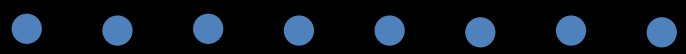
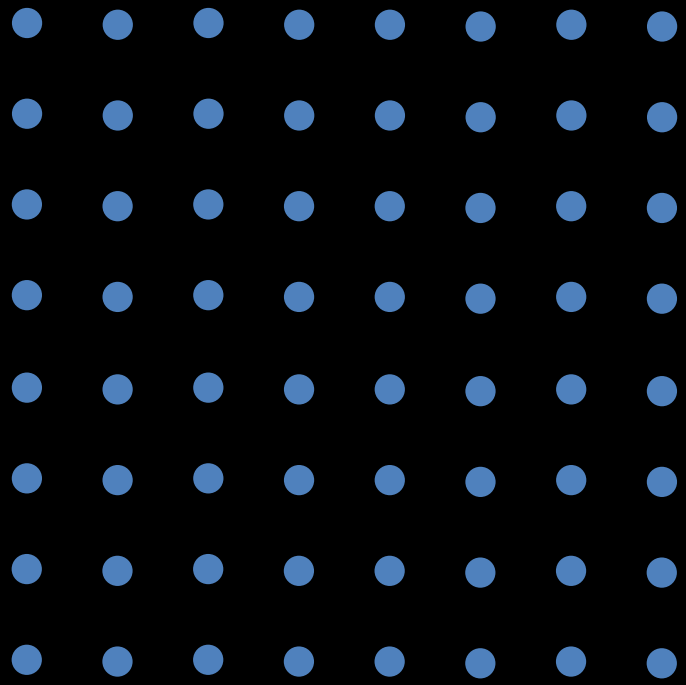
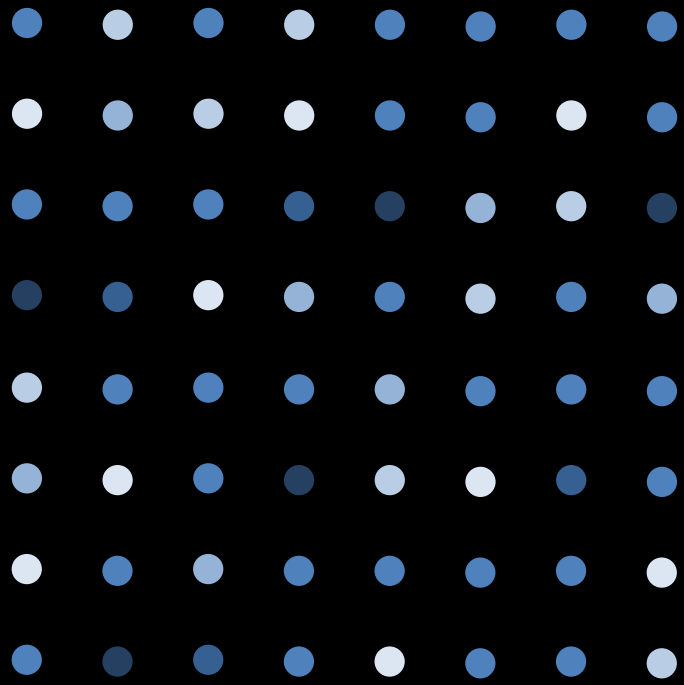
X. Wu and D. Chen:

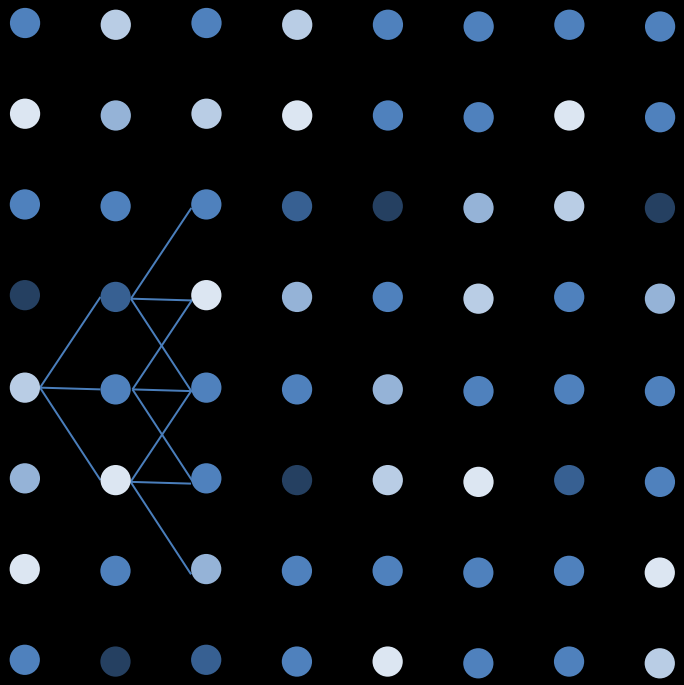
Optimal Net Surface Problems with Applications

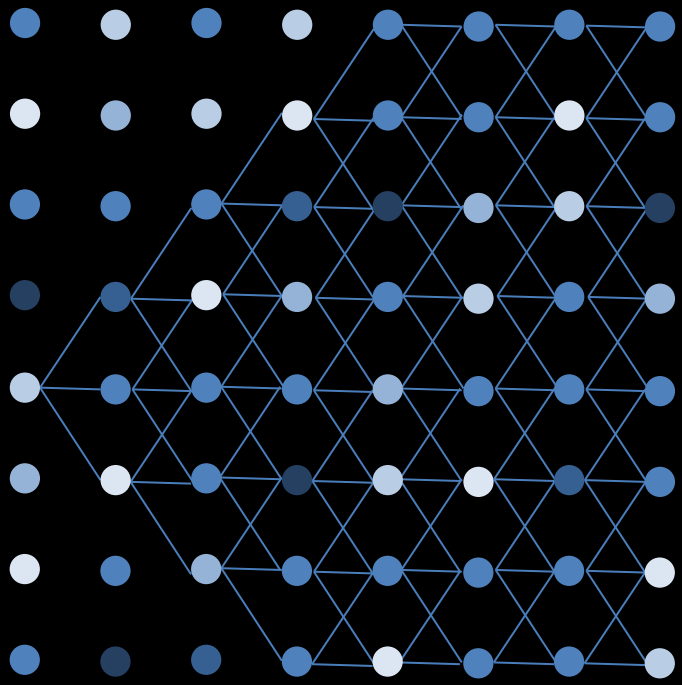
In: Automata, Languages and Programming,
Springer LNCS, vol 2380, pp 1029-1042, 2002.

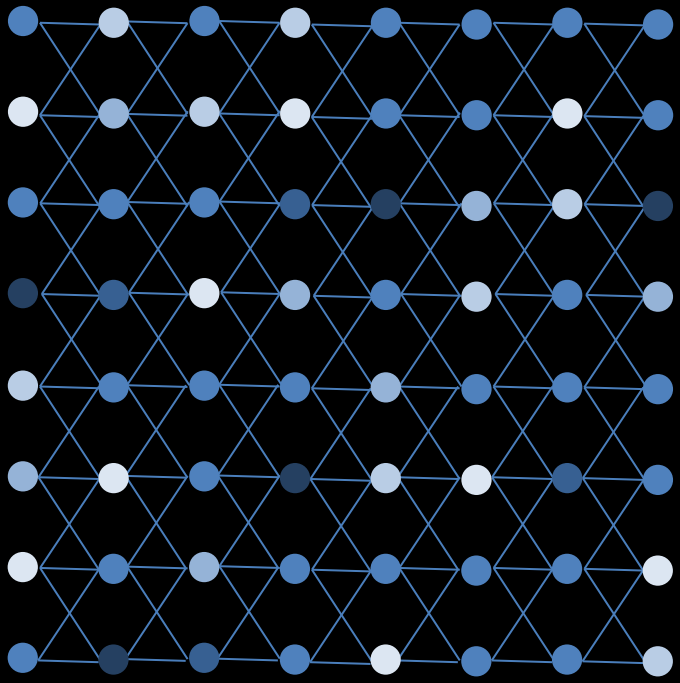


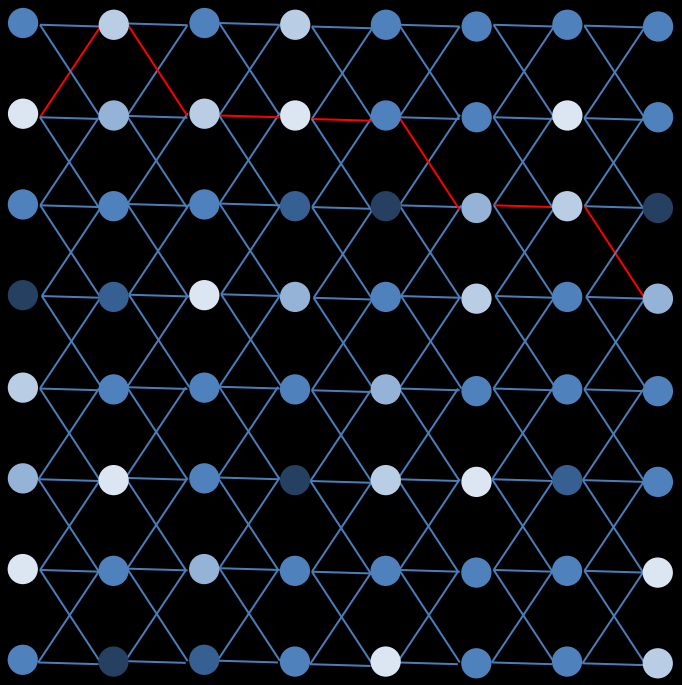


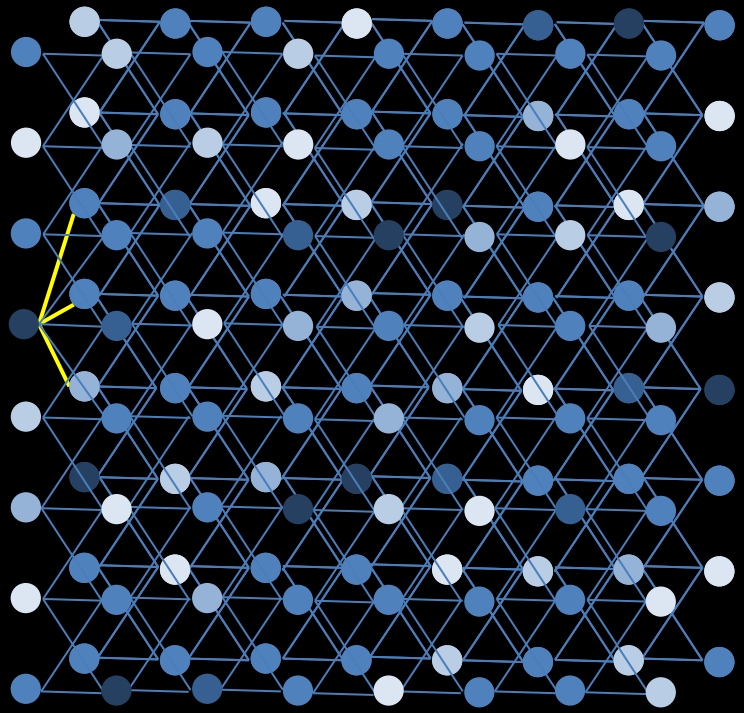




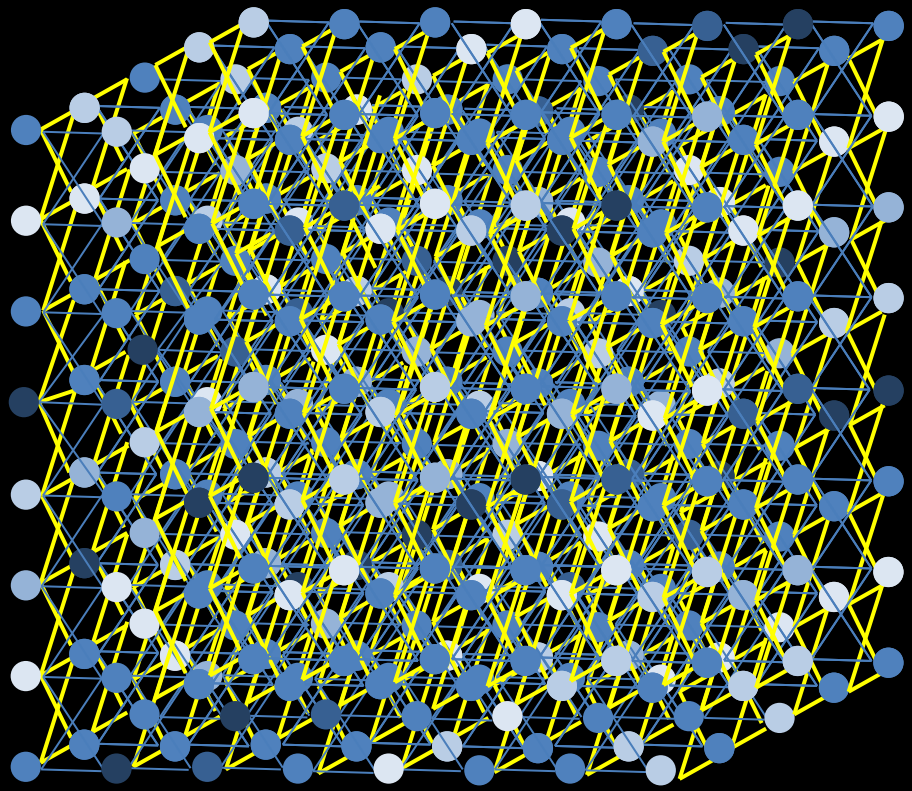




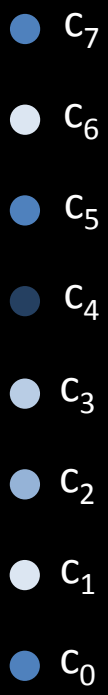

Optimal Path

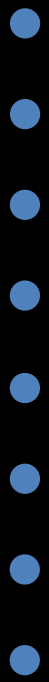


Optimal Surface = ?



