E2-212 MATRIX THEORY: ASSIGNMENT 2

Question 1. *Let* $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{m \times n}$. *If* $\text{rank}(\mathbf{A}) = r$ *and* $\text{rank}(\mathbf{B}) = k \leq r$, *prove that:*

$$r - k \leq \text{rank}(\mathbf{A} + \mathbf{B}) \leq r + k.$$

Question 2. *Let* $\mathbf{C}, \mathbf{D} \in \mathbb{C}^{n \times n}$.

(a) If the product $\mathbf{CD}$ *is invertible, are both* $\mathbf{C}$ *and* $\mathbf{D}$ *invertible?*

(b) If the sum $\mathbf{C} + \mathbf{D}$ *is invertible, are both* $\mathbf{C}$ *and* $\mathbf{D}$ *invertible?*