IIR Filter Design

Recall: Steps in designing an IIR Filter:

1. Convert the discrete-time design specifications into continuous-time specifications.
2. Design a continuous-time filter, that is, obtain a system function that satisfies the continuous-time specifications.
3. Convert to an appropriate system function, which meets the specifications, using a continuous-time to discrete-time transformation.

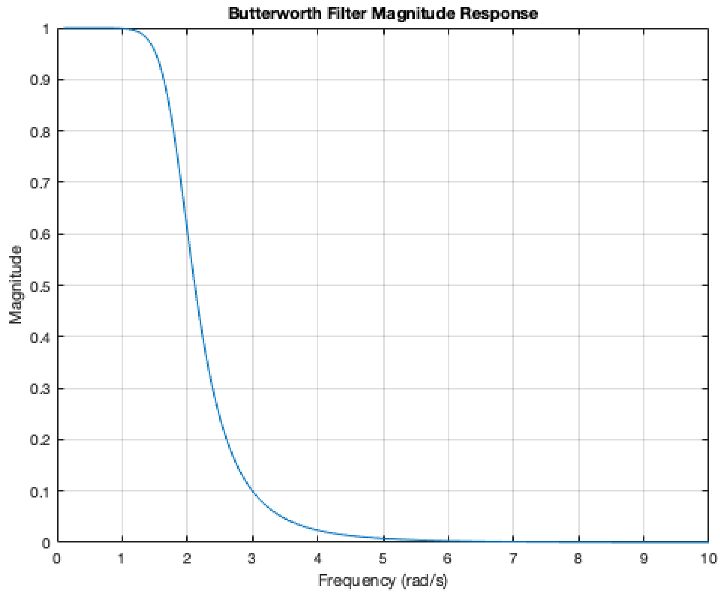
Example: Design an analog filter that satisfies the following specifications:

- $-6\text{ dB} \le 20\log_{10}|H(j\Omega)| \le 0, \qquad 0 \le |\Omega| \le 2\,\frac{\text{rad}}{\text{sec}}$
- $20\log_{10}|H(j\Omega)| \le -20\text{ dB}, \qquad 3\,\frac{\text{rad}}{\text{sec}} \le |\Omega| \le \infty$

Steps 1&2

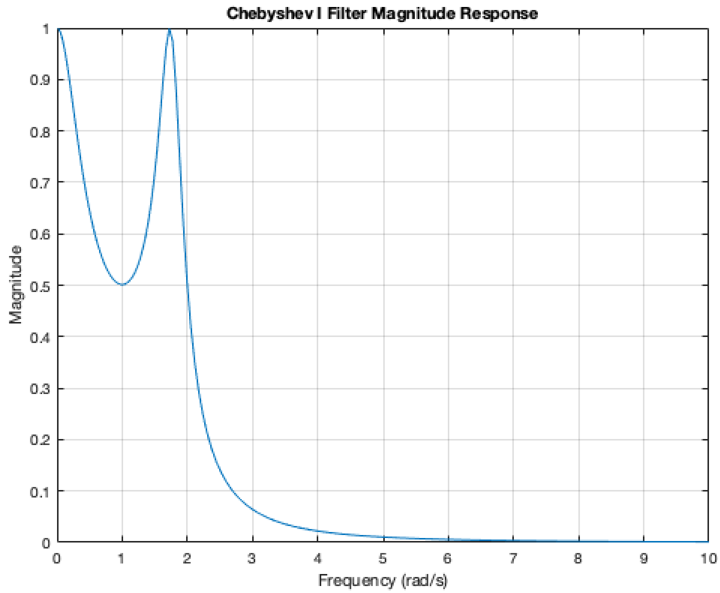
Using Butterworth Filter

```
[nButt, OmegaC] = buttord(2, 3, 6, 20, 's');
% nButt = order
% OmegaC = cutoff
% 's' denotes that this is an analog filter
[numButt denButt] = butter(nButt, OmegaC, 's');
[hButt, wButt] = freqs(numButt, denButt);
figure(); plot(wButt, abs(hButt));
title('Butterworth Filter Magnitude Response')
xlabel('Frequency (rad/s)')
ylabel('Magnitude')
grid on
```