Step 1: Surface ⇔ Non-Empty Closed Set

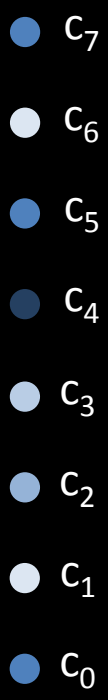




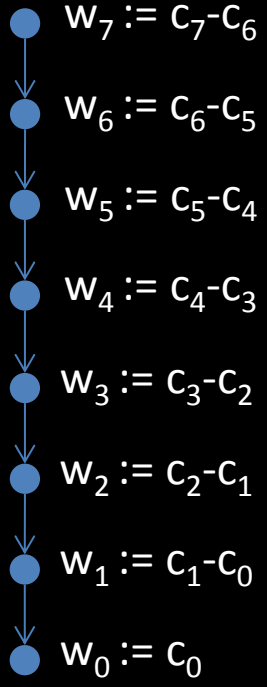
Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

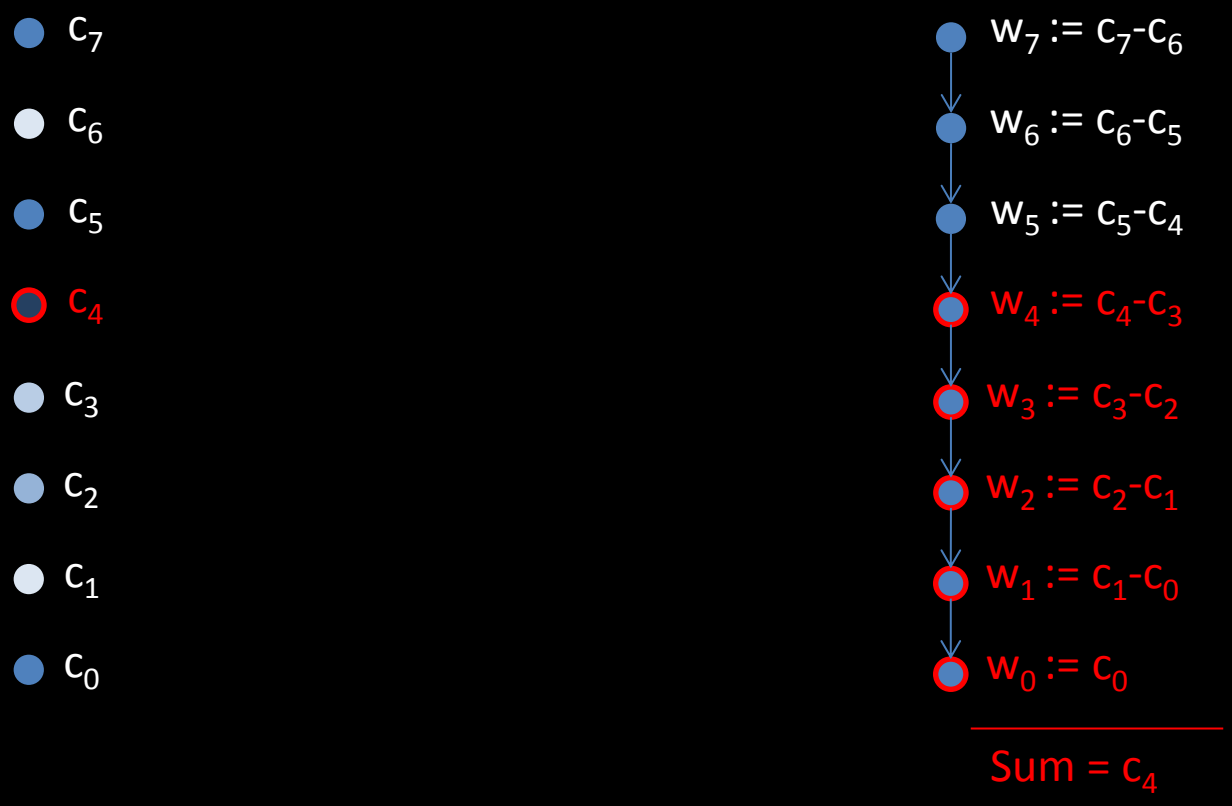


Surface



Non-Empty Closed Set

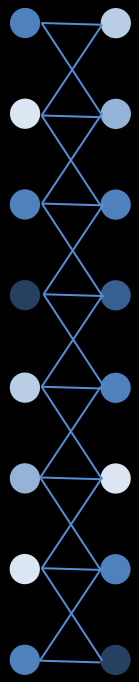
Step 1: Surface ⇔ Non-Empty Closed Set

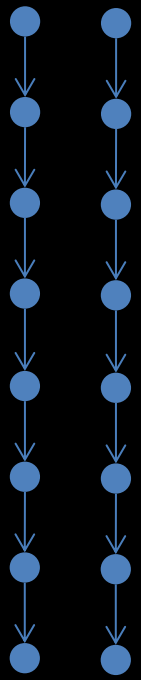


Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

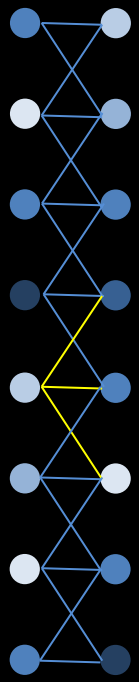


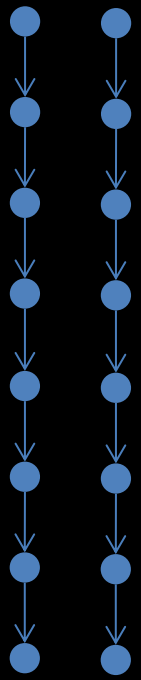


Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

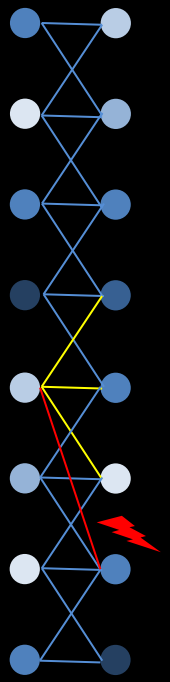


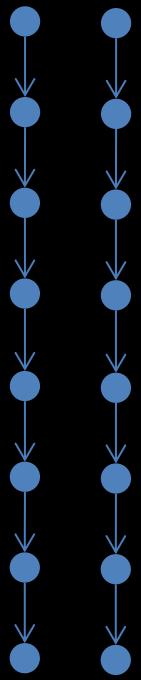


Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

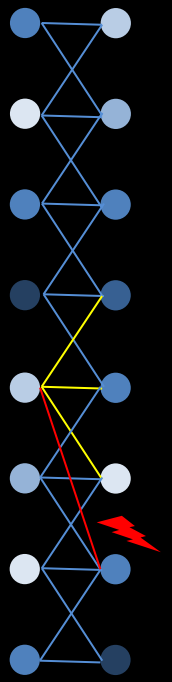


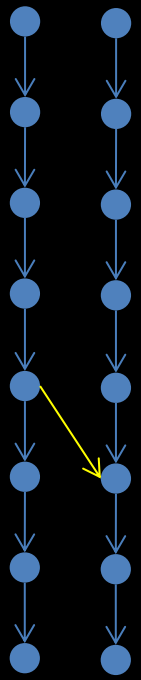


Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

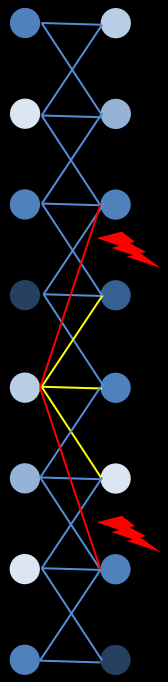


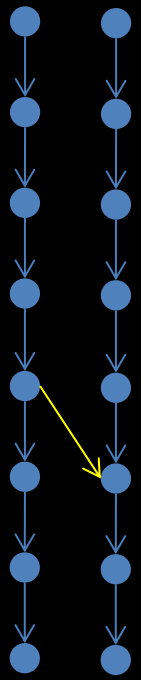


Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

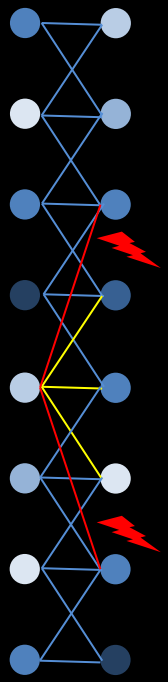


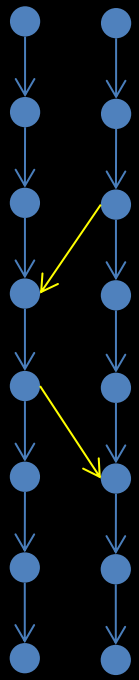


Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

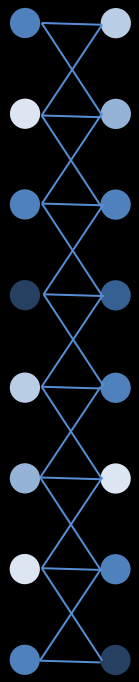


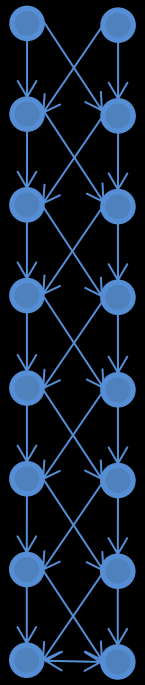


Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set



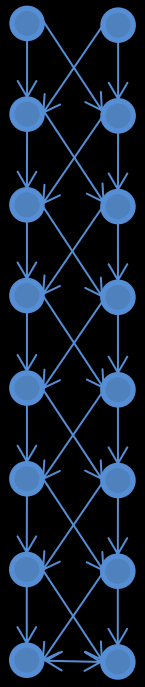


Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

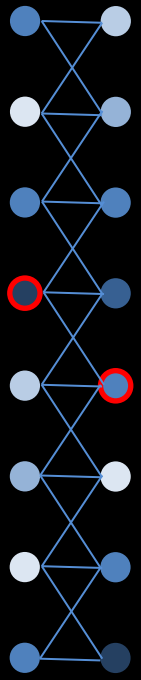
any surface corresponds to
a non-empty closed set
with same cost



Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set



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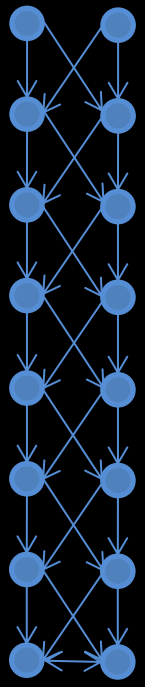

