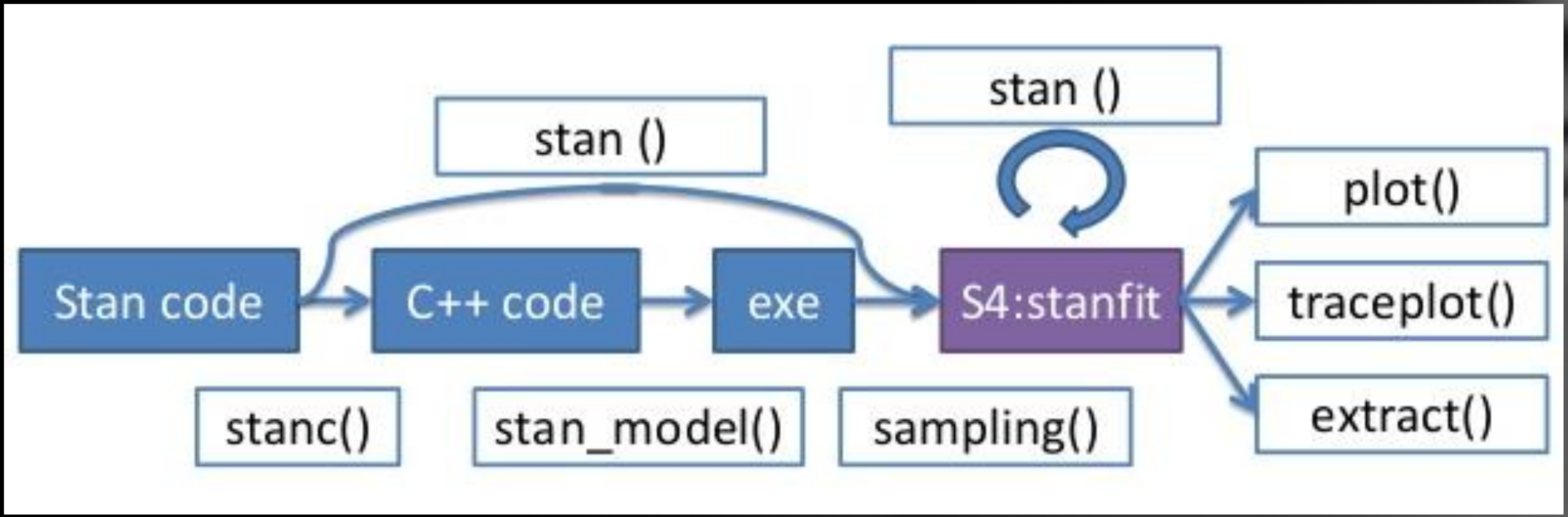



```
model
{
  for (i in 1:N)
  {
    y[i] ~ dnorm(mu[i], tau)
    mu[i] <- a[g[i]] * x[i] + b[g[i]]
  }

  for (j in 1:K)
  {
    a[j] ~ dnorm(mu.a, tau.a)
    b[j] ~ dnorm(mu.b, tau.b)
  }
    mu.a ~ dnorm(0, 0.0001)
    mu.b ~ dnorm(0, 0.0001)

    tau <- pow(sigma, -2)
    sigma ~ dunif(0, 1000)

    tau.a <- pow(sigma.a, -2)
    tau.b <- pow(sigma.b, -2)
    sigma.a ~ dunif(0, 1000)
    sigma.b ~ dunif(0, 1000)
}
```

