INTRODUCTION

The concept of fuzzy set with initiated by Zadeh in 1965. It has opened up keen insights and applications in a wide range of scientific fields. since its inception, the theory of fuzzy subsets has developed in many directions and found applications in a wide variety of fields. the study of fuzzy subsets and its applications to varies mathematics contexts has given rise to what is now commonly called fuzzy mathematics. fuzzy algebra is an important branch of fuzzy mathematics. fuzzy ideas have applied other algebraic structure such as group , rings modules ,vector spaces and topologies

In this paper, we introduce the concept of fuzzy PS- ideal of PS – algebra under homomorphism and some of its properties. we proud that $_\beta$ is a fuzzy PS- ideal (PS-subalgebra) of a PS- algebra x iff μ_β is a fuzzy (PS- sub algebra) of XxX. Where μ_β is the strongest fuzzy β relation.

In this chapter I we discussed about some basics definition which are needed for the further chapter.

In this chapter 2 we discussed about a homeomorphism and cartesian product of fuzzy PS- algebra

In this chapter 3 we discussed about a homeomorphism on fuzzy translation and fuzzy multiplication.

CHAPTER –1

BASIC DEFINITIONS

In this chapter we discussed some basics definition which are needed for the further chapter.

DEFINITION :1.1

Set is a collotion of well – defined objects.

DEFINITION :1.2

Let A and B two sets. we say that A is contained in B. if each element of A is also an element B. we say that A is called a **Subset** of B.

Example:

i. The set Z of all integers is a subset of Q. The set of all rational numbers.
ii. The set of Q of all rational numbers is a subset of R. The set of all real numbers.

DEFINITION: 1.3

A set that contains n elements is called **a null set or an empty set.**

DEFINITION :1.4

Let A and B be any two sets. then the **Cartesian product** of A and B is denoted by A x B is A Set of all order pair <a,b> where a ∈ A,b ∈ B.

i.e., AxB = {<a,b> a ∈ A , b ∈ B}

A = {{1,2,} {3}}, B{(a,b) (c,d)}

A x B = { ({1,2})(a,b), ({1,2 }(c.d)), {((a,b), ({3}(c,d))}

DEFINITION: 1.5

Let μ_{α}^{T} and δ_{α}^{T} fuzzy set in X. Then cartesian product μ_{α}^{T} and δ_{α}^{T}:

XxX → [0,1] is defined by

$$(\mu_{\alpha}^{T} \times \delta_{\alpha}^{T})(x,y) = \min\{\mu_{\alpha}^{T}(x), \delta_{\alpha}^{T}(y)\} \text{ for all } x,y \in X$$

DEFINITION: 1.6

Let X be a set and f be a mapping f: XxX→ X. Then f is called **Binary operation on X.**

In general a mapping f : X^n → x is called n-ary operation and n is called the order of the operation

For n = 1, f: x → x is called a binary operation.

Example:

X = 1,5

XxX = (1,1) (1,5) (5,1) (5,5)

X = 0,1,2 a binary operation '*' on A.

*	0	1	2
0	0	1	2
1	0	0	1
2	2	0	1

DEFINITION: 1.7

A **function** is a set of order pair in which no two orders pairs have the same first co-ordinates and different second co-ordinates.

i. The domin of a relation is the set of all first co-ordinates of the ordered pairs.
ii. The range of a relation is the set of all second co- ordinated of the ordered pairs.

DEFINITION :1.8

A **group** $< G,* >$ is an non empty set with a binary operation '*' satisfying following conditions.

(i) Closure Axioms

$\forall$ a,b $\in$ G

a*b $\in$ G

(ii) Associative law:

$\forall$ a,b,c $\in$ G

a*(b*c) = (a*b)*c

(iii) Existence of Identity:

There exist an element e $\in$ G such that

a*e = e*a = a $\forall$ a $\in$ G

(iv) Inverse:

There exist an elemant $a^{-1} \in$ G Such that ,

$a*a^{-1} = a = a^{-1}*a \quad \forall\ a \in G$

Example:

(Z, +) is a Group. (Integers)

DEFINITION :1.9

A non – empty set **"R" together with two binary operations called addition and multiplication denoted by ''+'' and ''.''** respectively.

I.e.) $\forall$ a,b $\in$ R

We have a + b $\in$ R then this algebraic structure (R,+, ,) is called a **Ring.**

Such that a, b, c in R.

(i) **Closure law:**

$\forall$ a,b $\in$ R

a+b $\in$ R

(ii) Associative law

$$(a + b) + c = (a + b) + c$$

$$a. (b + c) = (a . b) + c$$

(iii) Identity law:

$$a * e = e * a$$

(iv) Inverse law:

$$A+(-a) = (-a) + a=0$$

(v) Commutative law:

$$a+b = b+a$$

(vi) Distributive law:

$$a. (b + c) = a. b + a. c$$

$$(a + b) .c = a. c + b. c$$

(vii) Ring with unit element:

If a ring R there exists an element $1 \in R$, such that $1 . a = a . 1 = 1$. then R is called ring.

DEFINITION: 1.10

A nonempty subset of a ring R is called a **subring** of R. If S is a ring with respect to addition and multiplication in R.

DEFINITION 1.11:

A non empty subset U of R is said to be **right ideal** of " R'' if

i. U is a sub group of R under addition.

ii. $\forall u \in U, r \in R \Rightarrow ur \in U$.

DEFINITION 1.12:

A,non – empty subset u and R is said to be **a left ideal** of " R'' if,

1. U is a sub group of “R” under addition .
2. $\forall$ u $\in$ U, r $\in$ R $\Rightarrow$ ru $\in$ U.

DEFINITION :1.13

A **non -empty set** is a set containing one or more elements. Any set other then the empty set ϕ is therefore a non–empty set. Non-empty set are sometimes also called non-void sets.

DEFINITION :1.14

An associative ring A is called an **Algebra** over F if A Is a vector space over F such that for all a,b $\in$ A and a $\in$ F.

$\alpha(ab) = (\alpha a)\, b = a(\alpha b)$

DEFINITION :1.15

A Sub-algebra is a subset of an algebra, closed under all its operations and carrying the induced operations.

DEFINITION :1.16

Let X be a PS-algebra and I be a subset of X. then I is called a PS-ideals of X. If it satisfies the following conditions:

i. 0 $\in$ 1

ii. y *x $\in$ I and y $\in$ I $\Rightarrow$ X $\in$ I

DEFINITION: 1.17

Let X be a non-empty set. **A fuzzy subset** of the set X is a mapping.

$\mu : X \rightarrow [0,1]$

DEFINITION: 1.18

An **algebraic expression** is an expression that contains one or more numbers, one or more variables and one or more arithmetic operations.