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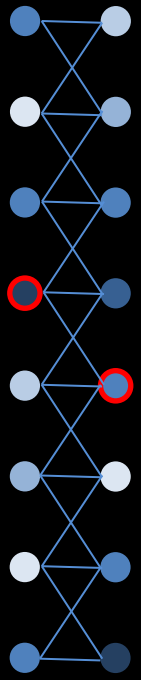

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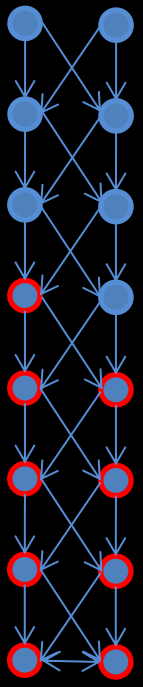

Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

any surface corresponds to
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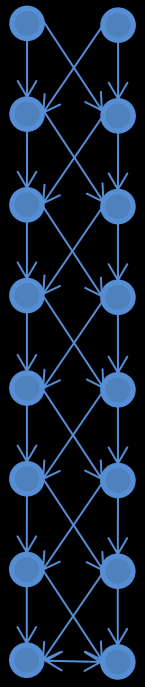
with same cost

any non-empty closed set
corresponds to a surface
with same cost



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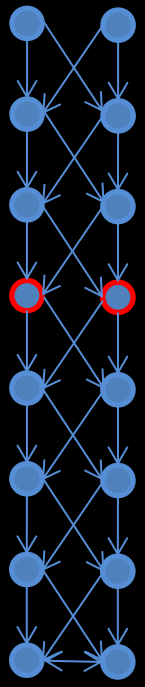


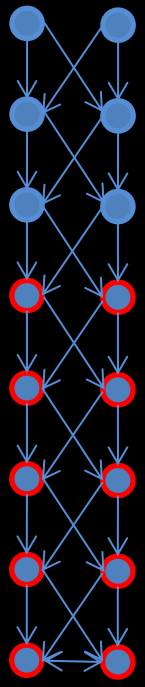
Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

any surface corresponds to a non-empty closed set with same cost

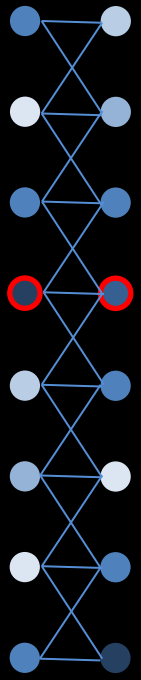
any non-empty closed set corresponds to a surface with same cost



Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set



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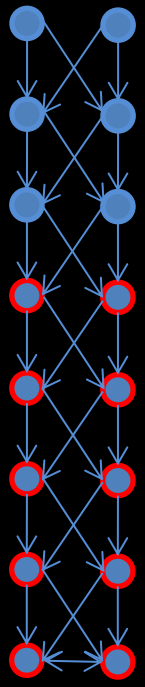

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Non-Empty Closed Set

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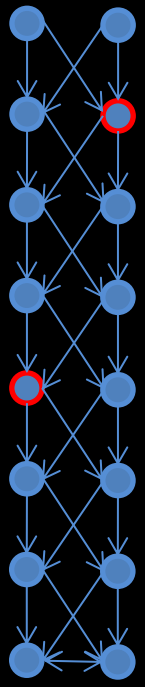
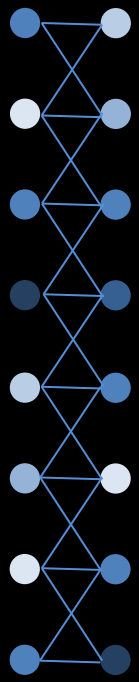


Surface

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set



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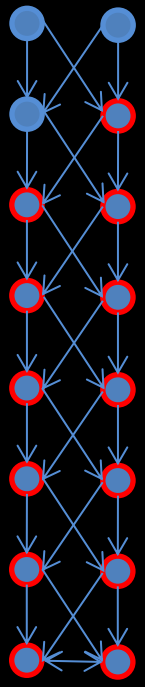

Surface

Non-Empty Closed Set

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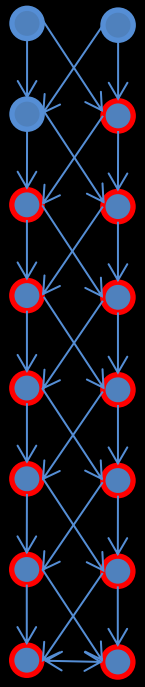

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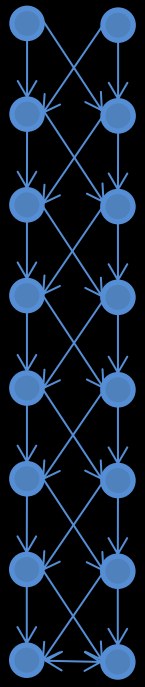
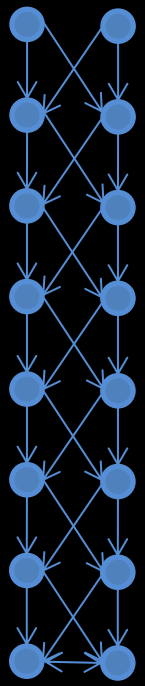


Minimal Surface Problem ⇔ Minimum Non-Empty Closed Set Problem

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut



Finite Cost s-t-Cut

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut

Finite Cost s-t-Cut

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut

Finite Cost s-t-Cut

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

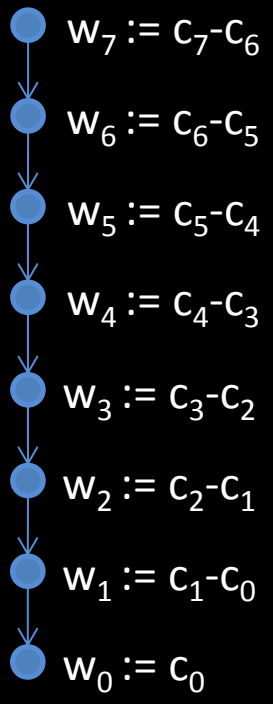
Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut

Finite Cost s-t-Cut

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut

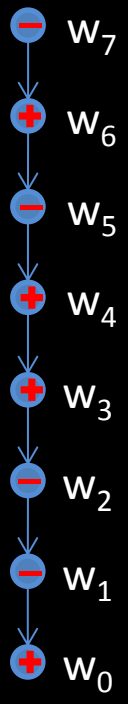


Finite Cost s-t-Cut

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut

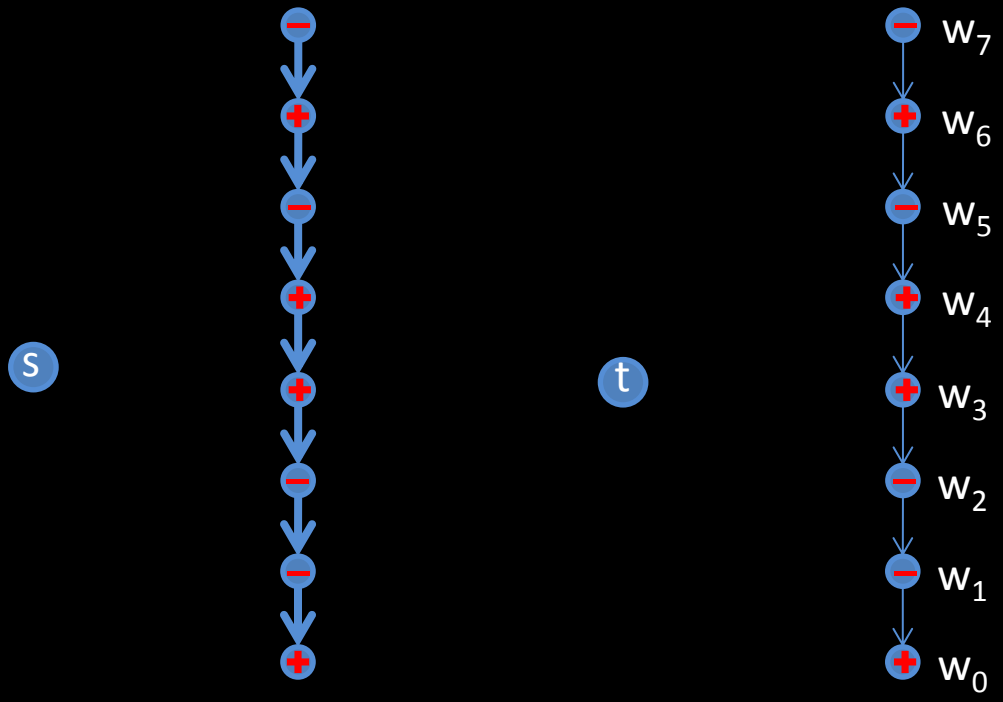


Finite Cost s-t-Cut

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut

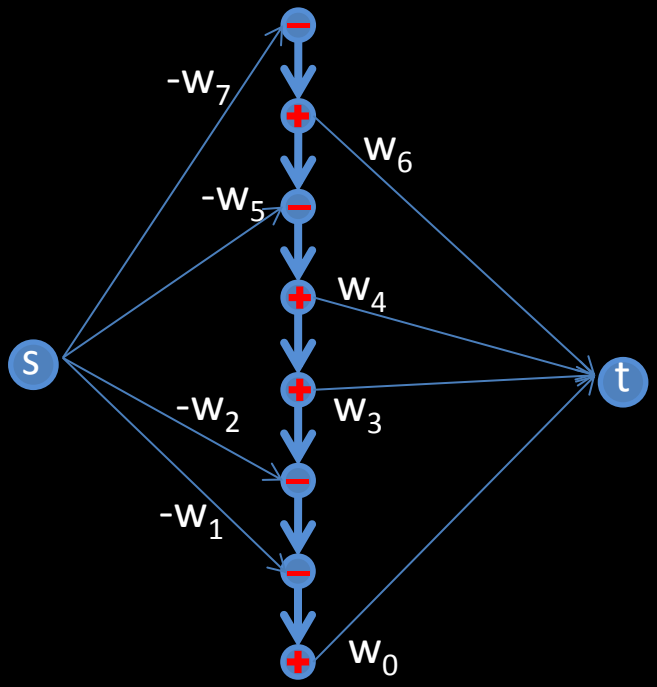


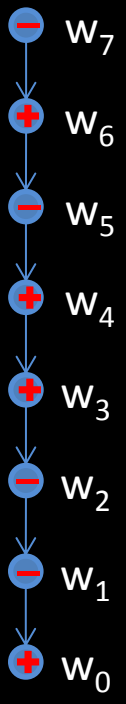
Finite Cost s-t-Cut

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut



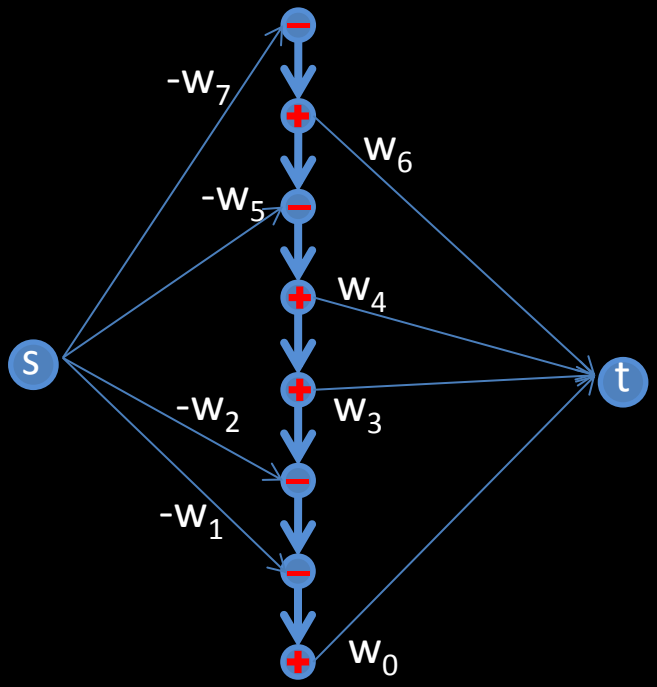


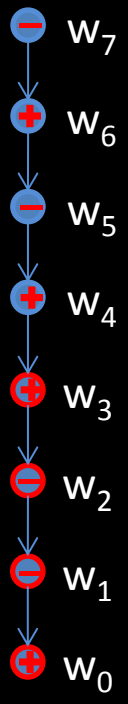
Finite Cost s-t-Cut

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut



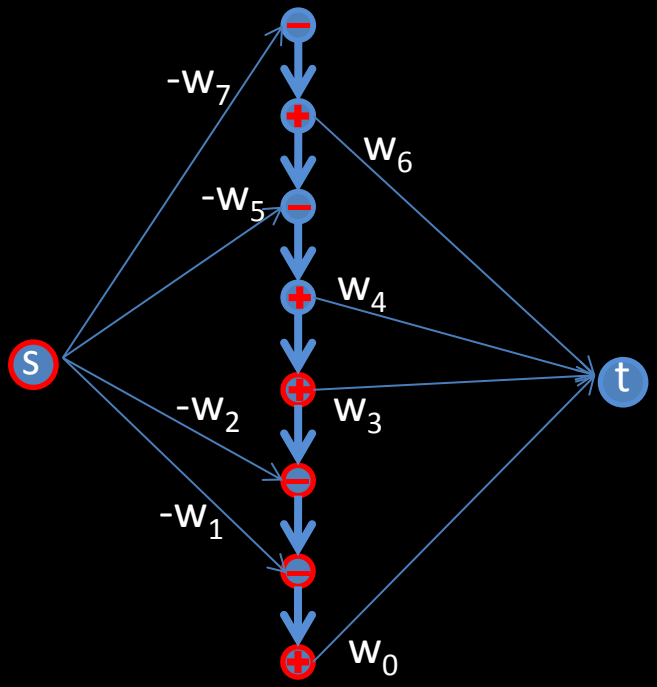


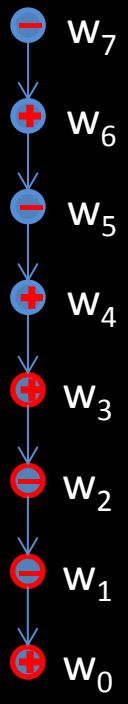
Finite Cost s-t-Cut

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut



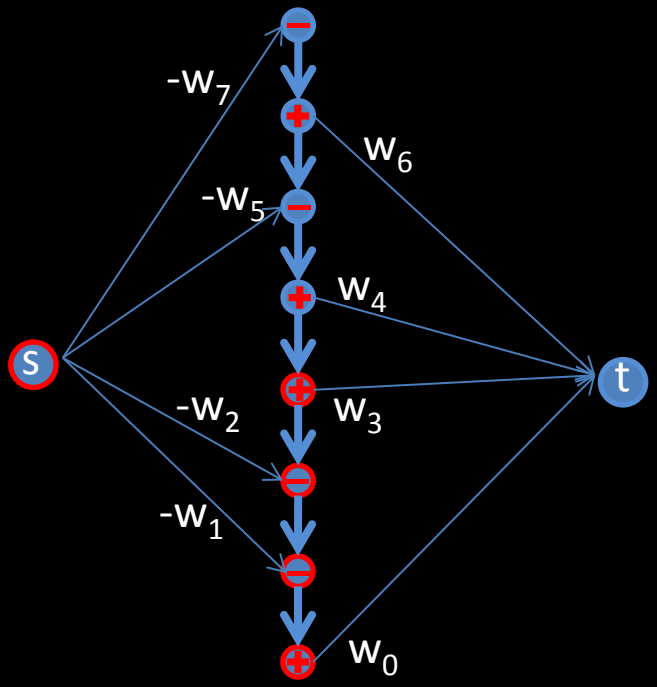


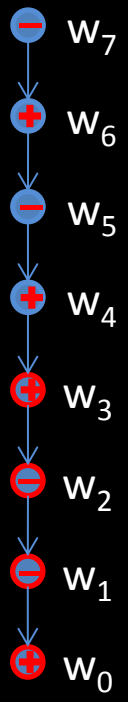
Finite Cost s-t-Cut

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut



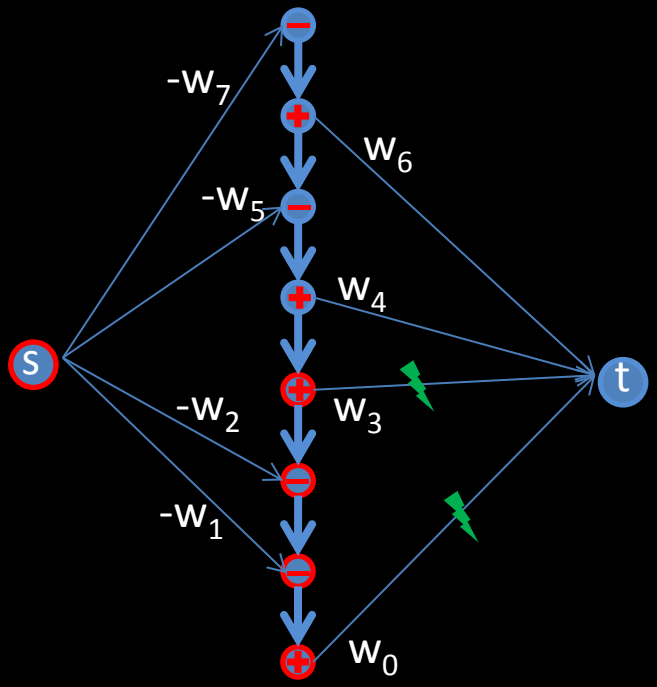


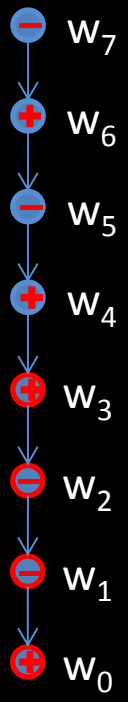
Cut:

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





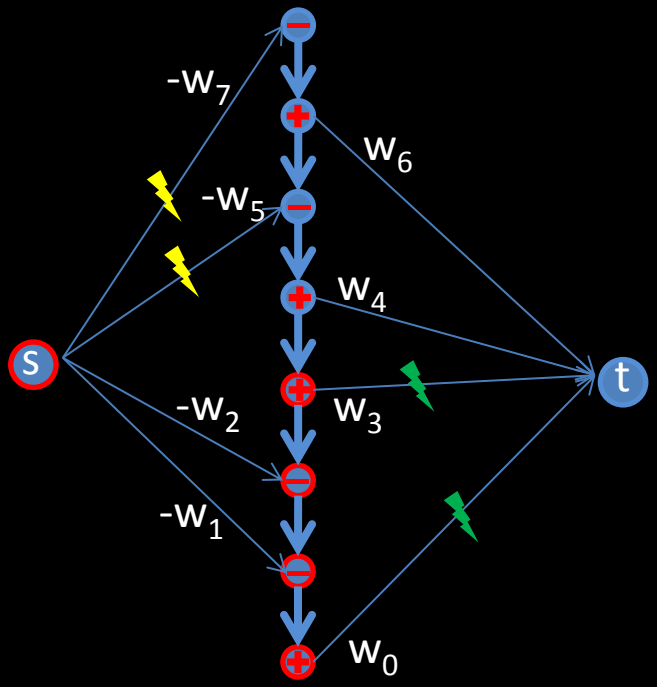
Cut:

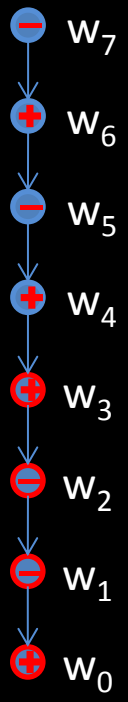
Source set to t: $w(V^+ \cap S)$

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





Cut:

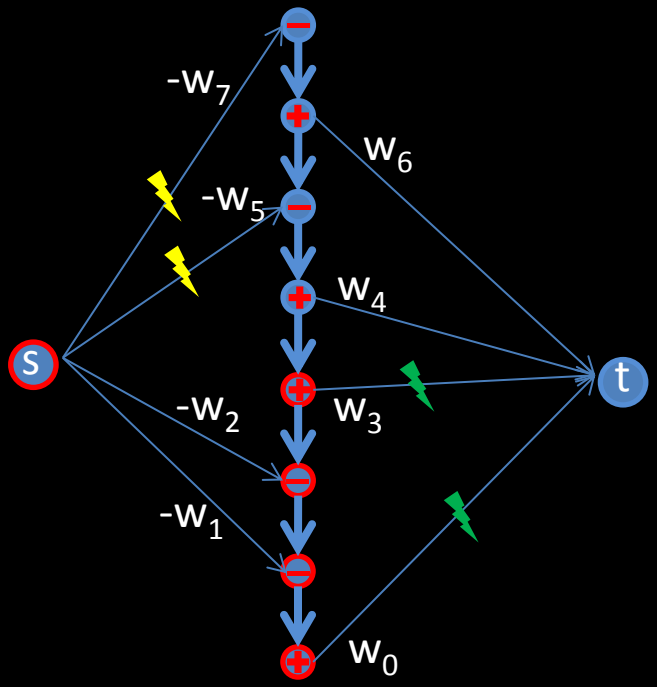
Source set to t: $w(V^+ \cap S)$

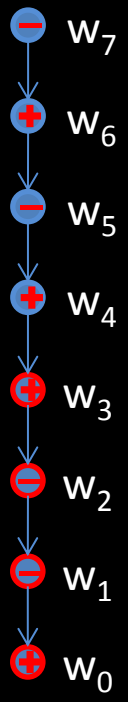
s to Terminal set: $-w(V^- \cap T)$

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





Cut:

Source set to t: $w(V^+ \cap S)$

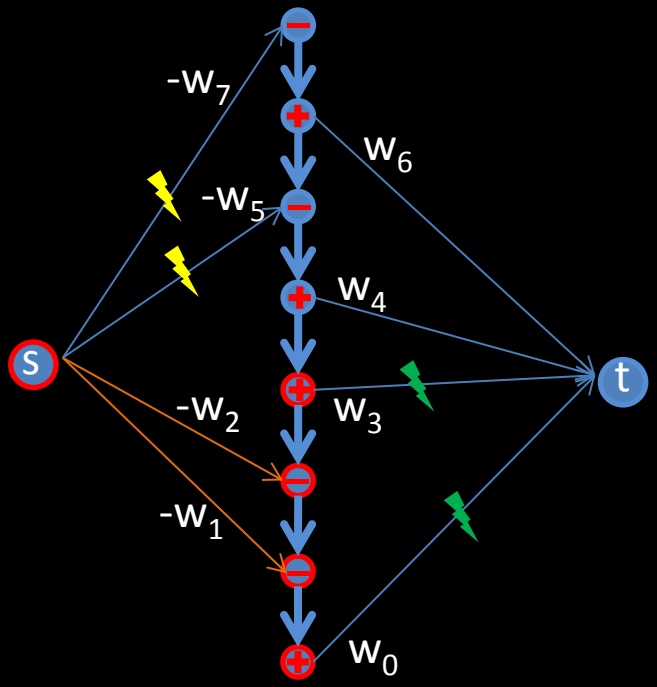
s to Terminal set: $-w(V^- \cap T)$

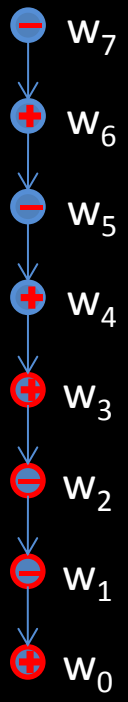
Cost of cut: $w(V^+ \cap S) - w(V^- \cap T)$

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





Cut:

