E1 222 Stochastic Models and Applications

Problem Sheet 2–2

1. Two fair dice are rolled and X is the maximum of the two numbers. Specify a reasonable probability space and then define this random variable as a function on the Ω. Derive its probability mass function.

2. A fair dice is rolled repeatedly till the sum of all numbers obtained exceeds 6. Let X denote the number of rolls needed. Find the values of $F_X(1)$, $F_X(7)$ and $F_X(2)$.

3. Let X be geometric. Calculate probabilities of the events (i). $[X \leq 10]$, (ii). $[X = 3 \ \text{ or } \ 5 \leq X \leq 7]$.

4. Let X be a rv with density function
$$f(x) = cx^3, \quad \text{if} \quad 0 \leq x \leq 1.$$
($f(x)$ is zero for all other values of x). Find the value of c and the distribution function of X. Find $P[X > 0.5]$.

5. Let X be exponential random variable. Calculate probabilities of (i). $[|X| \leq 3]$, (ii). $[X \leq 4 \ \text{ or } \ X \geq 10]$.

6. Let X be a continuous random variable with uniform density over $(-1,\ 1)$. Find the density (or mass function) of the random variables: (a.) $U = (X+1)/2$, (b.) $U = \frac{X}{1+X}$, (c.) $U = g(X)$ where $g(x) = -1$ if $x > 0$, $g(x) = 0$ if $x = 0$, and $g(x) = 1$ if $x < 0$.

7. Let X be a continuous random variable having uniform density over $[0,\ 3]$. Let $Y = (X-1)^2$. Find the density of Y.

8. Let X be a random variable, g be some density function and ϕ a differentiable strictly increasing function on $(-\infty,\ \infty)$. Suppose that
$$P[X \leq x] = \int_{-\infty}^{\phi(x)} g(z)\, dz.$$
Show that the density of $Y = \phi(X)$ is $g(y)$.