dapper: An R Package for studying differentially private algorithms

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Abstract This paper serves as a reference and introduction on using the *gdpR* R package. The goal of this package is to provide some tools for exploring the impact of different privacy regimes on a Bayesian analysis. A strength of this framework is the ability to target the exact posterior in settings where the likelihood is too complex to analytically express.

1 Introduction

Differential privacy provides a rigorous framework for protecting confidential information. One of it's goal is to allow the widespread dissemination of summary statistics while hiding sensitive characteristics of the data. For example, a data aggregation service might collect salary information for the purpose of helping its users negotiate salaries. Such information is typically considered both sensitive. In this scenario, there is a strong desire to simultaneously keep the salary information of individuals anonymous and make public, aggregate salary data so others can negotiate better salaries.

In the context of statistical models, the bulk of DP research has concentrated on protecting privacy by either adding noise to a model parameter estimate or adding noise to the estimating process itself. Adding noise directly to the data is a less studied approach and is the case which this paper addresses. In both cases, the added noise induces biases which must be accounted for in order to perform sound inference. This paper will go over the dapper package. It implements the Bayesian framework described in [-]. The intent of the dapper package is to help privacy researchers evaluate how their analysis might be affected by directly infusing data with noise.

The rest of this article is organized as follows. Section 2 covers the necessary background to understand the mathematical notation and ideas used throughout the paper. Section 3 goes over the main algorithm without going into mathematical detail, for specifics see [Ju et al.]. Section 4 provides an overview of the dapper package and discusses important implementation details. Section 5 contains two example of how one might use the package to analyze the impact of adding noise for privacy.

2 Background

Let $x = (x_1, \ldots, x_n) \in \mathcal{X}^n$ represent a confidential database containing n records. Usually the goal of collecting data is to learn some characteristic about the underlying population. To accomplish this task, a common approach is to assume the population is represented by some statistical model $f(\cdot \mid \theta)$. It is often the case that some function of θ has relevant meaning to the scientific question at hand. In this setting, learning characteristics of a population reduces to learning about θ.

In the Bayesian framework, this is accomplished by drawing samples from the posterior $p(\theta \mid x) \propto f(x \mid \theta)p(\theta)$. For large data sets, it is common to work with a summary statistic $s = s(x)$ that has much smaller dimension than the original data. Doing so can greatly simplify calculations. In general, there can be information loss with using summary statistics, but for models with a sufficient statistic, there is no loss. In the context of privacy, providing a summary statistic can offer a level of anonymity.

Differential Privacy

While a summary statistic can already anonymize the data, it is still possible to deduce information about an individual entry depending on the distribution of x. For additional anonymity, one idea is to add noise to the summary statistic s. More formally we write $s_{dp} \sim \eta(\cdot \mid x)$. Here, s_{dp} is the noise infused version of s and η is a known noise infusion process. The privacy mechansim η is said to be ϵ-differentially private [Dwork] if for all values of s_{dp}, and all "neighboring" databases $(x, x') \in \mathcal{X}^n \times \mathcal{X}^n$ differing by one record (denoated by $d(x, x')$), the probability ratio is bounded:

$$\frac{\eta(s_{dp} \mid x)}{\eta(s_{dp} \mid x')} \leq \exp(\epsilon), \quad \epsilon > 0.$$

The parameter ϵ is called the privacy loss budget, and controls how informative s_{dp} is about x. Larger values of ϵ correspond to less noise being added.

Data Augmentation

3 Methodology

Statistical regression models typically assume no covariate measurement error. In the presence of such errors, standard estimators can exhibit significant bias. Historically, mis-measured covariates were often associated with data collected issues. However, In the context of privacy, confidential data is purposely infused with noise in order to preserve anonymity.

One approach, to account for the added noise, is to treat the confidential data as latent quantities within a statistical model. In such settings, it is common to conduct inference by specifying a complete likelihood. Once the complete likelihood has been specified, parameter estimation can be done using the EM algorithm or its Bayesian analogue, the data augmentation method. In the case of gdpr, inference is done via data augmentation. A notable benefit of the Bayesian approach is that both uncertainty quantification and estimation are done simultaneously. The EM approach only provides an estimate.

