E2-212 MATRIX THEORY: ASSIGNMENT 6

Question 1. *Let* $\mathbf{A} \in \mathbb{C}^{n\times n}$ *and* $U \subseteq \mathbb{C}^n$ *be an* $\mathbf{A}-$*invariant subspace, i.e.,* $\mathbf{A}\mathbf{x} \in U \ \forall \mathbf{x} \in U$.

(a) Prove that there exists a vector $\mathbf{u} \in U$ *and* $\lambda \in \mathbb{C}$ *such that* $\mathbf{A}\mathbf{u} = \lambda\mathbf{u}$. *(4 points)*

(b) Let $\mathbf{0} \neq \mathbf{x} \in \mathbb{C}^n$ *and* k *be the smallest integer such that* $\left\{\mathbf{x}, \mathbf{A}\mathbf{x}, \ldots, \mathbf{A}^k\mathbf{x}\right\}$ *is a dependent set. Prove that* $V = \operatorname{span}\left\{\mathbf{x}, \mathbf{A}\mathbf{x}, \ldots, \mathbf{A}^{k-1}\mathbf{x}\right\}$ *is* $\mathbf{A}-$*invariant.* *(2 points)*

Question 2. *For* $\mathbf{C} \in \mathbb{C}^{n\times n}$, *prove that* $\det(\exp(\mathbf{C})) = \exp(\operatorname{Tr}(\mathbf{C}))$. *(4 points)*