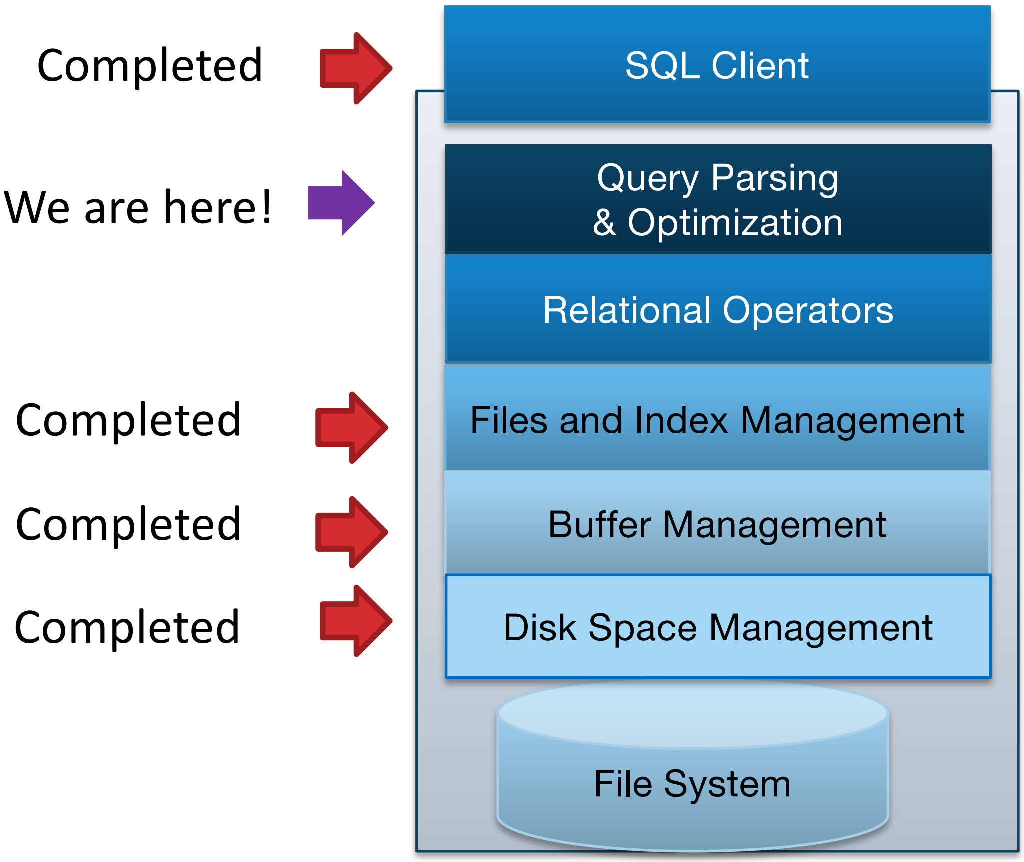
# Sorting and Hashing

## Algorithms and Costs

Alvin Cheung
Fall 2024

Reading: R & G Chapters 9.1, 13.1-3, 13.4.2

# Architecture of a DBMS: What we've learned




Parsing: take cs164!

# Query Execution

- We'll now study how to implement each query operator
  - Filtering, group by, join, etc

- Let's first focus on two fundamental operations: sorting and hashing. Goals:
  - Learn about algorithms
  - Understand their costs

# Why Sort?

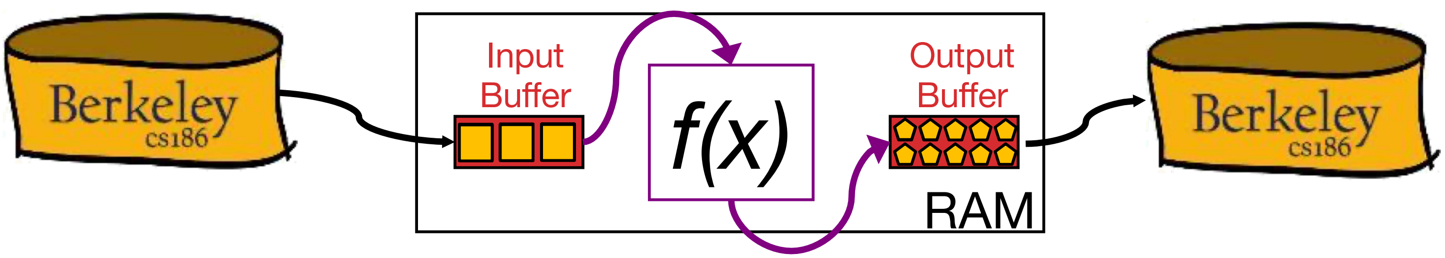
- "Rendezvous"
  - Eliminating duplicates (`DISTINCT`)
  - Grouping for summarization (`GROUP BY`)
  - Upcoming sort-merge join algorithm
- Explicitly requested: ordering
  - For ordered outputs (`ORDER BY`)
  - First step in bulk-loading tree indexes
- Problem: sort 100GB of data with 1GB of RAM
  - why not virtual memory?

