ASSIGNMENT III
PARTIAL DIFFERENTIAL EQUATIONS

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This homework can be presented in pairs.

1. Apply separation of variables to find the solution of heat distribution through a solid block

$$\begin{aligned} u_t &= c^2\nabla^2 u. \\ u(0,y,z,t) &= k_1 \\ u(a,y,z,t) &= k_2 \\ u(x,0,z,t) &= k_3 \\ u(x,b,z,t) &= k_4 \\ u(x,y,0,t) &= k_5 \\ u(x,y,c,t) &= k_6 \\ u(x,y,z,0) &= f(x) \end{aligned}$$

where $0 < x < a, 0 < y < b, 0 < z < c$.

2. Apply separation of variables to solve the one dimensional wave equation, for the finite vibrating string with fixed ends, i.e.,

$$\begin{aligned} u_{tt} &= c^2 u_{xx}. \\ u(0,t) &= 0 \\ u(l,t) &= 0 \\ u(x,0) &= \phi(x) \\ u_t(x,0) &= \psi(x) \end{aligned}$$

where $0 < x < l$, $t > o$ and $\phi(x), \psi(x)$ are well defined for $x \in (0,l)$. Explain in full detail every step in your process.

3. Prove that:

- $\int_{-L}^{L} \sin\left(\frac{m\pi}{L}x\right) \sin\left(\frac{n\pi}{L}x\right) dx = L\delta_{nm}$.
- $\int_{-L}^{L} \cos\left(\frac{m\pi}{L}x\right) \cos\left(\frac{n\pi}{L}x\right) dx = L\delta_{nm}$.
- $\int_{-L}^{L} \sin\left(\frac{m\pi}{L}x\right) \cos\left(\frac{n\pi}{L}x\right) dx = 0$.

4. A metal bar with $\alpha^2 = 0{,}75\text{cm}^2/\text{s}$ of length 10cm is heated to a uniform temperature of 100°C. At $t = 0$ the ends of the bar are immersed in an icy bath at 0°C and from that moment on the ends remain at this temperature. Heat is not allowed to escape through the side surface of the bar. Find an expression for the temperature at every point on the bar at each instant.

5. The wave equation describing the transverse vibrations of a stretched membrane under tension T and having a uniform surface density ρ is

$$T\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = \rho\frac{\partial^2 u}{\partial t^2}$$

Find a separable solution appropriate to a membrane stretched on a frame of length a and width b, showing that the natural angular frequencies of such a membrane are given by

$$\omega^2 = \frac{\pi^2 T}{\rho}\left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right)$$

where n and m are any positive integers.