dapper: An R Package for studying differentially private algorithms

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Abstract This paper serves as a reference and introduction on using the dapper R package. The goal of this package is to provide some tools for exploring the impact of different privacy regimes on a Bayesian analysis. A strength of this framework is the ability to target the exact posterior in settings where the likelihood is too complex to analytically express.

1 Introduction

Differential privacy provides a rigorous framework for protecting confidential information. One of it's goal is to allow the widespread dissemination of summary statistics while hiding sensitive characteristics of the data. For example, a data aggregation service might collect salary information for the purpose of helping its users negotiate salaries. Such information is typically considered both sensitive. In this scenario, there is a strong desire to simultaneously keep the salary information of individuals anonymous and make queries of the salary database publicly available.

In the context of statistical models, considerable effort has been put in ensuring privacy is achieved by using two methods. The first method consist of first fitting the model to the confidential data set and then adding noise to its output. The second method injects noise at some point in the model fitting process. The fitting process in many models can be viewed as minimizing a loss function. Privacy can be achieved by adding random perturbations to the loss function. See (Ji, Lipton, and Elkan 2014) for a survey that covers the two methods applied to variety of commonly used machine learning models. On the other hand, adding noise directly to the data is a less studied approach and is the case which this paper addresses.

Statistical regression models typically assume no measurement error in the observed covariate data. In the presence of such errors, standard estimators can exhibit significant bias (Gong 2022). Therefore, fitting standard statistical models after adding noise directly to the data for privacy can lead to incorrect inference. Adjusting models to take into account noisy covariates has a rich history spanning several decades. For textbook length treatments see (Yi 2017; Carroll et al. 2006). Prior work mostly focuses on methods which did not require fully specifying the measurement error model, since this was often unknown. However, in differential privacy, the measurement error model is exactly known. This difference, makes feasible some ideas which the measurement error community has not previously considered (Smith 2011; Karwa, Kifer, and Slavković 2015).

One approach, to account for the added noise, is to treat the confidential data as latent quantities within a statistical model. In such settings, it is common to conduct inference by specifying a complete likelihood. Once the complete likelihood has been specified, parameter estimation can be done using the EM algorithm or its Bayesian analogue, the data augmentation method. In the case of dapper, inference is done using data augmentation as described in (Ju et al. 2022). A notable benefit of the Bayesian approach is that both uncertainty quantification and estimation are done simultaneously. The EM approach only provides an estimate.

The rest of this article is organized as follows. Section 2 covers the necessary background to understand the mathematical notation and ideas used throughout the paper. Section 3 goes over the main algorithm without going into mathematical detail, for specifics see (Ju et al. 2022). Section 4 provides an overview of the dapper package and discusses important implementation details. Section 5 contains two example of how one might use the package to analyze the impact of adding noise for privacy.

2 Background

Let $x = (x_1, \ldots, x_n) \in \mathcal{X}^n$ represent a confidential database containing n records. Usually the goal of collecting data is to learn some characteristic about the underlying population. To accomplish this task, a common approach is to assume the population is represented by some statistical model $f(\cdot \mid \theta)$. It is often the case that some function of θ has relevant meaning to the scientific question at hand. In this setting, learning characteristics of a population reduces to learning about θ.

In the Bayesian framework, this is accomplished by drawing samples from the posterior $p(\theta \mid x) \propto f(x \mid \theta)p(\theta)$. For large data sets, it is common to work with a summary statistic $s = s(x)$ that has much smaller dimension than the original data. Doing so can greatly simplify calculations. In general, there can be information loss with using summary statistics, but for models with a sufficient statistic, there is no loss. In the context of privacy, providing a summary statistic can offer a level of anonymity.

