

# Assignment 7



## 1 Markov Chain Monte Carlo

Write a simple MCMC routine to produce  $N$  draws  $\{x_i\}$  from the scaled/shifted Student-t distribution

$$T(x|\nu, \sigma, \mu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\nu\pi}\sigma} \left(1 + \frac{1}{\nu} \left(\frac{x-\mu}{\sigma}\right)^2\right)^{-\frac{\nu+1}{2}},$$

where the parameters  $\nu, \sigma, \mu$  are fixed here, and they define the properties of the distribution. As a concrete example, set them to  $\nu = 3, \sigma = 1, \mu = 1.5$ . For the proposal distribution you could use a multivariate normal random variable<sup>1</sup>.

- (30 Points) Run your code trying four different jump size scalings,  $\alpha = 0.01, \alpha = 0.1, \alpha = 1$  and  $\alpha = 10$ . Visually inspect the chains. Which one appears to be exploring the distribution most effectively?
- (15 Points) Using the same previous 4 sets of jumps sizes, run your code for  $N = 1000$  iterations and use the output to produce histograms of the  $\{x_i\}$ . Plot them against the target distribution  $T(x|\nu, \sigma, \mu)$ . Which distribution looks the best?
- (5 Points) Have your code compute the acceptance fraction for the proposed jumps (i.e., the fraction of the proposed jumps that are accepted). Does a high acceptance rate necessarily mean efficient exploration of the distribution?

<sup>1</sup>[https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.multivariate\\_normal.html](https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.multivariate_normal.html)