Computational Methods of Optimization
Final Exam-Part 2(25th Jan,2021)

Start Time: 10:30 AM End Time : 12:00 Noon

Instructions

- Answer all questions
- See upload instructions in the form

Question:	1	2	3	Total
Points:	10	10	10	30
Score:				

1. Consider the gradient projection algorithm

$$\mathbf{x}^{(k+1)} = P_C\left(\mathbf{x}^{(k)} - \alpha\nabla f(\mathbf{x}^{(k)})\right)$$

for

$$\min_{\mathbf{x}\in C} f(\mathbf{x})$$

where $P_C(\mathbf{z})$ is the projection of the point $\mathbf{z}$ on the convex set C.

(a) (5 points) At a feasible point $\mathbf{x}^{(k)}$ suppose we use

$$\mathbf{x}^{(k+1)} = P_C\left(\mathbf{x}^{(k)} + \alpha\mathbf{u}\right)$$

where $\mathbf{u} \in \mathbb{R}^d$. For what values of $\mathbf{u}$ is $\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}$ a *feasible descent direction.*

(b) (5 points) State an upper-bound on the stepsize if the derivative of f was continuous with Lipschitz constant L

2. Consider the following problem

$$p^* = min_{x,y\in\mathbb{R}} f(x,y)\left(\equiv x^2 - y^2 + 2(x+y)\right)$$

subject to $x^2 + y^2 = 1$.

(a) (3 points) Define the dual function, $g(\mu)$ where μ is a dual variable? What is the domain of the function

(b) (2 points) State the optimality criteria of the dual optimization problem

(c) Let d^* be the optimal value of the dual problem.

i. (1 point) Is $p^* = d^*$? A. Yes B. No

ii. (4 points) Give reasons

3. Consider the following Linear Program

$$min_{x_1,x_2} x_1 + x_2 \text{ subject to} x_1 + 2x_2 \leq 4, x_2 \leq 1$$

$$x_1, x_2 \geq 0$$

(a) (2 points) Express the problem in the standard form?

$$min_z c^\top z, \text{subject to} Az = b, z \geq 0$$

Clearly State A, b, c

(b) (2 points) Find the Basis and BFS, $\hat{z}$, corresponding to the point where the constraints $x_1+2x_2 \leq 4$, and $x_2 \leq 1$ are active.

(c) (3 points) Is the BFS optimal? Give reasons

(d) (3 points) Find a new BFS using the simplex method? Identify the new basis vector and the vector which is leaving?