Differential Privacy

While a summary statistic can already anonymize the data, it is still possible to deduce information about an individual entry depending on the distribution of x. For additional anonymity, one idea is to add noise to the summary statistic s. More formally we write $s_{dp} \sim \eta(\cdot \mid x)$. Here, s_{dp} is the noise infused version of s and η is a known noise infusion process. The privacy mechanism η is said to be ϵ-differentially private (Dwork and Roth 2013) if for all values of s_{dp}, and all "neighboring" databases $(x, x') \in \mathcal{X}^n \times \mathcal{X}^n$ differing by one record (denoated by $d(x, x')$), the probability ratio is bounded:

$$\frac{\eta(s_{dp} \mid x)}{\eta(s_{dp} \mid x')} \leq \exp(\epsilon), \quad \epsilon > 0.$$

The parameter ϵ is called the privacy loss budget, and controls how informative s_{dp} is about x. Larger values of ϵ correspond to less noise being added.

Data Augmentation

The idea behind data augmentation is to run a Gibbs sampler on a coupling of two random variables where one of the marginals is the target distribution. Suppose we wish to sample from a density f_A which is difficult. The data augmentation method instead considers sampling from a joint distribution $f(a, b)$. Since we are ultimately interested in samples from f_A, the joint distribution should be chosen so that (i) the marginal distribution with respects to a is f_A and (ii) $f(a \mid b)$ and $f(b \mid a)$ are easy to sample from. The choice f is not unique and can require some foresight.

3 Methodology

Given data s_{dp}, the goal of Bayesian inference is to sample from the posterior distribution $p(\theta \mid s_{dp})$. Since the observed likelihood, $p(s_{dp} \mid \theta)$ often has no simple closed form expression, most standard sampling schemes do not apply. To conduct privacy aware Bayesian inference, the dapper package implements the data augmentation algorithm which allows us to sample from $p(\theta \mid s_{dp})$ without needing to specify $p(s_{dp} \mid \theta)$.

The algorithm considers the joint distribution $p(\theta, x \mid s_{dp})$ and alternates sampling from the two distributions

- $p(\theta \mid x, s_{dp})$
- $p(x \mid \theta, s_{dp})$

Since s_{dp} is derived from x, we have $p(\theta \mid x, s_{dp}) = p(\theta \mid x)$ which is just the usual posterior distribution given the confidential data x. The dapper package assumes the user has access to a sampler for $p(\theta \mid x)$. This can come from any R package such as fmcmc. For the second distribution, $p(x \mid \theta, s_{dp})$, may only be known up to a constant. The dapper package samples from this distribution by running a Gibbs like sampler. Each of the n components of x is individually updated. However unlike the standard Gibbs sampler, each component is updated using a Metropolis-Hasting algorithm. This method is sometimes called the Metropolis within Gibbs sampler (Robert and Casella 2004).

In some cases, sampling from $p(x \mid \theta, s_{dp})$ can be made more efficient when the summary statistic can be written as the sum of individual contributions from each observation. More precisely, we say a statistic satisfies the record additivity property if $\eta(s_{dp} \mid x) = g(s_{dp}, \sum_{i=1}^{n} t_i(x_i, s_{dp}))$ for some known and tractable functions $g, t_1, \ldots, t_n$.

The algorithm is in the following pseudo code:

1. Sample θ^{t+1} from $p(\cdot \mid x^{(t)})$.
2. Sample from $p(x \mid \theta, s_{dp})$ using a three step process
    - Propose $x_i^* \sim f(\cdot \mid \theta)$.
    - If s satisfies the record additive property then update $s(x^*, s_{dp}) = t(x, s_{dp}) - t_i(x_i, s_{dp}) + t_i(x_i^*, s_{dp})$.
    - Accept the proposed state with probability $\alpha(x_i^* \mid x_i, x_{-i}, \theta)$ given by:

$$\alpha(x_i^* \mid x_i, x_{-i}, \theta) = \min\left\{\frac{\eta(s_{dp} \mid x_i^*, x_{-i})}{\eta(s_{dp} \mid x_i, x_{-i})}\right\}.$$

4 Using dapper

The package is structured around the two functions `dapper_sample` and `new_privacy`. The first function is used to draw samples from the posterior. The second function is used to create the privacy model. Since the input to these functions are R functions, there is a great deal of freedom left up to user. The next two sections describe in detail the inputs into these functions and highlight some considerations that should be taken into account in order avoid slow or unexpected behavior.