The data augmentation approach considers the joint distribution $p(\theta, x \mid s_{dp})$.

$$p(\theta, x \mid s_{dp}) \propto p(\theta) f(x \mid \theta) \eta(s_{dp} \mid x).$$

The problem now is to figure out how to sample for this joint posterior. This is accomplished using a Metropolis-within-Gibbs sampler. Each

4 Using dapper

The package is structured around the two functions `gdp_sample` and `new_privacy`. The first function is used to draw samples from the posterior. The second function is used to create the privacy model. Since the input to these functions are R functions, there is a great deal of freedom left up to user. The next two sections describe in detail the inputs into these functions and highlight some considerations that should be taken into account in order avoid slow or unexpected behavior.

(not yet sure where to put this tid bit, here looks about right) Before delving into the specifics of each component, it is necessary to clearly define how the confidential data is represented. Internally, the confidential database is encoded as a 2D matrix. There are often multiple ways of doing this. For example, if our data consist of 100 responses from a two question, yes/no, survey. Then we can either encode the data as a 2×2 matrix, or a 100×2 matrix. Both are mathematically equivalent, but the 2×2 matrix will be much more memory efficient. In general, the representation that uses the least amount of memory should be used. Correctly specifying the privacy model will require a consistent representation among all components.

Sampling

The main function in **gdpR** is the `gdp_sample` function. The call signature of the function is:

```
gdp_sample(data_model, sdp, nobs, init_par, niter = 2000, warmup = floor(niter / 2),
           chains = 1, varnames = NULL)
```

The three required inputs into `gdp_sample` function are the privacy model (`data_model`), the value of the observed privatized statistic (`sdp`), and the total number of observations in the complete data (`nobs`) [MAKE SURE NOTATION IS INTRODUCED]. The **gdpR** package is best suited for problems where the complete data can be represented in tabular form. This is because internally, it is represented as a matrix.

The optional arguments are the number of mcmc draws (`niter`), the burn in period (`warmup`), number of chains (`chains`) and character vector that names the parameters. Running multiple chains can be done in parallel using the **furrr** package. Additionally, progress can be monitored using the **progressr** package.

The `data_model` input is a `privacy` object that can be constructed using the `new_privacy` constructor. The process of constructing a `privacy` object will be discussed in the next section.

Privacy Model

Creating a privacy model is done using the `new_privacy` constructor. The main arguments consist of the four components as outlined in the methodology section.

Algorithm 1 One iteration of the privacy-aware Metropolis-within-Gibbs sampler

1. Conditional update of $p(\theta \mid x)$ using the kernel from Assumption 1.

2. For each $i = 1, 2, \ldots,$ sequentially update $x_i \mid x_{-i}, \theta, s_{dp}$.

   (a) Propose $x_i^\star \sim f(\cdot \mid \theta)$.

   (b) Update $t(x^\star, s_{dp}) = t(x, s_{dp}) - t_i(x_i, s_{dp}) + t_i(x_i^\star, s_{dp})$ according to Assumption 2.

   (c) Accept the proposed state with probability $\alpha(x_i^\star \mid x_i, x_{-i}, \theta)$ given by:

$$\alpha(x_i^\star \mid x_i, x_{-i}, \theta) = \min\left\{\frac{n(s_{dp} \mid x_i^\star, x_{-i})}{n(s_{dp} \mid x_i, x_{-i})}, 1\right\} = \min\left\{\frac{g(s_{dp}, t(x^\star, s_{dp}))}{g(s_{dp}, t(x, s_{dp}))}, 1\right\}. \quad (5)$$

Figure 1: A nice image.

```
new_privacy(post_smpl = NULL, lik_smpl = NULL, ll_priv_mech = NULL,
            st_calc = NULL, add = FALSE, npar = NULL)
```

The internal implementation of the DA algorithm in `gdp_sample` requires some care in how each component is constructed.

- `lik_smpl` is an R function that samples from the likelihood. Its call signature should be `lik_smpl(theta)` where `theta` is a vector representing the likelihood model parameters being estimated. This function must work with the supplied initial parameter provide in the `init_par` argument of `gdp_sample` and its output should be a $n \times k$ matrix. k the dimension of the complete data table.
- `post_smpl` is a function which represents the posterior sampler. It should have the call signature `post_smpl(dmat, theta)`. Where `dmat` is the complete data. This sampler can be generated by wrapping mcmc samplers generated from other R packages (e.g. **rstan**, **fmcmc**, **adaptMCMC**). If using this approach, it is recommended to avoid using packages such as **mcmc** whose implementation clashes with `gdp_sample`. In the case of **mcmc**, the Metropolis-Hastings loop is implemented in C which incurs a very large overhead in `gdp_sample` since it is reinitialized every iteration. In general, repeatedly calling an R function that hooks into C code is slow. (NOT QUITE ACCURATE FIX LATER)
- `ll_priv_mech` is an R function that represents the log-likelihood of $\eta(s_{sdp} \mid x)$. The function can output the log likelihood up to an additive constant.
- `st_calc` is an R function which calculates the summary statistic. The optional argument `add` is a flag which represents whether T is additive or not.

