E0 230 : Computational Methods for Optimization
Tutorial 4

1. Suppose f is a stritly convex function, and C is a compact convex set. What can you say about x^*, where
$$x^* = \arg\max_C f(x)?$$
What if $f(\cdot)$ is only convex, and not strictly convex?

2. Consider the problem
$$\begin{aligned} \arg\min \quad & \frac{1}{2}\|w\|^2 + C\sum_i s_i \\ \text{such that} \quad & y_i(w^T x_i + b) \geq 1 - s_i \\ & s_i \geq 0 \end{aligned}$$
where $C > 0,\ y_i \in \{\pm 1\},\ x_i \in \mathbb{R}^n$ are given scalars. Find the dual of this problem.

3. Consider the problem
$$m_\infty = \arg\min_m \sum_i \|x_i - m\|_\infty.$$
Can you reformulate this problem as a linear program? If so, what is the dual?

4. Consider the problem
$$\begin{aligned} \min \quad & -x - y \\ \text{such that:} \quad & x \leq 2 \\ & y \leq 1 \\ & 2x + 3y \leq 6 \\ & x, y \geq 0. \end{aligned}$$
What are the vertices of the feasible set, and what are the values of the cost function at these points? Next, use the simplex algorithm to solve this problem. Lastly, describe the solution if we changed the cost to $-\frac{1}{3}x - \frac{1}{2}y$?

5. Consider the polytope $S = \{x : c_i^T x + b_i \leq 0,\ i = 1, ..., M\}$. What is the projection of a point z onto $\{x : c_1^T x + b_1 \leq 0\}$. What is the projection onto S? Can you think of an algorithm that uses the first result to efficiently compute the projection of a point onto S?

6. **(Optional)** Consider a convex set S that is the intersection of 2 convex sets C and D. Suppose we want to compute the projection of a point onto S but can't do so efficiently; however, we can efficiently compute projections onto C and D. Find an algorithm to compute the projection of a point onto S using the projections onto C and D. Show that this algorithm converges.