portada de la presentación

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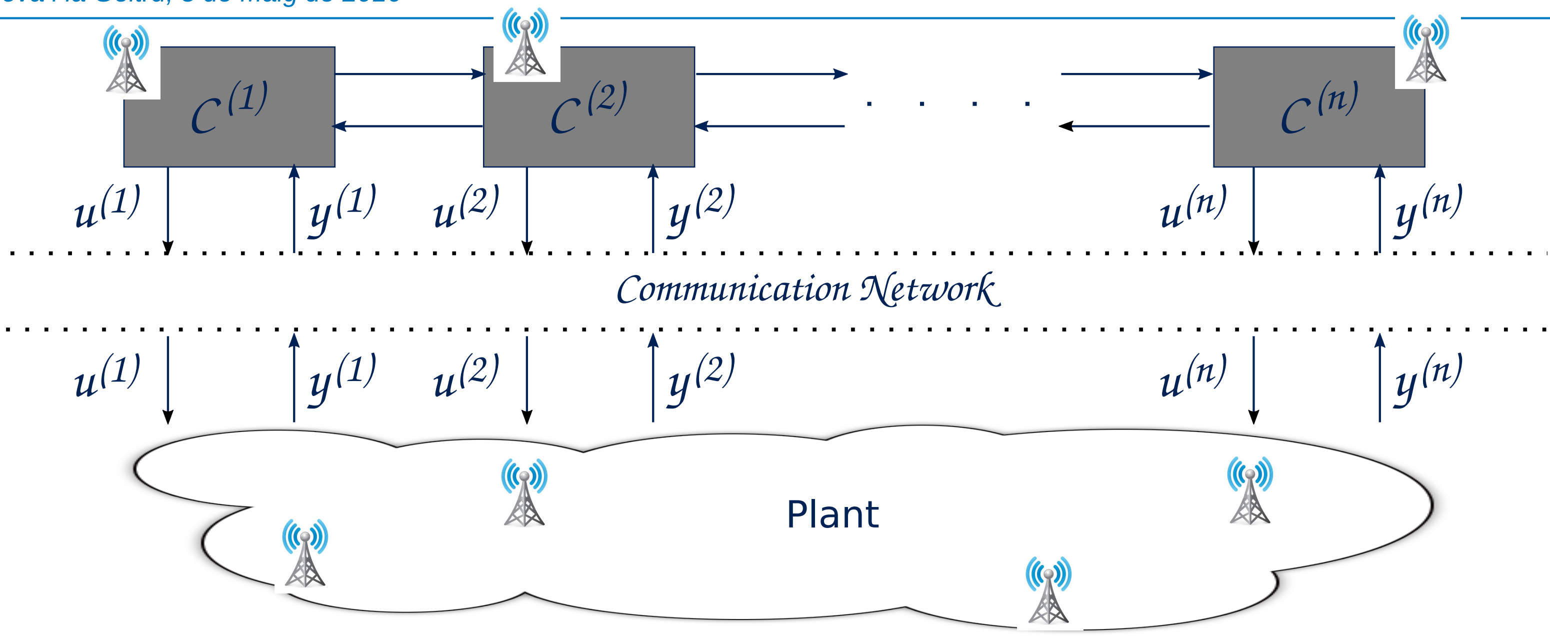
- Bas van Loon, TU/e
- Jos van Schendel, TU/e
- ...

Control over networks

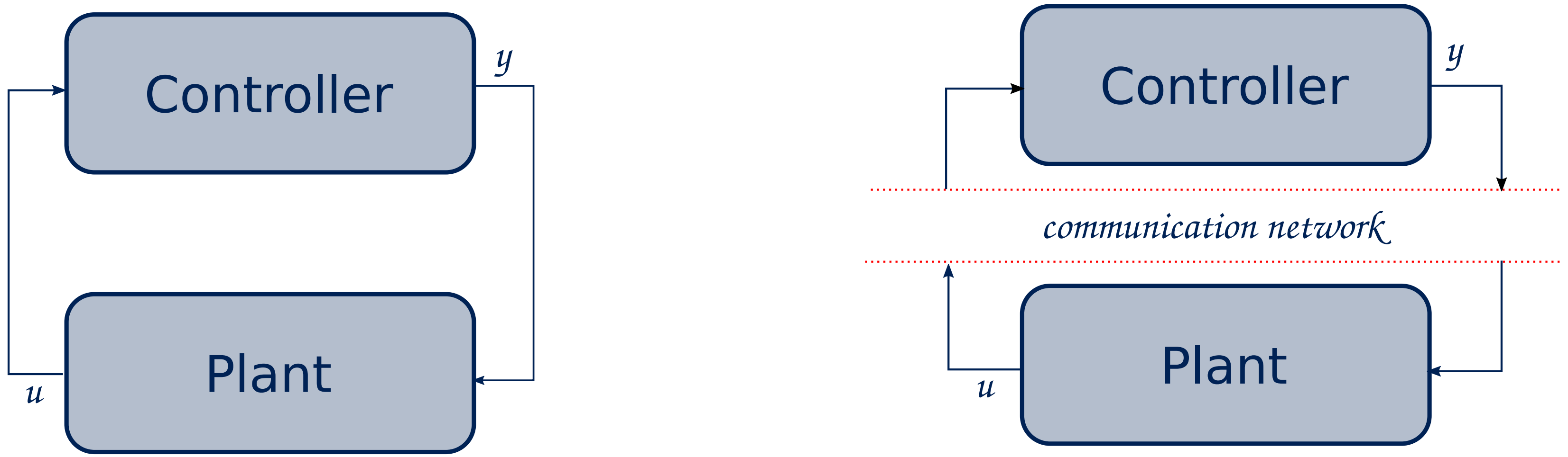


Networked Control Systems

To network...

- Ease of installation and maintenance
- Large flexibility
- Deployment in harsh environments
- Lower costs
- Less wires (less wear, less disturbances, less weight) in case of WSN

Control over networks



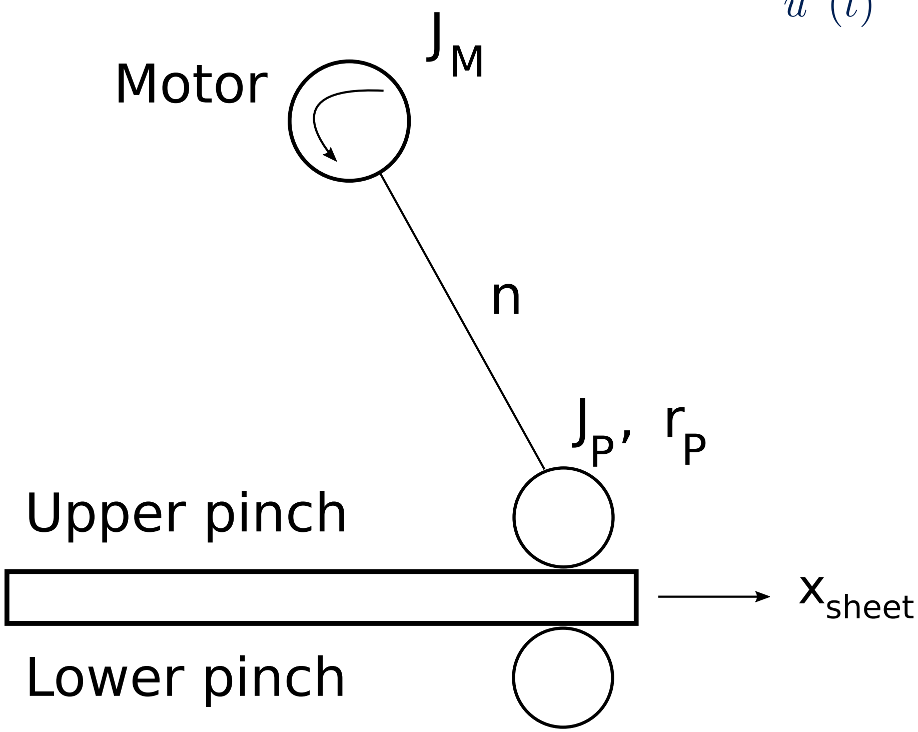
To network... or not to network:

(i) Varying sampling/transmission interval
(ii) Varying communication delays
(iii) Packet loss
(iv) Communication constraints through shared network
(v) Quantization

These (uncertain) effects influence stability and performance

Motivating example

The influence of time-varying dalays on stability...



$$\dot{x}(t) = Ax(t) + Bu^*(t)$$
$$u^*(t) = u_k, \quad for \quad t \; \epsilon \; [s_k + \tau_k \; , \; s_{k+1} + \tau_{k+1})$$

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \frac{nr_p}{J_M + n^2 J_P} \end{pmatrix}$$

$$x(t) = \begin{pmatrix} x_s(t) \\ \dot{x}_s(t) \end{pmatrix}$$

$$u_k = -\hat{K} x_k$$

[Cloosterman et al, CDC 2006, IEEE Trans. Aut. Control, 2009]

Motivating example

$$h = 1ms$$

$$T^a = 0.2ms$$

$$T^b = 0.6ms$$

$$\hat{K} = (K_1 \;\; K_2)$$

$$= (50 \;\; 11.8)$$

graficas

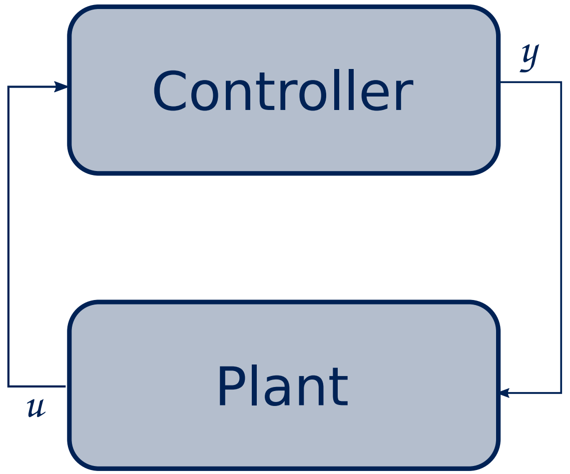
graficas

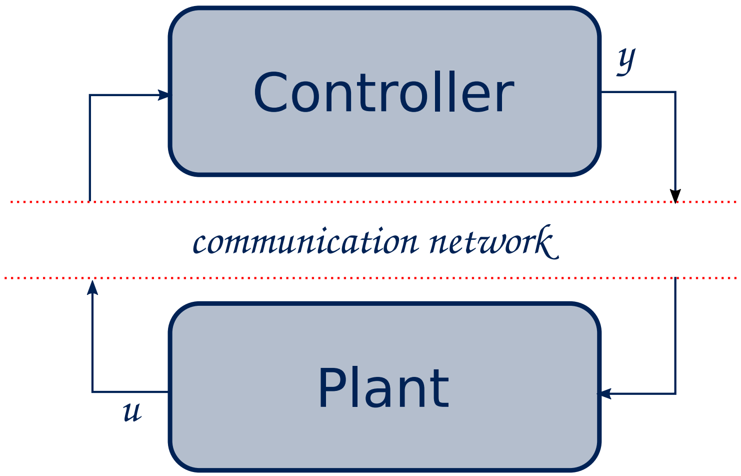
Switching sequence:

$\tau^a, \tau^b, \tau^a, \tau^b, \ldots$

Also varying sampling intervals h_k might show similar (destabilizing) effects

Control over networks





To network... or not to network:

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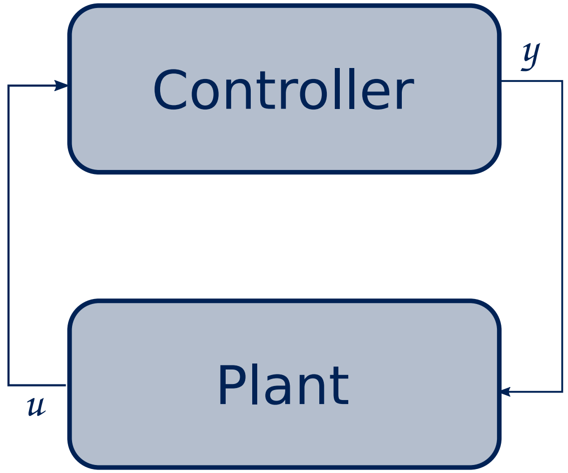
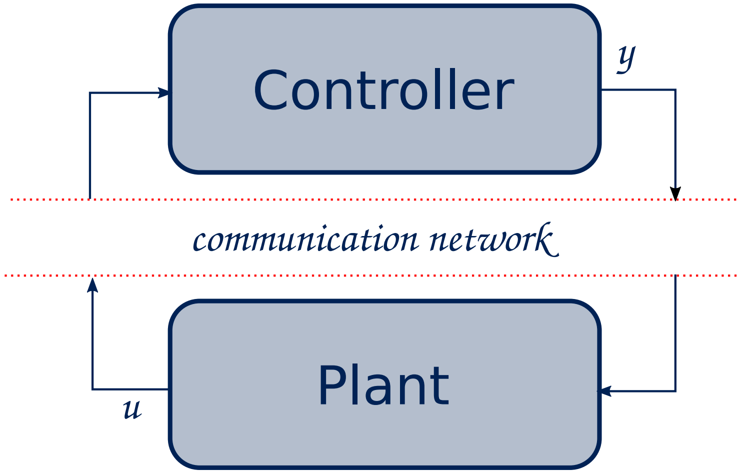
Goal: Quantitative understanding of effects on stability & performance

Control over networks





To network... or not to network:

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(ii) Varying communication delays
(iii) Packet loss
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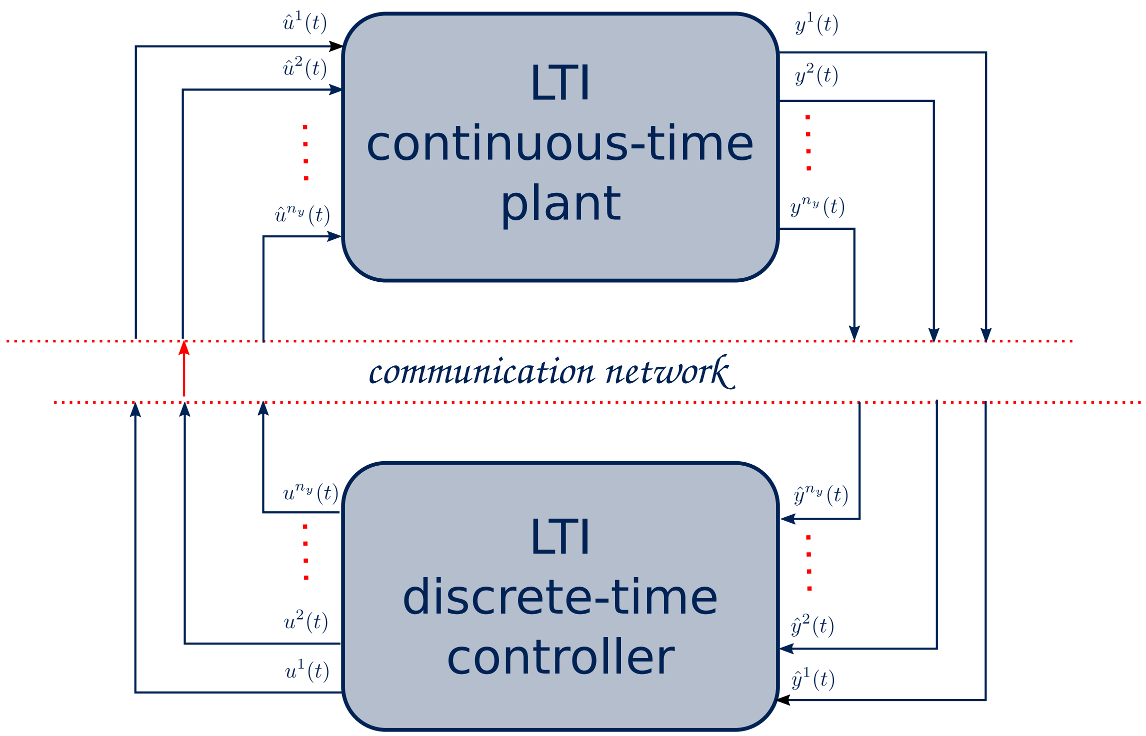
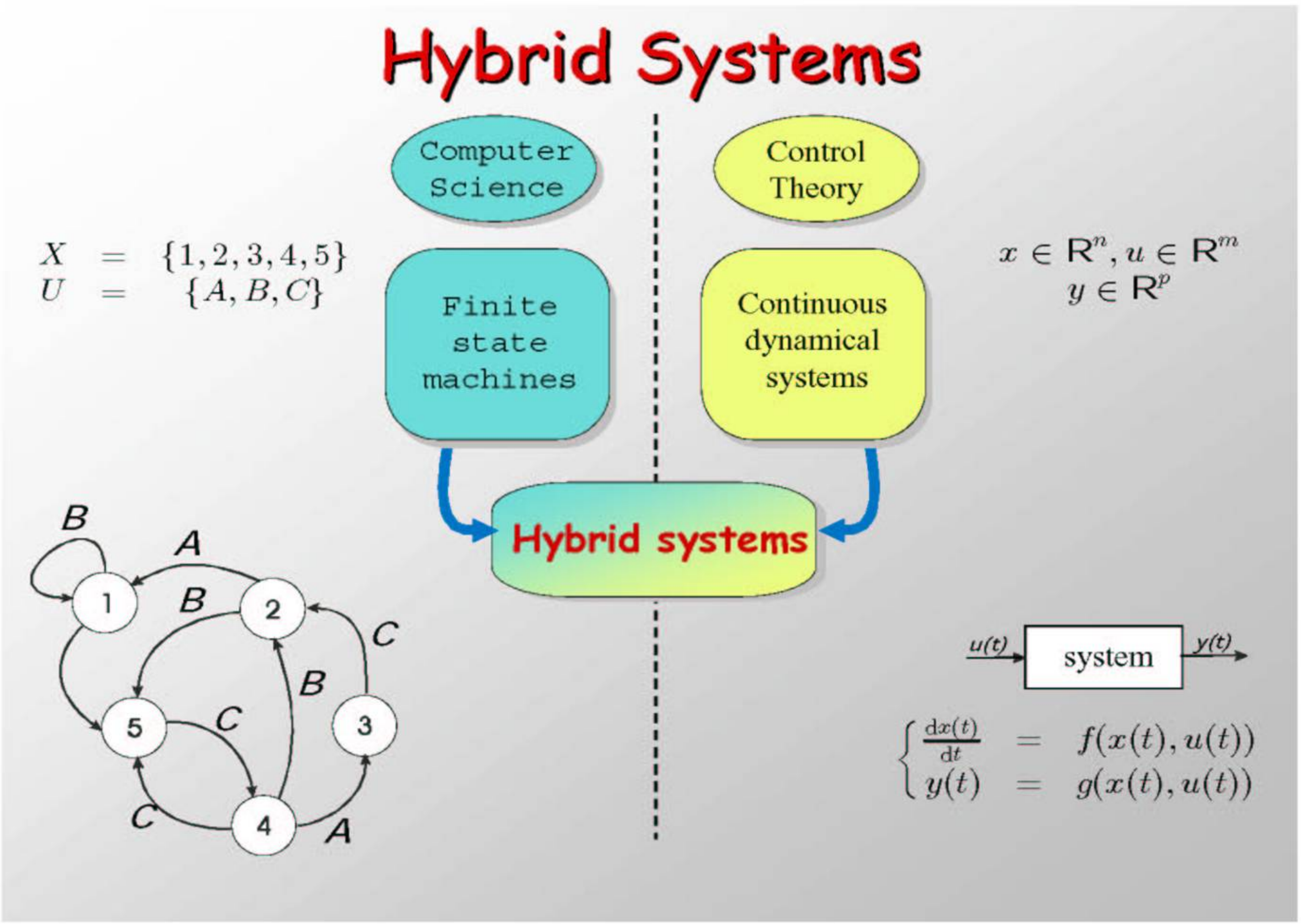
Goal: Quantitative understanding of effects on stability & performance

Communication constraints

- Network is divided into sensor and actuator nodes
- Only one node can access the network simultaneously
- This gives rise to the problem of scheduling: protocols



Hybrid systems



Outline Lectures I-V

Main focus: Stability of NCS under network-induced imperfections

- [Lec. I]: Simple NCS analysis problems and solutions (varying delays, sampling intervals)