Source set to t: $w(V^+ \cap S)$

s to Terminal set: $-w(V^- \cap T)$

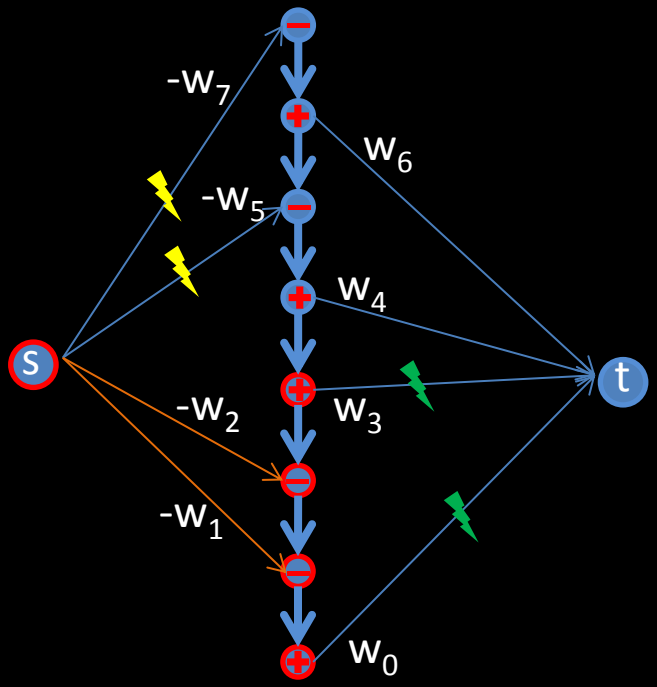
Cost of cut: $w(V^+ \cap S) - w(V^- \cap T)$

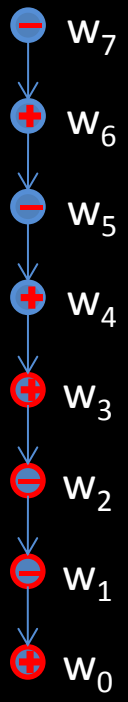
$= w(V^+ \cap S) + w(V^- \cap S) - w(V^- \cap T) - w(V^- \cap S)$

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





Cut:

Source set to t: $w(V^+ \cap S)$

s to Terminal set: $-w(V^- \cap T)$

Cost of cut: $w(V^+ \cap S) - w(V^- \cap T)$

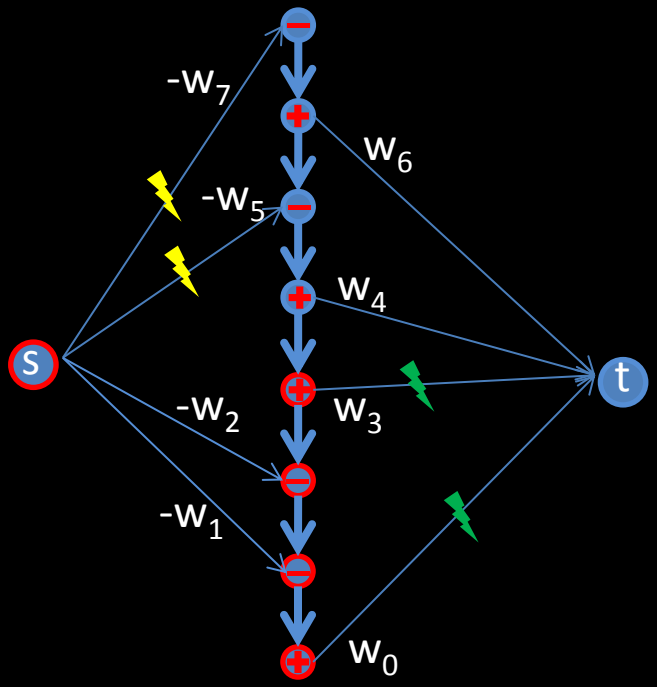
$= w(V^+ \cap S) + w(V^- \cap S) - w(V^- \cap T) - w(V^- \cap S)$

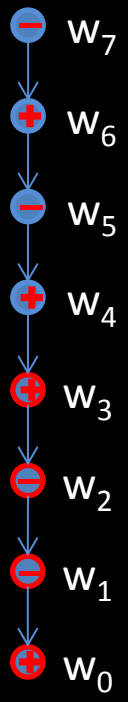
$= w(S) - w(V^-)$

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





Cut:

Source set to t: $w(V^+ \cap S)$

s to Terminal set: $-w(V^- \cap T)$

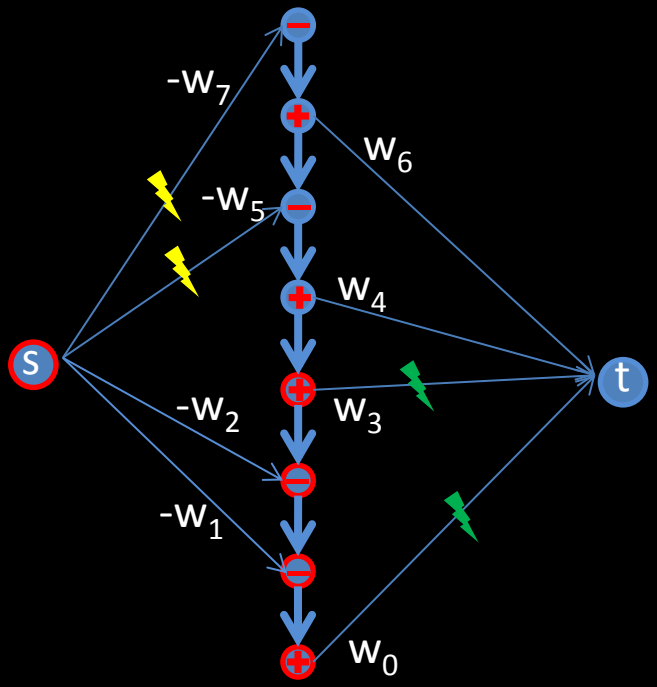
Cost of cut: $w(V^+ \cap S) - w(V^- \cap T)$

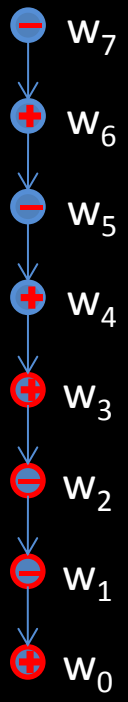
$= w(S) - w(V^-)$

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





any non-empty closed set
corresponds to a
finite cost s-t cut
with constant cost difference

Cut:

Source set to t: $w(V^+ \cap S)$

s to Terminal set: $-w(V^- \cap T)$

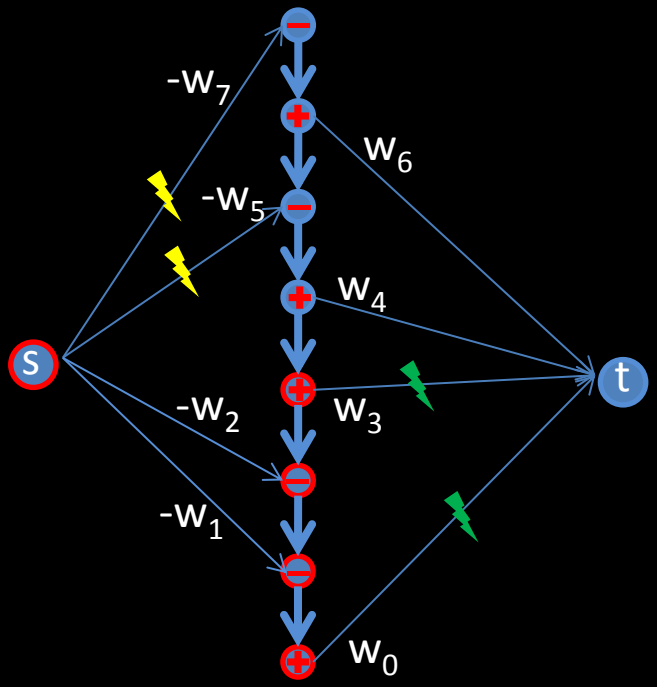
Cost of cut: $w(V^+ \cap S) - w(V^- \cap T)$

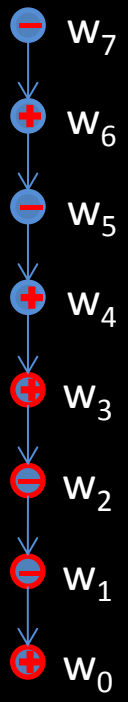
$= w(S) - w(V^-)$

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





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with constant cost difference

Cut:

Source set to t: $w(V^+ \cap S)$

s to Terminal set: $-w(V^- \cap T)$

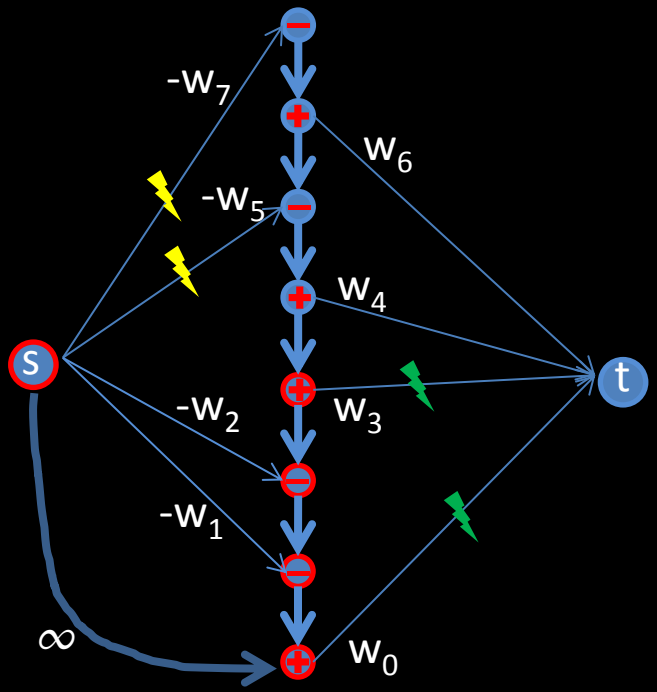
Cost of cut: $w(V^+ \cap S) - w(V^- \cap T)$

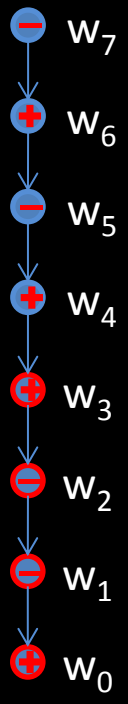
$= w(S) - w(V^-)$

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





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any finite cost s-t cut
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non-empty closed set
with constant cost difference

Cut:

Source set to t: $w(V^+ \cap S)$

s to Terminal set: $-w(V^- \cap T)$

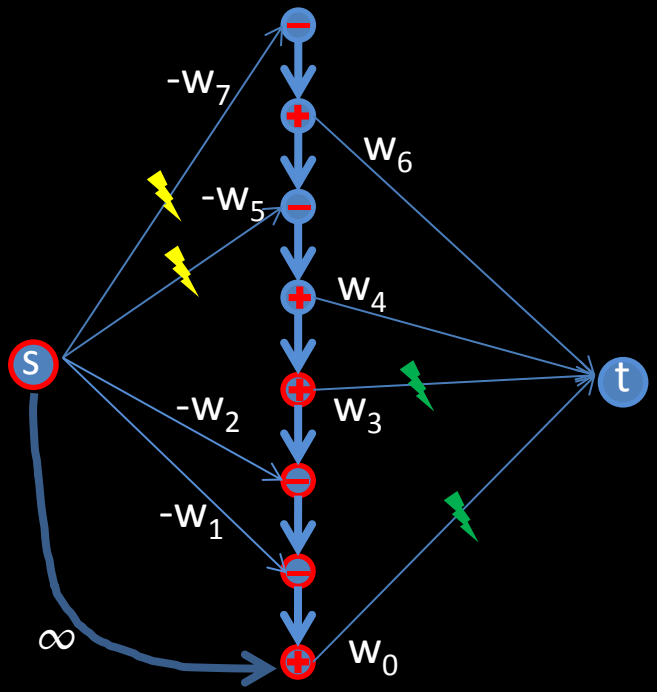
Cost of cut: $w(V^+ \cap S) - w(V^- \cap T)$

