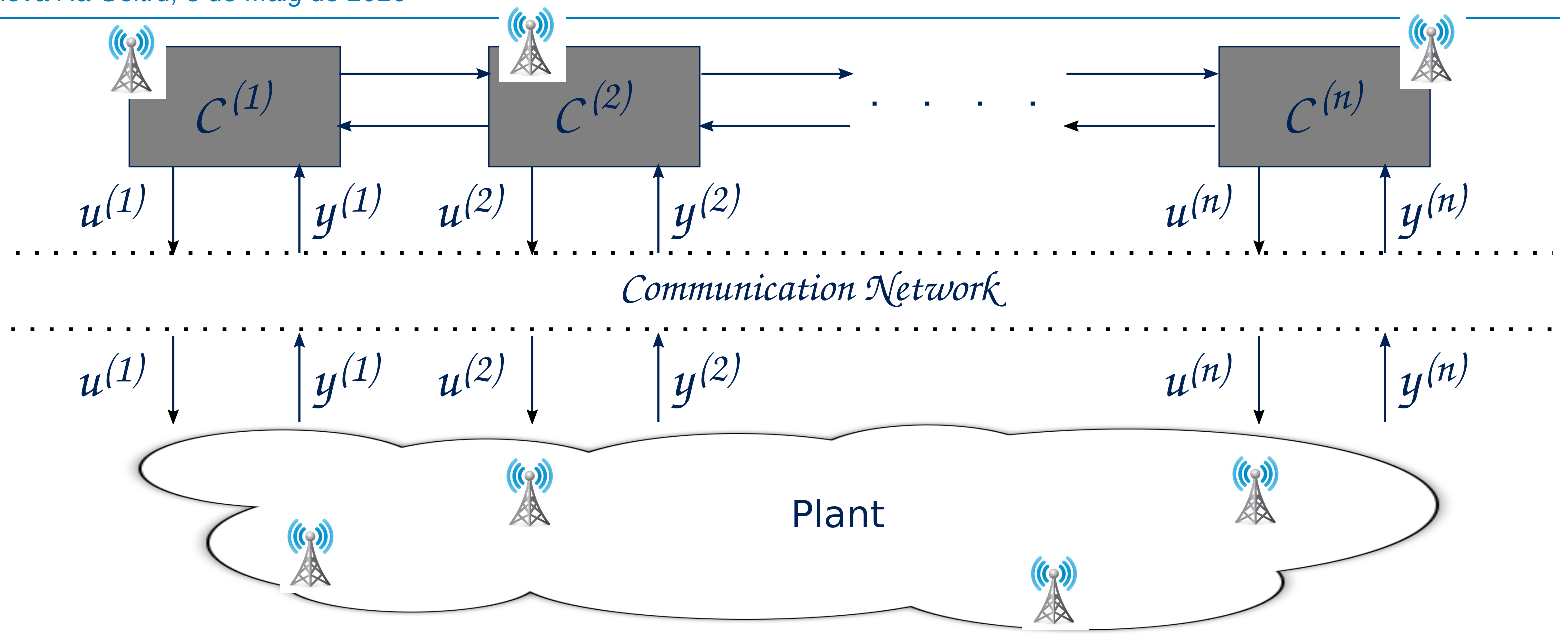
Control over networks

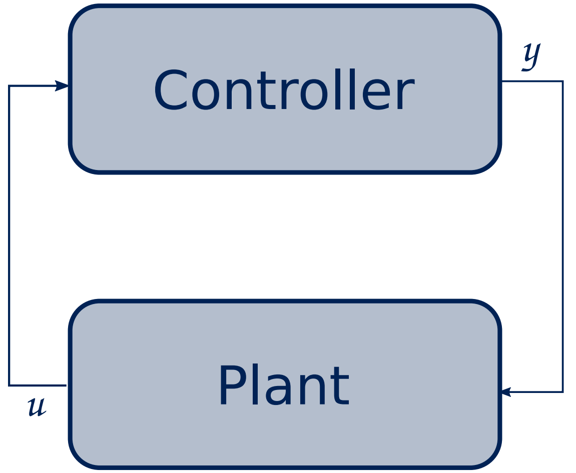


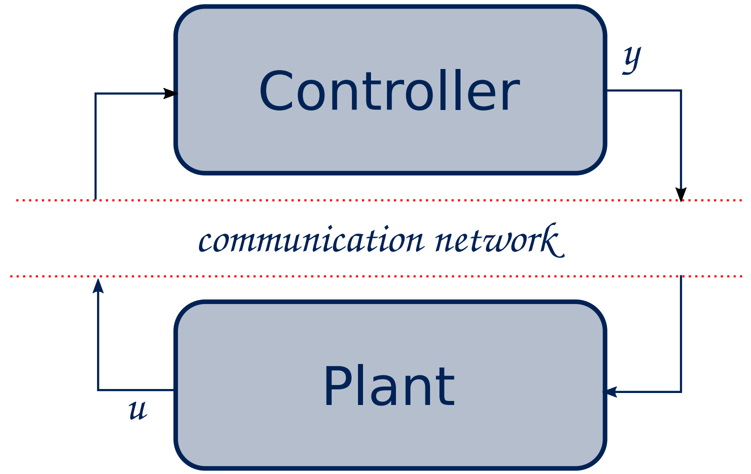
Networked Control Systems

To network...

- Ease of installation and maintenance
- Large flexibility
- Deployment in harsh environments
- Lower costs
- Less wires (less wear, less disturbances, less weight) in case of WSN

Control over networks



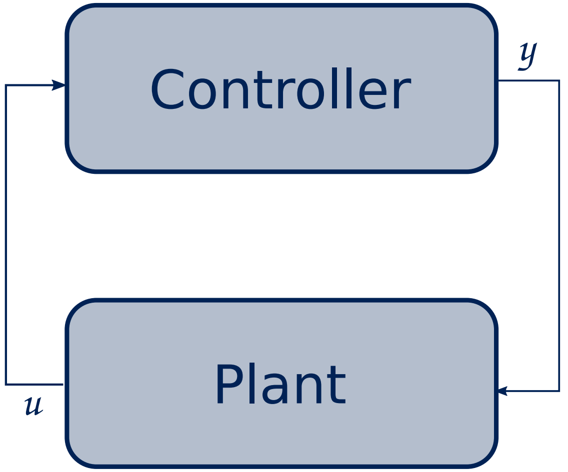


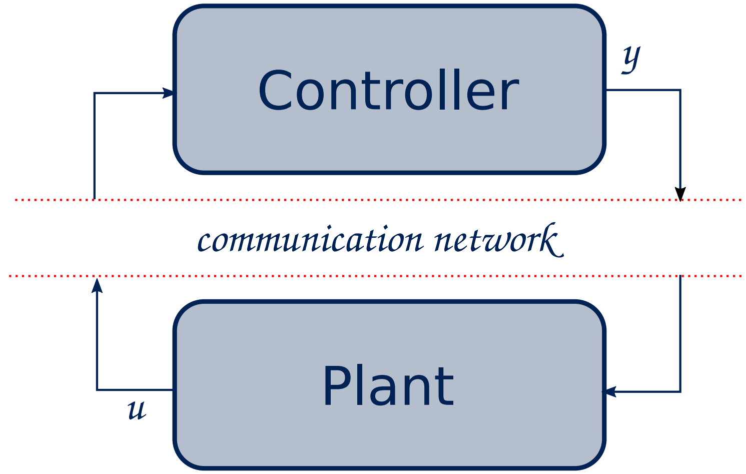
To network... or not to network:

(i) Varying sampling/transmission interval
(ii) Varying communication delays
(iii) Packet loss
(iv) Communication constraints through shared network
(v) Quantization