Recopilación de funciones

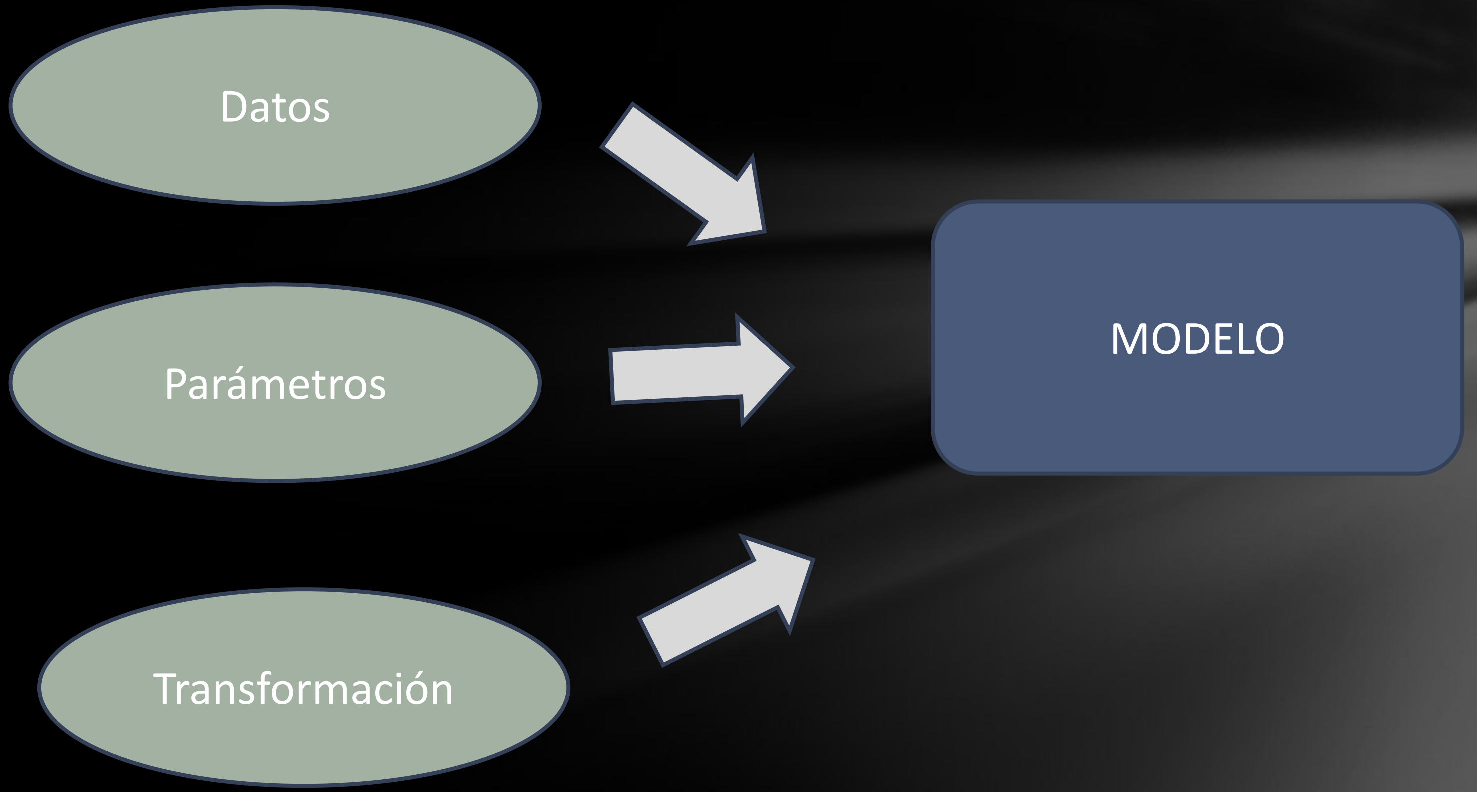
Distribution	Density	Distribution	Quantile
Bernoulli	dbern	pbern	qbern
Beta	dbeta	pbeta	qbeta
Binomial	dbin	pbin	qbin
Chi-square	dchisqr	pchisqr	qchisqr
Double exponential	ddexp	pdexp	qdexp
Exponential	dexp	pexp	qexp
F	df	pf	qf
Gamma	dgamma	pgamma	qgamma
Generalized gamma	dgen.gamma	pgen.gamma	qgen.gamma
Noncentral hypergeometric	dhyper	phyper	qhyper
Logistic	dlogis	plogis	qlogis
Log-normal	dlnorm	plnorm	qlnorm
Negative binomial	dnegbin	pnegbin	qnegbin
Noncentral Chi-square	dnchisqr	pnchisqr	qnchisqr
Normal	dnorm	pnorm	qnorm
Pareto	dpar	ppar	qpar
Poisson	dpois	ppois	qpois
Student t	dt	pt	qt
Weibull	dweib	pweib	qweib