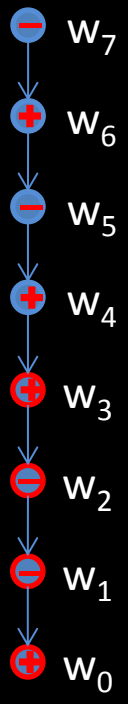
$= w(S) - w(V^-)$

Non-Empty Closed Set

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





any non-empty closed set
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any finite cost s-t cut
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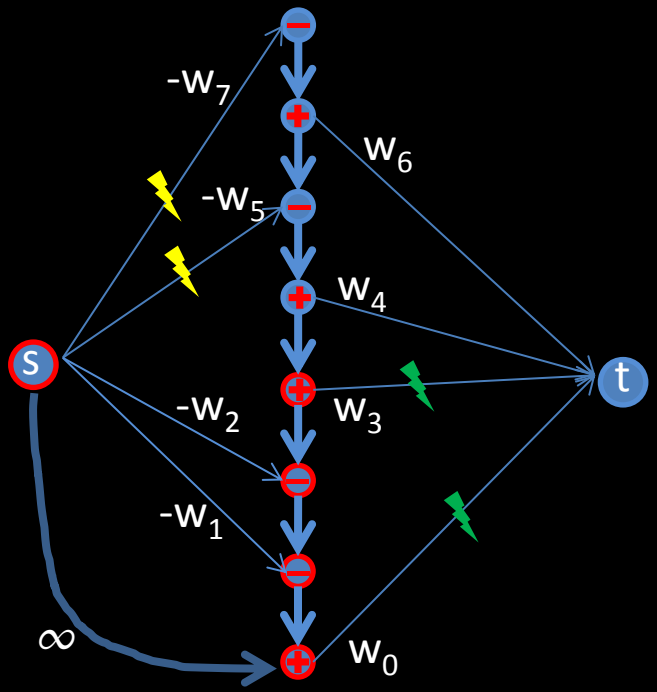
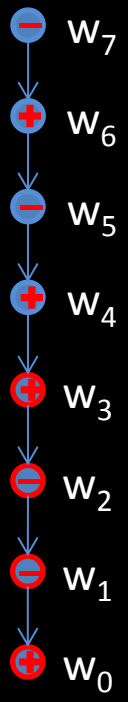
non-empty closed set
with constant cost difference

Minimum s-t Cut Problem ⇔ Minimum Non-Empty Closed Set Problem

Step 1: Surface ⇔ Non-Empty Closed Set

Step 2: Non-empty Closed Set ⇔ Finite Cost s-t-Cut





any non-empty closed set
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finite cost s-t cut
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any finite cost s-t cut
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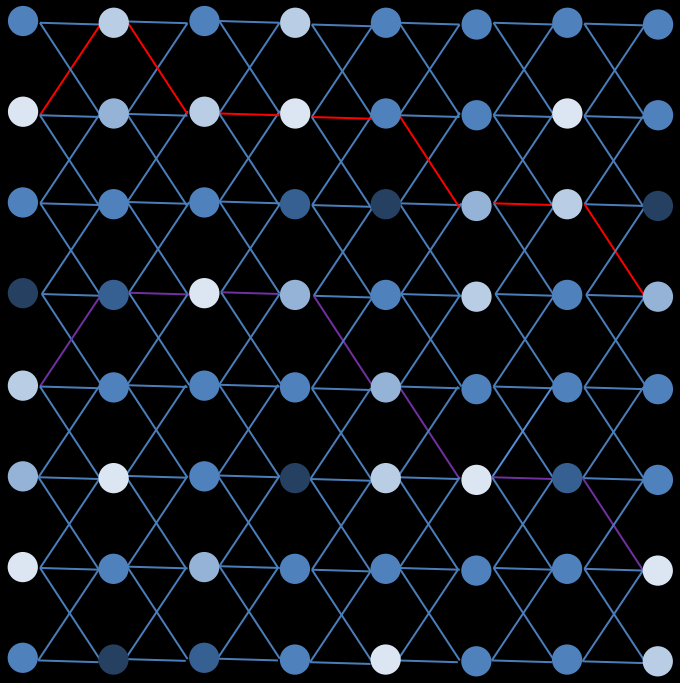
non-empty closed set
with constant cost difference

