E2-212 MATRIX THEORY: ASSIGNMENT 11

Question 1. *Let $\mathbf{x}, \mathbf{y} \in \mathbb{C}^n$, and $\mathbf{A} \in \mathbb{C}^{n\times n}$. Let $\mathbf{B} = \mathbf{I} - \mathbf{A}$ and $\mathbf{x}_0 \in \mathbb{C}^n$ be an arbitrary vector. Given $\mathbf{A}$ and $\mathbf{y}$, we wish to solve the linear system of equations $\mathbf{A}\mathbf{x} = \mathbf{y}$ as follows:* *(7 points)*

$$\text{For } i = 1, 2, 3, \ldots,$$
$$\text{Do}: \mathbf{x}_i = \mathbf{B}\mathbf{x}_{i-1} + \mathbf{y}$$

(a) Let $\varepsilon_i = \mathbf{x}_i - \mathbf{x}$ be the error in the ith iteration. Show that $\varepsilon_i = \mathbf{B}^i(\mathbf{x}_0 - \mathbf{x})$.
(b) Conclude that if $\rho(\mathbf{B}) < 1$, then this algorithm works (in the sense that $\mathbf{x}_i \to \mathbf{x}$ as $i \to \infty$).
(c) Use the Gersgorin theorem to show that a hermitian $\mathbf{A}$ must be strictly diagonally dominant and $|||\mathbf{A}|||_\infty < 2$ to ensure that this algorithm works.

Question 2. *Let $\mathbf{y}$ and $\mathbf{x}$ be unit ℓ_2-norm left and right eigenvectors respectively of $\mathbf{C} \in \mathbb{C}^{n\times n}$ corresponding to a simple eigenvalue λ. Let $S(\lambda) \triangleq |\mathbf{y}^H\mathbf{x}|$. Prove that $S(\lambda) \neq 0$.* *(3 points)*
(Hint for proof by contradiction: Show that $\mathbf{x} \in \mathcal{R}(\mathbf{C} - \lambda\mathbf{I})$. Also, there can exist only 1 linearly independent generalized eigenvector corresponding to simple eigenvalues.)