STAN



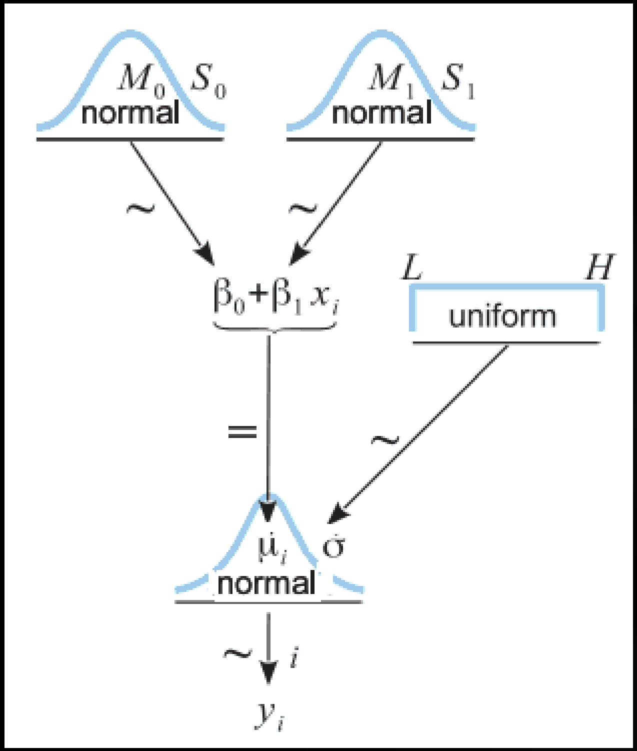
stan ()
stan ()
plot()
Stan code
C++ code
exe
S4:stanfit
traceplot()
stanc()
stan_model()
sampling()
extract()

STAN



¡Stan es un lenguaje imperativo!

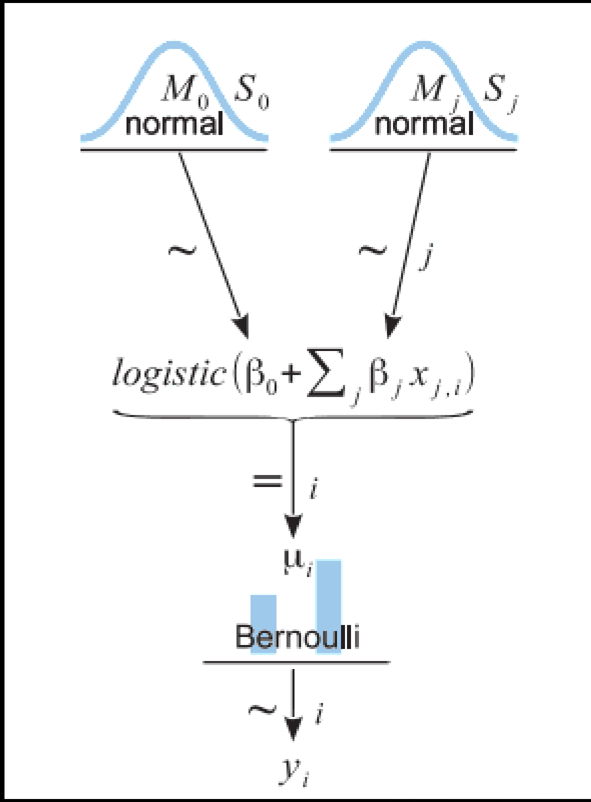
STAN – Regresión Lineal




```
data {
  int<lower=0> N;
  vector[N] x;
  vector[N] y;
}
parameters {
  real alpha;
  real beta;
  real<lower=0> sigma;
}
model {
  for (n in 1:N)
    y[n] ~ normal(alpha + beta * x[n], sigma);
}
```

```
model {
  y ~ normal(alpha + beta * x, sigma);
}
```

