Computational Methods of Optimization Third Midterm(30th Jan, 2021)

Time: 60 minutes

Instructions

- Answer all questions
- $\bullet\,$ See upload instructions in the form

Question:	1	2	3	Total
Points:	10	10	10	30
Score:				

- 1. (10 points) Mark the correct choices. All questions carry equal marks
 - (a) For $x_1, x_2 \ge 0$

$$x_1 + x_2 - x_3 - 3x_4 + 6x_5 = 2, x_2 + x_3 + 5x_4 + 6x_5 = 2$$

The solution $x_1 = x_3 = x_4 = 0, x_2 = 8, x_5 = -1$ is A. Basic Solution B. Basic Feasible solution C. Not a Basic solution D. a feasible solution

- (b) The optimum of an LP occurs at $A = [1,0,0,2]^{\top}$, and $B = [0,1,0,3]^{\top}$. The optimum also occurs at A. $[3,0,0,3]^{\top}$ B. $[\frac{1}{2},\frac{1}{2},0,\frac{5}{2}]^{\top}$ C. $[1,2,0,3]^{\top}$ D. none of the above E. a feasible solution
- (c) The following LP has

$$min_{x_1,x_2} - 2x_1 + 6x_2$$
, s.t. $x_1 \ge x_2, -x_1 + 3x_2 \ge 5, x_1, x_2 \ge 0$

A. unique solution B. unbounded solution C. Non-unique solution

(d) Consider the LP

$$max_{x_1,x_2}5x_1 + 2x_2$$
 s.t. $x_1 + x_2 \le 3, 2x_1 + 3x_2 \ge 5, x_1, x_2 \ge 0$

Which of the following primal- dual solutions are optimal A. $x_1 = 3, x_2 = 1; y_1 = 4, y_2 = 1$ B. $x_1 = 4, x_2 = 1, y_1 = 1, y_2 = 0$ C. $x_1 = 3, x_2 = 0, y_1 = 5, y_2 = 0$ D. none of the above

(e) The optimal value of the dual of the following problem is

$$\max_{x_1,x_2,x_3} 2x_1 + x_2 + 3x_3$$
, s.t. $x_1 - x_2 + x_3 \ge 5, x_1, x_2, x_3 \ge 0$

A. -1 B. -3 C. none of the above D. -5

2. Consider the following problem

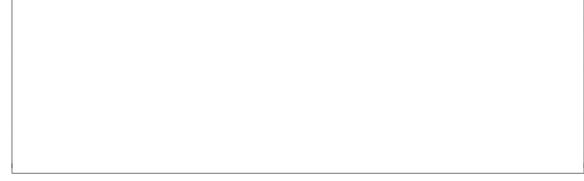
$$min_{\mathbf{x}} \frac{1}{2} \|\mathbf{x}\|^2$$
 subject to $a_i^{\top} \mathbf{x} = b_i$

 $a_i \in \mathbb{R}^d, b_i \in \mathbb{R}, i = \{1, \dots, m\}, d > m.$ Suppose $a_i^{\top} a_j = 0, i \neq j.$

(a) (5 points) Show that one iteration of Gradient Projection Algorithm for this problem can be stated

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - \alpha R \mathbf{x}^{(k)}$$

where R is a $d \times d$ matrix.



(b) (3 points) Find R

(c)	(2 points) For a constant stepsize find the number of iterations in which the algorithm converges. Justify.
Con	nsider the application of Active set strategy to the problem
	$min_{\mathbf{x}}f(\mathbf{x})\left(\equiv rac{1}{2}\mathbf{x}^ op Q\mathbf{x} + h^ op \mathbf{x} ight)$
	$\text{ ject to } a_i^\top \mathbf{x} \geq b_i, \ i = \{1, \dots, m\}.$
(a)	(5 points) Let $\mathbf{x}^{(k)}, W_k$ be a feasible point and $W_k = \{1, 2, 3\}$ be the working set. The following information is available
	$\nabla f(\mathbf{x}^{(k)}) = [1, -1, 1]^{\top}, Q = Diag[1, 2, 3], a_1 = [1, 0, 1]^{\top}, a_2 = [0, 1, 0]^{\top}, a_3 = [1, 0, -1]^{\top}$
	Determine if $\mathbf{x}^{(k)}$ is the minimum of $f(\mathbf{x})$ over active constraints defined over W_k .

3.