# Out-of-Core Algorithms

- "Out of core"?
  - RAM used to be called core memory
  - Out of core = algorithms that can process data larger than RAM
- Two themes
  1. Single-pass streaming data through RAM
  2. Divide (into RAM-sized chunks) and Conquer

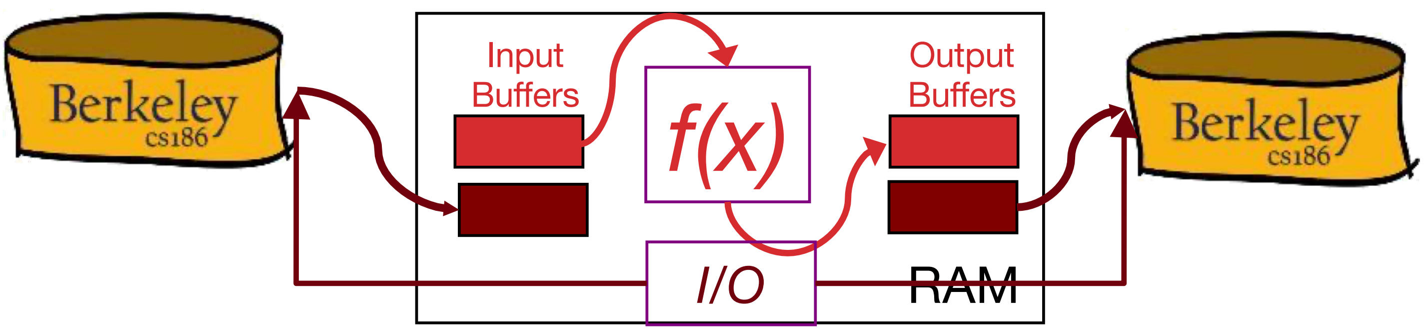
# Single-pass Streaming

- Simple case: “Map”
  - Goal: Compute f(x) for each record, write out the result
  - Challenge: minimize RAM usage and disk read/write calls
- Approach
  - Read a chunk from INPUT to an Input Buffer
  - Write f(x) for each item to an Output Buffer
  - When Input Buffer is consumed, read another chunk
  - When Output Buffer fills, write it to OUTPUT



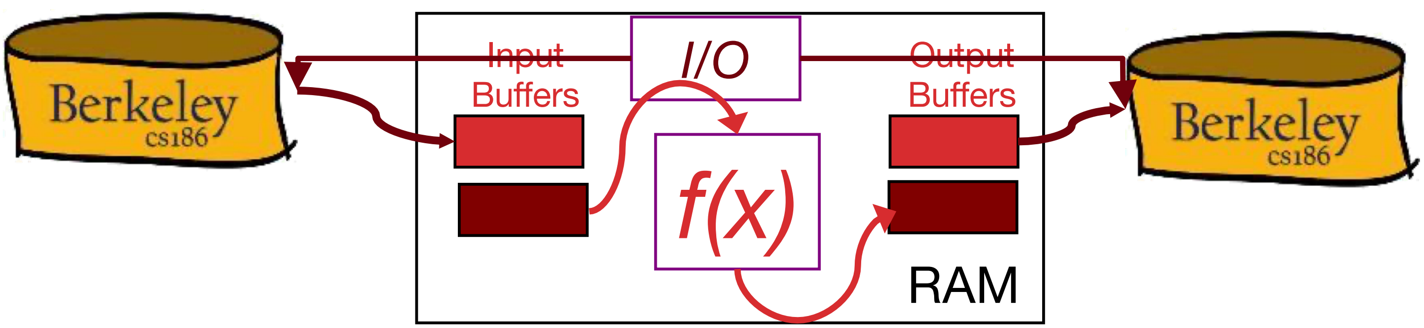
# Better: Double Buffering pt 1

- **Main thread** runs f(x) on one pair I/O buffers
- Second **I/O thread** drains/fills unused I/O buffers in parallel
  - Why is parallelism available?
  - Theme: I/O handling usually deserves its own thread
- Main thread ready for a new buffer? Swap!



