

## E2-212 MATRIX THEORY: ASSIGNMENT 3

**Question 1.** Let  $\mathbf{A} \in \mathbb{R}^{m \times n}$  and  $\|\cdot\|$  be a vector norm on  $\mathbb{R}^m$ . Define  $\|\mathbf{x}\|_{\mathbf{A}} \triangleq \|\mathbf{Ax}\|$ .  
Prove or disprove the following:

(6 points)

- (a)  $\|\cdot\|_{\mathbf{A}}$  is a norm on  $\mathbb{R}^n$  when  $\text{rank}(\mathbf{A}) = n$ .
- (b)  $\|\cdot\|_{\mathbf{A}}$  is a norm on  $\mathbb{R}^n$  when  $\text{rank}(\mathbf{A}) = k < n$ .

**Question 2.** Let  $\mathbf{C} \in \mathbb{C}^{m \times n}$ ,  $\mathbf{D} \in \mathbb{C}^{n \times m}$ . Prove or disprove:

(4 points)

- (a)  $\det(\mathbf{CD}) = \det(\mathbf{DC})$ , when  $m = n$ .
- (b)  $\det(\mathbf{CD}) = \det(\mathbf{DC})$ , when  $m > n$ .