E2-212 MATRIX THEORY: ASSIGNMENT 10

Question 1. *Let* $\lambda, a \in \mathbb{R}, \mathbf{y} \in \mathbb{C}^n$, *and* $\mathbf{A} = \begin{bmatrix} \lambda \mathbf{I}_n & \mathbf{y} \\ \mathbf{y}^H & a \end{bmatrix} \in \mathbb{C}^{(n+1)\times(n+1)}$. *(6 points)*

(a) Use Cauchy interlacing theorem to show λ *is an eigenvalue of* $\mathbf{A}$ *with multiplicity at least* $n-1$.

(b) Find the other two eigenvalues of $\mathbf{A}$.

(Hint: If $\mathbf{M} = \begin{bmatrix} \mathbf{X}_{p\times p} & \mathbf{Y}_{p\times q} \\ \mathbf{W}_{q\times p} & \mathbf{Z}_{q\times q} \end{bmatrix}$ *and* $\mathbf{X}$ *is invertible, then* $det(\mathbf{M}) = det(\mathbf{X})det(\mathbf{Z} - \mathbf{W}\mathbf{X}^{-1}\mathbf{Y})$.*)*

Question 2. *Let* $\mathbf{A} \in \mathbb{C}^{n\times n}$ *be Hermitian and* a_{ij} *be the* (i,j)*th element of* $\mathbf{A}$. *(4 points)*

(a) Prove that $\lambda_{min}(\mathbf{A}) \le a_{ii} \le \lambda_{max}(\mathbf{A}),\ \forall\ i = 1, 2, \dots, n$.

(b) If $\mathbf{A}$ *is positive definite, prove that* $a_{ii} > 0,\ \forall\ i = 1, 2, \dots, n$.