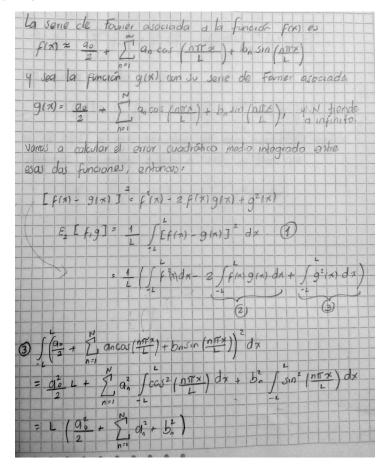
Taller 5

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1. Proof that.

$$a_0 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \le \frac{1}{2T} \int_{-T}^{T} f(x)^2 dx.$$

Known as the Bessel's inequality.



2. Write in full detail the proof, of Dirichlet theorem for Fourier series sketched in class.

My F on f' are personese continuous burtons on the whomal -T S X S T and outside the interest the function is well defined had it is periodic with point 27. Her & Mr. a representation in a Justin f (x) = ao + \(\sum_{\text{an}} \left(a_n \cos(\frac{m\pi}{\pi} x) + b_n \sin(\frac{m\pi}{\pi} x) \right) \] where the angipuits are given by an= = = | f(x) (05 (" + x) dx, h=== T F(x) SIN (my x) dx do= = F(x) dx The Jacob Suss, carroge to f(x), it my point reduce the fundament in continue and communinge by $\frac{1}{2}\left[f(x')+f(x)\right]$ when the function is throutinesse. But Fin the best part let f be some precious continues buston on the island will defined, the letter Town recognition are good to orsar be Justil and In

FIN = De + [[an cra(n x) + bn sn (n x x)] They he seemed point fix the with posted seem of the some in 5 (x) = = = a0 + 2 (ax(05(k+) + bx 51-(kx)) a== free resched dt, b=== free free dt @ by substitute a is a and bottom for my about. Visiter that Sh(kx) sh(kt) = = [(US(k(x-0)-Cu)(k(x+t))] (05(kg) 205(k0 = 12(205(k(x-6))+ (3(k(x+6))) = (05(K(x-0) there (use 5, (x) = = (05/k(E-x)) dt

War we define the Brocket Knul by D(0: = + E (15 (40)) with they would me evente Sn(x) on 5、(めこう) F(も Dn(モース) dt War we weeken die allestitution unt. - x, 5, (10) = = (F(x+4) D, (1) du on D(a) has a period 27, the on F(x+4), the Sn(x) = = Fray On(u) du. of Q(+11) = O(11), Exploring lay - is an the privatory equitor, 5,00=-= (f(2-u) D,(u) du, = 1 (* (200) Pp (W) du

Wan we add the two provous First we altain, 2 5,000 = = [F (x + w) + F(x - w)] D, (w) du == = [[(x+w) + [(x-w) Dn(w) du Water that of Ton(10) du = of its check the sent. $\frac{1}{2} \int_{-\infty}^{\infty} D_{n}(u) du = \frac{1}{2} \int_{-\infty}^{\infty} \left[\frac{1}{2} + \sum_{k=1}^{\infty} cos(k u) \right] du$ $=\frac{1}{2}\left[\frac{2}{3}+0\right]$ Was we substead 5, 10 - F(x), we altain $\int_{\mathbf{N}} (x) - f(x) = \frac{1}{\pi} \int_{-\pi}^{\pi} \left[f(x+u) + f(x-u) - 2f(x) \right] Q_{\eta}(\mathbf{a}) da,$ Buy Frome identifier we have $O_n(u) = \frac{1}{2} + \sum_{k=1}^{n} \cos(ku) = \frac{S_{in}(n + \frac{1}{2})u}{2S_{in}(\frac{u}{2})}$

In (1) to an be didnere 5,00- fex = } [sn(n+1) ug (w)du 9 (x4) = f(x+4) + f(x-4) - 2 ((2)) even to prove that the reser convey, therefore we much to proof lim 5,00 = 400) In single single + =) ada = 0, from they (g (u) cos (2) - Sh (nu) du + (g (u) sh (m) cos (nu) du = 6 by rue se A=== (g(u)-5/2 (1/2) + (05 (nu) du, $B_{n} = \frac{2}{\pi} \int_{0}^{\pi} g(u) \cos\left(\frac{u}{2}\right) \cdot \sin(nu) du,$ howeve thelds, let = (An +Bn) = 0 By Bord's hegualdy han > 0, B > 0

