RWTH Aachen
Faculty of Computer Science
Chair of Combinatorial Optimization

Bachelor Thesis in Computer Science

Min-Max and Regret Spanning Trees

Complexity and Approximation under Discrete and Interval Uncertainty

Submitted by

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Abstract

Spanning tree problems connect all network nodes at minimum cost, but edge costs are often uncertain due to market fluctuations, estimation errors, or operational changes. This thesis studies robust spanning trees under two uncertainty models (discrete scenarios, interval ranges) and two objectives (Min-Max, Min-Max Regret), classifying computational complexity and approximability across this design space.

We provide a self-contained treatment from first principles. After establishing MST foundations with five complete proofs (fundamental cycle and cut lemmas, three optimality criteria via exchange arguments), we systematically analyse four problem variants. For intervals, we prove extremal lemmas showing worst cases occur at boundaries: Min-Max becomes polynomial whilst Regret remains NP-hard with a 2-approximation. For discrete scenarios, we prove weak NP-hardness for K=2 via partition reduction and survey results for larger K including pseudo-polynomial algorithms and approximation schemes.

The thesis delivers nine complete proofs including two representatives (partition reduction, 2-approximation algorithm) and synthesises eight problem variants in a comprehensive classification table. A fixed four-vertex micro-graph illustrates concepts throughout. This work balances mathematical rigour with pedagogical clarity, serving as a focused entry point into robust combinatorial optimisation.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation

1.2 Research Questions and Scope

1.3 Contributions

1.4 Thesis Structure

Chapter 2

Foundations

2.1 Graphs, Trees, and Notation

2.2 Minimum Spanning Tree Optimality

Lemma 1 (Fundamental Cycle).

Proof. □

Lemma 2 (Fundamental Cut).

Proof. □

Theorem 1 (MST Cycle Criterion).

Proof. □

Theorem 2 (MST Cut Criterion).

Proof. □

Theorem 3 (MST Characterisation).

Proof. □

2.3 Kruskal and Prim: Algorithmic Remarks

2.4 Uncertainty Models and Robust Optimisation

2.5 Algorithmic Preliminaries

Summary

Chapter 3

Min-Max Spanning Tree

3.1 Problem Formulation

3.2 Interval Worst-Case Characterisation

Lemma 3 (Interval Extremal Cost)**.**

Proof. □

3.3 Complexity and Approximation

3.3.1 Discrete Scenarios: Constant K

Theorem 4 (Partition Reduction)**.**

Proof. □

Theorem 5 (Pseudo-Polynomial Algorithm)**.**

3.3.2 Discrete Scenarios: Unbounded K

Theorem 6 (Strong NP-Hardness)**.**

3.4 Discussion

Summary

Chapter 4

Min-Max Regret Spanning Tree

4.1 Regret Formulation

4.2 Interval Regret Extremal Behaviour

Lemma 4 (Interval Extremal Regret)**.**

Proof. □

4.3 Complexity for Discrete Scenarios

4.3.1 Constant K

Theorem 7 (K=2 Weak NP-Hardness)**.**

Proof. □

Theorem 8 (Pseudo-Polynomial for Constant K)**.**

4.3.2 Unbounded K

Theorem 9 (Strong NP-Hardness)**.**

4.4 Interval Regret Approximation

Theorem 10 (Interval NP-Hardness)**.**

Theorem 11 (2-Approximation via Midpoint)**.**

Lemma 5 (Lower Bound on Regret)**.**

Proof. □

Lemma 6 (Upper Bound Relating Solutions)**.**

Proof. □

Proof of Theorem 11. □

4.5 Discussion

Summary

Chapter 5

Synthesis and Example Gallery

Chapter 6

Conclusion and Outlook

6.1 Summary

6.1.1 Thesis Achievements

6.1.2 Core Findings

6.2 Limitations

6.2.1 Scope Restrictions

6.2.2 Proofs and Coverage

6.2.3 Micro-Graph and Empirical Work

6.3 Outlook

6.3.1 Extensions in Uncertainty Modelling

6.3.2 Extensions in Solution Concepts

6.3.3 Open Problems

Appendix A

Notation

Affidavit / Eidesstattliche Versicherung

I hereby declare that I have written this thesis independently and that I have not used any sources or aids other than those indicated. All passages which are quoted or closely paraphrased from other works are marked as such. This thesis has not been submitted in the same or a substantially similar form to any other examination board.

Place, Date Signature