  Discrete-time modeling framework

- [Lec. II]: NCS under communication constraints (incl. I) & dropouts

  Discrete-time modeling framework

- [Lec. III]: "Guided problem solving"
- [Lec. IV]: NCS under communication constraints

  Continuous-time modeling framework (emulation)

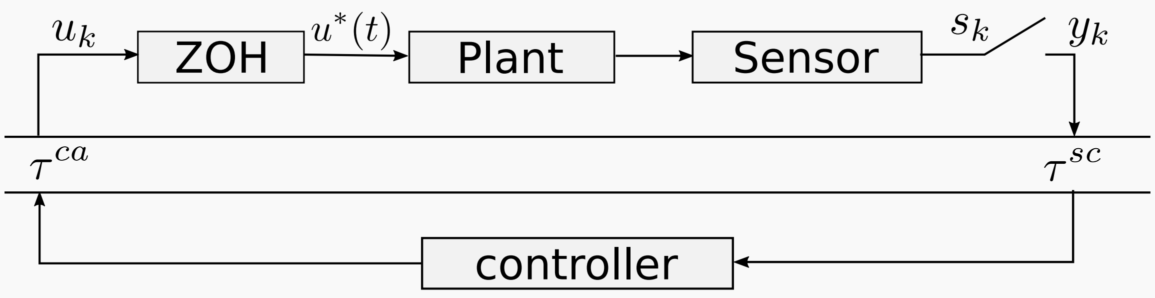
- [Lec. V]: "Guided problem solving"

Outline Lec. I & II

- Provide simple NCS analysis problems and solutions
  - Only one network-induced imperfection
- Sampled-data system
  - Constant interval/delay: linear discrete-time system
- Sampled-data system with varying intervals/delays:
  - finite number: d-t switched linear system
  - Constant interval/delay: d-t linear parameter-varying (LPV) system
- Stepping stone towards more complex situations (incl. cc) ⟶
- NCS with cc: d-t switched linear parameter-varying (SLPV) system
- NCS with packet loss:
  - Upperbound on number of subsequent drops
  - Stochastic models: Bernoulli/Gilbert-Elliot

Constant delays & Sampling intervals

Sampled-data system with delay



Sampled-data Control System with constant delay

Assumptions:

- Time-driven sensor (sampling times: $s_k = kh$)
- Event-driven controller
- Event-driven actuator
- Static controller

- Sensor-to-controller delay τ^{sc}
- Controller-to-controller delay τ^{ca}
- Computational delay τ^{c}
- Constant delay: $\tau = \tau^{sc} + \tau^{ca} + \tau^{c}$
- $0 \leq \tau \leq h$

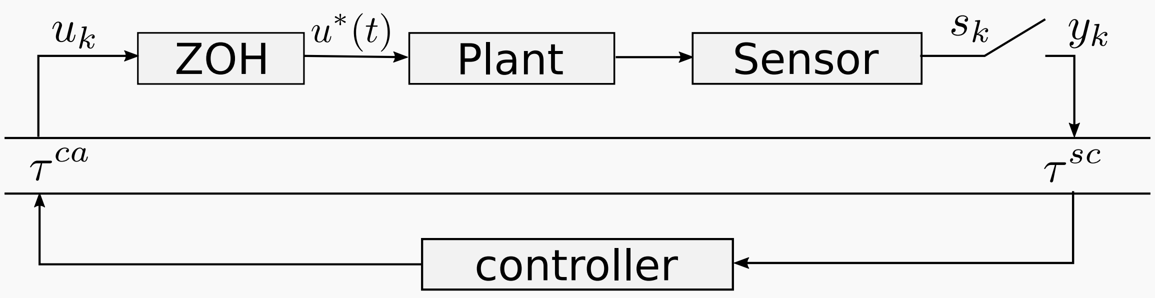
Sampled-data system with delay

Modeling

- Continuous-time, sampled-data dynamics of the linear plant:

$$\dot{x}(t) = Ax(t) + Bu^*(t)$$

$$u^*(t) = u_k, \quad \text{for} \quad t \in [s_k + \tau, \quad s_{k+1} + \tau)$$



Sampled-data Control System with constant delay

Sampled-data system with delay

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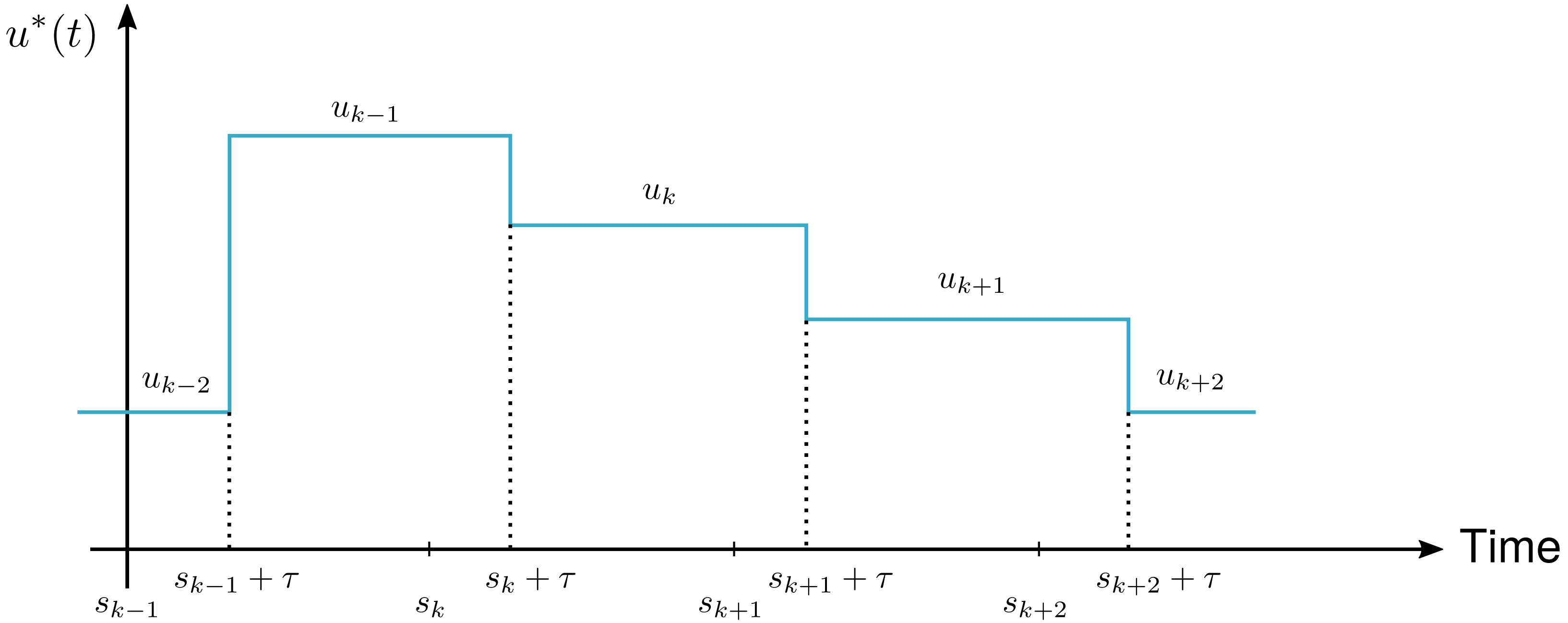

Sampled-data system with delay

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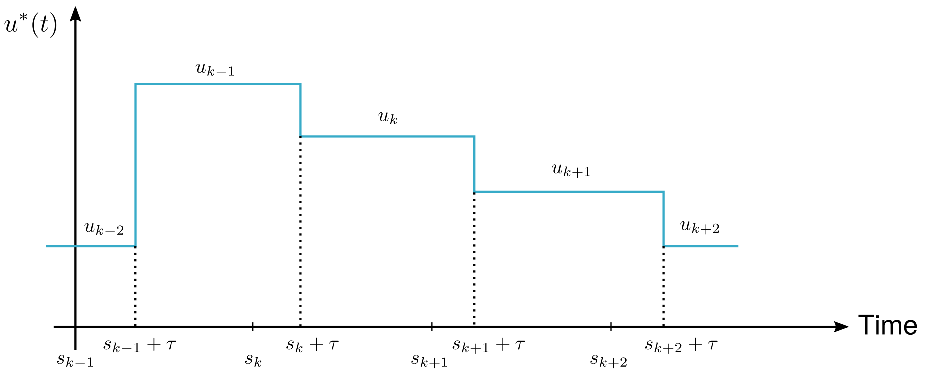

$$x_k := x(s_k)$$

$$x_{k+1} = e^{Ah}x_k + \int_\tau^h e^{A(h-s)}dsB \quad u_k + \int_0^\tau e^{A(h-s)}dsB \quad u_{k-1}$$

$$x_{k+1} = e^{Ah}x_k + \int_0^{h-\tau} e^{As}dsB \quad u_k + \int_{h-\tau}^{h} e^{As}dsB \quad u_{k-1}$$

Lifted NCS model

- Exact discretisation

$$x_{k+1} = e^{Ah}x_k + \int_0^{h-\tau} e^{As}dsB \quad u_k + \int_{h-\tau}^{h} e^{As}dsB \quad u_{k-1}$$

- Using the extended state vector $\xi_k = \begin{pmatrix} u_k \\ u_{k-1} \end{pmatrix}$ we obtain the lifted model

$$\xi_{k+1} = \begin{pmatrix} x_{k+1} \\ u_k \end{pmatrix} = \underbrace{\begin{pmatrix} e^{Ah} & \int_0^{h-\tau} e^{As}dsB \\ 0 & 0 \end{pmatrix}}_{=:F(h,\tau)} + \underbrace{\begin{pmatrix} \int_0^{h-\tau} e^{As}dsB \\ I \end{pmatrix}}_{=:G(h,\tau)} u_k$$

Lifted NCS model

- Exact discretisation

$$x_{k+1} = e^{Ah} x_k + \int_0^{h-\tau} e^{As} ds B \quad u_k + \int_{h-\tau}^{h} e^{As} ds B \quad u_{k-1}$$