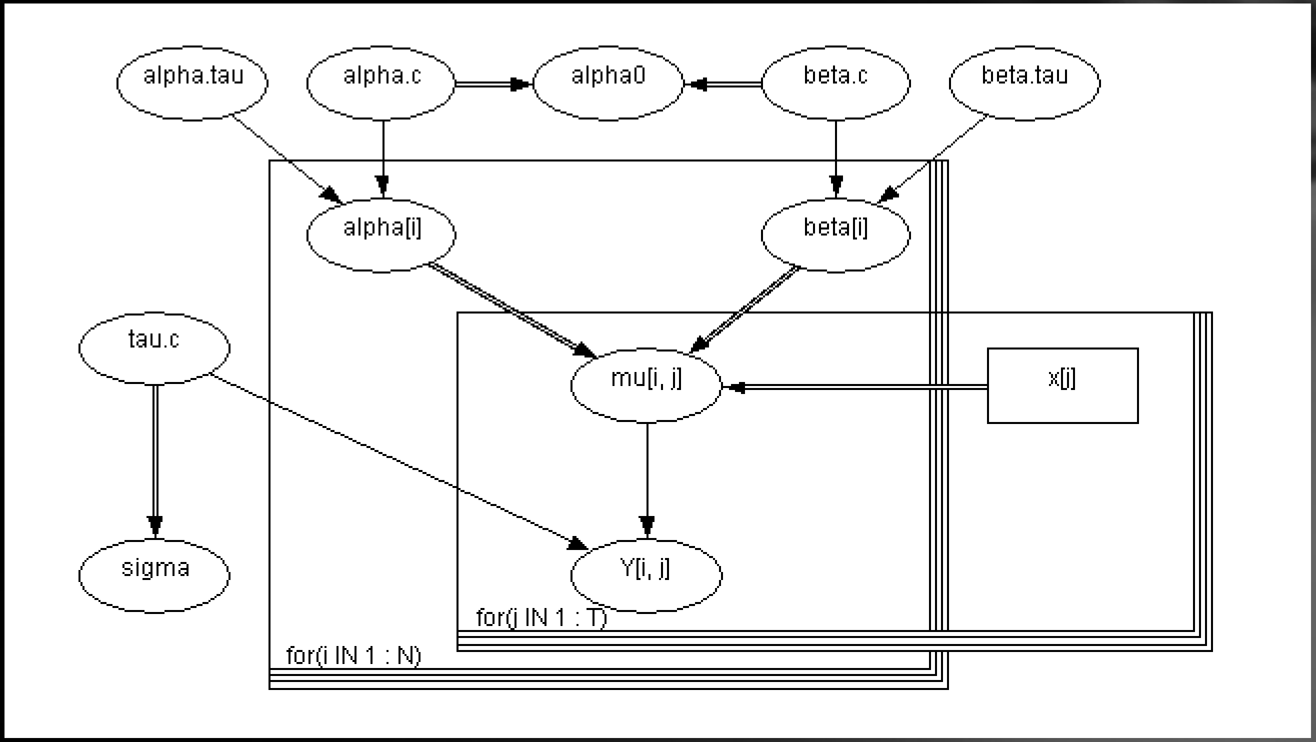
STAN – Regresión Logística




```
data {
  int<lower=0> N;
  vector[N] x;
  int<lower=0,upper=1> y[N];
}
parameters {
  real alpha;
  real beta;
}
model {
  y ~ bernoulli_logit(alpha + beta * x);
}
```