These (uncertain) effects influence stability and performance

Control over networks





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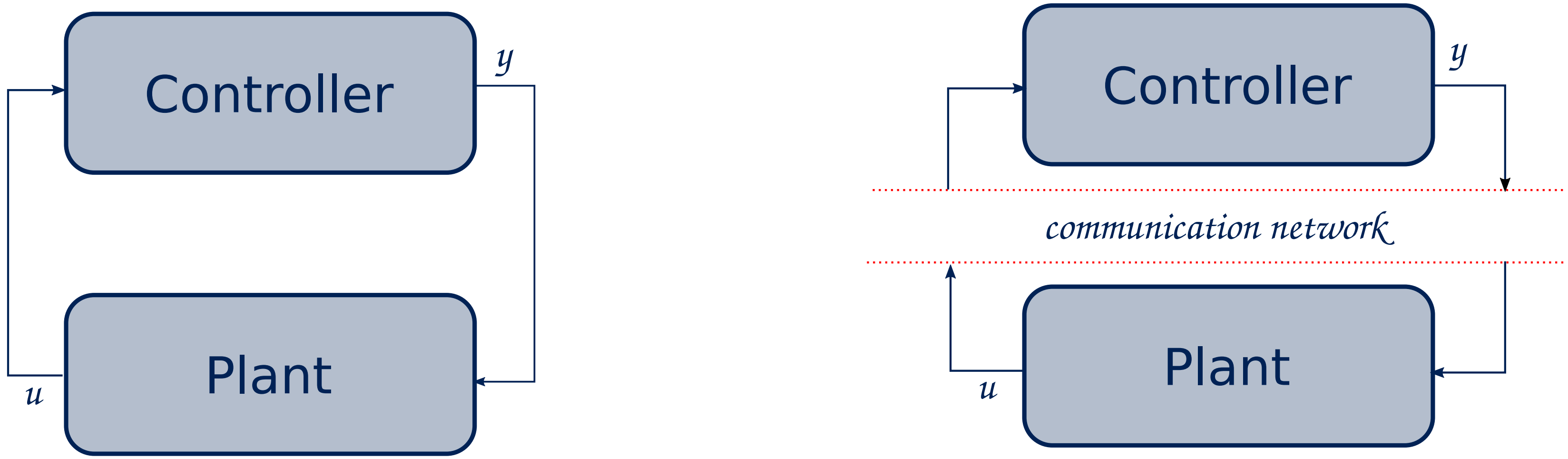
Goal: Quantitative understanding of effects on stability & performance

