Assignment 9

1 Parameter Estimation using a MCMC

- (*5 Points*) Use the best mixing version you found from the code from Assignment 7 to produce a large number of independent samples from the scaled/shifted Student-t distribution with $\nu = 3$, $\mu = 1.5$ and $\sigma = 1$. In this case, the samples $\{x_i\}$ are the data, d, and the model parameters are ν, μ and σ. The goal of this HW is to infer the parameters ν, μ and σ that produce the data.

- (*25 Points*) Write a MCMC to explore the produce the posterior distribution functions for these parameters. The likelihood is given by

$$p\left(d|\nu,\sigma,\mu\right) = \prod_{i=1}^{N} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}\sigma}\left(1+\frac{1}{\nu}\left(\frac{x_i-\mu}{\sigma}\right)^2\right)^{-\frac{\nu+1}{2}}.$$

  Take the priors to be uniform in the ranges $\nu \in [0.1, 10]$, $\mu \in [-5, 5]$ and $\sigma \in [0.1, 10]$. For the proposal distribution use a multi-variate Gaussian[1].

- For $N = 100$ and $N = 1000$ data samples, and always make sure that the posterior distributions for the parameters recover the prior distributions on the parameters.
  - (*10 Points*) Produce marginalized posterior distribution functions for the parameters.
  - (*10 Points*) Look at 2-d scatter plots of ν versus σ etc to get a sense of the parameter correlations.

[1] You can use:
from scipy.stats import multivariate_normal