

E1 222 Stochastic Models and Applications

Problem Sheet 2-1

1. State whether the following sequences of sets are monotone.
 - (i). $A_k = [0, 1 + \frac{(-1)^k}{k}]$, $k = 1, 2, \dots$
 - (ii). $A_k = [1/k, 1]$, $k = 1, 2, \dots$
2. Let $A_k = (-1/k, 1]$, $k = 1, 2, \dots$. Let $B = \cap_{k=1}^{\infty} A_k$. For any $x < 0$, show that there is a K such that $x \notin A_K$. For any x , such that $0 < x < 1$, show that $x \in A_k$, $\forall k$. Now determine what B is.
3. Let $A_k = [1/k, 1]$, $k = 1, 2, \dots$. Let $B = \cup_{k=1}^{\infty} A_k$. For any $0 < x < 1$, show that there is a K such that $x \in A_K$. Now determine what B is.
4. Let (Ω, \mathcal{F}, P) be a probability space and let $A_1, A_2 \in \mathcal{F}$. Consider the following random variable:

$$\begin{aligned} X(\omega) &= -1 && \text{if } \omega \in A_1 \\ &= +1 && \text{if } \omega \in A_1^c A_2 \\ &= 0 && \text{if } \omega \in A_1^c A_2^c \end{aligned}$$

What is the event $[X < 0.5]$? Find the distribution function of X .

5. Consider the probability space with $\Omega = [0, 1]$ and the usual probability assignment (where probability of an interval is the length of the interval). Define X by $X(\omega) = 2\omega$ if $0 \leq \omega \leq 0.5$, and $X(\omega) = 2\omega - 0.5$ if $0.5 < \omega \leq 1$. What is the event $[X \in (0.5, 0.75)]$? Find the distribution function of X .
6. Let X be a random variable with $P[X = a] = 0$. Express $P[|X| \geq a]$ in terms of the distribution function of X .