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- In cloop with extended-state feedback $u_k = -K\xi_k = -\hat{K} x_k - K_u u_{k-1}$:

$$\xi_{k+1} = \underbrace{\begin{pmatrix} e^{Ah} - \int_0^{h-\tau} e^{Ah} ds B \hat{K} & \int_0^{h-\tau} e^{As} ds B - \int_0^{h-\tau} e^{As} ds B K_u \\ -\hat{K} & -K_u \end{pmatrix}}_{=:H(h,\tau)} \xi_k$$

- Exponentially satable iff $H(h,\tau)$ Schur, i.e. all eigenvalues within open unit circle

Varying delays and sampling intervals

- Finite number of values
- $h_k \in \{h^1, h^2, ..., h^M\}$
- $\tau_k \in \{\tau^1, \tau^2, ..., \tau^L\}$

Sampled-data system

Time-Varying delays and sampling intervals

- Previously: Constant delays and sampling intervals $\xi_{k+1} = \left(x_k^T u_{k-1}^T\right)^T$:

$$\xi_{k+1} = H(h,\tau)\xi_k \quad \text{with}$$

$$H(h,\tau) = \begin{pmatrix} e^{Ah} - \int_0^{h-\tau} e^{Ah} ds B\hat{K} & \int_0^{h-\tau} e^{As} ds B - \int_0^{h-\tau} e^{As} ds B K_u \\ -\hat{K} & -K_u \end{pmatrix}$$

Sampled-data system

Time-Varying delays and sampling intervals

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- Now: $h_k \in \left\{h^1, h^2, ..., h^M\right\}$ and $\tau_k \in \left\{\tau^1, \tau^2, ..., \tau^L\right\}$
- Discrete-time sampled-data dynamics in extended-state formulation

$$\xi_{k+1} = H(h_k, \tau_k)\xi_k$$

- Swithced linear systemns:
  - Depending on h_k and τ_k a different linear model is active!

Sampled-data system

Switched linear system: $\xi_{k+1} = H(h_k \tau_k)\xi_k$

with $h_k \in \{h^1, h^2, ..., h^M\}$ and $\tau_k \in \{\tau^1, \tau^2, ..., \tau^L\}$

- SLS exponentially stable if Lyapunov funciton $V(\xi) = \xi^T P \xi$ is found, i.e.
  - V is positive definite, i.e. $\xi^T P \xi > 0$ when $\xi \neq 0$ and
  - Decrease of V along trajectories (irrespective of h_k, τ_k)

$$V(\xi_{k+1}) < V(\xi_k) \quad \Leftrightarrow \quad \xi_{k+1}^T P \xi_{k+1} < \xi_k^T P \xi_k, \xi_k \neq 0$$

or

$$\xi_k^T \left[H(h_k, \tau_k)^T P H(h_k, \tau_k) - P \right] \xi_k < 0 \text{ for } \xi_k \neq 0$$

Sampled-data system

Switched linear system: $\xi_{k+1} = H(h_k \tau_k)\xi_k$

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- Gives rise to linear matrix inequalities (LMIs)

$$P \succ 0 \qquad \text{(positive definite)}$$

$$H(h, \tau)^T P H(h, \tau) - P \succ 0 \qquad h_k \in \{h^1, h^2, ..., h^M\}, \quad \tau_k \in \{\tau^1, \tau^2, ..., \tau^L\}$$

- "Appendix" LMIs

Sampled-data system

Switched linear system: $\xi_{k+1} = H(h_k \tau_k)\xi_k$

with $h_k \in \{h^1, h^2, ..., h^M\}$ and $\tau_k \in \{\tau^1, \tau^2, ..., \tau^L\}$

- SLS exponentially stable if

$$P \succ 0 \qquad \text{(positive definite)}$$

$$H(h,\tau)^T P H(h,\tau) - P \succ 0 \qquad h_k \in \{h^1, h^2, ..., h^M\}, \ \tau_k \in \{\tau^1, \tau^2, ..., \tau^L\}$$

- LMIs can be solved using efficient LMI solvers (e.g. in Matlab)
- LMIs based on a common quadratic Lyapunov function.
- Results using switched/multiple Lyapunov functions exist $V(x,h,\tau) = x^T P_{ij} x$, when $h = h^i, i = 1, ..., M$ and $\tau = \tau^j, j = 1, ..., L$
- See, e.g., [Daafouz et al, TAC, 2002]

From finite to infinite!

- Varying sampling intervals $h_k \in [h_{min}, h_{max}]$
- Varying delays $\tau_k \in [\tau_{min}, \tau_{max}]$

NCS with time-varying τ_k and h_k

Discrete-time LPV models

$$\xi_{k+1} = H(h_k \tau_k)\xi_k$$

with

$$H(h,\tau) = \begin{pmatrix} e^{Ah} - \int_0^{h-\tau} e^{Ah} ds B\hat{K} & \int_0^{h-\tau} e^{As} ds B - \int_0^{h-\tau} e^{As} ds B K_u \\ -\hat{K} & -K_u \end{pmatrix}$$

- Varying sampling intervals $h_k \in [h_{min}, h_{max}]$
- Varying delays $\tau_k \in [\tau_{min}, \tau_{max}]$
- Linear parameter-varying (LPV) system of the "nasty" kind

NCS with time-varying τ_k and h_k

Discrete-time LPV models

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- Varying sampling intervals $h_k \in [h_{min}, h_{max}]$
- Varying delays $\tau_k \in [\tau_{min}, \tau_{max}]$
- Linear parameter-varying (LPV) system of the "nasty" kind
- Example

$$A = \begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix} \quad \text{then} \quad e^{Ah} = \begin{pmatrix} e^{2h}\cos 3h & e^{2h}\sin 3h \\ -e^{3h}\sin 3h & e^{2h}\cos 3h \end{pmatrix}$$

- Parameters appear in a nonlinear / exponential manner!
- Uncommon in LPV literature!

Stability problem

Parameter-varying NCS model:

$$\xi_{k+1} = H(\tau_k)\xi_k$$

Stable NCS model for $\tau_k \in [\tau_{min}, \tau_{max}]$??

Stability problem

Parameter-varying NCS model:

$$\xi_{k+1} = H(\tau_k)\xi_k$$

Stable NCS model for $\tau_k \in [\tau_{min}, \tau_{max}]$??

Sufficient condition: Lyapunov function $V(\xi) = \xi^T P \xi$ with $P \succ 0$ and

$$H(\tau)^T P H(\tau) - P \prec 0 \quad \text{for all} \quad \tau \in [\tau_{min}, \tau_{max}] \qquad (*)$$

Stability problem

Parameter-varying NCS model:

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Stable NCS model for $\tau_k \in [\tau_{min}, \tau_{max}]$??

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$$H(\tau)^T P H(\tau) - P \prec 0 \quad \text{for all} \quad \tau \in [\tau_{min}, \tau_{max}] \qquad (*)$$

- PROBLEM: Infinite set of Linear Matrix Inequalities (LMIs)

  $\Rightarrow$ Recognized in the literature as a tough problem

  (see e.g. Hespanha et al, Proc. IEEE, 2007): "From a numerical perspective it is generally not simple to find a matrix P that satisfies $(*)$ for all values of τ in the given interval."

- How to arrive at a finite number of LMIs?

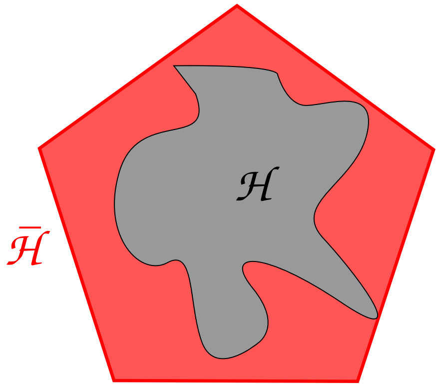
Polytopic overapproximations

Parameter-varying NCS model:

$$\xi_{k+1} = H(\tau_k)\xi_k$$

- Basic idea: embed the uncertainty matrix set $\{H(\tau) \mid \tau \in [\tau_{min}, \tau_{max}]\}$ in a polytopic matrix set with vertices $H_i, i = 1, ..., N$:

$$\mathcal{H} := \{H(\tau) \mid \tau \in [\tau_{min}, \tau_{max}]\} \subseteq \bar{\mathcal{H}} := \text{convex hull}\,\{H_1, ..., H_N\}$$



chose vertices such that
uncertainly set is embedded

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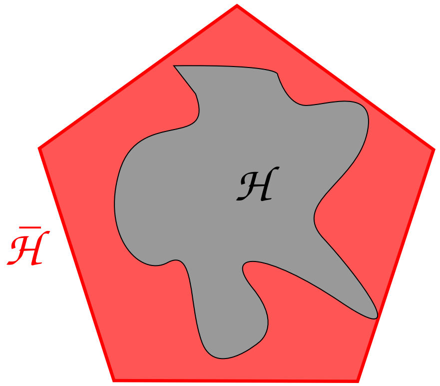
$$= \left\{ \sum_{i=1}^{N} \lambda_i H_i \mid \lambda = [\lambda_1 ... \lambda_N]^T \in \Lambda \right\}$$

with $\Lambda = \left\{ \lambda \in R^N | \lambda_i \geq 0, i = 1, ..., N, \sum_{i=1}^{N} \lambda_i = 1 \right\}$



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Polytopic overapproximations

Parameter-varying NCS model:

$$\xi_{k+1} = H(\tau_k)\xi_k$$

- Polytopic overapproximation:

$$\mathcal{H} := \ \{H(\tau) \mid \tau \in [\tau_{min}, \tau_{max}]\} \subseteq \overline{\mathcal{H}} := \text{convex hull}\, \{H_1, ..., H_N\}$$

- Polytopic model:

$$\xi_{k+1} = \left(\sum_{i=1}^{N} \lambda_{k,i} H_i \right) \xi_k, \quad \lambda_k \in \Lambda \qquad (**)$$

Polytopic overapproximations

Parameter-varying NCS model:

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- Polytopic overapproximation:

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Lemma: Equivalent:

- $V(\xi) = \xi^T P \xi$ a Lyapunov function for polytopic LPV system (**)
- Finite number of LMIs satisfied:

$$H_i^T P H_i - P \prec 0, \ i = 1, ..., N \text{ and } P \succ 0$$

Relaxation: Parameter-dependent LF $V(x_k, \lambda_k) = \sum_{i=1}^{N} \lambda_{k,i} x_k^T P_i x_k$ avaliable