5 Examples

2x2 Contingency Table

A common procedure when analyzing contingency tables is to estimate the odds ratio. Something something about safetab to connect back to DP (dont forget citation!). As a demonstration, we analyze the UC Berkeley admissions data, which is often used as an illustrative example of Simpson's paradox. The question is whether the data suggest there is bias against females during the college admissions process. Below is a table of the aggregate admissions result from six departments based on sex.

	Male	Female		Male	Female
Admitted	1198	557	Admitted	1135	473
Rejected	1493	1278	Rejected	1511	1438

Below we walk through the process of defining a privacy model.

1. `lik_smpl`: Conditional on the table total, the table counts follow a multinomial distribution. We can easily draw from this distribution using the `rmultinom` function in the `base` stats package. Note, in this example, the return value of one sample from `rmultinom` is a 4×1 matrix. In order to conform with `dapper_sample` we must convert the matrix to a vector.

   ```
   lik_smpl <- function(theta) {
     t(rmultinom(1, 4526, theta))
   }
   ```

2. `post_smpl`: Given confidential data X we can derive the posterior analytically using a Dirichlet prior. In this example, we use a flat prior which corresponds to Dirch(1) distribution. A sample from the Dirichlet distribution can be generated using the gamma distribution via the following relation (INSERT)

   ```
   post_smpl <- function(dmat, theta) {
     x <- c(dmat)
     t1 <- rgamma(length(theta), x + 1, 1)
     t1/sum(t1)
   }
   ```

3. `st_calc`: The complete data can be represented in two ways. Micro vs cell totals. (what section to introduce?) This function must return a vector.

```
st_calc <- function(dmat) {
  c(dmat)
}
```

4. `ll_priv_mech`: Privacy Mechanism Guassian white noise is added to each cell total. Hence given confidential data $(n_{11}, n_{22}, n_{12}, n_{21})$

$$\eta(s_{dp} \mid x) = \prod \phi(s_{sd}; n_{ij}, 100^2)$$

```
ll_priv_mech <- function(sdp, x) {
  dnorm(sdp - x, mean = 0, sd = 100, log = TRUE)
}
```

Once privacy model has been defined we can run `gdp_sample`

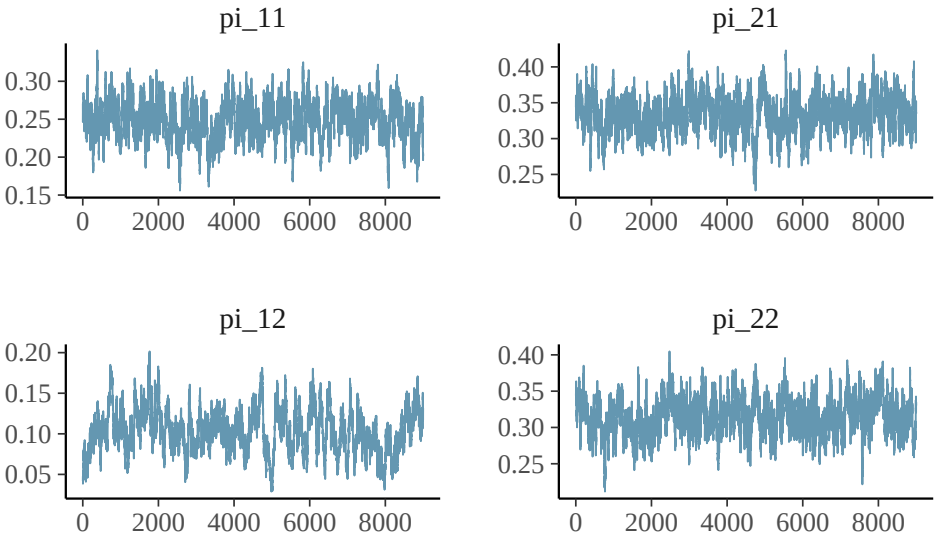
```
library(DPloglin)
dmod <- new_privacy(post_smpl = post_smpl,
                    lik_smpl = lik_smpl,
                    ll_priv_mech = ll_priv_mech,
                    st_calc = st_calc,
                    add = FALSE,
                    npar = 4,
                    varnames = c("pi_11", "pi_21", "pi_12", "pi_22"))

dp_out <- dapper_sample(dmod,
                  sdp = c(adm_prv),
                  niter = 10000,
                  warmup = 1000,
                  chains = 1,
                  init_par = rep(.25,4))
```

