E2-212 MATRIX THEORY: ASSIGNMENT 5

Question 1. *Let $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{n \times n}$, $\mathbf{A}$ be invertible, and $\mathbf{A} + \mathbf{B}$ be singular.*
For any matrix norm $\|\|\cdot\|\|$, show that $\|\|\mathbf{B}\|\| \geq 1/\|\|\mathbf{A}^{-1}\|\|$. *(3 points)*

Question 2. *Prove or disprove the following properties of the spectral radius $\rho(\cdot)$ on $\mathbb{C}^{n \times n}$:*

(a) Non-negativity
(b) Positivity
(c) Homogeneity
(d) Triangle inequality
(e) Submultiplicativity

Is the spectral radius $\rho(\cdot)$ a matrix norm on $\mathbb{C}^{n \times n}$?
Note: A formal proof is required when proving any of the above properties. For disproving, a simple counter-example is enough. *(7 points)*