Example:

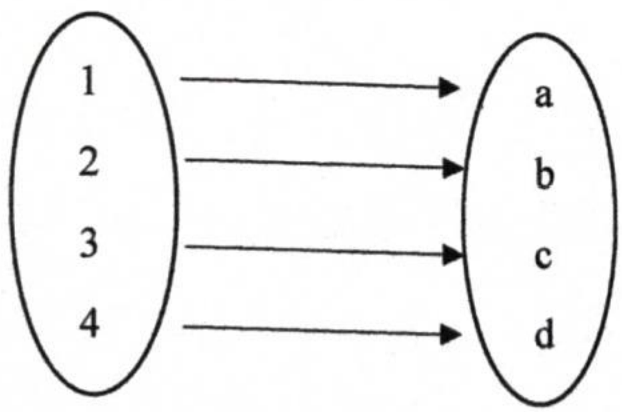
X + 2=5

DEFINITION: 1.19

A relation is a set of **ordered pairs** (x ,y).

Example:

The set {(1,a) (2,b) (3,c) (4,d)) is a relation.



DEFINITION :1.20

A system < G,*> Where G is a non- empty set and * is a binary composition on G is called a **semigroup**.

If it satisfies the following operation .

(i)Closure law:

$\forall$ (a,b) $\in$ G

a*b $\in$ G

(ii) Associative law:

∀ a,b,c ∈ G

a(b*c) = (a*b)*c

Every group is a semi – group but every semi- group may or may not be group .

DEFINITION :1.21

Let μ and δ be the fuzzy sets in X. The Cartesian product.

μ X δ : X x X → [0,1] is defined by,

(μ X δ) (x,y) = min {μ(u) ,δ (y)} for all x,y ∈ X.

DEFINITION :1.22

Let β be a fuzzy subset of X. the strongest fuzzy β-relation on PS- algebra X is the fuzzy subset μ_β of XxX given by,

μ_β (x,y) = min {β(x) , β(y) } for all x,y ∈ X.

CHAPTER-II

HOMOMORPHISM AND CARTESIAN PRODUCT OF FUZZY PS-ALGEBRA

In this chapter we discussed about a homomorphism and cartesian product of fuzzy PS-algebra.

2.1 FUZZY PS-ALGEBRA

DEFINITION :2.1.1

A non -empty set X with a constant 0 and a binary operation * is called **PS-algebra.**

If its satisfies the following axioms.

i. $x*x = 0$
ii. $x*0 = 0$
iii. $x*y = 0$ and $y * x= 0 \Rightarrow x=y$ for all $x,y \in X$.

DEFINITION: 2.1.2

Let x be a PS-algebra a . fuzzy set μ in x is called a **fuzzy PS- ideal** of X it satisfies the following condition.

a. $\mu (0) \geq \mu (x)$

b. $\mu (x) \geq \min \{\mu (y* x), \mu (y) \}$ for all $x,y \in X$.

DEFINITION :2.1.3

A fuzzy set μ in a PS-algebra. X is called a **fuzzy PS- sub algebra** of X. if $\mu (x*y) \geq \min \{(x) ,\mu(y) \}$ for all $x,y \in X$.

THEOREM :2.1.4

Let X be a PS- algebra. μ is a fuzzy PS- ideal of X iff μ is a fuzzy PS- sub algebra of X.

PROOF:

Let μ be a fuzzy PS- ideal.

TO PROVE:

μ is a fuzzy PS- sub algebra of X.

By definition of fuzzy PS- ideal, $\mu(x) \geq \min\{\mu(y*x), \mu(y)\}$, for all $x,y \in X$

Now, $\mu(x*y) \geq \min\{\mu(y*(x*y)), \mu(y)\}$, for all $x,y \in X$.

$= \min[\mu(0), \mu(y)\}$

$\geq \min\{\mu(x), \mu(y)\}$

$\Rightarrow$ μ is a fuzzy PS- sub algebra of x.

Conversely, let μ be a fuzzy PS- sub algebra of x.

TO PROVE:

μ is a fuzzy PS- ideal of X.

Now $\mu(0) = \mu(x*x)$

$\geq \min\{\mu(x), \mu(x)\}$

$= \mu(x)$

$\Rightarrow \mu(0) \geq \mu(x)$

And $\mu(x) \geq \mu(y)$

$= \min(\mu(0), \mu(y))$

$= \min\{\mu(y*x), \mu(y)\}$

$\Rightarrow \mu(x) \geq \min\{\mu(y*x), \mu(y),\}$ for all $x, y \in X$.

Hence u is a fuzzy PS-ideal of X.

Hence the proof.

DEFINITION: 2.1.5

$A \cap B$ denotes the **Intersection** of A and B and consists of the elements in both A and B.

Example:

Let A = {1,3,5,9} and B = {3,5,10}.

Then, A ∩ B = {3,5}

THEOREM: 2.1.6

The intersection of any set of fuzzy PS- ideals in PS- algebra X is also a fuzzy PS- ideal.

PROOF:

Let $\{\mu_i\}$ be a family of fuzzy PS-ideals of PS-algebra X. Then for any $x,y \in X$.

$(\cap \mu_i)(0) = \text{Inf}\,(=0))$

$\geq \inf(\mu_i(x))$

$= (\cap \mu_i)(x)$

And $(\cap \mu_i)(x) = \inf(\mu_i\, x))$

$\geq \text{Inf}(\min\{\mu_i(y * x), \mu_i(y)\}$

$= \min\{\text{Inf}(\mu_i(y*x)), \text{Inf}(\mu_i(y))\}$

$= \min\{(\cap \mu_i)(y*x),.\ (\cap \mu_i)(y)\}$

Hence the proof.

DEFINITION 2.1.7

A **fuzzy set M** is an arbitrary set X is a function with domain x valuers in [0, 1]

THEOREM: 2.1.8

A fuzzy set μ of a PS-algebra X is a fuzzy PS-sub algebra iff for every $t \in [0,1]$ μ^t is either empty or a PS - sub algebra of X.

PROOF:

Assume that μ is a fuzzy PS - sub algebra of X and $\mu^t \neq \phi$

Then for any $x, y \in \mu^t$, we have $\mu(x) = \mu(y) = t$

$\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$

$= \min\{t, t\} = t$

Therefore $x*y \in \mu$.

Hence μ^t is a PS-sub algebra of X.

Conversely, assume that μ^t is a PS-sub algebra of X.

Let $x, y \in X$.

Take $t = \min\{\mu(x), \mu(y)\}$

Then by assumption μ' is a PS-sub algebra of X, $x * y \in \mu^t$

$\mu(x*y) \geq t = \min\{\mu(x), \mu(y)\}$

Hence μ is a fuzzy PS- sub algebra of X.

Hence the proved

THEOREM: 2.1.9

Any sub algebra of a PS-algebra X can be realized as a level sub algebra of some fuzzy PS-sub algebra of X.

PROOF:

Let μ be sub algebra of the given PS-algebra X.

Let μ be a fuzzy set in X defined by

$\mu(x) =$ t, if $x \in A$

0, if $x \in A$

where $t \in [0, 1]$ is fixed. It is clear that $\mu^t = A$.

Now we prove such defined μ is a fuzzy PS- sub algebra of X.

Let $x, y \in X$.

If $x, y \in A$, then $x * y \in A$.

Hence $\mu(x) = \mu(y) = \mu(x*y) = t$ and $\mu(x*y) \geq \min\{\mu(x), \mu(y)\}$

If $x, y \notin A$, then $\mu(x) = \mu(y)=0$ and $\mu(x*y) \geq \min\{\mu(x), \mu(y)\} = 0$.