results can be quickly summarized using the `summary` function

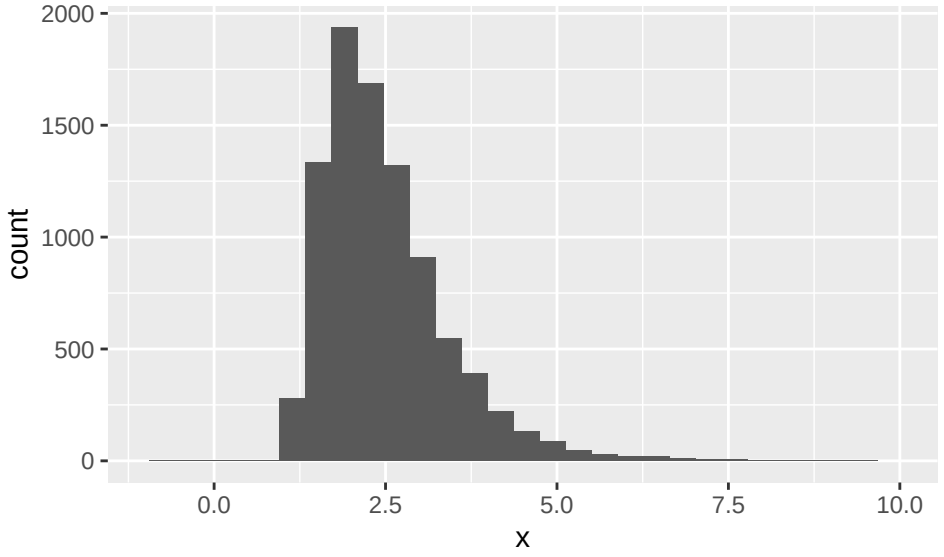
```
#> # A tibble: 4 x 10
#>   variable  mean median     sd    mad     q5   q95  rhat ess_bulk ess_tail
#>   <chr>    <num>  <num>  <num>  <num>  <num> <num> <num>    <num>    <num>
#> 1 pi_11    0.248  0.249 0.0254 0.0257 0.206  0.288  1.00    191.      388.
#> 2 pi_21    0.334  0.333 0.0261 0.0254 0.291  0.377  1.00    210.      329.
#> 3 pi_12    0.103  0.103 0.0272 0.0271 0.0586 0.149  1.02     76.0     159.
#> 4 pi_22    0.315  0.315 0.0256 0.0261 0.275  0.358  1.00    164.      460.
```

Diagnostic checks can be done using the **Bayesplot** package.



log odds distribution

```
#>     2.5%      50%    97.5%
#> 1.297400 2.289552 4.785425
```



(Insert gdp Analysis Here)

For clean data, a estimate for the odds ratio and a confidence interval can be constructed using Woolf's method (i.e. A wald confidence interval). It uses the fact that the log of the odds ratio is approximately normal for large sample sizes.

$$\log\left(\frac{n_{11}\cdot n_{22}}{n_{12}\cdot n_{21}}\right) \pm z_{\alpha/2}\sqrt{\frac{1}{n_{11}}+\frac{1}{n_{22}}+\frac{1}{n_{12}}+\frac{1}{n_{21}}}$$

```
or_confint <- function(x, alpha) {
  or <- log(x[1] * x[4]/ (x[2] * x[3]))
  se <- sqrt(sum(1/x))
  c(or - qnorm(alpha/2) * se, or + qnorm(alpha/2) * se)
}

#clean data
exp(or_confint(x, .95))

#> [1] 1.848471 1.833718

#privitized data
exp(or_confint(sdp, .95))

#> [1] 2.293115 2.274220
```

Logistic Regression

6 Summary

This package is cool. You should install it.

7 References

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