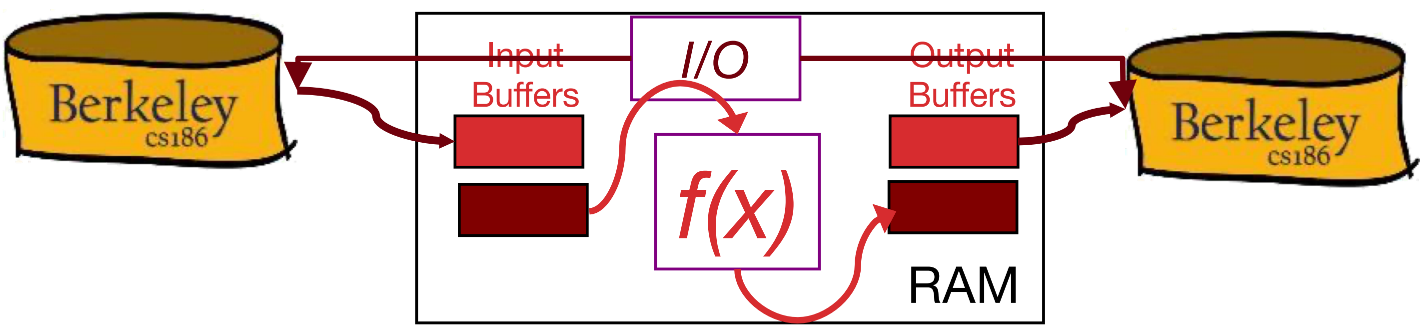
# Better: Double Buffering pt 2

- **Main thread** runs f(x) on one pair I/O buffers
- Second **I/O thread** drains/fills unused I/O buffers in parallel
  - Why is parallelism available?
  - Theme: I/O handling usually deserves its own thread
- Main thread ready for a new buffer? Swap!



# Double Buffering applies to all streams

- Usable in any of the subsequent discussion
  - Assuming you have RAM buffers to spare!
  - But for simplicity we won't bring this up again.



# Sorting & Hashing: Formal Specs

**Sorting**

- Produce an output file $F_S$
  - with contents $R$ ***stored in order by a given sorting criterion***

**Hashing**

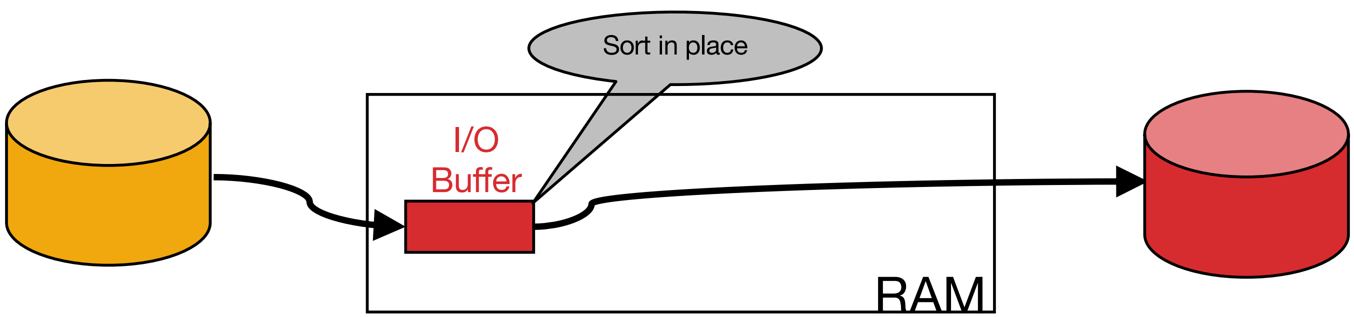
- Produce an output file $F_H$
  - with contents $R$, ***arranged on disk so that no two records that have the same value are separated by a record with a different value.***
  - I.e. matching records are always "stored consecutively" in $F_H$.
  - Recall our goal of using this to eliminate duplicates using only sequential scans