Minimum s-t Cut Problem ⇔ Minimum Non-Empty Closed Set Problem

⇔ Minimal Surface Problem

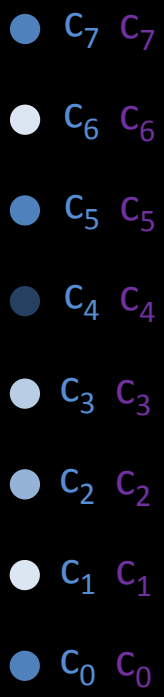
PART II

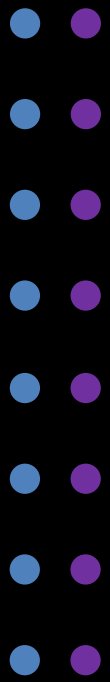
2 Surfaces with min/max distance



Two surfaces
min 2 apart
max 3 apart
Optimal total cost

2 Surfaces ⇔ 1 Non-Empty Closed Set

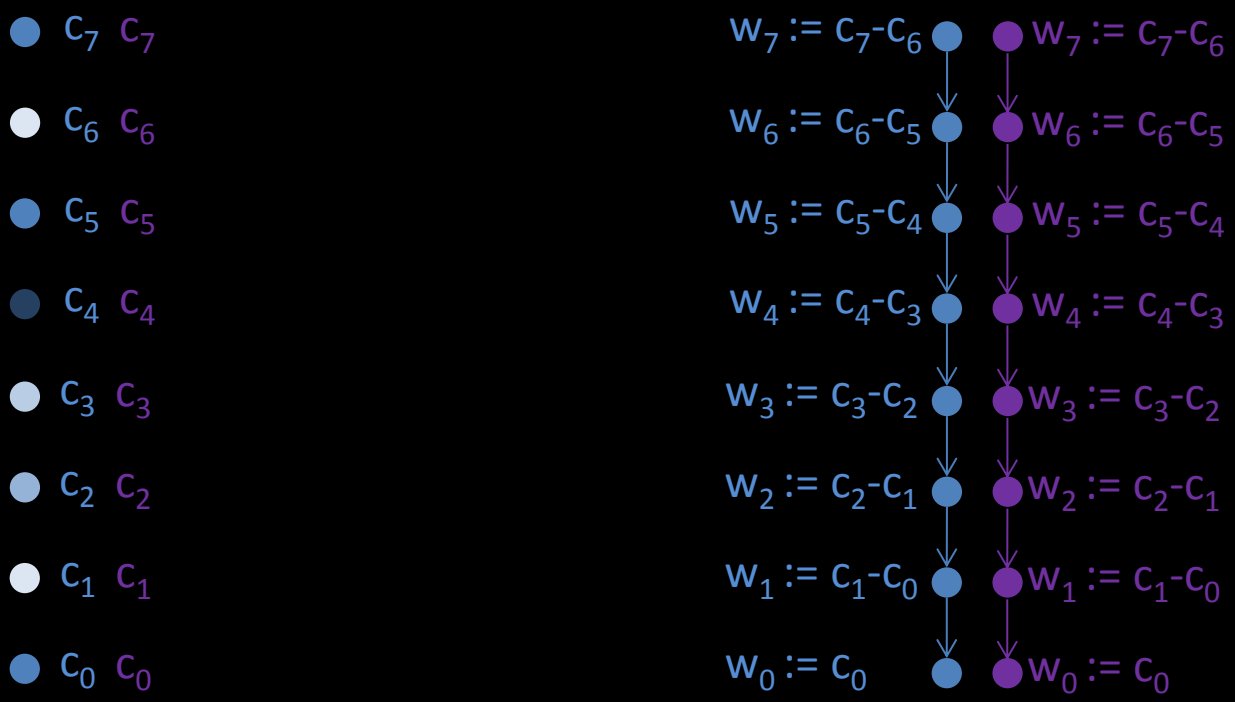




Two Surfaces

Non-Empty Closed Set

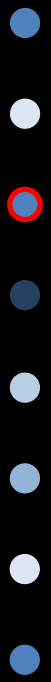
2 Surfaces ⇔ 1 Non-Empty Closed Set

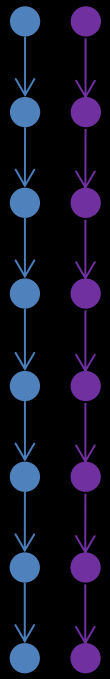


Two Surfaces

Non-Empty Closed Set

2 Surfaces ⇔ 1 Non-Empty Closed Set

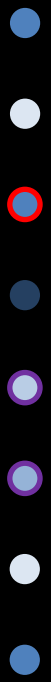


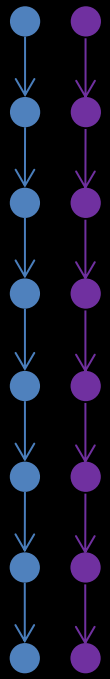


Two Surfaces

Non-Empty Closed Set

2 Surfaces ⇔ 1 Non-Empty Closed Set

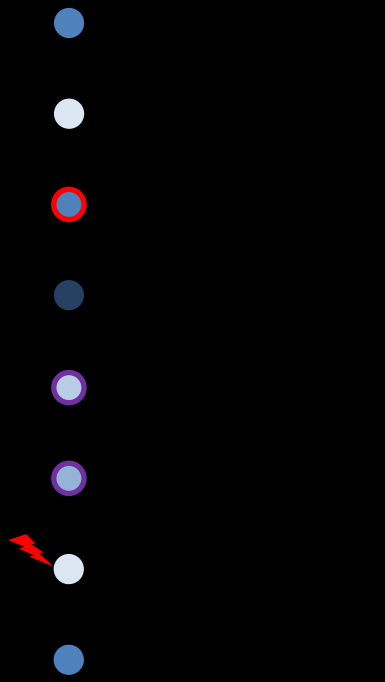


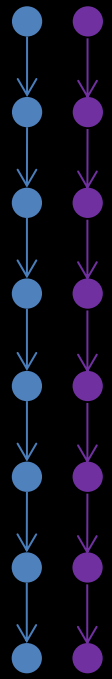


Two Surfaces

Non-Empty Closed Set

2 Surfaces ⇔ 1 Non-Empty Closed Set

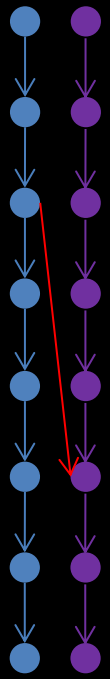




Two Surfaces

Non-Empty Closed Set

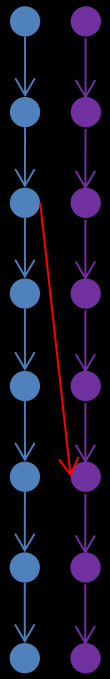
2 Surfaces ⇔ 1 Non-Empty Closed Set



Two Surfaces

Non-Empty Closed Set

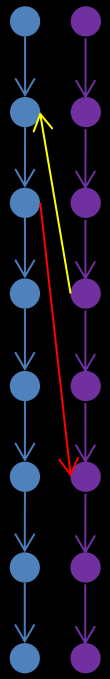
2 Surfaces ⇔ 1 Non-Empty Closed Set



Two Surfaces

Non-Empty Closed Set

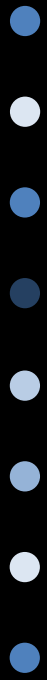
2 Surfaces ⇔ 1 Non-Empty Closed Set

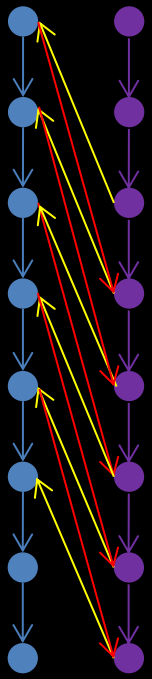


Two Surfaces

Non-Empty Closed Set

2 Surfaces ⇔ 1 Non-Empty Closed Set



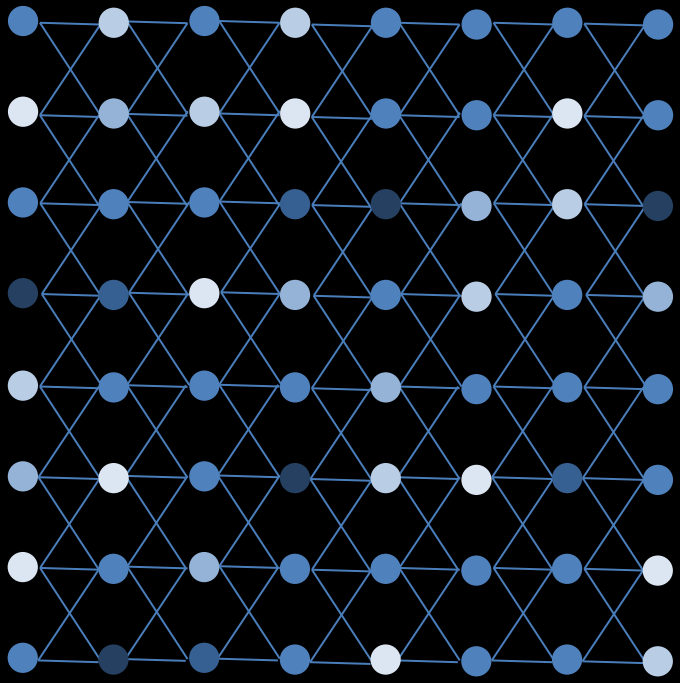


Two Surfaces

Non-Empty Closed Set

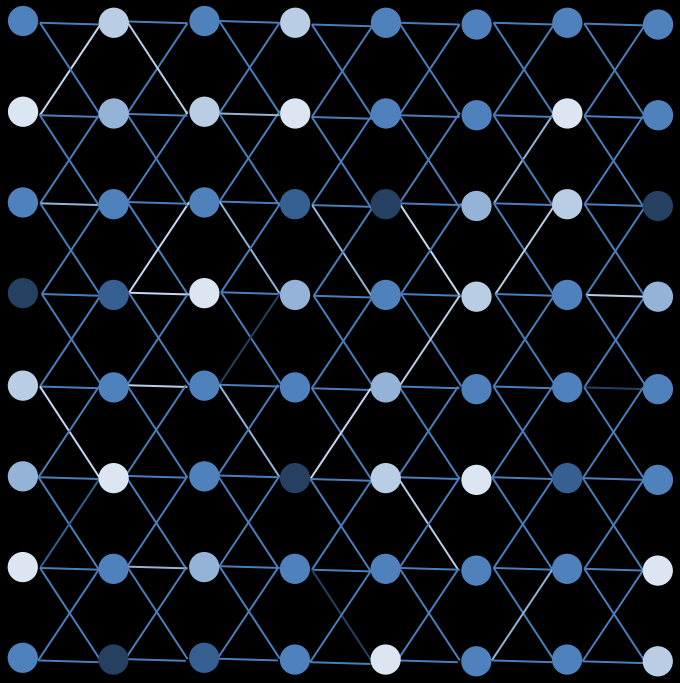
PART III

1 Surface with vertex- and edge costs



PART III

1 Surface with vertex- and edge costs

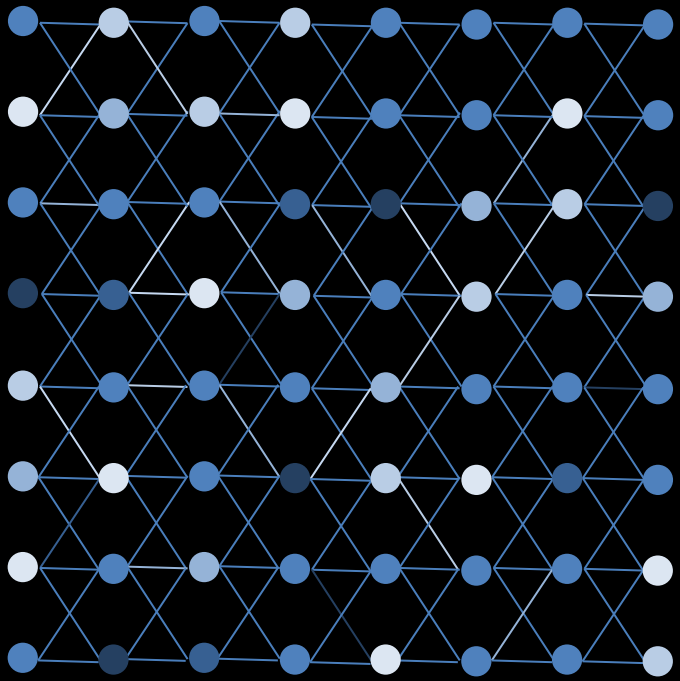


NP hard in general

But…

PART III

1 Surface with vertex- and edge costs

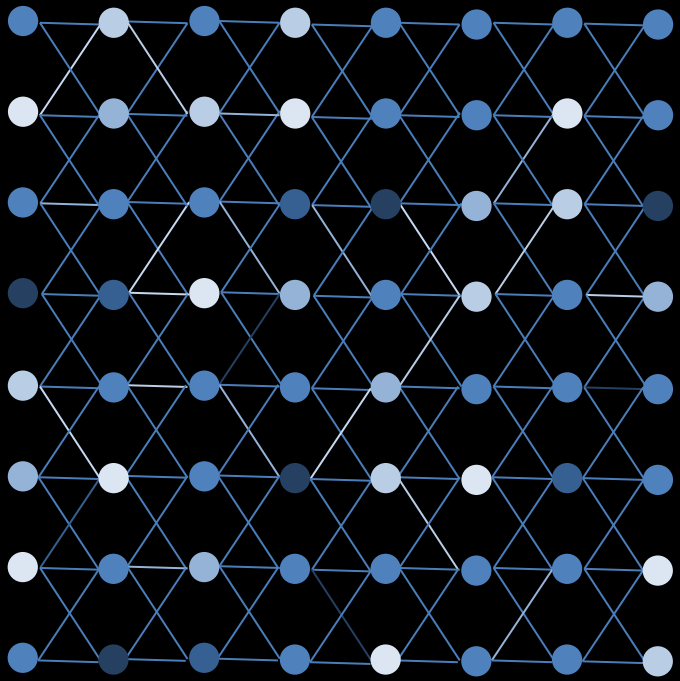


NP hard in general

But…

PART III

1 Surface with vertex- and edge costs



NP hard in general

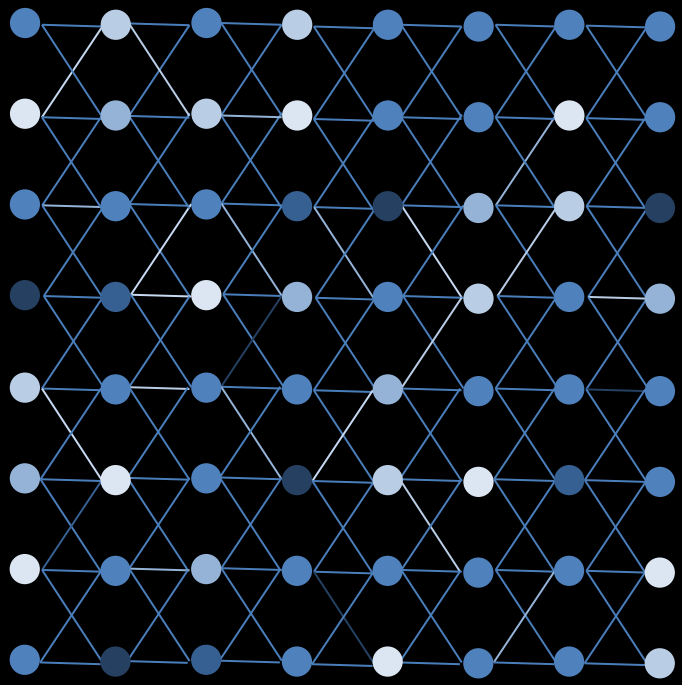
But…

EdgeCost(k,k+d) := f(d)

f convex, non-decreasing

PART III

1 Surface with vertex- and edge costs



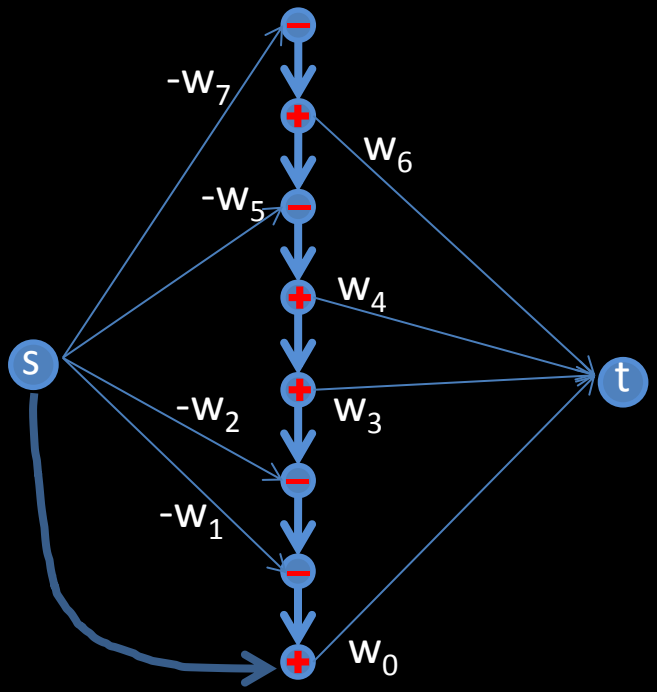
NP hard in general

But…

EdgeCost(k,k+d) := f(d)

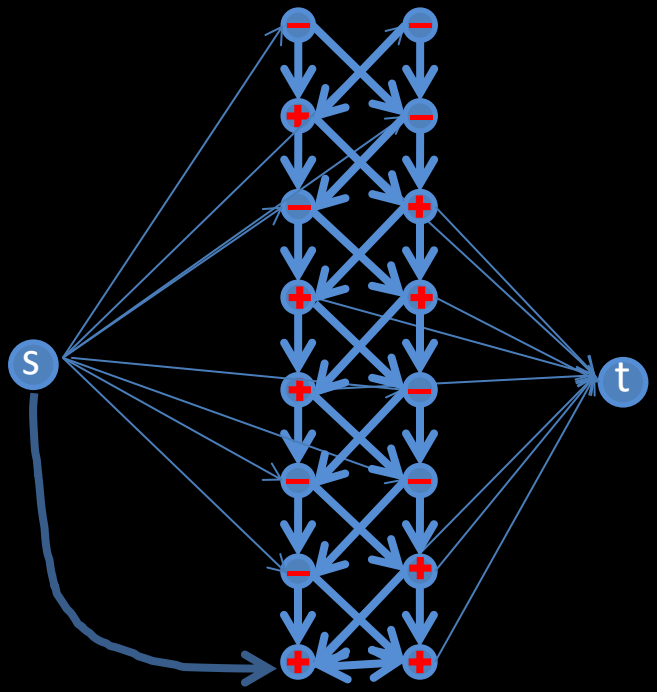
f convex, non-decreasing

Example: f(d):=α•d



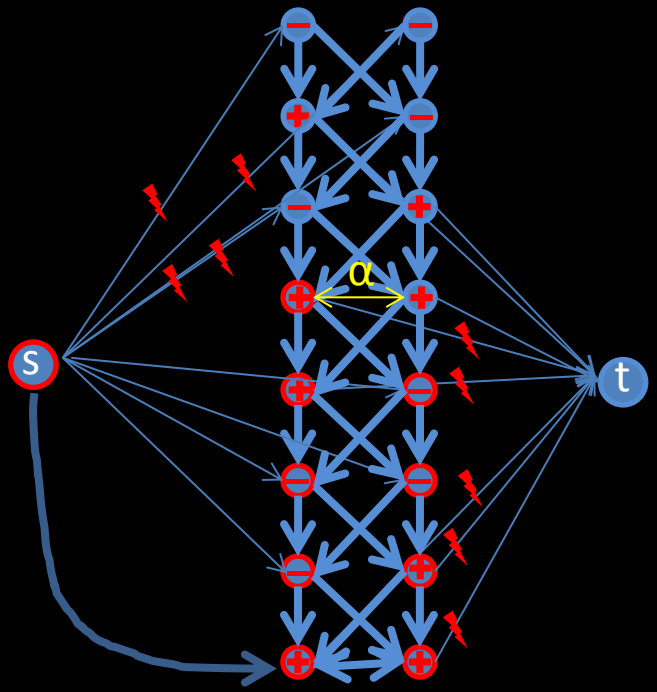
Finite Cost s-t-Cut

EdgeCost(k,k+d) := f(d)
f convex, non-decreasing
Example: f(d):=α•d



Finite Cost s-t-Cut

EdgeCost(k,k+d) := f(d)
f convex, non-decreasing
Example: f(d):=α•d



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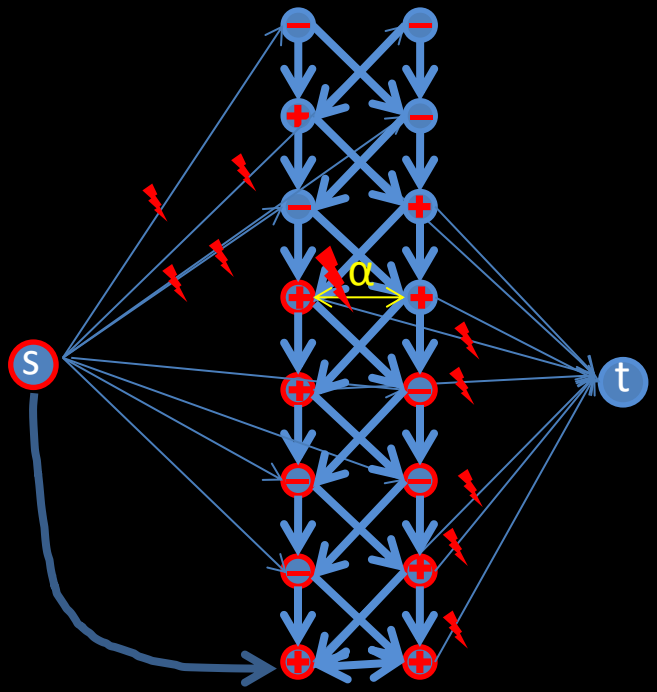