[Daafouz & Bernussou, SCL 2001]

Polytopic overapproximations

Parameter-varying NCS model:

$$\xi_{k+1} = H(\tau_k)\xi_k$$

- Sometimes overapproximation contains uncertainty

$$\mathcal{H} \subseteq \overline{\mathcal{H}} = \left\{ \sum_{i=1}^{N} \lambda_i (H_i + B_i \Delta C_i) \mid \lambda = [\lambda_1 ... \lambda_N]^T \in \Lambda,\ \Delta \in \boldsymbol{\Delta} \right\}$$

- Polytopic model with uncertainty:

$$\xi_{k+1} = \left(\sum_{i=1}^{N} \lambda_{k,i} (H_i + B_i \Delta_k C_i) \right) \xi_k, \quad \lambda_k \in \Lambda \qquad \Delta_k \in \boldsymbol{\Delta}$$

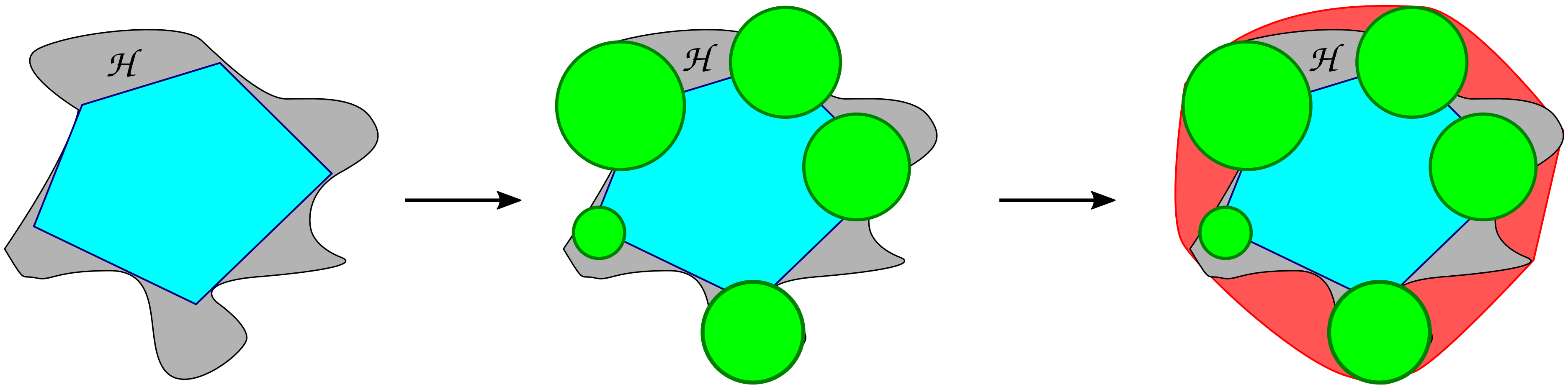
- LMI based stability conditions using the full-block S-procedure [Scherer 1999],see [Donkers et al, TAC, 2011]

Polytopic overapproximations

How to get them?

- In the literature many ways exist to compute the matrix exponential e^{As}, see `Nineteen Dubious Ways to Compute the Exponential of a Matrix, Twenty-Five Years Later, Moler et. al. 2003`

- There are several (dubious?) ways to overapproximate the uncertain 'matrix exponential'-related terms



Choose vertices in uncertainty set

Add norm-bounded uncertainty

Embedding of the uncertainty set

Overapproximation methods

- Methods for obtaining polytopic overapproximations based on
  - Interval matrices
    (Cloosterman, van de Wouw, Heemels, Nijmeijer, CDC 2006)
  - Real Jordan form
    (Cloosterman, van de Wouw, Heemels, Nijmeijer, CDC 2007, TAC 2009, Automatica 2010)
  - Cayley-Hamilton theorem
    (Gielen et al, Automatica 2010)
  - Taylor series
    (Hetel, Daafouz, Iung, TAC 2006)
  - Gridding (and norm bounding of approximation error)
    (Balluchi et al, HSCC 2005), (Fujioka, ACC 2008), (Suh, Automatica 2008), (Skaf, Boyd, TAC 2009), (Dritsas et al, IJC 2009)
  - Gridding with interpolation
    (Donkers et al, HSCC 2009), (Donkers et al, TAC 2011)

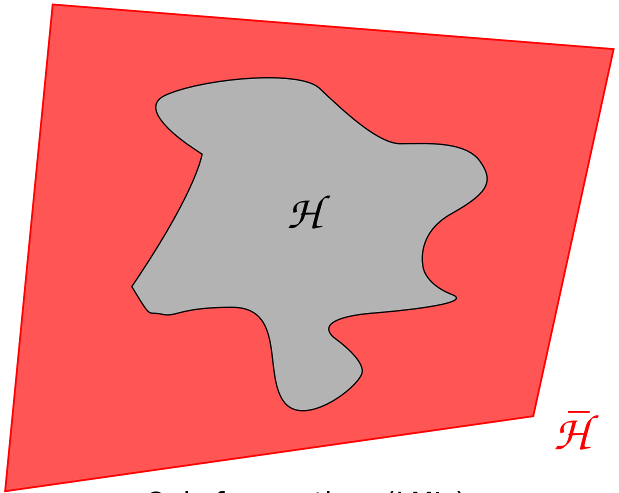
Overapproximation methods

How to get them?

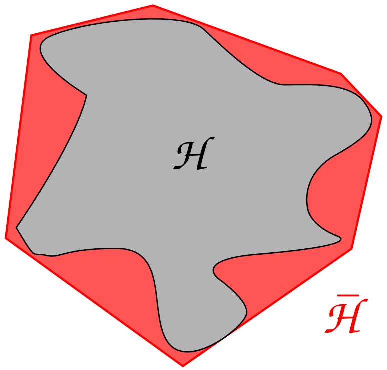
- Important criteria for methods of overapproximation:
  - Accuracy/conservatism:

    How 'tight' is the overapproximation?
  - Computational complexity:

    How many vertices do we need (number of LMIs to be solved)?



Only few vertices (LMIs),
but large polytopic set



Tight approximation,
but many vertices (LMIs)

- Comparison in `[Heemels et al, HSCC 2010]`

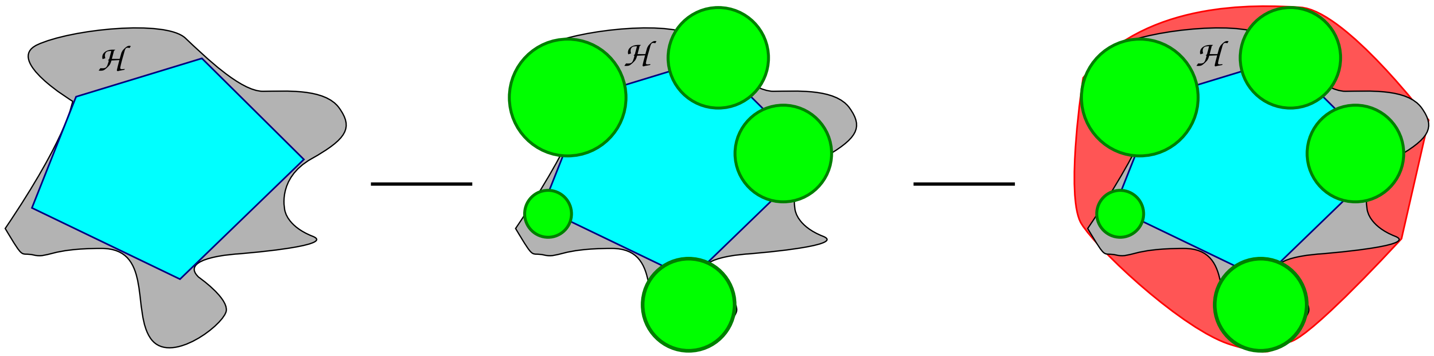
Overapproximation methods

Comparison

→ "Winner" Gridding with interpolation

(Donkers et al, HSCC 2009), (Donkers et al, TAC 2011)

$$\mathcal{H} = \{H(\tau) \mid \tau \in [\tau_{min}, \tau_{max}]\}$$



Choose vertices in uncertainty set — Add norm-bounded uncertainty — Embedding of the uncertainty set

- Overapproximation can be made arbitrarily tight (from a stability point of view), while having control on complexity through number of grid points
- [Heemels et al, HSCC 2010; Donkers et al, HSCC 2009; Donkers et al, TAC 2011]

Overapproximation methods

Remarks and Extensions

- Approach for stability analysis also applicable for the case with
  - Variations in the sampling interval $h_k \in [h_{min}, h_{max}]$
  - Large delays $\tau_k > h_k$
  - Packet dropouts (modelled as prolongations of delay)
- See [Cloosterman et al, TAC 2009, Automatica 2010]
- Other approaches towards modeling packet dropouts:
  - Modelled as prolongation of maximal sampling interval [Garcia-Rivera, Automatica 2007]
  - Modelled using separate (hybrid) model [van Schendel et al, ACC 2010]

Overapproximation methods

Remarks and Extensions

- Design extended state feedback $u_k = K\xi_k = K_x x_k + K_u u_{k-1}$

  [Cloosterman et al, TAC 2009]

- Design state feedback $u_k = [K \ 0]\xi_k = K_x x_k$

  [Cloosterman et al, Automatica 2010]

- Design decentralized output-feedback controllers

  [Bauer et al, ACC 2010 & submitted]

- Usage of parameter-dependent Lyapunov functions more general than discrete-time Lyapunov-Krasovskii methods

  [Cloosterman et al, Automatica 2010]

Summary so far

- Stability analysis problems and solutions for NCS
  - Constant delays/s.i.: Linear system
  - Varying but finite: Switched linear system
  - Varying infinite: LPV system
- Additional take-away observations:
  - Infinite values: polytopic approximations
- Next: include communication constraints / scheduling protocols based on