If at most one of $x, y \in A$, then at least on of $\mu(x)$ and $\mu(y)$ is equal to 0.

Therefore, $\min\{\mu(x), \mu(y)\} = 0$ so that $\mu(x * y) \geq 0$, which completes the proof.

As a generalization of theorem 2.1.10, we prove the following theorem

Hence the proved.

THEOREM: 2.1.10

Let X be a PS - algebra. Then given any chain of sub algebra $So \subset S1 \subset S2 \subset \ldots \subset S_r = X$, there exists a fuzzy sub algebra μ of X whose level sub algebras are exactly the sub algebras of this chain.

PROOF:

Consider a set of numbers $t_0 > t_1 > t_2 > \ldots\ldots\ldots > t_r$, where each $t_i \in [0,1]$.

Let $\mu : X \rightarrow [0,1]$ be a fuzzy set defined by $\mu(s_0) = to$ and

$\mu(s_i - s_{i-1}) = t_i$, $0 < i \leq r$.

We claim that μ is a fuzzy sub algebra of X. Let $x, y \in X$.

Then we classify it into two cases as follows:

Case (1): Let $x, y \in S_i - S_{i-1}$. Then by the definition of μ, $\mu(x) = t_i = (y)$.

Since Si is a sub algebra, it follows that $x*y \in Si$,and so either $x*y \in S_i - S_{i-1}$ (or) $x * y \in S_{i-1}$

In any case, we conclude that $\mu(x * y) > t_i = \min \{\mu(x), \mu(y)\}$.

Case (2): For i>j, Let $x \in S_i - S_{i-1}$ and $y = S_j - S_{j-1}$

Then $\mu(x) = ti$; $\mu(y) = t_j$ and x*y e Si, since S; is a subalgebra of X and B $S_j \subset S_{i.}$

Hence $\mu(x * y) > tj = \min \{\mu(x), \mu(y)\}$

Thus u is a fuzzy subalgebra of X.

From the definition of μ, it follows that $Im(\mu) = \{ t_0, t_1, t_2, \ldots\ldots t_r \}$.

Hence the level subalgebras of μ are given by the chain of subalgebras.

$\mu_{t0} \subset \mu_{t1} \subset \mu_{t2} \subset \ldots\ldots\subset \mu tr = X$. Now $\mu t_0 = \{x \in X / \mu(x) \geq t_0\} = So$.

Finally, we prove that $\mu t_i = s_i$ for $0 < i \leq r$.

Clearly $S_i \subseteq \mu_{ti.}$

If $x \in \mu_{ti}$, then $u(x) \geq t_i$ which implies that $x \in S_j$ for j> i.

Hence $\mu(x) \in \{ t_1, t_2, \ldots\ldots t_i\}$ and $x \in S_k$ for some $k \leq i$.

As S_k s_i , it follows that $x \in Si$.

$\Rightarrow \mu ti = Si$ for $0 < I < r$.

Hence the proved.

THEOREM: 2.1.11

Two level sub algebras μ^s, μ^t ($s < t$) of a fuzzy PS- sub algebras are equal if there is no $x \in X$ such that $s \leq \mu(x) < t$.

PROOF:

Let $\mu^s = \mu^t$ for some $s < t$.

If there exist $x \in X$ such that $s \leq \mu(x) < t$, then μ^t is a proper subset of μ^s, which is a contradiction.

Conversely, assume that there is no $x \in X$ such that $s \leq \mu(x) < t$,

since $s < t$, $\mu^t \subseteq \mu^s$.

If $x \in \mu$ then $u(x) \geq s$ and so $u(x) \geq t$, because $\mu(x)$ does not lie between s and t.

Hence $x \in \mu^t$, which gives $\mu^t \subseteq \mu^s$.

Hence the proved.

DEFINITION:2.1.12

Let A be a fuzzy subgroup of S. For $t \in [0,1]$, the set $A_t = \{x \in S \mid A(x) \geq t$ is called a level subset of the fuzzy subset A.

THEOREM:2.1.13

Let u be a fuzzy set in a PS-algebra X and let $t \in \text{Im}(\mu)$. Then μ is a fuzzy PS-ideal of X. if and only if the level subset μ^t is a PS-ideal of X, which is called a level PS-ideal of X.

PROOF:

Assume that μ is a fuzzy PS-ideal of X.

Clearly $0 \in \mu^t$

Let $y * x \in \mu^t$ and $y \in \mu^t$.

Then $\mu(y*x) \geq t$ and $\mu(y) \geq t$

Now μ (x)$\geq$ min {μ (y*x), μ (y)} $\geq$ min (t,t) =t.

Hence the level subset μ^t is a PS-ideal of X.

Conversely assume that, the level subset u' is a PS-ideal of X, for any t $\in$ [0,1].

Suppose assume that there exist some $x_0 \in$ X such that $\mu(0) < \mu(x_0)$

Take s = ½ [$\mu(0) + \mu(x_0)$]

$\mu(0) > s > \mu$ (x0)

$\Rightarrow x_0 \in \mu^s$ and $0 \notin \mu^s$, a contradiction, since u is a PS-ideal of X.

Therefore, $\mu(0) \geq \mu$ (x) for all x $\in$ X.

If possible, assume that $x_0, y_0 \in$ X such that $\mu(x_0)$ <min { $\mu(y_0*x_0)$, μ (y_0)}

Take s = ½ [u(x)+min ($\mu(y_0*x_0)$. $\mu(y_0)$}]

$\Rightarrow s > \mu(x_0)$ and s <min {$u(y_0*x_0)$, $\mu(y_0)$}

$\Rightarrow s > \mu$(xo), s < μ (y_0*x_0) and s<$\mu(y_0)$.

$\Rightarrow x_0 \notin \mu^s$, a contradiction, since μ^{s}'' is a PS-ideal of X

Therefore μ(x)$\geq$ min {μ (y* x), μ (y)}, for any x, y $\in$ X.

Hence the proved.

2.2 HOMOMORPHISM ON FUZZY PS-ALGEBRAS

DEFINITION: 2.2.1

Let (X,*, 0) and (Y,*, 0) be PS-algebras. A mapping f : X $\rightarrow$ Y is said to be a **Homomorphism**. If

f(x*y) f(x)* f(y) for all x, y $\in$ X

DEFINITION: 2.2.2

Let f: X $\rightarrow$ X be an **Endomorphism** and μ be a fuzzy set in X. We define a new fuzzy set in X by μ_f in X as

$\mu_r(x) = \mu$ (f(x)) for all x in X.

THEOREM:2.2.3

Let f be an endomorphism of a PS-algebra X. If u is a fuzzy PS- ideal of X. Then so is μ_f.

PROOF:

Let μ is a fuzzy PS- ideal of X.

Now, $\mu_f (x) = \mu (f(x)) \leq \mu ((0)) = \mu_f(0)$

For all $x \in X$.

$\mu_f (0) \geq \mu_f (x)$

Let $x, y \in X$.

