E2-212 MATRIX THEORY: ASSIGNMENT 9

Question 1. *Let* $\mathbf{M}_{n\times n}$ *be a matrix which when multiplied with a vector* $\mathbf{x}_{n\times 1}$ *produces zeros on components* $[k+1:n]$*. Further, let* $x_k \neq 0$ *and* $\mathbf{e}_k$ *be the* k*-th column of* $\mathbf{I}_n$*.* *(6 points)*

(a) Write down the elements of the $\mathbf{M}$ *in terms of the elements of* $\mathbf{x}$*.*
(b) Verify that $\mathbf{M}$ *can be written as* $\mathbf{I}_n - \mathbf{t}\mathbf{e}_k^T$*. What are the elements of* $\mathbf{t}$*?*
(c) Obtain an expression for $\mathbf{M}^{-1}$*.*

$$\left(\text{Hint : Verify that } (\mathbf{I}_n + \mathbf{u}\mathbf{v}^T)^{-1} = \mathbf{I}_n - \frac{\mathbf{u}\mathbf{v}^T}{1+\mathbf{v}^T\mathbf{u}},\ 1+\mathbf{v}^T\mathbf{u} \neq 0\right)$$

Question 2. *Let* $\mathbf{A} \in \mathbb{R}^{n\times n}$ *be a normal matrix and* $\mathbf{B} \in \mathbb{R}^{n\times n}$ *be a nilpotent matrix. If* $\mathbf{A}+\mathbf{B} = \mathbf{I}_n$*, then show that* $\mathbf{A} = \mathbf{I}_n$*.* *(4 points)*