Ratas – Regresión Lineal jerárquica

30 ratas son pesadas durante cinco semanas



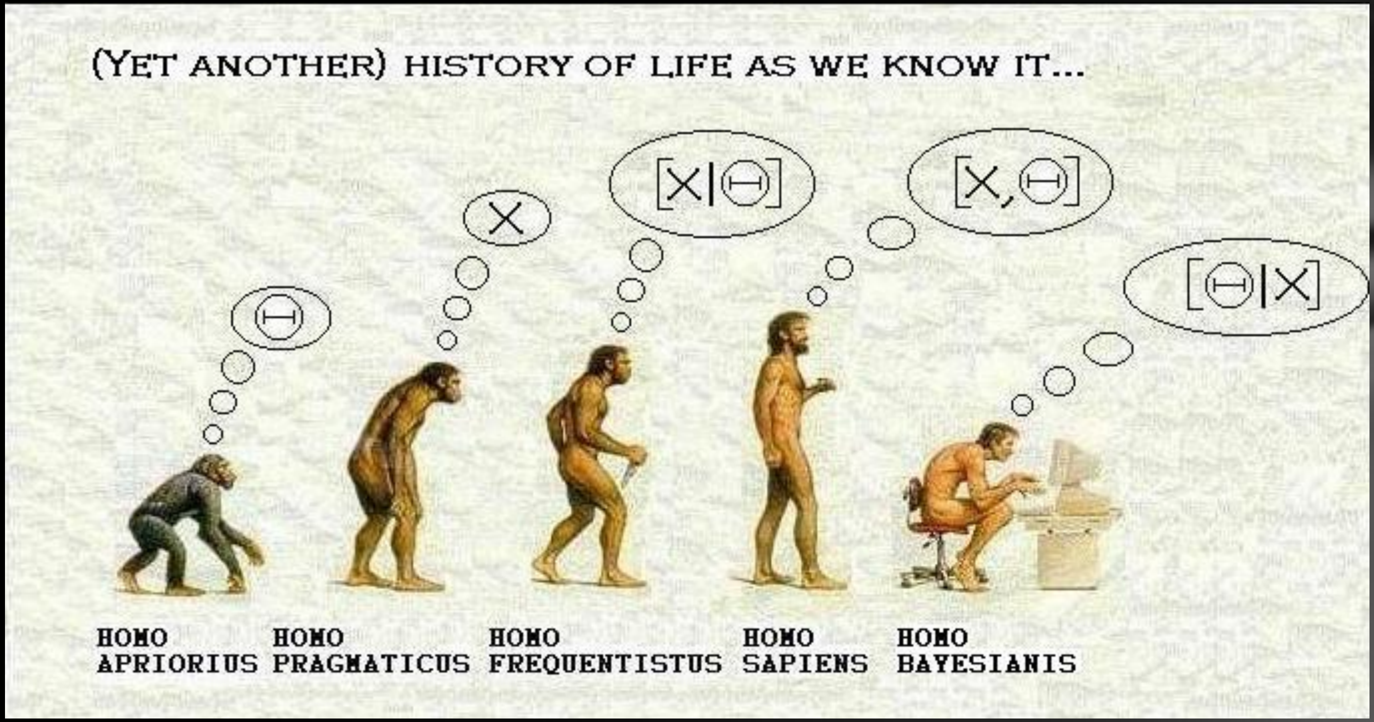
BUGS

```
model
{
  for( i in 1 : N ) {
    for( j in 1 : T ) {
      Y[i , j] ~ dnorm(mu[i , j],tau.c)
      mu[i , j] <- alpha[i] + beta[i] * (x[j] - xbar)
      culmative.Y[i , j] <- culmative(Y[i , j], Y[i , j])
      post.pv.Y[i , j] <- post.p.value(Y[i , j])
      prior.pv.Y[i , j] <- prior.p.value(Y[i , j])
      replicate.post.Y[i , j] <- replicate.post(Y[i , j])
      pv.post.Y[i , j] <- step(Y[i , j] - replicate.post.Y[i , j])
      replicate.prior.Y[i , j] <- replicate.prior(Y[i , j])
      pv.prior.Y[i , j] <- step(Y[i , j] - replicate.prior.Y[i , j])
    }
    alpha[i] ~ dnorm(alpha.c,alpha.tau)
    beta[i] ~ dnorm(beta.c,beta.tau)
  }
  tau.c ~ dgamma(0.001,0.001)
  sigma <- 1 / sqrt(tau.c)
  alpha.c ~ dnorm(0.0,1.0E-6)
  alpha.tau ~ dgamma(0.001,0.001)
  beta.c ~ dnorm(0.0,1.0E-6)
  beta.tau ~ dgamma(0.001,0.001)
  alpha0 <- alpha.c - xbar * beta.c
}
```

STAN

```
data {
  int<lower=0> N;
  int<lower=0> T;
  real x[T];
  real y[N,T];
  real xbar;
}
parameters {
  real alpha[N];
  real beta[N];

  real mu_alpha;
  real mu_beta;

  real<lower=0> sigmasq_y;
  real<lower=0> sigmasq_alpha;
  real<lower=0> sigmasq_beta;
}
transformed parameters {
  real<lower=0> sigma_y;
  real<lower=0> sigma_alpha;
  real<lower=0> sigma_beta;

  sigma_y <- sqrt(sigmasq_y);
  sigma_alpha <- sqrt(sigmasq_alpha);
  sigma_beta <- sqrt(sigmasq_beta);
}
model {
  mu_alpha ~ normal(0, 100);
  mu_beta ~ normal(0, 100);
  sigmasq_y ~ inv_gamma(0.001, 0.001);
  sigmasq_alpha ~ inv_gamma(0.001, 0.001);
  sigmasq_beta ~ inv_gamma(0.001, 0.001);
  alpha ~ normal(mu_alpha, sigma_alpha); // vectorized
  beta ~ normal(mu_beta, sigma_beta);  // vectorized
  for (n in 1:N)
    for (t in 1:T)
      y[n,t] ~ normal(alpha[n] + beta[n] * (x[t] - xbar), sigma_y);
}
```

Conclusión


(YET ANOTHER) HISTORY OF LIFE AS WE KNOW IT...
[Θ]
[X]
[X|Θ]
[X,Θ]
[Θ|X]
HOMO APRIORIUS
HOMO PRAGMATICUS
HOMO FREQUENTISTUS
HOMO SAPIENS
HOMO BAYESIANIS