

## E1 222 Stochastic Models and Applications

### Problem Sheet 2–3

1. Let  $X$  be a random variable uniformly distributed over  $\{0, 1, \dots, N\}$ . Find  $E[X]$ .
2. A darts board consists of concentric circles with radius  $\frac{k}{n}$ ,  $k = 1, 2, \dots, n$ . Thus there are  $n$  annular regions. A dart is thrown randomly. If it hits the  $k^{th}$  annular region we get  $1/(2k - 1)$  rupees. What is the expected amount one gets if a dart is thrown randomly.
3. Let  $X$  an exponential rv. Find  $EX^3$ .
4. Let  $X$  be a rv with density function

$$f(x) = cx(1 - x), \quad \text{if } 0 \leq x \leq 1.$$

( $f(x)$  is zero for all other values of  $x$ ). Let  $Y = 2X^3 - 3X^2 + 3X + 5$ . Find the value of  $c$  and  $E[Y]$ .

5. The price of some commodity is Rs. 2 per gram this week. Next week the price would be either Rs.1 per gm or Rs. 4 per gram, each with probability 0.5. You have a capital of Rs.1000. What would be your strategy if (i) you want to maximize expected amount of money with you (next week), (ii) you want to maximize the expected quantity of the commodity with you.
6. Children from a school went to a picnic in four buses. Different buses carried different number of students. Define two random variables,  $X, Y$ , as follows. We select one of the four drivers at random and  $X$  is the number of students in the bus driven by that driver. We select a student at random and  $Y$  is the number of students in the bus in which the selected student travelled. Can you say whether  $EX > EY$  or  $EY > EX$  (or the information given is not sufficient to decide which of  $EX, EY$  is greater)?