

## E2-212 MATRIX THEORY: ASSIGNMENT 8

**Question 1.** Let  $\mathbf{A} \in \mathbb{C}^{n \times n}$  be given. Define  $K_\lambda \triangleq \{\mathbf{x} \in \mathbb{C}^n \mid (\mathbf{A} - \lambda \mathbf{I}_n)^p \mathbf{x} = \mathbf{0} \text{ for some } p \in \mathbb{Z}^+\}$ . If  $\lambda$  is an eigenvalue of  $\mathbf{A}$  with algebraic multiplicity  $a_\lambda$ , then prove: (5 points)

- (a)  $K_\lambda$  is an  $\mathbf{A}$ -invariant subspace of  $\mathbb{C}^n$ .
- (b)  $\dim(K_\lambda) \leq a_\lambda$ .

**Question 2.** Let  $\mathbf{B} \in \mathbb{C}^{n \times n}$ . Prove or disprove: (5 points)

- (a) If  $\mathbf{B}$  is normal, then  $\mathcal{R}(\mathbf{B}) \perp \mathcal{N}(\mathbf{B})$ .
- (b) If  $\mathcal{R}(\mathbf{B}) \perp \mathcal{N}(\mathbf{B})$ , then  $\mathbf{B}$  is normal.