Computational Methods of Optimization
Third Midterm(30th Jan, 2021)

Time: 60 minutes

Instructions

- Answer all questions
- See upload instructions in the form

Question:	1	2	3	Total
Points:	10	10	10	30
Score:				

1. (10 points) Mark the correct choices. All questions carry equal marks

   (a) For $x_1, x_2 \geq 0$

   $$x_1 + x_2 - x_3 - 3x_4 + 6x_5 = 2, x_2 + x_3 + 5x_4 + 6x_5 = 2$$

   The solution $x_1 = x_3 = x_4 = 0, x_2 = 8, x_5 = -1$ is A. Basic Solution B. Basic Feasible solution C. Not a Basic solution D. a feasible solution

   (b) The optimum of an LP occurs at $A = [1, 0, 0, 2]^\top$, and $B = [0, 1, 0, 3]^\top$. The optimum also occurs at A. $[3, 0, 0, 3]^\top$ B. $[\frac{1}{2}, \frac{1}{2}, 0, \frac{5}{2}]^\top$ C. $[1, 2, 0, 3]^\top$ D. none of the above E. a feasible solution

   (c) The following LP has

   $$min_{x_1,x_2} - 2x_1 + 6x_2, \text{ s.t. } x_1 \geq x_2, -x_1 + 3x_2 \geq 5, x_1, x_2 \geq 0$$

   A. unique solution B. unbounded solution C. Non-unique solution

   (d) Consider the LP

   $$max_{x_1,x_2} 5x_1 + 2x_2 \text{ s.t. } x_1 + x_2 \leq 3, 2x_1 + 3x_2 \geq 5, x_1, x_2 \geq 0$$

   Which of the following primal- dual solutions are optimal A. $x_1 = 3, x_2 = 1; y_1 = 4, y_2 = 1$ B. $x_1 = 4, x_2 = 1, y_1 = 1, y_2 = 0$ C. $x_1 = 3, x_2 = 0, y_1 = 5, y_2 = 0$ D. none of the above

   (e) The optimal value of the dual of the following problem is

   $$max_{x_1,x_2,x_3} 2x_1 + x_2 + 3x_3, \text{ s.t. } x_1 - x_2 + x_3 \geq 5, x_1, x_2, x_3 \geq 0$$

   A. -1 B. -3 C. none of the above D. -5

2. Consider the following problem

$$min_{\mathbf{x}} \frac{1}{2} \|\mathbf{x}\|^2 \quad \text{subject to } a_i^\top \mathbf{x} = b_i$$

$a_i \in \mathbb{R}^d, b_i \in \mathbb{R}, i = \{1, \ldots, m\}, d > m$. Suppose $a_i^\top a_j = 0, i \neq j$.

   (a) (5 points) Show that one iteration of Gradient Projection Algorithm for this problem can be stated as

   $$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - \alpha R \mathbf{x}^{(k)}$$

   where R is a $d \times d$ matrix.

   (b) (3 points) Find R

(c) (2 points) For a constant stepsize find the number of iterations in which the algorithm converges. Justify.

3. Consider the application of Active set strategy to the problem

$$min_{\mathbf{x}} f(\mathbf{x}) \left(\equiv \frac{1}{2}\mathbf{x}^\top Q\mathbf{x} + h^\top \mathbf{x}\right)$$

subject to $a_i^\top \mathbf{x} \geq b_i,\ i = \{1, \ldots, m\}$.

(a) (5 points) Let $\mathbf{x}^{(k)}, W_k$ be a feasible point and $W_k = \{1, 2, 3\}$ be the working set. The following information is available

$$\nabla f(\mathbf{x}^{(k)}) = [1, -1, 1]^\top, Q = Diag[1, 2, 3], a_1 = [1, 0, 1]^\top, a_2 = [0, 1, 0]^\top, a_3 = [1, 0, -1]^\top$$

Determine if $\mathbf{x}^{(k)}$ is the minimum of $f(\mathbf{x})$ over active constraints defined over W_k.

(b) (5 points) In case $\mathbf{x}^{(k)}$ is not optimal find a point which is optimal over W_k. In case $\mathbf{x}^{(k)}$ is optimal check if it is globally optimal. If it is not globally optimal find a feasible descent direction at $\mathbf{x}^{(k)}$.