Given:

- A file $F$:
  - containing a multiset of records $R$
  - consuming **N** blocks of storage
- Two "scratch" disks
  - each with >> N blocks of free storage
- A fixed amount of space in RAM
  - memory capacity equivalent to **B** blocks of disk

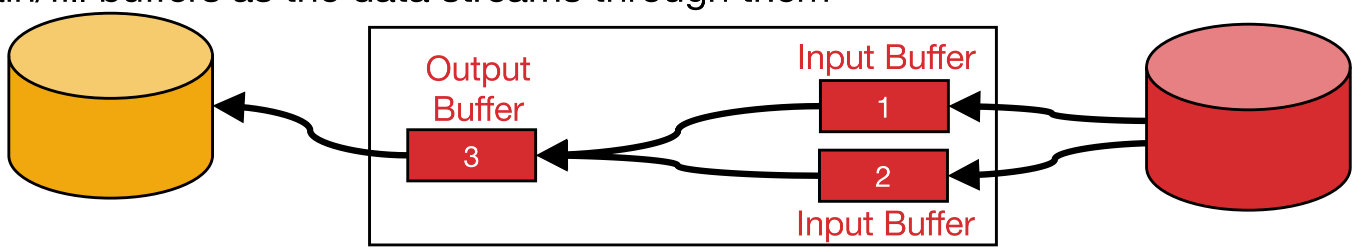
# Sorting: 2-Way (a strawman)

- Pass 1 (conquer a batch):
  - read a page, sort it, write it.
  - only one buffer page is used
  - a repeated “batch job”
  - results in N sorted blocks

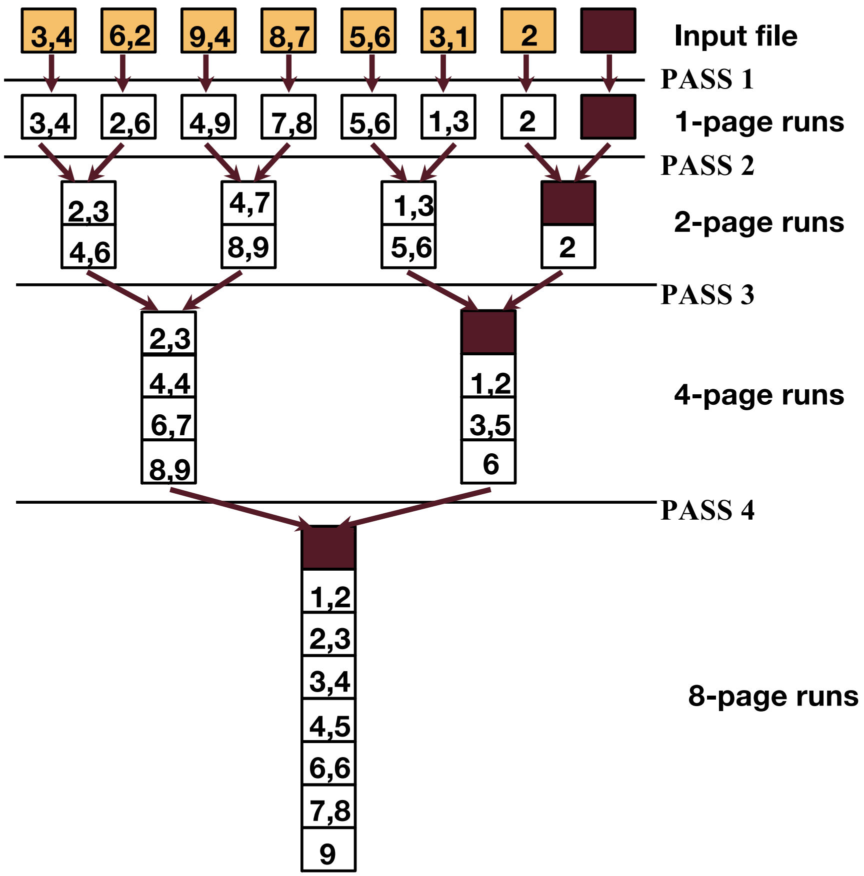


# Sorting: 2-Way (a strawman), cont

- Pass 1 (conquer a batch):
  - read a page, sort it, write it.
  - only one buffer page is used
  - a repeated "batch job"
  - results in N sorted blocks
- Pass 2, 3, 4, …, etc. (merge via streaming):
  - requires 3 buffer pages
    - note: this has nothing to do with double buffering!
  - merge pairs of runs into runs twice as long
  - a streaming algorithm, as in the previous slide!
    - Drain/fill buffers as the data streams through them