Control over networks



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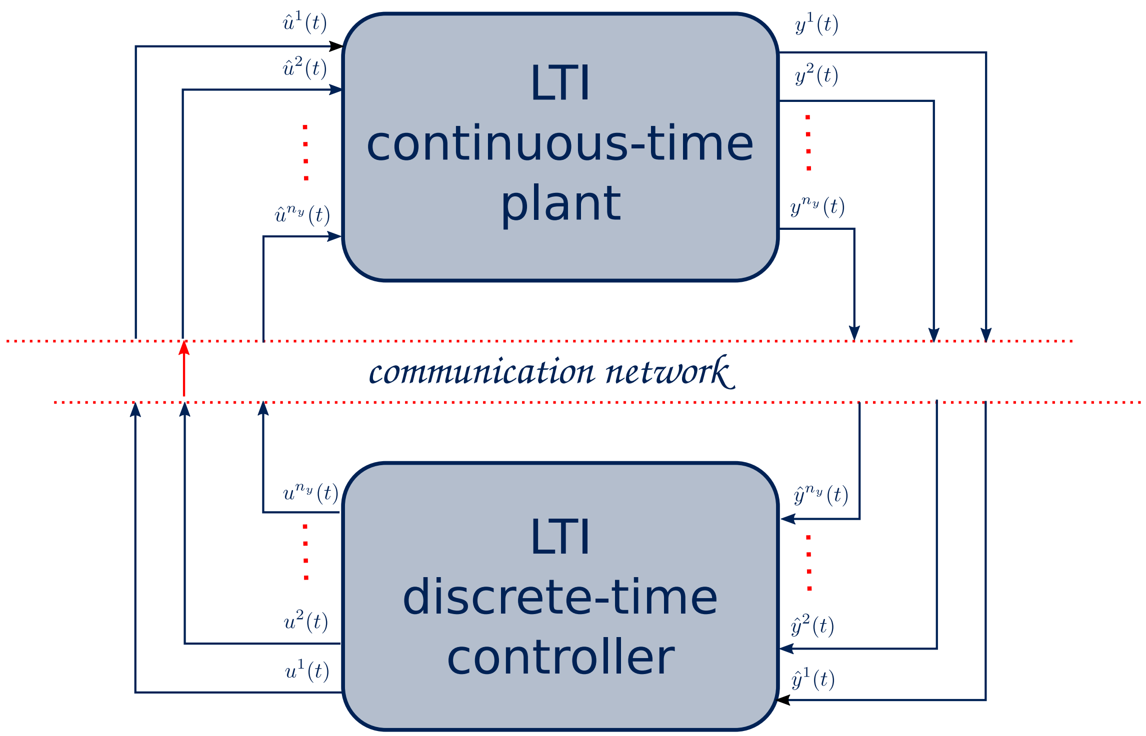
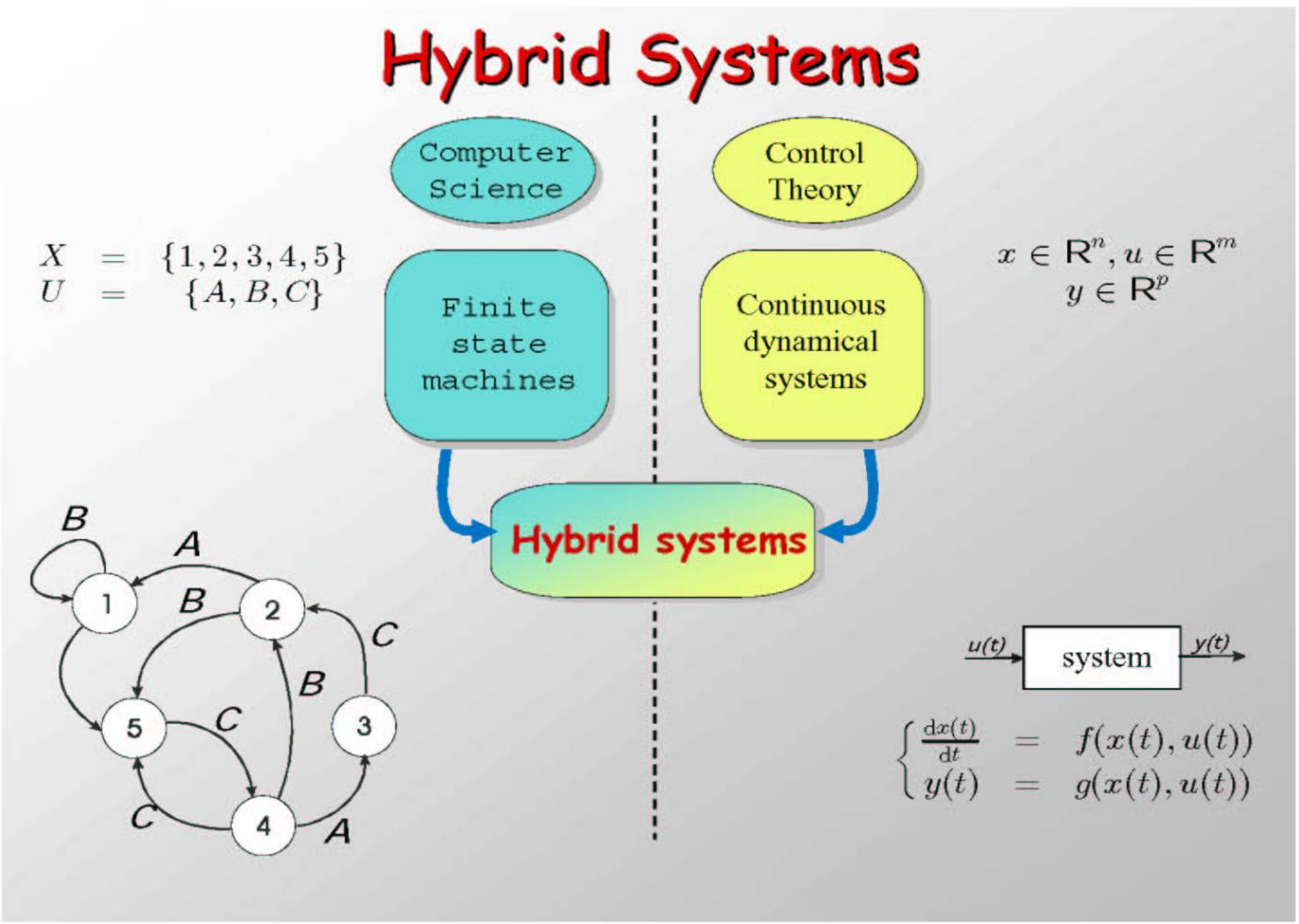
Goal: Quantitative understanding of effects on stability & performance

Communication constraints

- Network is divided into sensor and actuator nodes