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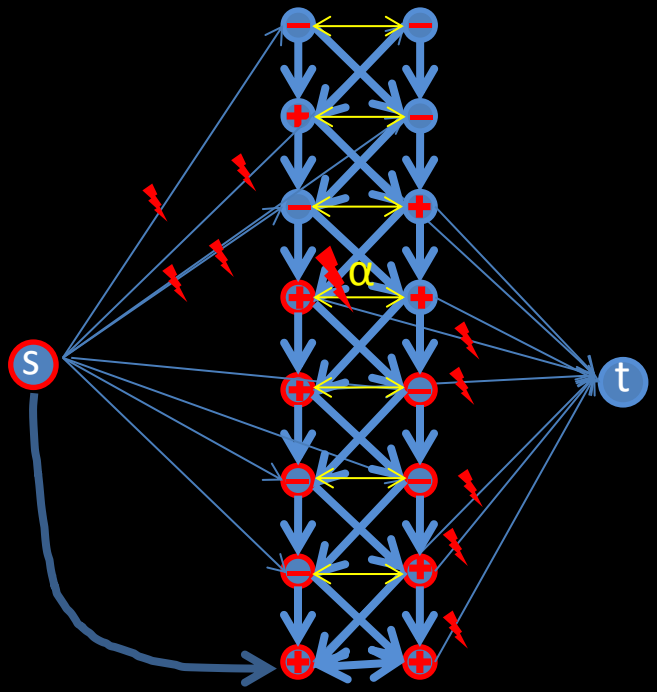

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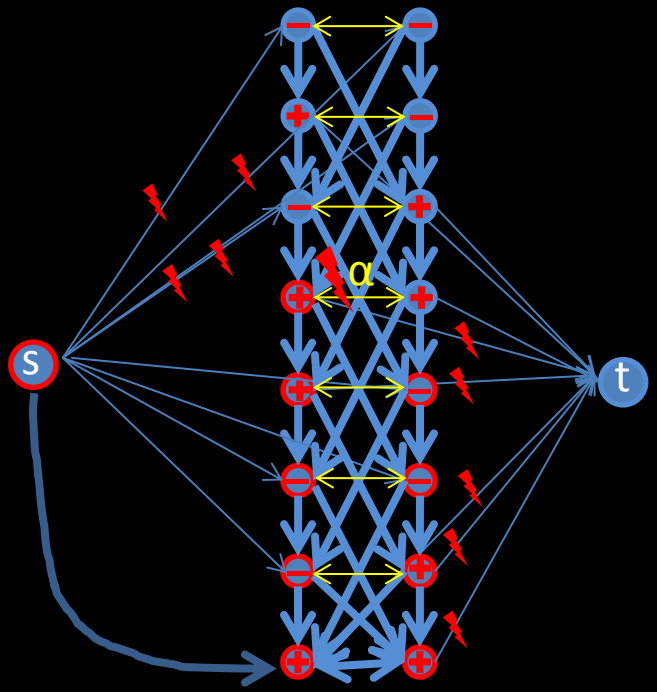

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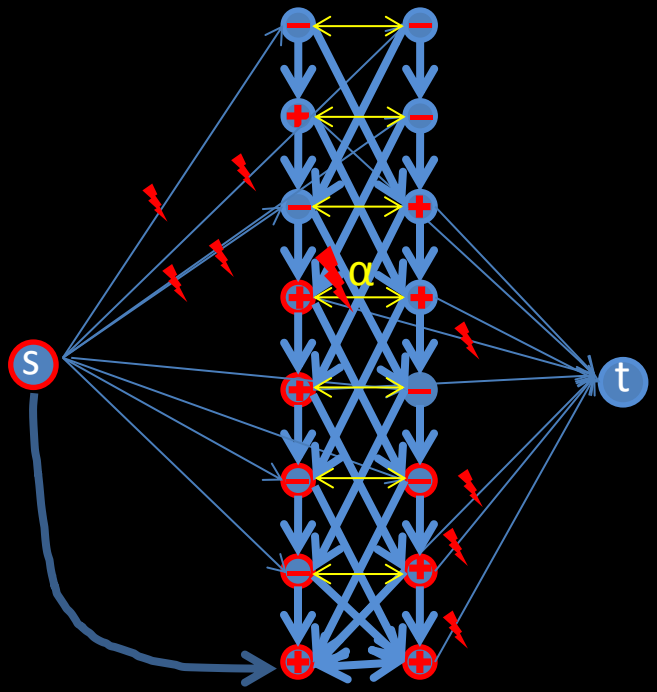

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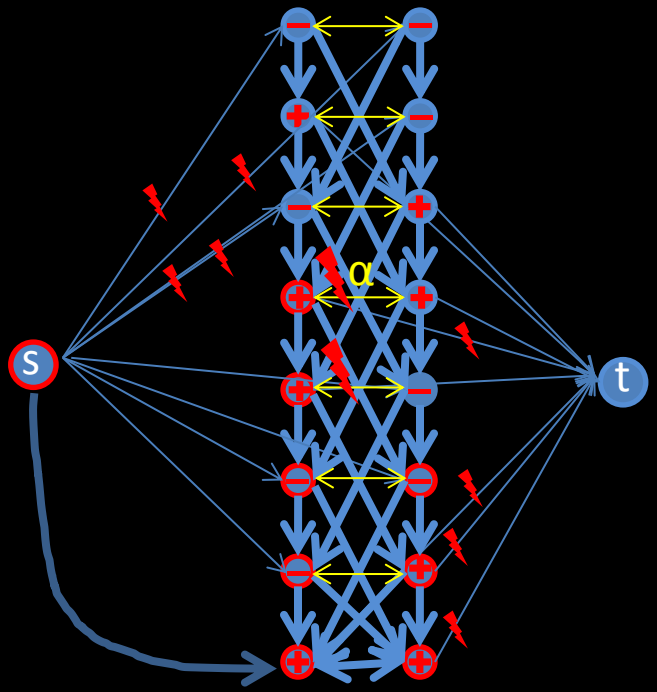

Finite Cost s-t-Cut

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f convex, non-decreasing
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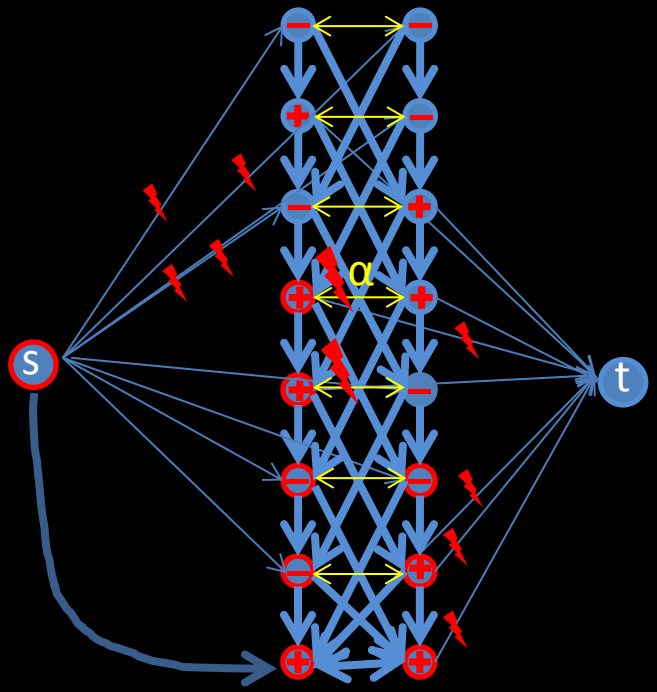
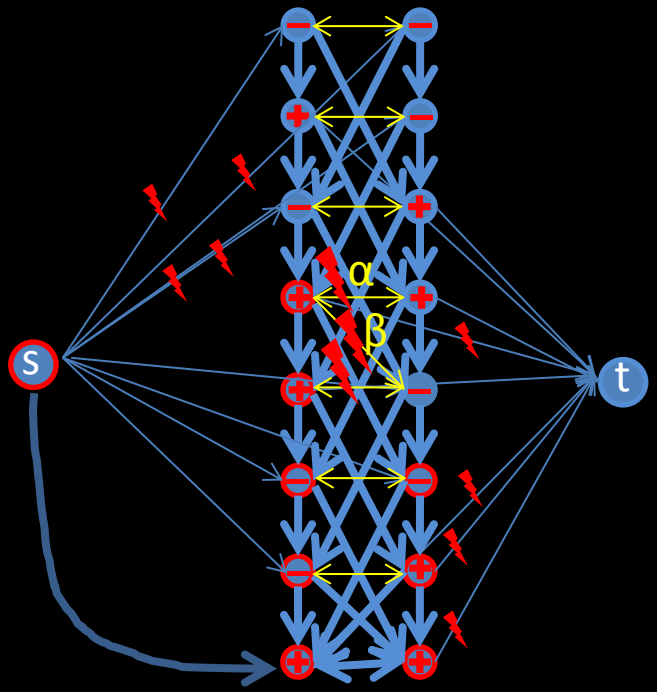


f(1)=α
f(2)=2α

Finite Cost s-t-Cut

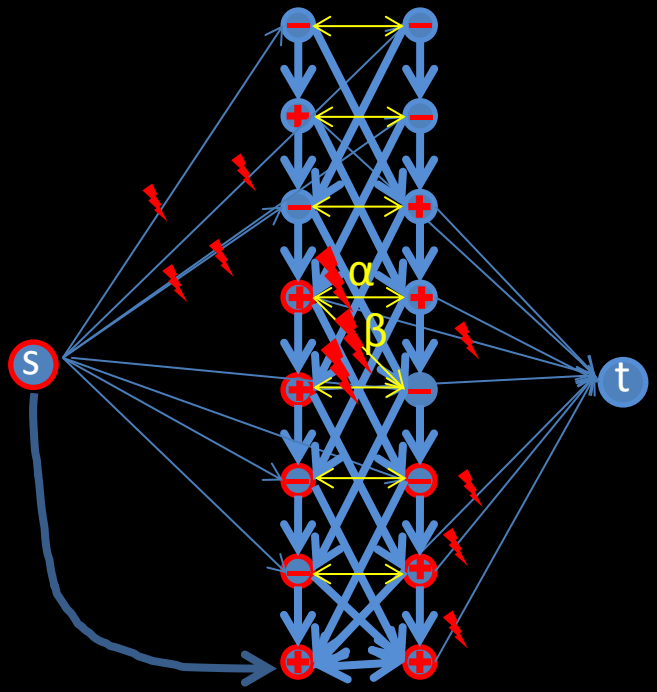
EdgeCost(k,k+d) := f(d)
f convex, non-decreasing
Example: f(d):=α•d



f(1)=α
f(2)=2α+β

Finite Cost s-t-Cut

EdgeCost(k,k+d) := f(d)
f convex, non-decreasing
Example: f(d):=α•d



Finite Cost s-t-Cut

f(1)=α
f(2)=2α+β
f(3)= 3α+2β+γ
…

EdgeCost(k,k+d) := f(d)
f convex, non-decreasing
Example: f(d):=α•d