Then, $\mu_f (x) = \mu (f(x))$

$\geq \min \{\mu(f(y)* f(x)), \mu(f(y))\}$

$= \min (\mu(f(y*x)),\mu (f(y))\}$

$= \min \{\mu_f (y*x), \mu_f(y)\}$

$\mu_f(x) \geq \min \{\mu*x), \mu_f(Y)\}$

Hence μ_f is a fuzzy PS-ideal of X.

Hence the proved.

THEOREM: 2.2.4

Let: $X \rightarrow Y$ be an epimorphism of **PS-algebra**. If μ_f is a fuzzy PS-ideal of X. then μ is a fuzzy PS-ideal of Y.

PROOF:

Let μ_f is a fuzzy PS- ideal of X and let $y \in Y$.

Then there exists $x \in X$.

Such that f(x)=y.

Now, $\mu(0)= \mu(f(0))$

$= \mu_f (0) \geq \mu_f(x)$

$=\mu_f (f(x)) = \mu (y)$

$\therefore \mu (0) \geq \mu (y)$

Let $y_1, y_2 \in Y$,

$\mu\ (y_1) = \mu\ (f(x)) = \mu(f(y))$

$\geq \min\ \{\ \mu(x_2 * x_1).\ \mu f(x_2))$

$= \min\{\ \mu(f(x_2 * x_1)),\ \mu(f(x_2))\}$

$= \min\ (\mu\ (f(x_2)\ f(x_1)),\ (f(x_2))\}$

$= \min\ (\mu\ (y_2 * y_1),\ \mu(y_2)\}$

$\mu(y_1) \geq \min\ (\mu\ (y_2 * y_1),\ \mu(y_2)\}$

Hence μ is a fuzzy PS-ideals of Y.

Hence the proved.

THEOREM: 2.2.5

Let f: X → Y be a homomorphism of PS-algebra. If u is a fuzzy PS-ideal of Y, then μ_f , is a fuzzy PS-ideal of X.

PROOF:

Let u is a fuzzy PS-ideal of Y and let x, y ∈ X

Then, $\mu_f(0) = \mu(f(0))$

$\geq \mu\ (f(x))\ \mu_f\ (x)$

$\mu_f\ (0) \geq \mu\ (x)$

Also $\mu,(0) = \mu(x)$

$\geq \min(\ \mu_f\,(y) * f(x),\ \mu f(y))$

$= \min\ (\mu f\ (y * x),\ \mu f(y))$

$= \min\ \{\mu_f(y * x).\ \mu_f\ (y)\}$

$\mu_f\,(x) \geq \min\ \{\ \mu_f\,(y * x).\mu_f\ (y)\}$

Hence μ_f is a fuzzy ps-ideals of X.

Hence the proved.

THEOREM: 2.2.6

Let f: X → Y be a homomorphism of PS-algebra X into a PS-algebra Y. If u is a fuzzy PS-sub algebra of Y, then the pre-image of u denoted by $f^{-1}(\mu)$ defined as $\{f^{1}(\mu)\}(x) = \mu(f(x))\ \forall\ x \in X$ is a fuzzy PS-sub algebra of X.

PROOF:

Let μ is a fuzzy PS-ideal of Y and let $x, y \in X$

Now, $(f^{-1}(\mu))(x * y) = \mu(f(x * y))$

$= \mu(f(x) * f(y))$

$\geq \min\{\mu(f(x)), \mu(f(y))\}$

$\{f^{-1}(\mu)\}(x * y) = \min\{\{f^{-1}(\mu)\}(x), (f^{-1'}(\mu)\}(y)\}$

Hence $f^{-1}(\mu)$ is a fuzzy PS-sub algebra of X.

Hence the proved.

THEOREM: 2.2.7

If μ be a fuzzy PS-sub algebra of X. Then μ_f is also a fuzzy PS-sub algebra of X.

PROOF:

Let μ, be a fuzzy PS-sub algebra of X. Let $x, y \in X$.

Now, $\mu f(x*y) = \mu(f(x*y))$

$= \mu(f(x)*f(y))$

$\geq \min\{\mu(f(x)), \mu(f(y))\}$

$\mu_f(x*y) = \min\{\mu(f(x)), \mu(f(y))\}$

μ_f is a fuzzy PS-subalgebra of X.

Hence the proved

2.3 CARTESIAN PRODUCT OF FUZZY PS-IDEALS OF PS – ALGEBRAS

DEFINITION: 2.3.1

Let μ and & be the fuzzy sets in X. The Cartesian product $\mu \times \delta: X \times X \rightarrow [0.1]$ is defined by $(\mu \times \delta)(x, y) = \min(\mu(x), \delta(y)\}$, for all $x, y \in X$.

DEFINITION:2.3.2

Let be a fuzzy subset of X. The strongest fuzzy β - relation on PS-algebra X is the fuzzy subset μβ of XxX given by $\mu ß (x, y) = \min(\beta(x). \beta(Y))$, for all $x, y \in X$.

THEOREM:2.3.3

If μ and δ are fuzzy PS-ideals in a PS-algebra X, then μ x δ is a fuzzy PS- ideal in XxX.

PROOF:

Let $(X_1, X_2) \in X x X$.

$$(\mu \times \delta)(0,0) = \min(\mu(0), \delta(0))$$
$$\geq \min\{\mu(x_1), \delta(x_2))$$
$$= (\mu \times \delta)(X_1, X_2)$$

$$\Rightarrow (\mu \times \delta)(0,0) \geq (\mu \times \delta)(X_1, X_2)$$

Let $(x_1 x_2), (y_1, y_2) \in X \times X$.

Now, $(\mu \times \delta)(x_1, x_2) = \min\{(\mu(x_1), \delta(x_2)\}$

$$\geq \min\{\min(\mu(y_1 * x_1), \mu(y1)\}, \min\{\delta(y_2 * x_2), \delta(y2)\}$$
$$= \min(\min\{\mu(y_1,* x_1), \delta(y_2 * x_2))\}, \min(\mu(y_1), \delta(y_2)\}$$
$$= \min\{(\mu \times \delta)((y_1. Y_2) * (x_1, x_2)), (\mu \times \delta)(y_1. y_2))\}$$

$\therefore \quad (\mu \times \delta)(X_1, X_2) \geq \min\{(\mu \times \delta)((y_1, y_2) * X_1, X_2)), (\mu \times \delta)(y_1, y_2)\}$

Hence μ x δ is a fuzzy PS- ideal in X x X.

Hence the proved.

THEOREM: 2.3.4

Let μ and & be fuzzy sets in a PS-algebra X such that $\mu \times \delta$ is a fuzzy PS- idea of XxX. Then

i. Either $\mu(0) \geq \mu(x)$ or $\delta(0) \geq \delta(x)$ for all $x \in X$
ii. If $\mu(0) \geq (x)$ for all $x \in X$, then either $\delta(0) \geq (0)$ or $\delta(0) \geq \delta$
iii. If $\delta(0)) \geq \delta(x)$ for all $x \in X$, then either $\mu(0) \geq \mu(x)$ or $\mu(0) \geq \delta(x)$

PROOF:

Let μ x & be a fuzzy PS-ideal of XXX.