  `[Donkers et al, TAC 2011]`

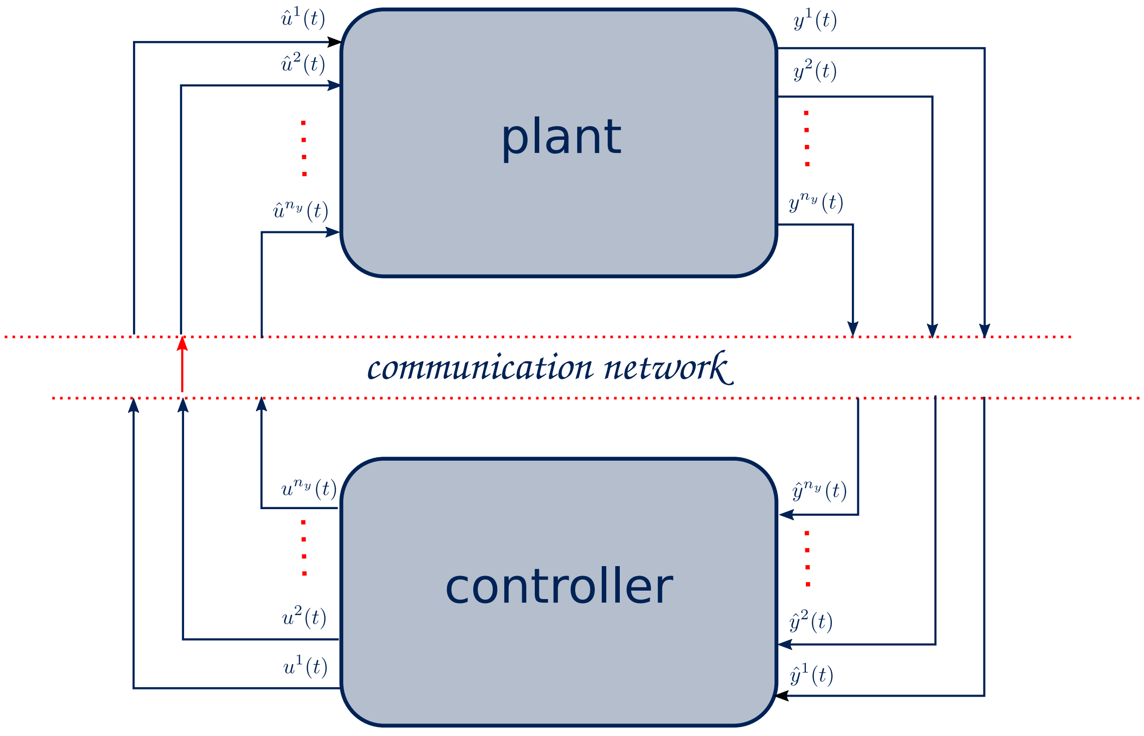
Communication Constraints & Protocols

- [Donkers, Heemels, vdWouw, Hetel, TAC 2011]

Including scheduling ...

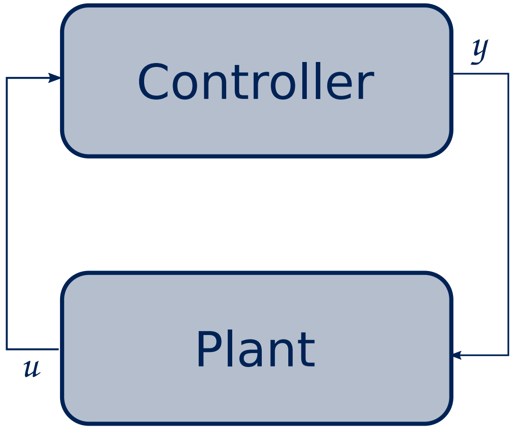
Communication constraints:

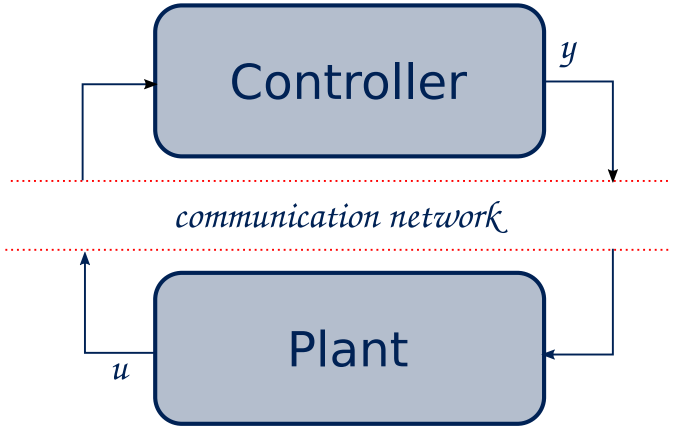
- Network is divided into sensor and actuator nodes
- Only one node can access the network simultaneously
- This gives rise to the problem of scheduling: protocols



Including scheduling ...

Networked control systems





- Communication constraints
- Varying transmission intervals
- Delays

For ease of exposition: only varying transmission intervals (no delays)

- [Donkers, Heemels, vdWouw, Hetel, TAC 2011]

Including scheduling ...

System Description

- Plant and controller are given by

$$\begin{cases} \dot{x}^P(t) = A^P x^P(t) + B^P \hat{u}(t), \quad \hat{u}(t) = \hat{u}_k := \hat{u}(t_k) \quad \forall t \in [t_k, t_{k+1}) \\ y(t) = C^P x^P(t) \end{cases}$$

$$\begin{cases} x^c_{k+1} = A^c x^c_k + B^c \hat{y}_k \\ u_k = C^c x^c_k + D^c \hat{y}_{k-1} \end{cases}$$

- Transmission/sampling times t_k and $y_k = y(t_k)$

- Transmission intervals $h_k := t_{k+1} - t_k$ in $[\underline{h}, \overline{h}]$

Including scheduling ...

System Description

- Plant and controller are given by

$$\begin{cases} \dot{x}^P(t) = A^P x^P(t) + B^P \hat{u}(t), \quad \hat{u}(t) = \hat{u}_k := \hat{u}(t_k) \quad \forall t \in [t_k, t_{k+1}) \\ y(t) = C^P x^P(t) \end{cases}$$

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- Data exchange is constrained, i.e., for $z = (y, u)$ and $\hat{z} = (\hat{y}, \hat{u})$

$$\hat{z}_k = \hat{z}(t_k^+) = \Gamma_{\sigma_k} z_k + (I - \Gamma_{\sigma_k})\hat{z}_{k-1}$$

Including scheduling ...

System Description

- Plant and controller are given by

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Example: Two sensors $z^1 = y^1$ and $z^2 = y^2$ use network.

Node 1 communicates at $t_k(\sigma_k = 1)$

$$\hat{z}_k = \hat{z}(t_k^+) = \begin{pmatrix} z_k^1 \\ \hat{z}_{k-1}^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} z_k + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \hat{z}_{k-1}$$
$$= \Gamma_{\sigma_k} z_k \qquad + (I - \Gamma_{\sigma_k}) \hat{z}_{k-1}$$

Including scheduling ...

Discrete-time NCS model

- Exact discretisation results in a SLPV system

$$\xi_{k+1} = H(\sigma_k, h_k)\xi_k \quad \text{with}$$

$$H(\sigma_k, h_k) = \begin{bmatrix} e^{A^P h_k} + E^p_{h_k} B^p D^c C^p & E^p_{h_k} B^p C^c & E^p_{h_k} B^p D^c & E^p_{h_k} B^p (I - \Gamma^u_{\sigma_k}) \\ B^c C^p & A^c & B^c (I - \Gamma^u_{\sigma_k}) & 0 \\ C^p (I - e^{A^P h_k} + E^p_{h_k} B^p D^c C^p) & -C^P E^p_{h_k} B^p C^c & I - \Gamma_{\sigma_k} - C^p E^p_{h_k} B^p D^c & -C^p E^p_{h_k} B^p (I - \Gamma^u_{\sigma_k}) \\ -C^c B^c C^p & C^c (I - A^c) & D^c \Gamma^u_{\sigma_k} - C^c B^c (I - \Gamma^u_{\sigma_k}) & I - \Gamma^u_{\sigma_k} \end{bmatrix}$$

where $E^p_{h_k} = \iint_0^{h_k} e^{A^p s} ds$ and $\xi = (x^p, x^c, e)$ with $e = \hat{z} - z$

  - Uncertainty: unknown time-varying transmission intervals
  - Switching: communication constraints: RR &TOD protocols

Protocols as Switching Functions

$$\begin{cases} x_{k+1}^c = A^c x_k^c + B^c \hat{y}_k \\ u_k = C^c x_k^c + D^c \hat{y}_{k-1} \end{cases}$$

- Periodic protocols:
  - Round-Robin protocol

$$\sigma_k = \begin{cases} 1, & \text{if } k = 0,2,4,6... \\ 2, & \text{if } k = 1,3,5... \end{cases}$$

- Quadratic protocols
  - Try-Once-Discard (TOD) or Maximun-Error-First (MEF) protocol:

$$\sigma_k = arg\ max\{||e_k^1||, ||e_k^2||\}$$

with $e_k^i = \hat{z}_{k-1}^i - z_k$ network-induced error for node i at time t_k

  - In general with $\xi_k = (x_k^p, x_k^c, e_k)$:

$$\sigma_k = arg\ min\{\xi_k^p Q_1 \xi_k, ..., \xi_k^p Q_l \xi_k\}$$

Stability analysis

Discrete-time NCS model

- Exact discretisation results in a SLPV system

$$\xi_{k+1} = H(\sigma_k, h_k)\xi_k \quad \text{with}$$

$$H(\sigma_k,h_k) = \begin{bmatrix} e^{A^P h_k} + E^p_{h_k} B^p D^c C^p & E^p_{h_k} B^p C^c & E^p_{h_k} B^p D^c & E^p_{h_k} B^p (I - \Gamma^u_{\sigma_k}) \\ B^c C^p & A^c & B^c(I - \Gamma^u_{\sigma_k}) & 0 \\ C^p(I - e^{A^P h_k} + E^p_{h_k} B^p D^c C^p) & -C^P E^p_{h_k} B^p C^c & I - \Gamma_{\sigma_k} - C^p E^p_{h_k} B^p D^c & -C^p E^p_{h_k} B^p (I - \Gamma^u_{\sigma_k}) \\ -C^c B^c C^p & C^c(I - A^c) & D^c \Gamma^u_{\sigma_k} - C^c B^c (I - \Gamma^u_{\sigma_k}) & I - \Gamma^u_{\sigma_k} \end{bmatrix}$$

where $E^p_{h_k} = \iint_0^{h_k} e^{A^p s} ds$ and $\xi = (x^p, x^c, e)$ with $e = \hat{z} - z$

- Uncertainty: unknown time-varying transmission intervals
- Switching: communication constraints: RR &TOD protocols

After some work (polytopic overapproximations & dealing with protocols):

- Arbitrarily tight LMI conditions for existence of parameter-dependent quadratic Lyapunvos functions.