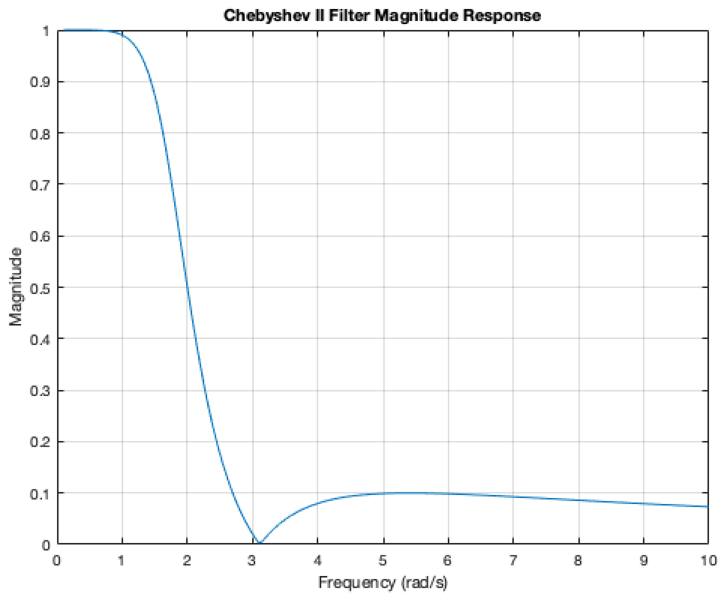
Using Chebyshev-I Filter

```
[nCheb1, OmegaC]= cheb1ord(2, 3, 6, 20, 's');
[numCheb1, denCheb1] = cheby1(nCheb1, 6, OmegaC, 's');
[hCheb1, wCheb1] = freqs(numCheb1, denCheb1);
figure(); plot(wCheb1, abs(hCheb1));
title('Chebyshev I Filter Magnitude Response')
xlabel('Frequency (rad/s)')
ylabel('Magnitude')
grid on
```



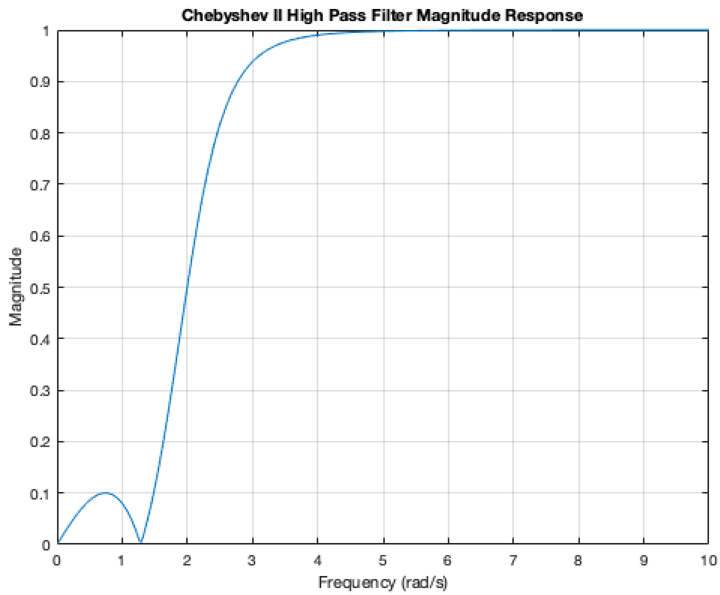
Using Chebyshev-II Filter

```
[nCheb2, OmegaC]= cheb2ord(2, 3, 6, 20, 's');
[numCheb2, denCheb2] = cheby2(nCheb2, 20, OmegaC, 's');
[hCheb2, wCheb2] = freqs(numCheb2, denCheb2);
figure(); plot(wCheb2, abs(hCheb2));
title('Chebyshev II Filter Magnitude Response')
xlabel('Frequency (rad/s)')
ylabel('Magnitude')
grid on
```



Conversion from low pass to high pass

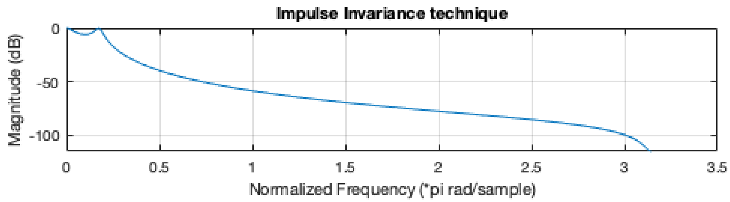
```
w0 = 4; % new cut-off frequency
[numHP, denHP] = lp2hp(numCheb2, denCheb2, w0);
[hHP, wHP] = freqs(numHP, denHP);
figure(); plot(wHP, abs(hHP));
title('Chebyshev II High Pass Filter Magnitude Response')
xlabel('Frequency (rad/s)')
ylabel('Magnitude')
grid on
```