Before delving into the specifics of each component, it is necessary to clearly define how the confidential data is represented. Internally, the confidential database is encoded as a 2D matrix. There are often multiple ways of doing this. For example, if our data consist of 100 responses from a two question, yes/no, survey. Then we can either encode the data as a 2×2 matrix, or a 100×2 matrix. Both are mathematically equivalent, but the 2×2 matrix will be much more memory efficient. In general, the representation that uses the least amount of memory should be used. Correctly specifying the privacy model will require a consistent representation among all components.

Sampling

The main function in **dapper** is the `gdp_sample` function. The call signature of the function is:

```
gdp_sample(data_model, sdp, nobs, init_par, niter = 2000, warmup = floor(niter / 2),
           chains = 1, varnames = NULL)
```

The three required inputs into `gdp_sample` function are the privacy model (`data_model`), the value of the observed privatized statistic (`sdp`), and the total number of observations in the complete data (`nobs`). The dapper package is best suited for problems where the complete data can be represented in tabular form. This is because internally, it is represented as a matrix.

The optional arguments are the number of mcmc draws (`niter`), the burn in period (`warmup`), number of chains (`chains`) and character vector that names the parameters. Running multiple chains can be done in parallel using the **furrr** package. Additionally, progress can be monitored using the **progressr** package.

The `data_model` input is a `privacy` object that can be constructed using the `new_privacy` constructor. The process of constructing a `privacy` object will be discussed in the next section.

Privacy Model

Creating a privacy model is done using the `new_privacy` constructor. The main arguments consist of the four components as outlined in the methodology section.

```
new_privacy(post_smpl = NULL, lik_smpl = NULL, ll_priv_mech = NULL,
            st_calc = NULL, add = FALSE, npar = NULL)
```

The internal implementation of the DA algorithm in `dapper_sample` requires some care in how each component is constructed.

- `lik_smpl` is an R function that samples from the likelihood. Its call signature should be `lik_smpl(theta)` where `theta` is a vector representing the likelihood model parameters being estimated. This function must work with the supplied initial parameter provide in the `init_par` argument of `dapper_sample` function and its output should be a $n \times k$ matrix. k the dimension of the complete data table.
- `post_smpl` is a function which represents the posterior sampler. It should have the call signature `post_smpl(dmat, theta)`. Where `dmat` is the complete data. This sampler can be generated by wrapping mcmc samplers generated from other R packages (e.g. **rstan**, **fmcmc**, **adaptMCMC**). If using this approach, it is recommended to avoid using packages such as **mcmc** whose implementation clashes with `gdp_sample`. In the case of **mcmc**, the Metropolis-Hastings loop is implemented in C which incurs a very large overhead in `gdp_sample` since it is reinitialized every iteration. In general, repeatedly calling an R function that hooks into C code is slow. (NOT QUITE ACCURATE FIX LATER)

- `ll_priv_mech` is an R function that represents the log-likelihood of $\eta(s_{sdp} \mid x)$. The function can output the log likelihood up to an additive constant.
- `st_calc` is an R function which calculates the summary statistic. The optional argument `add` is a flag which represents whether T is additive or not.

5 Examples

2x2 Contingency Table

A common procedure when analyzing contingency tables is to estimate the odds ratio. Something something about safetab to connect back to DP (dont forget citation!). As a demonstration, we analyze the UC Berkeley admissions data, which is often used as an illustrative example of Simpson's paradox. The question is whether the data suggest there is bias against females during the college admissions process. Below is a table of the aggregate admissions result from six departments based on sex.

