E2 212 MATRIX THEORY: ASSIGNMENT 1

Question 1. For $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{B} \in \mathbb{R}^{n \times p}$, prove the following:

- (a) $\operatorname{rank}(\mathbf{AB}) \leq \min\{\operatorname{rank}(\mathbf{A}), \operatorname{rank}(\mathbf{B})\}.$ (b) $\operatorname{rank}(\mathbf{AA}^T) = \operatorname{rank}(\mathbf{A}) = \operatorname{rank}(\mathbf{A}^T\mathbf{A}).$