Chair of Combinatorial Optimization
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Min-Max and Min-Max Regret Spanning Trees

Concepts, Complexity, and Results in Robust Combinatorial Optimization

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Abstract

This thesis is a self-contained primer on robust spanning trees under cost uncertainty. We formalize and contrast the *Min-Max* and *Min-Max Regret* objectives for discrete scenario sets and interval uncertainty, prove the foundational tools used later (fundamental cut/cycle, MST optimality via cut/cycle criteria), and present a curated synthesis of reported complexity and approximability results. A running micro-graph and a compact TikZ gallery illustrate how robust solutions differ from nominal MSTs.

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2.2 Fundamental Cut Lemma

Lemma 1 (Fundamental Cut Lemma)**.** *Let T be a spanning tree and $e \in E(T)$. Then $\delta(X_e) \cap E(T) = \{e\}$.*

Proof. □

2.3 MST Optimality: Cycle and Cut Criteria

Theorem 1. *For a spanning tree T, the following are equivalent: (i) T is an MST; (ii) every non-tree edge is a most expensive edge on its fundamental cycle; (iii) every tree edge is among the cheapest edges in its fundamental cut.*

Proof sketch. □

2.4 Kruskal and Prim (Brief Recap)

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$$\min_{T \in \mathcal{T}} \max_{u \in \mathcal{U}} \sum_{e \in T} c_e^{(u)}$$

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3.2 Short Properties Used Later

Lemma 2 (Interval extremal cost for a fixed tree)**.** *For interval costs, the worst-case cost of a fixed tree is attained by setting each chosen edge to its upper bound.*

Proof. □

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Proof. □

4.4 Reported Complexity and Approximability

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Representative Full Proof (Optional)

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Notation Table

Symbol	Meaning
$G = (V, E)$	Undirected connected graph
c_e	Edge cost (nominal)
$\delta(X)$	Cut induced by $X \subseteq V$
$\mathcal{T}$	Set of spanning trees
C_e	Fundamental cycle of non-tree edge e w.r.t. T
X_e	Vertex side for the fundamental cut of $e \in E(T)$
$\mathcal{U}$	Uncertainty set of scenarios
$c_e^{(u)}$	Cost of e in scenario u
$\text{Regret}(T, u)$	Scenario regret

Affidavit / Eidesstattliche Versicherung

I hereby declare that I have written this thesis independently and that I have not used any sources or aids other than those indicated. All passages which are quoted or closely paraphrased from other works are marked as such. This thesis has not been submitted in the same or a substantially similar form to any other examination board.

Place, Date Signature