	Male	Female
Admitted	1198	557
Rejected	1493	1278

	Male	Female
Admitted	1135	473
Rejected	1511	1438

Below we walk through the process of defining a privacy model.

1. `lik_smpl`: Conditional on the table total, the table counts follow a multinomial distribution. We can easily draw from this distribution using the `rmultinom` function in the `base` stats package. Note, in this example, the return value of one sample from `rmultinom` is a 4×1 matrix. In order to conform with `dapper_sample` we must convert the matrix to a vector.

   ```
   lik_smpl <- function(theta) {
     t(rmultinom(1, 4526, theta))
   }
   ```

2. `post_smpl`: Given confidential data X we can derive the posterior analytically using a Dirichlet prior. In this example, we use a flat prior which corresponds to Dirch(1) distribution. A sample from the Dirichlet distribution can be generated using the gamma distribution via the following relation (INSERT)

   ```
   post_smpl <- function(dmat, theta) {
     x <- c(dmat)
     t1 <- rgamma(length(theta), x + 1, 1)
     t1/sum(t1)
   }
   ```

3. `st_calc`: The complete data can be represented in two ways. Micro vs cell totals. (what section to introduce?) This function must return a vector.

   ```
   st_calc <- function(dmat) {
     c(dmat)
   }
   ```

4. `ll_priv_mech`: Privacy Mechanism Guassian white noise is added to each cell total. Hence given confidential data $(n_{11}, n_{22}, n_{12}, n_{21})$

   $$\eta(s_{dp} \mid x) = \prod \phi(s_{sd}; n_{ij}, 100^2)$$

   ```
   ll_priv_mech <- function(sdp, x) {
     dnorm(sdp - x, mean = 0, sd = 100, log = TRUE)
   }
   ```

Once privacy model has been defined we can run `gdp_sample`

```
library(DPloglin)
dmod <- new_privacy(post_smpl = post_smpl,
                    lik_smpl = lik_smpl,
                    ll_priv_mech = ll_priv_mech,
```

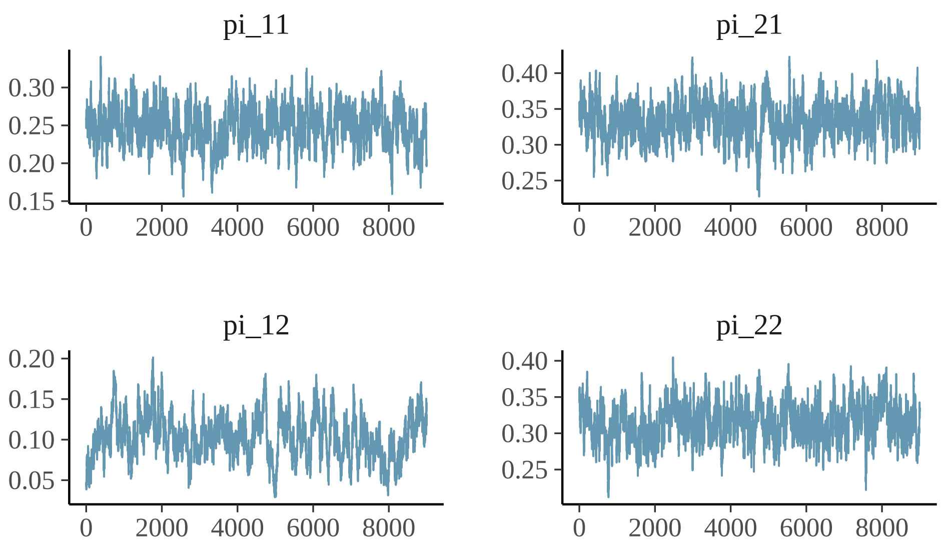
```
                          st_calc = st_calc,
                          add = FALSE,
                          npar = 4,
                          varnames = c("pi_11", "pi_21", "pi_12", "pi_22"))

dp_out <- dapper_sample(dmod,
                  sdp = c(adm_prv),
                  niter = 10000,
                  warmup = 1000,
                  chains = 1,
                  init_par = rep(.25,4))
```