(i) Suppose that $\mu(0) < (x)$ and $\delta(0)\ \delta(x)$ for some $x, y \in X$.

Then $(\mu \times \delta)(x, y) = \min\{\mu(x), \delta(y))$

$> \min\{\mu(0), \delta(0))$

$= (\mu \times \delta)(0,0)$, which is a contradiction.

Therefore $\mu(0) \geq \mu(x)$ or $\delta(0) \geq \delta(x)$, for all $x \in X$.

(ii)Assume that there exists $x, y \in X$ such that $\delta(0) < \mu(x)$ and $\delta(0) < \delta(x)$.

Then $(\mu \times \delta)(0,0)=\min(\mu(0), \delta(0)) = \delta(0)$

Now, $(\mu \times \delta)(x, y) = \min\{\mu(x),\delta(y)) > \delta(0) = (\mu \times \delta)(0,0)$ Which is a contradiction.

Hence, if $\mu(0)\geq u(x)$ for all $x \in X$, then either $\delta(0) \geq \mu(x)$ or $\delta(0) \geq \delta(x)$.

Similarly, we can prove that if $\delta(0) \geq \delta(x)\ \forall x \in X$, then either $\mu(0) \geq \mu(x)$ or $\mu(0) \geq \delta(x)$, which yields (iii).

Hence the proved.

THEOREM: 2.3.5

Let μ and δ be fuzzy sets in a PS-algebra X such that $\mu \times \delta$ is a fuzzy PS-ideal of X x X. Then either μ or δ is a fuzzy PS-ideal of X.

PROOF:

First, we prove that δ is a fuzzy PS-ideal of X.

Since by 4.3.4 (i), either $\mu(0) \geq \mu(x)$ or $\delta(0) \geq \delta(x)$ for all $x \in X$.

Assume that $\delta(0) \geq \delta(x)$ for all $x \in X$.

It follows from 4.3.4 (iii) that either $\mu(0) \geq \mu(x)$ or $\mu(0) \geq \delta(x)$.

If $\mu(0) \geq \delta(x)$, for any $x \in X$, then $\delta(x)= \min \{\mu(0), \delta(x)\} = (\mu \times \delta)(x,0)$

$$\delta(x) = (\mu \times \delta)(0,x))$$
$$\geq \min \{(\mu \times \delta)((0,y)*(0, x)), (\mu \times \delta)(0, y)\}$$
$$= \min \{(\mu \times \delta)((0*0), (y*x)) (\mu \times \delta)(0, y)\}$$
$$= \min ((\mu \times \delta)(0, (y*x)), (\mu \times \delta)(0, y)\}$$
$$= \min (\delta(y*x), \delta(y))$$

Hence & is a fuzzy PS-ideal of X.

Next, we will prove that μ is a fuzzy PS-ideal of X.

Let $\mu(0) \geq \mu(x)$. Since by theorem 4.3.4 (ii), either $\delta(0)\ \mu(x)$ or $\delta(0) \geq \delta(x)$.

Assume that $\delta(0) \geq \mu(x)$, then $\mu(x) = \min (\mu(x), \delta(0)) = (\mu \times \delta)(x,0)$

$$\mu(x) = (\mu \times \delta)(x,0)$$
$$\geq \min \{(\mu \times \delta)(y,0)*(x.0)), (\mu \times \delta)(y,0)\}$$
$$= \min \{(\mu \times \delta)((y*x).(0*0)), (\mu \times \delta)(y.0)\}$$
$$= \min \{(\mu \times \delta)((y*x),0),(\mu \times \delta)(y,0)$$
$$= \min \{\mu(y*x), u(y)\}$$

Hence μ is a fuzzy PS-ideal of X.

Hence the proved.

THEOREM: 2.3.6

If λ and μ are fuzzy PS-sub algebras of a PS-algebra X, then λ x u is also a fuzzy PS-sub algebra of XxX.

PROOF:

For any $x_1, x_2. Y_1, Y_2 \in X$.

$$(\lambda \times \mu)((x_1,y_1)*(x_2, y_2)) = (\lambda \times \mu)(x_1* x_2, y_1*y_2)$$
$$= \min (\lambda(x_1*x_2), \mu(y_1*y_2)\}$$
$$\geq \min (\min (\lambda(x_1), \lambda(x_2)\}, \min \{\mu(y_1),\mu(y_2)\}$$

$= \min (\min \{(\lambda x_1), \mu(y_1)), \min (\lambda(x_2), \mu(y_2)\}\}$

$= \min\{(\lambda x \mu) (x_1,y_1), (\lambda \text{ x } \mu) (x_2, y_2)\}$

Hence the proved.

THEOREM: 2.3.7

Let μβ be the strongest fuzzy β-relation on PS-algebra X, where β is a fuzzy set- of a PS-algebra X. If β is a fuzzy PS-ideal of X, then μβ is a fuzzy PS-ideal of XxX.

PROOF:

Let β be a fuzzy PS-ideal of a PS-algebra X.

Let $(x_1, x_2), (y_1, y_2) \in$ X x X.

Then $\mu\beta (0, 0) = \min ((\beta (0), \beta (0))$

$\geq \min (\beta (x_1), \beta(x_2))$

$= \mu\beta (x_1, x_2)$

$\Rightarrow \mu\beta (0, 0) \geq \mu\beta (x_1, x_2)$

And also $\mu\beta (x_1,x_2) = \min \{\beta(x_1),(x_2)\}$

$\geq \min \{\min \{\beta(y_1*x_1), \beta (y_1)\}, \min (\beta(y_2* x_2), \beta(y_2)\}\}$

$= \min \{\min (\beta(y_1 *x_1), \beta (y_2* x_2)), \min (\beta (y_1),\beta(y_2)\}\}$

$= \min (\mu\beta ((y_1*x_1), (y_2* x_2)), \mu\beta (y_1, y_2)\}$

$= \min \{\mu ß ((y_1, y_2) (x_1, x_2)), \mu\beta(y_1, y_2)\}$

$\Rightarrow \mu\beta (x_1, x_2) > \min (\mu ß ((y_1, y_2)* (x_1, x_2)), \mu\beta (y_1, y_2)\}$

Therefore μβ is a fuzzy PS-ideal of X x X.

Hence the proved.

THEOREM:2.3.8

If μβ is a fuzzy PS-ideal of X x X, then β is a fuzzy PS-ideal of a PS- algebra X

PROOF:

Let μβ is a fuzzy PS-ideal of X x X.

Then for all $(X_1, X_2), (y_1, y_2) \in$ X x X.

