# Assignment 3

## 1 Python Libraries

(*25 Points*) The Fourier Trigonometric Series of an integrable function $f(x)$ on $(0, P)$ is written as

$$\begin{aligned} f(x) \quad \sim \quad & \frac{a_0}{2} + \sum_{k=1}^{n} \left[ a_k \cos\left(\frac{2\pi k x}{P}\right) \right. \\ & \left. + b_k \sin\left(\frac{2\pi k x}{P}\right) \right], \end{aligned}$$

where[1]

$$\begin{aligned} a_0 &= \frac{2}{P} \int_0^P f(x)\, dx \\ a_k &= \frac{2}{P} \int_0^P f(x) \cos\left(\frac{2\pi k x}{P}\right) dx; \quad k \geq 1 \\ b_k &= \frac{2}{P} \int_0^P f(x) \sin\left(\frac{2\pi k x}{P}\right) dx. \end{aligned}$$

Write a Python function **FourierSeries(f,P,x,n)** that returns the values of the Fourier series of the function $f(x)$ at the point $x$. Plot the Fourier series of the following functions

$$\begin{aligned} f(x) &= 1 - x \\ f(x) &= x^2, \end{aligned}$$

using $P = 1$, but calculate the series on the interval $(0, P)$. Repeat the plot for $n = 6, 12, 36, 124$. What do you observe as $n$ becomes larger?

## 2 Mathematical Preliminaries

- (*10 Points*) Test the stability of the zero solution for the following system

$$\begin{aligned} x_1' &= -4x_1 + 8x_1 x_2^2 \\ x_2' &= -12x_1^2 x_2 - 6x_2 \end{aligned}$$

- (*15 Points*) Test the linear stability of the zero solution $x_1(t) = 0, x_2(t) = 0$ in the Lotka-Volterra population model, i.e.,

$$\begin{aligned} x_1' &= a x_1 - x_1 x_2 \\ x_2' &= -b x_2 + x_1 x_2, \end{aligned}$$

  for your favourite pair of integers $(a, b)$. *Suggestion:* For one of the two critial points will find, try to shift the origin to that critial point, by a simple coordinate transformation, and construct a Lyapunov function.

[1] Hint: Use **scipy.integrate.quad** to do the numerical integrations.