E2-212 MATRIX THEORY: ASSIGNMENT 7

Question 1. *Let* $\mathbf{A} \in \mathbb{C}^{n \times n}$ *be a Hermitian matrix and let* $\text{rank}(\mathbf{A}) = 1$. *Derive an expression for the eigenvalue decomposition of* $\mathbf{A}$. *(3 points)*

Question 2. *Let* $\mathbf{B} \in \mathbb{C}^{n \times n}$ *be such that* $\mathbf{B} = \mathbf{B}^H$, $\mathbf{x}_0 \in \mathbb{C}^n$ *be an arbitrary vector, and* $r \in \mathbb{R}$ *be such that* $\mathbf{C} = \mathbf{B} - r\mathbf{I}_n$ *is invertible. A variant of the power method is described as follows:*

$$\begin{aligned} &\text{For } i = 1, 2, 3, \ldots, \\ &\text{Do} : \mathbf{z}_i = (\mathbf{B} - r\mathbf{I}_n)^{-1}\mathbf{x}_{i-1} \\ &\mathbf{x}_i = \frac{\mathbf{z}_i}{\|\mathbf{z}_i\|_2} \\ &\sigma_i = \mathbf{x}_i^H \mathbf{B} \mathbf{x}_i \end{aligned}$$

(a) As $i \to \infty$, *what do* $\mathbf{x}_i$ *and* σ_i *converge to? (5 points)*
(b) Explain what happens to $\mathbf{x}_\infty$ *and* σ_∞ *when* r *is varied in* $(-\infty, \infty)$. *(2 points)*