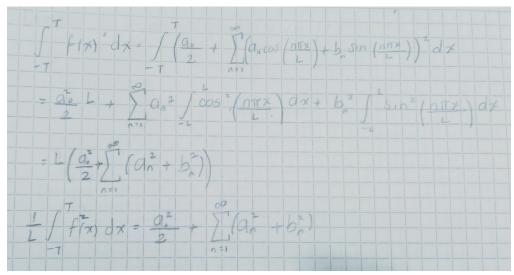
You best in the shept me bout Lm + (x'+4)- F(x) U-sot U 1.1m F(x=11)-F(X) $F(b) = \frac{F(x) + F(x)}{2}$

3 .

3. Supore that f is a 27-paradic prience smooth bustion. For bred I m [-4, 4] after $g(0) = \begin{cases} \frac{f(x+0) - f(x+1)}{2 \sin(\frac{1}{2})} & \text{if } 0 \in E \leq M \\ f'(x+1) & \text{if } 0 \in E = 0 \\ 0 & \text{if } -\pi \leq E \leq 0 \end{cases}$ a) show that g(0) = 0. by we approach bear 0 - we see that, 3(t) = 0 4 -7 5 & CO, Show that g(0+) = F'(x+), It me approach been Ut, we see the $g(\varepsilon) = \frac{f(x+\varepsilon) - f(x^{+})}{25!(\frac{\varepsilon}{2})}$ there appeals been ot,

4. Prove the Parseval identity

$$\frac{1}{2T} \int_{T}^{T} f(x)^{2} dx = a_{0}^{2} + \frac{1}{2} \sum_{n=1}^{\infty} (a_{n}^{2} + b_{n}^{2})$$



3, a	$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ penhe [- π , π]
Expo	andiendo la función f(x)=x con series de Founer: (6uta):
5,	$f(x)=x$ es pari esto implica que $a_0=a_n=0$ $b_h=\frac{2}{2}(-1)^{n+1}$
Ah	ora, usando la identidad de Raxeval se tiene:
	1 1 1 X3 1 T 7
	$\int_{0}^{1} \frac{1}{n^{2}} = \frac{1}{4\pi} \left(\frac{10^{3}}{3} - \frac{(-\pi)^{3}}{3} \right)$
	$= \frac{1}{4\pi} \cdot \frac{2\pi^3}{3} = \frac{\pi^2}{6}$

De	3. a Si	n=1	que	00 12n	1 1 1 2 1 1 2				
3.5 ①	ne i	(2m) ²	1 < 4	1 1	2	$\frac{\Pi^2}{6}$	π ² 24		
3.c②	m=0	1 (2m+4) ²	= \(\sum_{m=1}^{\infty} \)	1 m ²	2 N= 1	1 (2m) ²	$=\frac{\pi^2}{6}$	11 ² - 24	11 ²

5.

Fourier sories can be defined on other intervals besides -L = x & L.
Suppose that g(y) is defined for a = y = b. Represent g(y) using
periodic trigonometric functions with period b-a Determine formu-
las for the coefficients
$y = 0+b + b-a \times (1)$
Supongamos que existe fix) y su expansión en senes de Fourier:
$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n^{\pi x}/L) + \sum_{n=1}^{\infty} B_n \sin(n^{\pi x}/L) \qquad (2)$
y despejando a x de 111, tenemos lo siguiente.
$\chi = L[2y - b - a]$ (3)
4 lo sustituimos en (2), como tendiemos una función entérminos de g
g(y) = Ao + E Ances (nt/L. L(29-b-a) 6-a) + E Brin (nt/L. L(29-b-a) 6-a)
$= A_0 + \sum_{n=1}^{\infty} A_n \cos \left(n\pi \left(\frac{2y-b-a}{b-a} \right) + \sum_{n=1}^{\infty} B_n \sin \left(n\pi \left(\frac{2y-b-a}{b-a} \right) \right) - a \right)$
Frémonos que la períodos de sin (nax/L) y cos (nax/L) de giu
es L = b-a. enfunces, por las formulas de los coeficientes
de la serie de Founer, se tiene la signiente:
Ao = 1 /9(y) . 2L dy con b-adx = 2Ldy, dx = 2Ldy
$A_0 = \frac{1}{b-a} \int_a^b g(y) dy$