- Only one node can access the network simultaneously
- This gives rise to the problem of scheduling: protocols

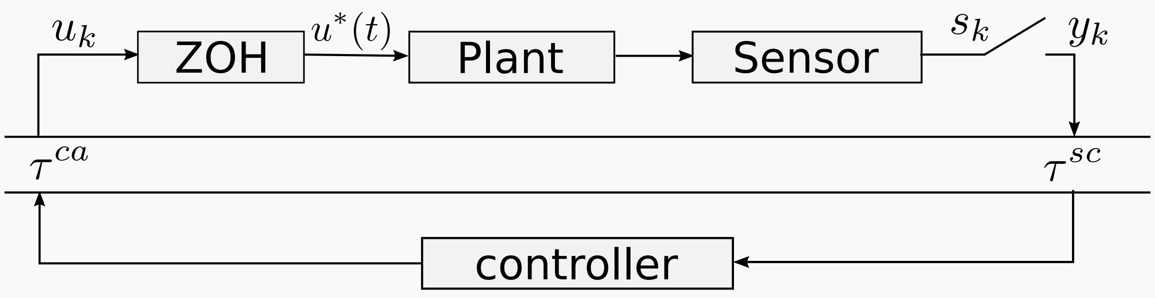


Hybrid systems



Constant delays & Sampling intervals

Sampled-data system with delay



Sampled-data Control System with constant delay

Assumptions:

- Time-driven sensor (sampling times: $s_k = kh$)
- Event-driven controller
- Event-driven actuator
- Static controller

- Sensor-to-controller delay τ^{sc}
- Controller-to-controller delay τ^{ca}
- Computational delay τ^{c}
- Constant delay: $\tau = \tau^{sc} + \tau^{ca} + \tau^{c}$
- $0 \leq \tau \leq h$

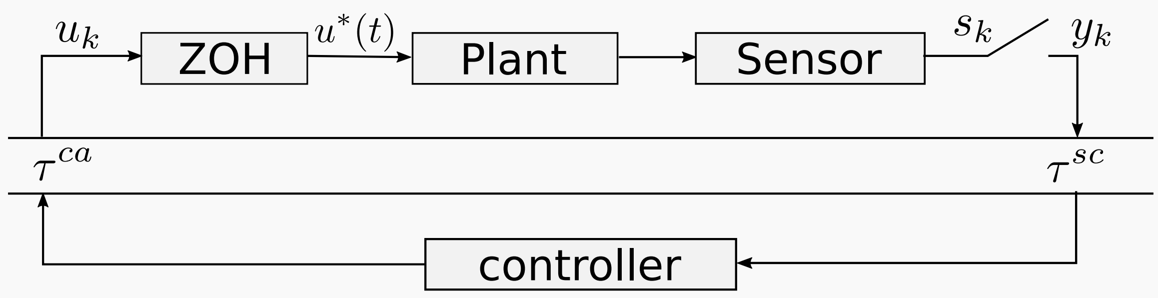
Sampled-data system with delay

Modeling

- Continuous-time, sampled-data dynamics of the linear plant:

$$\dot{x}(t) = Ax(t) + Bu^*(t)$$

$$u^*(t) = u_k, \quad \text{for} \quad t \in [s_k + \tau, \quad s_{k+1} + \tau)$$



Sampled-data Control System with constant delay

Sampled-data system with delay

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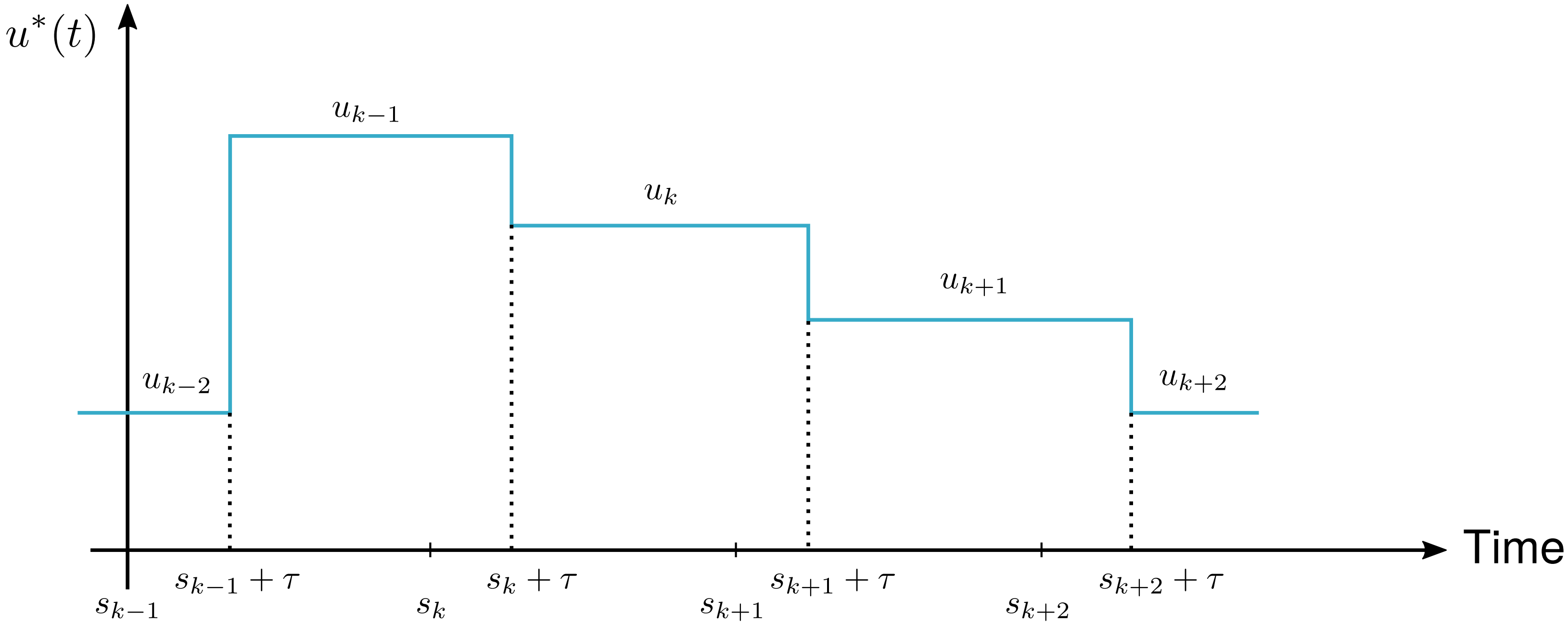

