

Data Augmentation for Privacy Aware Analysis

DP Group

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Abstract

This paper serves as a reference and introduction on using the **dadp** R package. The goal of this package is to provide some tools for exploring the impact of different privacy regimes on a Bayesian analysis. A strength of this framework is the ability to target the exact posterior in settings where the likelihood is too complex to analytically express.

Methodology

(Insert from DA paper?)

Using dadp

Introduce some basic notation (here or in intro) Consider combining this section with methodology.

Using the dadp package consist of specifying four components

1. $\pi(\theta \mid x)$.
2. $f(x \mid \theta)$.
3. $\eta(s_{dp} \mid x)$.
4. $T(x)$.

Differentially Private Simple Linear Regression

(Alabi et al. 2020)[1] considers adding noise to a sufficient static to create a differentially private algorithm for simple linear regression called **NoisyStats**

Suppose we would like to explore the potential impact of the NoisyStats mechanism on analysis. Assume the true data generating process is

$$\begin{aligned}x_i &\sim Unif(0, 1) \\ y_i &\sim N(-2 + 3x_i, 3^2)\end{aligned}$$

Algorithm 1 NoisyStats: $(\epsilon, 0)$ -DP Algorithm

```
1: Data:  $\{(x_i, y_i)\}_{i=1}^n$ 
2: Privacy Parameter:  $\epsilon$ 
3:  $\Delta_1 = \Delta_2 = (1 - 1/n)$   $\triangleright$  Set global sensitivity
4: Sample  $L_1 \sim \text{Lap}(0, 3\Delta_1/\epsilon)$ 
5: Sample  $L_2 \sim \text{Lap}(0, 3\Delta_2/\epsilon)$ 
6: if  $n\text{var}(x) + L_2 > 0$  then
7:    $\tilde{\alpha} = \frac{ncov(x, y) + L_1}{n\text{var}(x) + L_2}$ 
8:    $\Delta_3 = (1/n)(1 + |\tilde{\alpha}|)$ 
9:   Sample  $L_3 \sim \text{Lap}(0, 3\Delta_3/\epsilon)$ 
10:   $\tilde{\beta} = (\bar{y} - \tilde{\alpha}\bar{x}) + L_3$ 
11: return  $(\tilde{\alpha}, \tilde{\beta})$ 
12: return NA
```

Sampling from likelihood under complete data

likelihood function

$$\begin{aligned} f(y \mid x, \alpha, \beta) &= f(x \mid \mu_x, \sigma_x) f(y \mid x, \alpha, \beta) \\ &= \phi(x; \mu_x, \sigma_x) \phi(y; \alpha + \beta x, \sigma) \end{aligned}$$

```
lik_smp1 <- function(theta) {
  alpha <- theta[1]
  beta <- theta[2]
  mux <- theta[3]
  sx <- exp(theta[4])
  sy <- exp(theta[5])

  x <- rnorm(1, mean = mux, sd = sx)
  y <- rnorm(1, mean = alpha + beta * x, sd = sy)

  c(y, x)
}
```

Posterior given complete data

Statistic

NoisyStat computes three summary statistics

$$n\text{var}(x) = \frac{1}{n-1} \sum_{i=1}^n$$

A maybe...

As an example, use data from (Gelman). Problem consist of estimating the proportion of boys and girls. Data: 251,527 boys and 241,945 girls born in Paris from 1745 to 1770. Describe set up below

Privatize by adding noise, $\eta \sim N(0, 4000)$: [Use DP framework?]

```
n_g <- 241945
n_b <- 251527

eta <- rnorm(2, 0, 4000)

n_g + eta[1]

## [1] 240897.5

n_b + eta[2]

## [1] 252921
```

Sampling from likelihood under complete data

binomial distribution

$$f(x \mid \theta) = \binom{n}{n_g} \theta^{n_g} (1 - \theta)^{n - n_g}$$

```
lik_smpl <- function(theta) {
  t1 <- rbinom(1, 493472, theta)
  t2 <- 493472 - t1
  c(t1, t2)
}
```

Posterior given complete data

Using Jeffrey's prior $Beta(1/2, 1/2)$.

$$\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)} x^{\alpha-1} (1 - x)^{\beta-1}$$

conjugate model:

References

- [1] Daniel Alabi, Audra McMillan, Jayshree Sarathy, Adam Smith, and Salil Vadhan, *Differentially private simple linear regression*, (2020).