Step 3

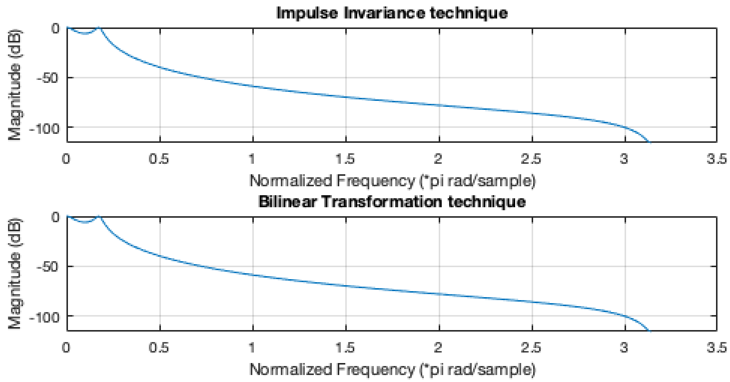
Conversion from Analog to Digital Filter using Impulse Invariance Technique

```
Fs = 10; % Sampling value
[numZa, denZa] = impinvar(numCheb1, denCheb1, Fs);
[Hza, Wza] = freqzM(numZa, denZa);
figure();
subplot(311); plot(Wza, Hza)
title('Impulse Invariance technique')
xlabel('Normalized Frequency (*pi rad/sample)')
ylabel('Magnitude (dB)')
grid on
```



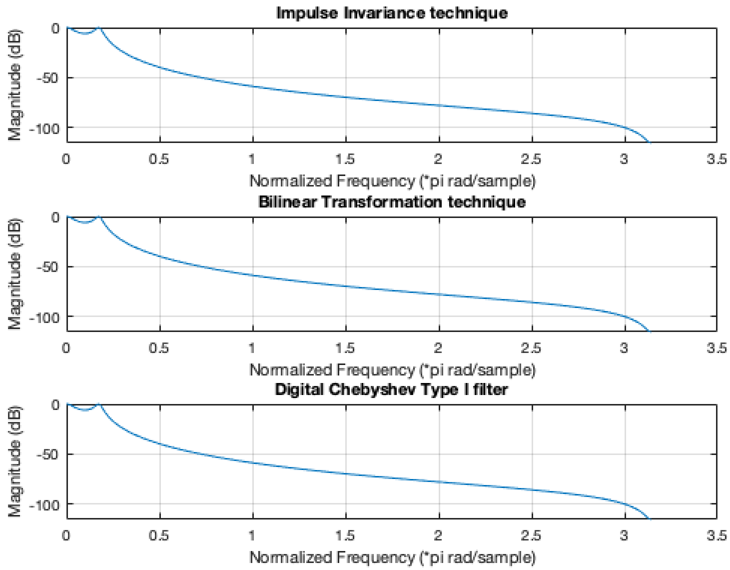
Conversion from Analog to Digital Filter using Bilinear Transform Technique

```
[BZb, AZb] = bilinear(numCheb1, denCheb1, Fs);
[Hzb, Wzb] = freqzM(BZb, AZb);
subplot(312); plot(Wzb, Hzb)
title('Bilinear Transformation technique')
xlabel('Normalized Frequency (*pi rad/sample)')
ylabel('Magnitude (dB)')
grid on
```



Alternative Solution: Construction of Digital IIR Filter using digital version of cheby1

```
[BZc AZc] = cheby1(nCheb1, 6, 2/(Fs/2));
[Hzc Wzc] = freqzM(BZc, AZc);
subplot(313); plot(Wzc, Hzc)
title('Digital Chebyshev Type I filter')
xlabel('Normalized Frequency (*pi rad/sample)')
ylabel('Magnitude (dB)')
grid on;
```



So far, we have only shown how to design a lowpass filter. Question is, how do design other types of filters? (highpass, bandpass, bandstop). This problem may be easily addressed by using the functions lp2hp(), lp2bp(), and lp2bs().

For this exercise, you are to create a Matlab script that solves the following.

1. Design an analog lowpass filter that has the following specifications:

$$A_p = 0.05\ dB, A_s = 10\ dB, transition\ region: 1500\ Hz - 2000\ Hz$$

a. What is the lowest-order Butterworth filter that satisfies the specifications? Show the magnitude response of your filter.

b. What is the lowest-order Chebyshev-I filter that satisfies the specifications? Show the magnitude response of your filter.

c. What is the lowest-order Chebyshev-II filter that satisfies the specifications? Show the magnitude response of your filter.

2. Convert the created Chebyshev-II filter in 1-c to the following specifications:

a. High pass filter with cut-off frequency of 1500 Hz. Show the magnitude response of your filter.

b. Band pass filter with cut-off frequencies at 500 Hz and 2000 Hz. Show the magnitude response of your filter.

c. Band stop filter with cut-off frequencies at 500 Hz and 2000 Hz. Show the magnitude response of your filter.

3. Consider the filter created in 1-c and Fs = 8000 Hz.

a. Convert the analog low pass filter into a discrete time filter using the impulse invariance technique. Show the magnitude response of your filter.

b. Convert the analog low pass filter into a discrete time filter using the bilinear transform technique. Show the magnitude response of your filter.

c. Convert the analog low pass filter into a discrete time filter using the digital version of cheby2. Show the magnitude response of your filter.