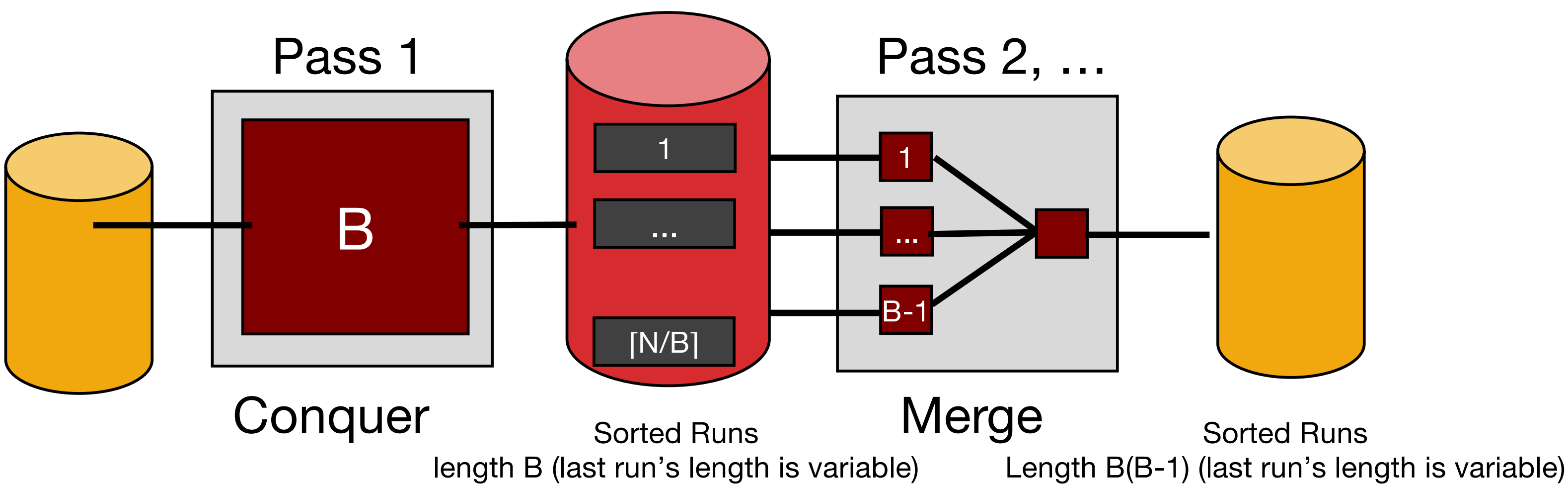
# Two-Way External Merge Sort




- Conquer and Merge:
  - sort subfiles and merge
- Each pass we read + write each page in file (2N)
- N pages in the file.
  - So, the number of passes is: $= \lceil \log_2 N \rceil + 1$
- So total I/O cost is: $2N(\lceil \log_2 N \rceil + 1)$

# General External Merge Sort

- We got more than 3 buffer pages. How can we utilize them?
  - Big batches in pass 1, many streams in merge passes
- To sort a file with N pages using B buffer pages:
  - Pass 1: use B buffer pages. Produce $\lceil N / B \rceil$ sorted runs of B pages each.
  - Pass 2, 3, …, etc.: merge B-1 runs at a time.



