ASSIGNMENT VI
PARTIAL DIFFERENTIAL EQUATIONS

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This homework can be presented in pairs. In each point do the pictures that you think help to present your results

1. Reproduce in full detail the problem solved in the video "Ultra-Mega Differential Equations Review Problem!!!!" (https://www.youtube.com/watch?v=yncPeXiRdck)

2. Establish the following properties for Bessel series

(a) $J_0(0) = 1, J_p(0) = 0$ if $p > 0$

(b) $J_n(x)$ is an even function if n is even, and odd if n is odd;

(c) $\text{lím}_{x\to 0^+} \dfrac{J_p(x)}{x^p} = \dfrac{1}{2^p\Gamma(p+1)}$

3. Proof the identities

(a) $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x} - \cos x\right]$

(b) $J_{-3/2}(x) = \sqrt{\frac{2}{\pi x}}\left[-\frac{\cos x}{x} - \sin x\right]$

4. Prove that

(a) $\cos(x) = J_0(x) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(x)$

(b) $\sin(x) = 2\sum_{k=0}^{\infty}(-1)^k J_{2k+1}(x)$

5. For $x, y > 0$

$$\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} = 2\int_0^{\pi/2} \cos^{2x-1}\theta \sin^{2y-1}\theta d\theta$$

Derive this useful formula as follows.

(a) Make the change of variables $u^2 = t$ in definition of Gamma function and obtain

$$\Gamma(x) = 2\int_0^\infty e^{-u^2}u^{2x-1}du, \quad x > 0$$

(b) Use (a) to show that for $x, y > 0$,

$$\Gamma(x)\Gamma(y) = 4\int_0^\infty \int_0^\infty e^{-\left(u^2+v^2\right)}u^{2x-1}v^{2y-1}dudv$$

(c) Change to polar coordinates in (b) $(u = r\cos\theta, v = r\sin\theta, \quad dudv = rdrd\theta)$ and obtain that for $x, y > 0$,

$$\Gamma(x)\Gamma(y) = 2\Gamma(x+y)\int_0^{\pi/2} \cos^{2x-1}\theta \sin^{2y-1}\theta d\theta$$