$\min\{\beta(0), \beta(0)\} = \mu\beta(0,0)$

$\geq \mu\beta(x_1, x_2)$

$= \min\{\beta(x_1), \beta(x_2))$

$\Rightarrow \min(\beta(0), \beta(0)) \geq \min\{\beta(x_1), \beta(x_2)\}$

$\Rightarrow \beta(0)\ \beta(x_1)$ or $\beta(0)\ \beta(x_2)$

Also, $\min(\beta(x_1), \beta(x_2)) = \mu\beta(x_1, x_2)$

$\geq \min(\mu\beta((y_1, y_2) * (x_1, x_2)).\ \mu\beta(y_1, y_2)$

$= \min(\mu\beta((y_1 * x_1), (y_2 * x_2)), \mu\beta(y_1, y_2)\}$

$= \min(\min(\beta(y_1 * x_1), \beta(y_2 * x_2)), \min(\beta(y_1), \beta(x_2)\}\}$

$= \min(\min\{\beta(y_1, x_1), \beta(y_1)), \min(\beta(y_2 * x_2), \beta(y_2)\}\}$

Put $x_2 = y_2 = 0$

We get $\beta(x_1) > \min\{\beta(y_1 * x_1), \beta(y_1)\}$

Hence β is a fuzzy PS-ideal of a PS-algebra X.

Hence the proved.

THEOREM: 2.3.9

If β is a fuzzy PS-sub algebra of a PS-algebra X, then μβ is a fuzzy PS- sub - algebra of X x X.

PROOF:

Let β be a fuzzy PS-sub algebra of a PS-algebra X.

Let $x_1, x_2, y_1, y_2 \in$ X.

Then $\mu\beta((x_1, y_1) * (x_2, y_2)) = \mu\beta(x_1 x_2, y_1 y_2)$

$= \min(\beta(x_1 * x_2), \beta(y_1 * y_2)\}$

$\geq \min\{\min(\beta(x_1), \beta(x_2)\}, \min(\beta(y_1), \beta(y_2)\})$

$= \min(\min(\beta(x_1), \beta(y1)), \min(\beta(x_2), \beta(y_2)\}\}$

$= \min(\mu\beta(x_1, y_1), \mu\beta(x_2, y_2))$

$\Rightarrow \mu\beta ((x_1,y_1)* (x_2, y2)) \geq \min \{\mu\beta (x_1,y_1), \mu\beta (x_2, y_2)\}$. Therefore $\mu\beta$ is a fuzzy PS-sub algebra of X x X.

Hence the proved.

THEOREM: 2.3.10

If $\mu\beta$ is a fuzzy PS-sub algebra of X x X, then β is a fuzzy PS- sub algebra of a PS - algebra X.

PROOF:

Let $x, y \in X$.

Now, $\beta (x * y) = \min \{\beta(x * y), \beta(x * y)\}$

$= \mu\beta ((x*y) (x*y))$

$\geq \min (\mu\beta (x*y), \mu\beta (x * y)\}$.

$= \min (\min (\beta(x), \beta(y)), \min (\beta(x), \beta(y)\}\}$

$= \min (\beta(x), \beta(y)\}$

$\Rightarrow \beta(x*y) \geq \min (\beta(x), \beta(y)\}$

$\therefore$ β is a fuzzy-PS - sub algebra of a PS - algebra X.

Hence the proved

CHAPTER-III

HOMOMORPHISM ON FUZZY TRANSLATION AND FUZZY MULTIPLICATION

In this chapter we discussed about a Homomorphism on fuzzy translation and fuzzy multiplication.

DEFINITION:3.1

Let f: X→X be an endomorphism and μ_α^M be an **fuzzy -multiplication** of μ in X. We define a new fuzzy set in X by $(\mu_\alpha^M)_f$ in X as

$$(\mu_\alpha^M)_f(x) = (\mu_\alpha^M)(f(x)) = \mu[f(x)]+\alpha, \ \forall\ x \in X.$$

THEOREM:3.2

Let f be an endomorphism of PS-algebra X. If u is a fuzzy PS-ideal of X, then so is $(\mu_\alpha^T\ )_f$

PROOF:

Let μ be a fuzzy PS-ideal of X.

Now, $((\mu_\alpha^T\ )_f\,(0) = (\mu_\alpha^T\ )[f(0)]$

$= \mu\,[f(0)]+\alpha$

$\geq\ \mu\,[f(x)]+a$

$= (\mu_\alpha^T\ )_f\,(f(x))$

$= (\mu_\alpha^T\ )_f\,(x)$

$\Rightarrow(\mu_\alpha^T\ )_f\,(0) \geq (\mu_\alpha^T\ )_f(x)$

Let x, y ∈ X.

Then $(\mu_\alpha^T\ )_f\,(x) = [f(x)]$

$= \mu\,[f(x)] + \alpha$

$= \min\{(\mu_\alpha^T\ )_f\,(y*x), (\mu_\alpha^T\ )_f\,(y)$

$\therefore\ (\mu_\alpha^T\ )_f\,(x) \geq \min\{(\mu_\alpha^T\ )_f\,(y*x),(\mu_\alpha^T\ )_f\,(y)\}$

Hence $(\mu_\alpha^T\ )$t is a fuzzy PS-ideal of X.

Hence the proved.

THEOREM:3.3

Let f: X→Y be an epimorphism of PS-algebra. If $(\mu_\alpha^T\ )_f$ is a fuzzy PS-ideals of X, then u is a fuzzy PS-ideals of Y.

PROOF:

Let $(\mu_\alpha^T\ )_f$ be a fuzzy PS-ideals of X and let y ∈ Y. Then there exists x ∈ X such that f(x) = y

Now, $\mu(0) + \alpha = (\mu_\alpha^T\ )$

$= \mu_\alpha^T\ [f(0)]$

$= (\mu_\alpha^T\ )_f\,(0)$

$\geq\ (\mu_\alpha^T\ )_f\,(x)$

$= (\mu_{\alpha}^{T})_f [f(x)]$

$= \mu (f(x)) + \alpha$

and so $\mu(0) \geq \mu (f(x)) = \mu(y)$

$\mu(0) \geq \mu(y)$

Let $y_1, y_2 \in Y$

$$\mu(y1)+ \alpha = (\mu_{\alpha}^{T})(y_1)$$
$$= \mu_{\alpha}^{T} (f(x_1))$$
$$= (\mu_{\alpha}^{T})_f (x_1)$$
$$\geq \min \{(\mu_{\alpha}^{T})_f (x_2 * x_1), (\mu_{\alpha}^{T})_f (x_2)\}$$
$$= \min \{\mu_{\alpha}^{T} [f(x_2 * x_1)], \mu_{\alpha}^{T} [f(x_2)]\}$$
$$= \min \{\mu_{\alpha}^{T} [f(x_2) \triangle f(x_1)], \mu_{\alpha}^{T} [f(x_2)]\}$$
$$= \min \{\mu_{\alpha}^{T} [y_2 \triangle y_1], \mu_{\alpha}^{T} [y_2]\}$$
$$= \min \{\mu (y_2 \triangle y_1) + \alpha_1\ \mu(y_2)+\alpha\}$$
$$= \min \{\mu (y_2 \triangle y_1), \mu(y_2)\} + \alpha$$
$$\therefore \quad \mu(y_1) \geq \min \{\mu (y_2 \triangle y_1), \mu (y_2))$$

Hence μ is a fuzzy PS-ideal of Y.

Hence the proved.

THEOREM: 3.4

Let f: X→Y be a homomorphism of PS-algebra. If μ is a fuzzy PS-ideals of Y, then $(\mu_{\alpha}^{T})_f$ is a fuzzy PS-ideals of X.