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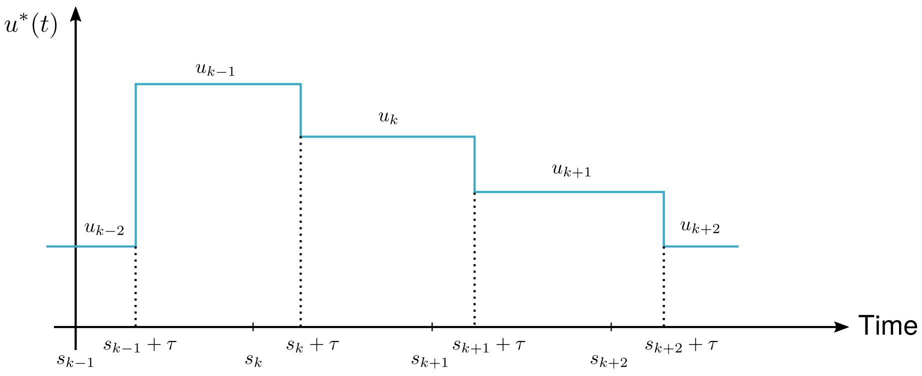

$$x_k := x(s_k)$$

$$x_{k+1} = e^{Ah} x_k + \int_{\tau}^{h} e^{A(h-s)} ds B \quad u_k + \int_{0}^{\tau} e^{A(h-s)} ds B \quad u_{k-1}$$

$$x_{k+1} = e^{Ah} x_k + \int_{0}^{h-\tau} e^{As} ds B \quad u_k + \int_{h-\tau}^{h} e^{As} ds B \quad u_{k-1}$$

Lifted NCS model

- Exact discretisation

$$x_{k+1} = e^{Ah}x_k + \int_0^{h-\tau} e^{As}dsB \quad u_k + \int_{h-\tau}^{h} e^{As}dsB \quad u_{k-1}$$

- Using the extended state vector $\xi_k = \begin{pmatrix} u_k \\ u_{k-1} \end{pmatrix}$ we obtain the lifted model

$$\xi_{k+1} = \begin{pmatrix} x_{k+1} \\ u_k \end{pmatrix} = \underbrace{\begin{pmatrix} e^{Ah} & \int_0^{h-\tau} e^{As}dsB \\ 0 & 0 \end{pmatrix}}_{=:F(h,\tau)} + \underbrace{\begin{pmatrix} \int_0^{h-\tau} e^{As}dsB \\ I \end{pmatrix}}_{=:G(h,\tau)} u_k$$

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- In cloop with extended-state feedback $u_k = -K\xi_k = -\hat{K} x_k - K_u u_{k-1}$:

$$\xi_{k+1} = \underbrace{\begin{pmatrix} e^{Ah} - \int_0^{h-\tau} e^{Ah} ds B \hat{K} & \int_0^{h-\tau} e^{As} ds B - \int_0^{h-\tau} e^{As} ds B K_u \\ -\hat{K} & -K_u \end{pmatrix}}_{=: H(h,\tau)} \xi_k$$

- Exponentially satable iff $H(h,\tau)$ Schur, i.e. all eigenvalues within open unit circle