results can be quickly summarized using the summary function

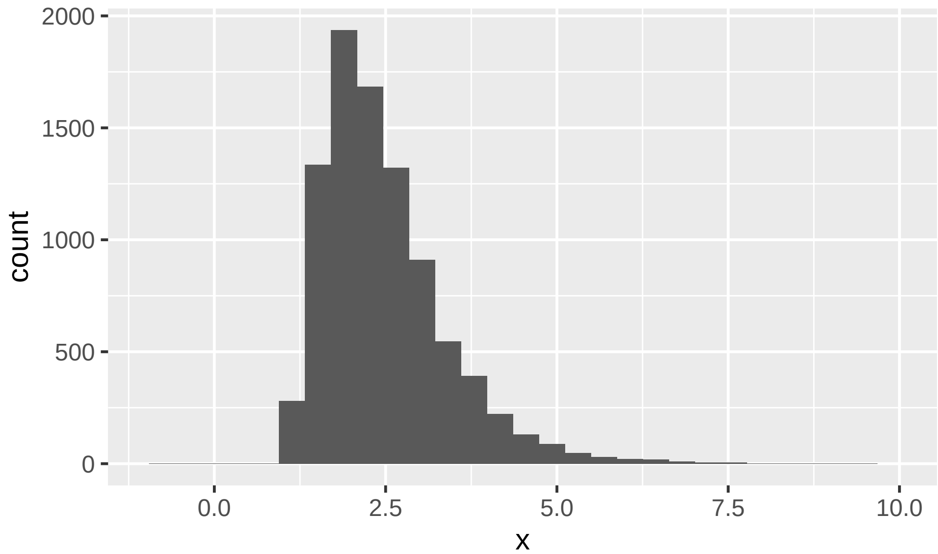
```
#> # A tibble: 4 x 10
#>   variable  mean median     sd    mad     q5   q95  rhat ess_bulk ess_tail
#>   <chr>    <num>  <num>  <num>  <num>  <num> <num> <num>    <num>    <num>
#> 1 pi_11    0.248  0.249 0.0254 0.0257 0.206  0.288  1.00     191.      388.
#> 2 pi_21    0.334  0.333 0.0261 0.0254 0.291  0.377  1.00     210.      329.
#> 3 pi_12    0.103  0.103 0.0272 0.0271 0.0586 0.149  1.02      76.0     159.
#> 4 pi_22    0.315  0.315 0.0256 0.0261 0.275  0.358  1.00     164.      460.
```

Diagnostic checks can be done using the **Bayesplot** package.



log odds distribution

```
#>     2.5%      50%    97.5%
#> 1.297400 2.289552 4.785425
```



(Insert gdp Analysis Here)

For clean data, a estimate for the odds ratio and a confidence interval can be constructed using Woolf's method (i.e. A wald confidence interval). It uses the fact that the log of the odds ratio is approximately normal for large sample sizes.

$$\log\left(\frac{n_{11}\cdot n_{22}}{n_{12}\cdot n_{21}}\right)\pm z_{\alpha/2}\sqrt{\frac{1}{n_{11}}+\frac{1}{n_{22}}+\frac{1}{n_{12}}+\frac{1}{n_{21}}}$$

```
or_confint <- function(x, alpha) {
  or <- log(x[1] * x[4]/ (x[2] * x[3]))
  se <- sqrt(sum(1/x))
  c(or - qnorm(alpha/2) * se, or + qnorm(alpha/2) * se)
}

#clean data
exp(or_confint(x, .95))

#> [1] 1.848471 1.833718

#privitized data
exp(or_confint(sdp, .95))

#> [1] 2.293115 2.274220
```

Logistic Regression

6 Summary

This package is cool. You should install it.

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