PROOF:

Let μ be a fuzzy PS-ideals of Y and let $x,y \in X$

Then, $(\mu_{\alpha}^{T})_f (0) = \mu_{\alpha}^{T} [f(0)]$

$$= \mu(f(0)) + \alpha$$
$$\geq \mu(f(x)) + \alpha$$
$$= \mu_{\alpha}^{T} [f(x)]$$
$$= (\mu_{\alpha}^{T})_f (x)$$

$\Rightarrow \quad (\mu_\alpha^T)_f (0) \geq (\mu_\alpha^T)_f (x)$

Also $(\mu_\alpha^T)_f (x) = \mu_\alpha^T [f(x)]$

$= \mu (f(x)) + \alpha$

$\geq \min \{\mu(f(y) \triangle f(x), \mu f(y))\} + \alpha$

$= \min\{\mu(fy*x)), \mu(f(y))\} + \alpha$

$= \min\{\mu(f(y*x)) + \alpha, \mu(f(y)) + \alpha\}$

$= \min \{\mu_\alpha^T [f(y*x)], \mu_\alpha^T [(f(y)]\}$

$= \min \{\mu_\alpha^T [f(y*x)], \mu_\alpha^T [f(y)]$

$= \min \{(\mu_\alpha^T)_f (y*x), (\mu_\alpha^T)_f (y)\}$

$= \min\{\mu (y_2 \triangle y_1), \mu(y_2)\} +\alpha$

$\therefore \quad (\mu_\alpha^T)_f (x) \geq \min \{\mu (y_2 \triangle y_1), \mu(y_2)\}+ \alpha$

Hence (4) is a fuzzy PS-ideal of X.

Hence the proved.

DEFINITION: 3.5

A fuzzy set μ in a PS-algebra. X is called a **fuzzy PS-sub algebra** of X. if $\mu (x* y) \geq \min \{(x), \mu (y)\}$ for all $x, y \in X$.

THEOREM:3.6

If μ be a fuzzy PS- sub algebra of X, then (u^t) is also a fuzzy PS-sub algebra of X

PROOF:

Let μ be a fuzzy PS-sub algebra of X. Let $x, y \in X$.

$\geq \min \{\mu (f(x)), \mu (f(y))\} + \alpha$

$= \min\{ \mu (f(x)) + \alpha, \mu(f(y)) + \alpha\}$

$= \min \{\mu_\alpha^T [f(x)], \mu_\alpha^T [f(y)]\}$

$= \min \{ (\mu_\alpha^T)_f (x), (\mu_\alpha^T)_f (y)\}$

$\Rightarrow (\mu_\alpha^T)_f \mu(x*y) \geq \min \{ (\mu_\alpha^T)_f (x), (\mu_\alpha^T)_f (y)\}$

Hence $(\mu_\alpha^T)_f$ is a fuzzy PS-sub algebra of X.

Hence the proved.

DEFINITION: 3.7

Let f:X→X be an endomorphism and μ_α^T be a **fuzzy α-translation** of μ in X. We define a new fuzzy set in X by $(\mu_\alpha^T)_f$ in X as

$(\mu_\alpha^T)_f (x) = (\mu_\alpha^T)(f(x)) = \mu[f(x)] + \alpha, \forall x \in X.$

THEOREM:3.8

Let f: X→Y be a homomorphism of PS-algebra X into a PS-algebra Y and μ_α^T be a fuzzy- α translation of μ, then the pre-image of μ_α^T denoted by $f^1(\mu_\alpha^T)$ is defined as $\{f^1(\mu_\alpha^T)\}(x) = \mu_\alpha^T(f(x)), \forall x \in X$. If μ is a fuzzy PS-sub algebra of Y, then $f^1(\mu_\alpha^T)$ is a fuzzy PS-sub algebra of X.

PROOF:

Let μ is a fuzzy PS-sub algebra of Y.

Let $x, y \in X$.

Now, $[f^1(\mu_\alpha^T)\}(x*y) = \mu_\alpha^T[(f(x*y)]$

$= \mu(f(x*y)) + \alpha$

$= \mu(f(x) \triangle f(y) + \alpha$

$\geq \min\{\mu(f(x)), \mu(f(y))\} + \alpha$

$= \min\{\mu(f(x)) + \alpha, \mu(f(y)) + \alpha\}$

$= \min\{\mu_\alpha^T[f(x)], \mu_\alpha^T[f(y)]\}$

$= \min\{(f^1(\mu_\alpha^T)\}(x), \{f^1(\mu_\alpha^T)\}(y)\}$

$\{f^1(\mu_\alpha^T)\}(x*y) \geq \min\{\{f^1(\mu_\alpha^T)\}(x), \{f^1(\mu_\alpha^T)\}(y)\}$

$f^1(\mu_\alpha^T)$ is a fuzzy PS-sub algebra of X.

Hence the proved.

THEOREM :3.9

Let f be an endomorphism of PS-algebra X. If u is a fuzzy PS-ideal of X, then so is $(\mu_\alpha^M)_f$.

PROOF:

Let μ be a fuzzy PS-ideal of X

Now, $(\mu_\alpha^M)_f (0) = \mu_\alpha^M [f(0)]$

$= \alpha\, \mu [f (0)]$

$\geq \alpha\, \mu [f(x)]$

$= (\mu_\alpha^M) [f(x)]$

$= (\mu_\alpha^M)_f (x)$

$\Rightarrow (\mu_\alpha^M)_f (0) \geq (\mu_\alpha^M)_f (x)$

Let x, y ∈ X.

Then $(\mu_\alpha^M)_f (x) = (\mu_\alpha^M) [f(x)]$

$= \alpha\, \mu [f(x)]$

$= \alpha \min \{ \mu(f(y * x)),\mu(f(y))\}$

$= \min\{a\, \mu [f(y*x)], a\, \mu (f(y))\}$

$= \min \{\mu_\alpha^M [f(y*x)], \mu_\alpha^M (f(y))\}$

$= \min \{(\mu_\alpha^M)_f (y*x), (\mu_\alpha^M)_f (y)\}$

$(\mu_\alpha^M)_f (x) \geq \min \{(\mu_\alpha^M)_f (y*x), (\mu_\alpha^M)_f (y)\}$

Hence (μ_a^m) is a fuzzy PS-ideal of X.

Hence Proved.

THEOREM: 3.10

Let f: X→Y be an endomorphism of PS-algebra. If (μ^m) is a fuzzy PS- ideals X, then u is a fuzzy PS-ideals of Y.

PROOF:

Let $(\mu_\alpha^M)_f$ be a fuzzy PS-ideal of X and let y ∈ Y .

Then there exists x € X such that f(x)=y.

Now,

$\alpha\ \mu(0) = \mu_\alpha^M\ (0)$

$= \mu_\alpha^M\ [f(0)]$

$= (\mu_\alpha^M)_f\ (0)$

$\geq (\mu_\alpha^M)_f\ (x)$

$= \mu_\alpha^M\ [f(x)]$

$= \alpha\ \mu[f(x)]$

$\Rightarrow \mu(0) \geq \mu[f(x)] = \mu(y).$

$\therefore\ \mu(0) - \mu(y).$

Let $y_1, y_2 \in Y$.

