Assignment 8

1 Markov Chain Monte Carlo

Write a simple MCMC routine to produce N draws $\{x_i\}$ from the scaled/shifted Student-t distribution

$$\pi\left(x|\nu,\sigma,\mu\right) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}\sigma}\left(1+\frac{1}{\nu}\left(\frac{x-\mu}{\sigma}\right)^2\right)^{-\frac{\nu+1}{2}},$$

where the parameters ν, σ, μ are fixed here, and they define the properties of the distribution. As a concrete example, set them to $\nu = 3, \mu = 1, \sigma = 1$. For the proposal distribution, use

$$q\left(y|x\right) = \frac{1}{\sqrt{2\pi\alpha}}e^{-\frac{(y-x)^2}{2\alpha^2}}.$$

- (*30 Points*) Run your code trying four different jump size scalings, $\alpha = 0.01$, $\alpha = 0.1$, $\alpha = 1$ and $\alpha = 10$. Visually inspect the chains. Which one appears to be exploring the distribution most effectively?
- (*15 Points*) Using the same 4 sets of jumps sizes, run your code for $N = 1000$ iterations and use the output to produce histograms of the $\{x_i\}$. Plot them against the target distribution $\pi\left(x|\nu,\sigma,\mu\right)$. Which distribution looks the best?
- (*5 Points*) Have your code compute the acceptance fraction for the proposed jumps (i.e. the fraction of the proposed jumps that are accepted). Does a high acceptance rate necessarily mean efficient exploration of the distribution?