

ORPHEUS MATHS

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$$\text{let } A = \begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

$$(A - \lambda I) \vec{v} = 0 \quad (1)$$

$$\Rightarrow \begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = 0 \text{ where } \lambda = \text{eigenvalues}$$

Determinant Equation let $\lambda = x$

$$x^4 + 13x^3 - 219x^2 - 835x + 3500 = 0$$

Solving the equation

$$x = -21, 12 \text{ to 2 decimal places}$$

Eigen Vector for this Eigen Value

Replacing λ with $-21, 12$ in (1)

$$\begin{pmatrix} 4 - (-21, 12) & 8 & -1 & -2 \\ -2 & -9 - (-21, 12) & -2 & -4 \\ 0 & 10 & 5 - (-21, 12) & -10 \\ -1 & -13 & -14 & -13 - (-21, 12) \end{pmatrix}$$

$$\begin{bmatrix} 25,12 & 8 & -1 & -2 \\ -2 & 12,12 & -2 & -4 \\ 0 & 10 & 26,12 & -10 \\ -1 & -13 & -14 & 8,12 \end{bmatrix}$$

Solving for $(A - \lambda I)\bar{V} = 0$

Augmented matrix $[A - \lambda I | 0]$:

$$\left(\begin{array}{cccc|c} 25,12 & 8 & -1 & -2 & 0 \\ -2 & 12,12 & -2 & -4 & 0 \\ 0 & 10 & 26,12 & -10 & 0 \\ -1 & -13 & -14 & 8,12 & 0 \end{array} \right) = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix}$$

Row Operations

Normalize R_1 by dividing by 25,12

$$\left(\begin{array}{cccc|c} 1 & 0,318 & -0,0398 & -0,0796 & 0 \end{array} \right)$$

Eliminate all rows below R_1

$$R_2 \rightarrow R_2 + 2 \times R_1 \quad R_4 = R_4 + R_1$$

After Elimination

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$$\left[\begin{array}{cccc|c} 1 & 0,318 & -0,0398 & -0,0796 & 1 & 0 \\ 0 & 12,756 & -2,0796 & -4,1592 & 1 & 0 \\ 0 & 10 & 26,12 & -10 & 1 & 0 \\ 0 & -12,682 & -14,0398 & 8,0404 & 1 & 0 \end{array} \right]$$

Normalize R_2 by dividing by 12,756

$$R_2 = (0 \quad 1 \quad -0,163 \quad -0,326 \quad | \quad 0)$$

Eliminate below R_2 :

$$R_3 \rightarrow R_3 - 10 \times R_2 \quad R_4 \rightarrow R_4 + 12,682 \times R_2$$

After Elimination

$$\left[\begin{array}{cccc|c} 1 & 0,318 & -0,0398 & -0,0796 & 1 & 0 \\ 0 & 1 & -0,163 & -0,326 & 1 & 0 \\ 0 & 0 & 27,75 & -6,74 & 1 & 0 \\ 0 & 0 & -16,107 & 3,906 & 1 & 0 \end{array} \right]$$

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Normalize R_3 by dividing by 27,75

$$R_3 = (0 \ 0 \ 1 \ -0,243 \ | \ 0)$$

Eliminate below R_3

$$R_4 \rightarrow R_4 + 16,107 \times R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0,318 & -0,0398 & -0,0796 & 0 \\ 0 & 1 & -0,163 & -0,326 & 0 \\ 0 & 0 & 1 & -0,243 & 0 \\ 0 & 0 & 0 & -0,005 & 0 \end{array} \right]$$

$$\begin{pmatrix} 1 & 0,318 & -0,0398 & -0,0796 \\ 0 & 1 & -0,163 & -0,326 \\ 0 & 0 & 1 & -243 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

let $(V_4 = t)$

$$R_3 : V_3 - 0,243V_4 = 0$$

$$V_3 = 0,243t$$

$$R_2: V_2 - 0,163V_3 - 0,326V_4 = 0$$

$$V_2 = 0,163(0,243t) + 0,326t$$

$$V_2 = 0,366t$$

$$R_1: V_1 + 0,318V_2 - 0,0398V_3 - 0,0796V_4 = 0$$

$$V_1 = -0,05t$$

EigenVector $t=1$

$$V = \begin{pmatrix} -0,05 \\ 0,366 \\ 0,243 \\ 1 \end{pmatrix} \quad \begin{array}{l} \text{EigenVector} \\ \text{for EigenValue} \\ -21,12 \end{array}$$