Stability analysis

Illustrative Example (batch reactor without delays)

- Results on bounds on the transmission interval (given for TOD protocol only)

Mehtod	**Range**
Simulation based, obtained in [1]	$h_k \in (\varepsilon, 0.07]$
Theoretical, obtained in [1]	$h_k \in (\varepsilon, 10^{-5}]$
Theoretical, obtained in [2]	$h_k \in (\varepsilon, 0.01]$
Theoretical, obtained in [3]	$h_k \in (\varepsilon, 0.0108]$
Newly obtained theoretical bound	$h_k \in [10^{-4}, 0.066]$

[1] Walsh et al, Trans. CST '02

[2] Nešić & Teel, TAC '04

[3] Carnevale et al, TAC '07

Illustrative Example (batch reactor without delays)

Now: varying delays, varying transmission intervals & comm. constraints:

gárfica!

[17] [Heemels, Teel, vdWouw, Nešić, TAC 2010]

Extensions

grafica!

- Delays and transmission intervals as random variables
- Stochastic protocols driven by Markov models
- [Donkers, Heemels, Bernardini, Bemporad & Shneer, ACC 2010/Automatica 2012?]

Extensions

- Controller synthesis

  [Cloosterman, vdWouw, Heemels, Nijmeijer, TAC 2009]

  [Cloosterman, Hetel, vdWouw, Heemels, Daafouz, Nijmeijer, Controller Synthesis for Networked Control Systems, Automatica 2010]

  [Bauer, Donkers, Heemels, vdWouw, ACC 2010]

  [Bernardini, Donkers, Bemporad & Heemels, NECSYS 2010]

- Discrete-time approach for nonlinear systems

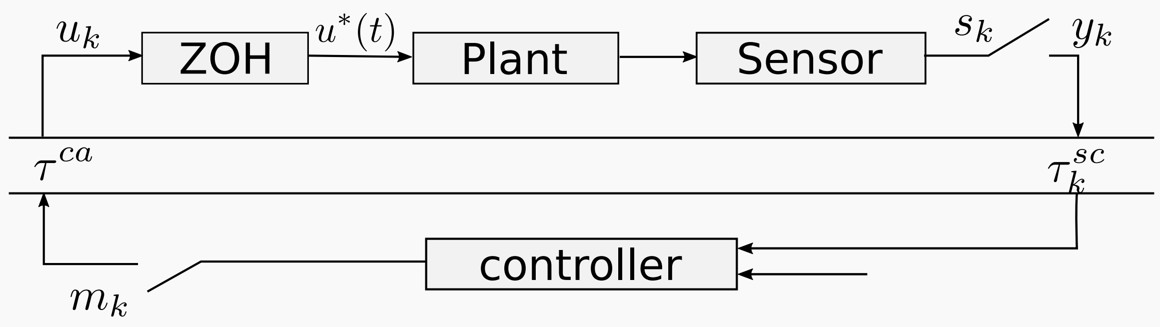
  [vdWouw, Nesic, Heemels, CDC 2010]

Summary so far

- Stability analysis problems and solutions for NCS
  - Constant delays/s.i.: Linear system
  - Varying but finite: Switched linear system
  - Varying infinite: LPV system
  - Incl. comm. constraints: Switched LPV system
- Additional take-away observations:
  - Infinite values: polytopic overapproximations
  - LMI-based anaylysis
  - Extensions involving stochastic models
  - Design control algorithms/protocols much to do!
- Next: dropouts...

Packet loss

Packet loss



Single-loop Networked Control System

Assumptions:

- Time-driven sensor (sampling times: $s_k = kh$)
- Event-driven controller
- Event-driven actuator
- Packet loss

$$m_k = \begin{cases} 1, & \text{no drop at } s_k \\ 2, & \text{drop at } s_k \end{cases}$$

- Sampling interval h
- ZOH= zero-order hold
- Value of output at kth sensor moment: $y_k := y(s_k)$
- u_k is control value computed on the basis of y_k
- $\tau_k^{sc} = \tau_k^{ca} = \tau_k^{c} = 0$

Packet loss

Modelling

- Continuous-time, sampled-data dynamics of the linear plant ("to zero"):

$$\dot{x}(t) = Ax(t) + Bu^*(t)$$

$$u^*(t) = \begin{cases} u_k = -\overline{K}x_k, & \text{when no drop: } m_k = 0 \\ 0, & \text{when drop: } m_k = 1 \end{cases}, \quad \text{for } t \in [s_k, s_{k+1}),$$

- To hold or to zero? `[Schenato, TAC 2009]`
- To hold:

$$u^*(t) = \begin{cases} u_k = -\overline{K}x_k, & \text{when no drop: } m_k = 0 \\ 0, & \text{when drop: } m_k = 1 \end{cases}, \quad \text{for } t \in [s_k, s_{k+1}),$$

- To hold: Use extended state and lifted models
- More advanced: Model-based predictions (e.g., `[Gommans, MSc Thesis 2011]`)

Packet loss

Modelling

- Continuous-time, sampled-data dynamics of the linear plant ("to zero"):

$$\dot{x}(t) = Ax(t) + Bu^*(t)$$

$$u^*(t) = \begin{cases} u_k = -\overline{K}x_k, & \text{when no drop: } m_k = 0 \\ 0, & \text{when drop: } m_k = 1 \end{cases}, \quad \text{for } t \in [s_k, s_{k+1}),$$

- Extract integration with sampling period h and $x_k = x(s_k)$ yields:

$$x_{k+1} = \begin{cases} \underbrace{[e^{Ah} - \iint_0^h e^{As} B\overline{K} ds]}_{=:A_0} x_k & \text{when no drop} \quad m_k = 0 \\ \underbrace{e^{Ah}}_{=:A_1} x_k & \text{when drop} \quad m_k = 1 \end{cases}$$

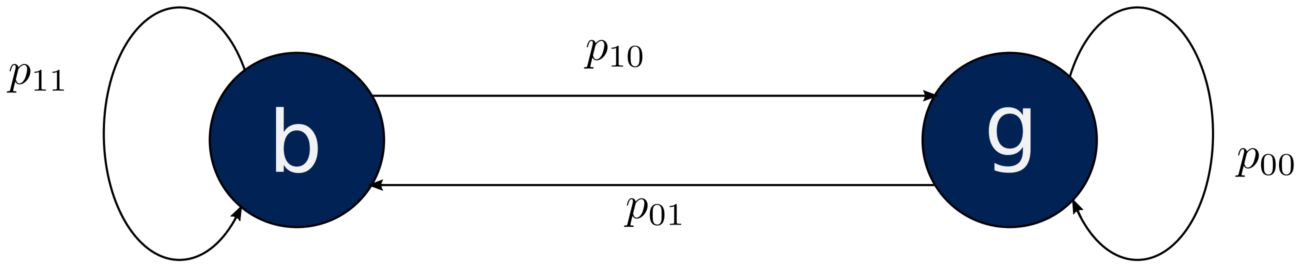
- Compact model: $x_{k+1} = A_{m_k} x_k$

Packet loss

Modelling

Three cases for dropout models:

- Maximun number of subsequent dropouts: δ
- Bernoulli models: Probability $\mathbb{P}(m_k = 1) = p$ of dropout with $0 \leq p \leq 1$.
- Gilbert-Elliot models: Probability of dropout based on status network
  - Probabilities $\mathbb{P}(m_k = j | m_{k-1} = i) = p_{ij}, \ i = 0, 1$
  - $m_{k-1} = 0$ is good (no drop), $m_{k-1} = 1$ is bad (drop)



- $p_{00} + p_{01}$ and $p_{10} + p_{11} = 1$

Packet loss: Upperbound drops

Modelling

- Model dropouts as prolongations of sampling interval
- Time-varying sampling intervals
- $h_1 \in \{h, 2h, 3h, ..., (\delta+1)h\}$
- $\widetilde{s}_{l+1} = \widetilde{s}_l + \widetilde{h}_l,\ l \in \mathbb{N}$ and $x_l := x(\widetilde{s}_{l+1})$
- Discrete-time model as in case of time-varying sampling intervals:

$$x_{l+1} = e^{Ah_l} x_l + \int_0^{h_l} e^{As} ds B u_l =: F(h_l) x_l + G(h_l) x_l$$

$$= [F(h_l) - G(h_l)\overline{K}] x_l$$

- Switched linear system
- Lyapunov-based analysis: LMIs as before
- Alternative modelling and analysis: see `[Van Schendel et al, ACC 2010]`

Bernoulli/Gilbert-Elliot

Packet loss modelling in NCS: Markovian Jump Linear Systems (JLS)

compact model: $x_{k+1} = A_{m_k} x_k$

- Bernoulli models: $\mathbb{P}(m_k = 1) = p$ (special case of Gilbert-Elliot)
- Gilbert-Elliot models: $\mathbb{P}(m_k = j | m_{k-1} = i) = p_{ij}, \; i = 0, 1$

Bernoulli/Gilbert-Elliot

Packet loss modelling in NCS: Markovian Jump Linear Systems (JLS)

compact model: $x_{k+1} = A_{m_k} x_k$

- Bernoulli models: $\mathbb{P}(m_k = 1) = p$ (special case of Gilbert-Elliot)
- Gilbert-Elliot models: $\mathbb{P}(m_k = j | m_{k-1} = i) = p_{ij},\ i = 0, 1$

Stochastic stability concepts:

- Mean square stability (MSS) it for all x_0 and all $m_{-1} : \lim_{k\to\infty} \mathbb{E}[||x_k||^2] = 0$
- Uniformly exponentially mean square stable (UEMSS) if there exist $c \geq 0$ and $0 \leq \rho \leq 1$ s.t. for all x_0 and all $m_{-1} : \mathbb{E}[||x_k||^2] \leq c\rho^k ||x_0||^2$
- Almost surely stable (ASS) if for all x_0 and $m_{-1} : \mathbb{P}[\lim_{k\to\infty} ||x_k|| = 0] = 1$