$\alpha\ \mu(y1) = \mu_\alpha^M\ [y1]$

$=(\mu_\alpha^M)[f(x1)]$

$=(\mu_\alpha^M)(x1)$

$\geq \min((\mu_\alpha^M)_f (x_2 * x_1), (\mu_\alpha^M)_f (x_2))$

$= \min\{\mu_\alpha^M [f(x_2 * x_1)], \mu_\alpha^M [f(x_2)]\}$

$= \min\{\mu_\alpha^M (f(x_2) \triangle f(x_1)], f(x_2))\}$

$= \min \mu_\alpha^M ([y_2 \triangle y_1], \mu_\alpha^M [y_2]\}$

$= \min\{\mu_\alpha^M [y_2 \triangle y_1], \mu_\alpha^M [y_2]\}$

$= \alpha \min\{\mu[y_2 \triangle y_1]. [y_2]\}$

$\mu(y_1) \geq \min\{u[y_2 \triangle y_1], \mu[y_2]\}$

Hence μ is a fuzzy PS-ideal of Y.

Hence the proved.

THEOREM: 3.11

If μ be a fuzzy PS-sub algebra of X then (μ_α^M) is also a fuzzy PS-sub algebra of X.

PROOF:

Let μ be a fuzzy PS-sub algebra of X.

Let x, y ∈ X.

Now,

$(\mu_\alpha^M)_f$ (x*y) = μ_α^M [f(x*y))

= α μ (f(x*y))

= α μ (f(x) f(y))

≥ α min (μ(f(x)), μ(f(y))}

= min{αμ(f(x)), αμ (f(y)

= min{ μ_α^M [f(x)], [f(y)]}

= min{ μ_α^M f (x), (μ_α^M) f(y)}

⇒ $(\mu_\alpha^M)_f$ (x*y) ≥ min ((μ_α^M) f(x), (μ_α^M)f(y))

Hence $(\mu_\alpha^M)_f$ is a fuzzy PS-sub algebra of X.

Hence the proved.

THEOREM:3.12

Let f: X→Y be a homomorphism of PS-algebra X into a PS-algebra Y and μ_α^M be a fuzzy- α translation of μ, then the pre-image of μ_α^M denoted by $f^{-1}(\mu_\alpha^M)$ is defined as {f-¹ (μ_α^M)}(x) = μ_α^M α(x)), ∀ x ∈ X. If μ is a fuzzy PS-sub algebra of Y, then f¹ $(\mu_\alpha^M$)is a fuzzy PS-sub algebra of X.

PROOF:

Let μ is a fuzzy PS-sub algebra of Y.

Let x, y ∈ X.

Now, {f-¹ (μ_α^M)} (x*y) = μα^M [f(x*y)]

=α μ (f(x*y))

=αμ (f(x) Δ f(y))

≥ α min {μ (f(x)), μ(f(y))}

= min { α μ (f(x)), μ (f(y)}

= min { μ_α^M αf (x)], μ_α^M [f(y)]}

= min {{ f^1 (μ_α^M)}(x), { f^1 (μ_α^M)}(y)}

$\Longrightarrow${f^{-1} (μ_α^M)} (x*y) $\geq$ min {{ f^1 (μ_α^M)(x)}μ_α^M, {f^1(μ_α^M)(y)}}

Hence f^1 (μ_α^M) is a fuzzy PS-sub algebra of X.

Hence the proved.

THEOREM: 3.13

If μ and δ are fuzzy PS-sub algebra of a PS-algebra X, then μ_α^T X δ_α^T is also a fuzzy PS-sub algebra of X-X.

PROOF:

For any $x_1, x_2, y_1, y_2 \in$ X.

(μ_α^T X δ_α^T)(x_1, y_1) * (x_2, y_2) = (μ_α^T X δ_α^T) (x_1* x_2, y_1 * y_2)

= min{μ_α^T (x_1*x_2), δ_α^T (y_1*y_2)}

= min{ μ (x_1*x_2) + α, δ(y_1*y_2) + α}

= min{ μ (x_1*x_2), δ(y_1*y_2) + α} + α

$\geq$ min{ min{ μ(x_1) * μ(x_2)}, min {δ(y_1)* δ(y_2) + α} }+ α

= min{ min{ μ (x_1) , μ x_2)} + α, min {δ(y_1), δ(y_2)} + α}

= min{ min{ μ (x_1) + α, μ (x_2) + α, min {δ(y_1)+ α, δ(y_2)} + α}

= min{ min{ μ_α^T (x_1) , μ_α^T (x_2)}, min{ δ_α^T (y_1) , δ_α^T (y_2)}

= min{ min{ μ_α^T (x_1) , δ_α^T (y_1)}, min{ μ_α^T (x_2), δ_α^T (y_2)}

= min{ μ_α^T X δ_α^T (x_1,y_1) , (μ_α^T X)(x_2 (μ_α^T X δ_α^T)((x_1,y_1)* (x_2,y_2))}

$\Longrightarrow$(μ_α^T δ_α^T) (x_1,y_1) *(x_2,y_2) $\geq$ min {(μ_α^T δ_α^T T (x_1,y_1), (μ_α^T X δ_α^T) (x_2,y_2)

Hence the proved.

CONCLUSION

In this article we discussed fuzzy PS-ideals and PS-sub algebras in fuzzy PS-algebra under homomorphism and Cartesian product.

Interestingly, the strongest fuzzy β- relation on PS-algebra concept has been discussed in Cartesian products and it adds on another dimension to the defined fuzzy PS-algebra. This concept can further be generalized to intuitionistic fuzzy set, interval valued fuzzy sets, Anti fuzzy sets for new results in future.

REFERENCES

[1]. **Kleki and S.Tanaka**, An introduction to the theory of BCK- algas, MathJaponica 23(1978) 1-20

[2] **Kiseki**, On BCK-algebras, Math Seminar Notes, 8(1980)125-130. [3]. T.Priya and T.Ramachandran, Fuzzy translation and fuzzy multiplication on PS- algebras, International journal of Innovation in Science and mathematics, 2(5) (2014) 485-489.

14). **T.Priya and T.Ramachandran**, Homomorphism of cartesian product of fuzzy PS-algebra, Applied Mathematics Sciences, 8, vol(67) (2014) 3321-3330

[5]. **T.Priya and T.Ramachandran**, Classification of PS-algebras, The International journal of Science and Technology, communicated.

[6] **T.Priya and T.Ramachandran,** Anti fuzzy PS-ideals in PS-algebra and its level subsets, International Journal of mathematical archive, vol 5, No.4 (2014) 1-7.

[7] **T.Senapati, M.Bhowmilk** and M.Pal, Fuzzy translation closed ideals B- algebras with interval valued membership function, International Joumal fuzzy mathematical archive, 1(2013) 79-91.

[8]. **Zadeh.L.A**. Fuzzy sets, Information and controls, 8 (1965) 338-353.