MSS $\Leftrightarrow$ UEMSS $\Rightarrow$ ASS

[Seiler and Sengupta, aCC 2001] & [Y. Ji, H. J. Chizeck, X. Feng, and K. Loparo, Contr.-Theory and Techn. J., 1991]

Packet loss: Bernoulli model

Modelling

- Extract integration with sampling period h and $x_k = x(s_k)$ yields:

$$x_{k+1} = A_{m_k} x_k = \begin{cases} \underbrace{[e^{Ah} - \iint_0^h e^{As} B\overline{K} ds]}_{=:A_0} x_k & \text{no drop: Probability } 1-p \\ \underbrace{e^{Ah}}_{=:A_1} x_k & \text{drop: Probability } p \end{cases}$$

- MSS: $V(x) = x^T P x$ satisfies $\mathbb{E}V(x_{k+1}) < V(x_k)$ for all $x_k \neq 0$:

$$(1-p)A_0^T P A_0 + p A_1^T P A_1 - P \prec 0 \quad \text{and} \quad P \succ 0$$

[Costa O. L. V., Fragoso M. D., "Stability Results for Discrete-TIme Linear Systems with Markovian Jumping Parameters", Journal of Mathematical Analysis and Applications, 179, p. 154-178, 1993]

Packet loss: Bernoulli model

Example

$$\dot{x}(t) = x(t) + u^*(t)$$

- Sampling interval: $h = ln2$
- Exact discretization: $x_{k+1} = 2x_k + u_k^*$ with $u_k^* = u^*(s_k)$
- Take $u_k = -1\frac{1}{2}x_k$ (in case of no drop)
- Consequently, in case of "to zero"

$$x_{k+1} = \begin{cases} 2x_k, & \text{when drop (probability } p\text{)} \\ \frac{1}{2}x_k, & \text{when no drop (probability } 1-p\text{)} \end{cases}$$

- $V(x) = x^2$ yields

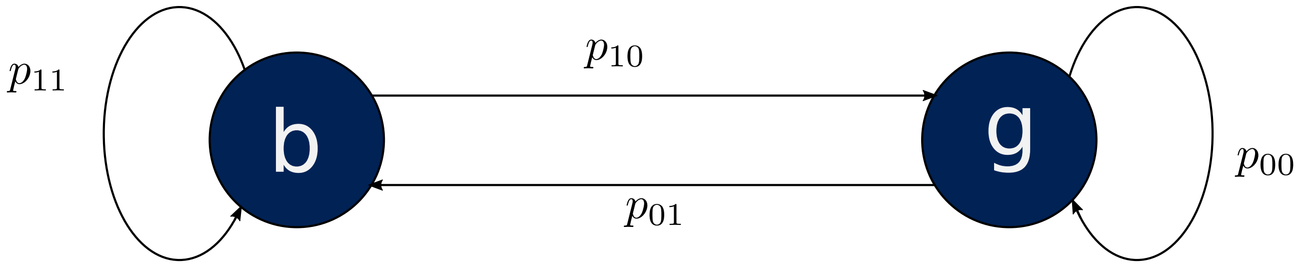
$$4p + (1-p)\frac{1}{4} - 1 < 0$$

- MSS: $0 \leq p < \frac{1}{5}$. Hence, max. 20% loss is OK

Example: when $u_k = -2x_k$ (in case of no drop). MSS for $0 \leq p < \frac{1}{4}$, but always ASS ($0 \leq p < 1$)

Packet loss: Gilbert-Elliot model

Modelling



$$x_{k+1} = A_{m_k} x_k \text{ with } \mathbb{P}(m_k = j | m_{k-1} = i) = p_{ij}$$

- MSS if and only if there are $P_i \succ 0, i = 0, 1$, such that

$$m_{k-1} = 0 : \quad p_{00} A_0^T P_0 A_0 + p_{01} A_1^T P_1 A_1 \prec P_0$$

$$m_{k-1} = 1 : \quad p_{10} A_0^T P_0 A_0 + p_{11} A_1^T P_1 A_1 \prec P_1$$

- Again LMI-based necessary and sufficient conditions
- Alternative LMI conditions avaliable (handouts)

[Seiler and Sengupta, ACC 2001] & [Y. Ji et al, 1991]

Summary Lec. I & II

- Stability analysis problems and solutions for NCS
  - Constant delays/s.i.: Linear system
  - Varying but finite: Switched linear system
  - Varying infinite: LPV system
  - Incl. comm. constraints: Switched LPV system
  - Dropouts: Switched or jump linear system
- Additional take-away observations:
  - Infinite values: polytopic overapproximations
  - Dropouts via prolongation sampling interval
- Several directions for further exploarion, a.o.,
  - Etochastic communication models & protocols
  - Design: Control, Protocol & Both

Outline Lectures I-V

Main focus: Stability of NCS under network-induced imperfections

- [Lec. I]: Simple NCS analysis problems and solutions (varying delays, sampling intervals)

  Discrete-time modeling framework

- [Lec. II]: NCS under communication constraints (incl. I) & dropouts

  Discrete-time modeling framework

- [Lec. III]: "Guided problem solving"

- [Lec. IV]: NCS under communication constraints

  Continuous-time modeling framework (emulation)

- [Lec. V]: "Guided problem solving"

Suggested reading material

M.C.F. Donkers, W.P.M.H. Heemels, N. van de Wouw and L.L. Hetel, Stability Analysis of Networked Control Systems Using a Switched Linear Systems Approach, IEEE Transactions on Automatic Control, 2011.

M.C.F. Donkers, W.P.M.H. Heemels, D. Bernardini, A. Bemporad and V. Shneer, Stability Analysis of Stochastic Networked Control Systems, American Control Conference 2010, Baltimore, USA, p. 3684-3689, Extended verison provisionally accepted for Automatica

W.P.M.H. Heemels, N. van de Wouw, R.H. Gielen, M.C.F. Donkers, L. Hetel, S. Olaru, M. Lazar, J Daafouz and S.I- Niculescu, Comparison of Overapproximation Methods for Stability Analysis of Networked Control Systems, Hybrid Systems: Computation and Control 2010, Stockholm, Sweden, p- 181-191.

M.B.G., Cloosterman, N. van de Wouw, W.P.M.H Heemels, H. Nijmeijer, "Stability of Networked Control Systems with Uncertain Time-varying Delays", IEEE Transactions on Automatic Control, 54(7), pp. 1575 -1580, 2009.

M.B.G. Cloosterman, L.L. Hetel, N. van de Wouw, W.P.M.H. Heemels, J. Daafouz, H. Nijmeijer, "Controller Synthesis for Networked Control Systems", Automatica, Volume 46, p. 1584-1594, 2010

R. Gielen, S. Olaru, M. Lazar, W.P.M.H. Heemels, N. van de Wouw, S. Niculescu, "On Polytopic inclusions as a Modeling Framework for Systems with Time-Varying Delays", Automatica, 46(3), 615-619, 2010.

D. Bernardini, M.C.F. Donkers, A. Bemporad and W.P.M.H. Heemels, "A Model Predictive Control Approach for Stochastic Networked Control Systems, 2nd IFAC Workshop on Distributed Estimation and Control in Networked Systems", Annecy, France (2010), p. 7-12.

N. van de Wouw, D. Nesic and W.P.M.H. Heemels, Stability Analysis for Nonlinear Networked Control Systems: A discrete-time Approach, 49th IEEE Conference on Decision and Control 2010, p. 7557-7563.

Matrices and inequalities

- A symmetric matrix $P \in \mathbb{R}^{nxn}$ is positive definite, if $x^T P x > 0$ for all $x \in \mathbb{R}^n$ with $x \neq 0$ .We write $P \succ 0$
- A symmetric matrix $P \in \mathbb{R}^{nxn}$ is positive semi-definite, if $x^T P x \geq 0$ for all $x \in \mathbb{R}^n$ We write $P \succeq 0$
- A symmetric matrix $P \in \mathbb{R}^{nxn}$ is negative definite, if $x^T P x < 0$ for all $x \in \mathbb{R}^n$ with $x \neq 0$ .We write $P \prec 0$
- A symmetric matrix $P \in \mathbb{R}^{nxn}$ is negative semi-definite, if $x^T P x \leq 0$ for all $x \in \mathbb{R}^n$ We write $P \preceq 0$

Matrices and inequalities

Characterisations in terms of eigenvalues and determinats of leading principal submatrices!

Equivalent (using symmetry of P):

- P positive definite ($x^T P x > 0$ for all $x \neq 0$)
- all eigenvalues are positive
- all leading principal minors $det P_{JJ} > 0$ for all $J = \{1, ..., j\}$ for $j = 1, ..., n.$

Equivalent (using symmetry of P):

- P positive semi-definite ($x^T P x \geq 0$ for all x)
- all eigenvalues are positive or 0
- all leading principal minors $det P_{JJ} \geq 0$ for all $J = \{1, ..., j\}$ for $j = 1, ..., n.$

Question: Why symmetry without loss of generality?

Matrices and inequalities

If P postive definite, then

- P is invertible (non-singular) Question: Why?
- P^{-1} is positive definite. Question: Why?

$$\lambda_{min}(P)||x||^2 \leq x^T P x \leq \lambda_{max}(P)||x||^2$$

where $||x||^2 = x^T x$ and $\lambda_{min}(P), \lambda_{max}(P)$ denote the smallest and largest eigenvalue of P

Partial ordering on matrices:

- $P \succ Q$ means that $P - Q \succ 0$ $(P - Q$ is a positive definite matrix).

"Appendix: LMIs"

Linear matrix inequalities (LMIs)

- Given the ordering induced by "positive definiteness" we can formulate inequalities in terms of matrices
- Example $V(x) = x`Px$ Lyap. function for $x_{k+1} = Ax_k$ yields

$$P \succ Q \quad \text{and} \quad A'PA - P \prec 0$$

- linear MI as the matrices we solve for appear linearly (no P^2, P_1AP_2)
- Important property of LMIs:
  - There are efficient numerical algorithms to solve LMIs
  - They can be used for many analysis and synthesis problems for linear, switched linear and piecewise linear systems
- Example linear systems: linear H_∞ -control can be solved via LMIs