# Cost of External Merge Sort

- Number of passes: $1 + \lceil \log_{B-1} \lceil N / B \rceil \rceil$
- Total I/Os = (I/Os per pass) * (# of passes) = 2*N * ($1 + \lceil \log_{B-1} \lceil N / B \rceil \rceil$)

- E.g., with 5 buffer pages, to sort 108 page file:
  - Pass 1: $\lceil 108 / 5 \rceil$ = 22 sorted runs of 5 pages each
    - last run is only 3 pages
  - Pass 2: $\lceil 22 / 4 \rceil$ = 6 sorted runs of 20 pages each
    - last run is only 8 pages
  - Pass 3: $\lceil 6 / 4 \rceil$ 2 sorted runs, 80 pages and 28 pages
  - Pass 4: Sorted file of 108 pages

Formula check: $1 + \lceil \log_4 22 \rceil = 1+3$ → 4 passes √

# # of Passes of External Sort

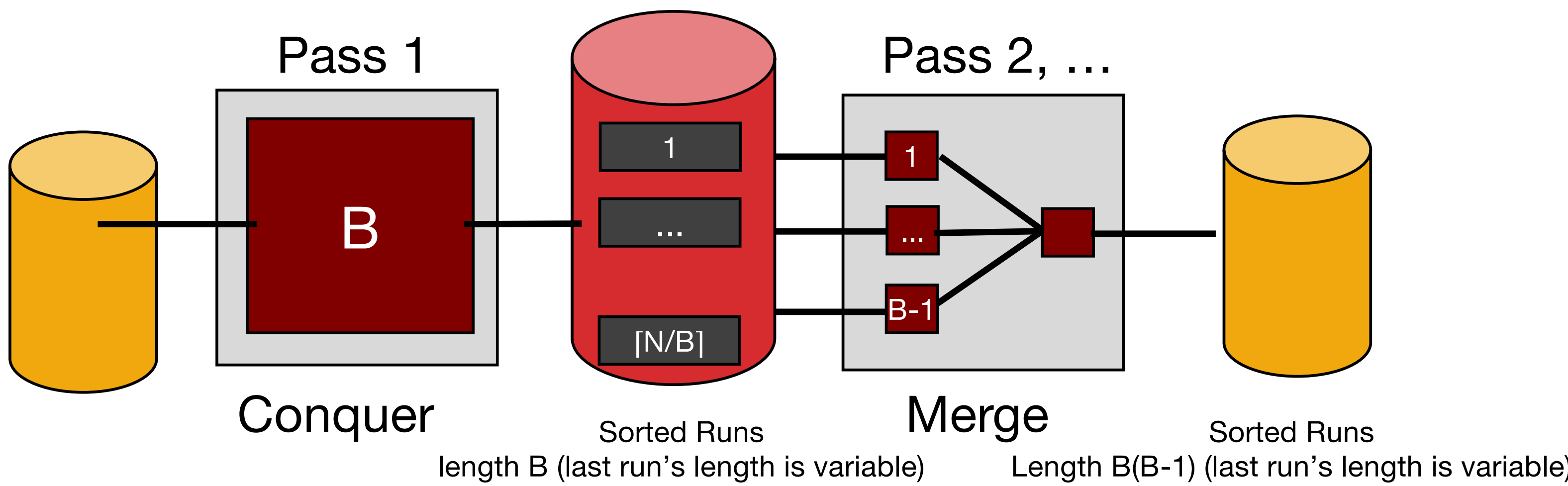
(Total I/O is 2N * # of passes)

| N | B=3 | B=5 | B=9 | B=17 | B=129 | B=257 |
|---|---|---|---|---|---|---|
| 100 | 7 | 4 | 3 | 2 | 1 | 1 |
| 1,000 | 10 | 5 | 4 | 3 | 2 | 2 |
| 10,000 | 13 | 7 | 5 | 4 | 2 | 2 |
| 100,000 | 17 | 9 | 6 | 5 | 3 | 3 |
| 1,000,000 | 20 | 10 | 7 | 5 | 3 | 3 |
| 10,000,000 | 23 | 12 | 8 | 6 | 4 | 3 |
| 100,000,000 | 26 | 14 | 9 | 7 | 4 | 4 |
| 1,000,000,000 | 30 | 15 | 10 | 8 | 5 | 4 |

Few runs can already sort large amounts of data!

# Memory Requirement for External Sorting

- How big of a table can we sort in exactly two passes?
  - Each "sorted run" after Pass 1 is of size B
  - Can merge up to B-1 sorted runs in Phase 2
- Answer: B(B-1) ~ $B^2$ data in two passes, using size B space
  - Sort X amount of data in about $\mathrm{B} = \sqrt{X}$ space (if we run only 2 passes)



# Announcements

- MT1 tomorrow evening

- Project 3 will be posted later today
  - 2 parts!
  - You can totally start this after MT1
  - Focus on MT1 first!

# Alternative: Hashing

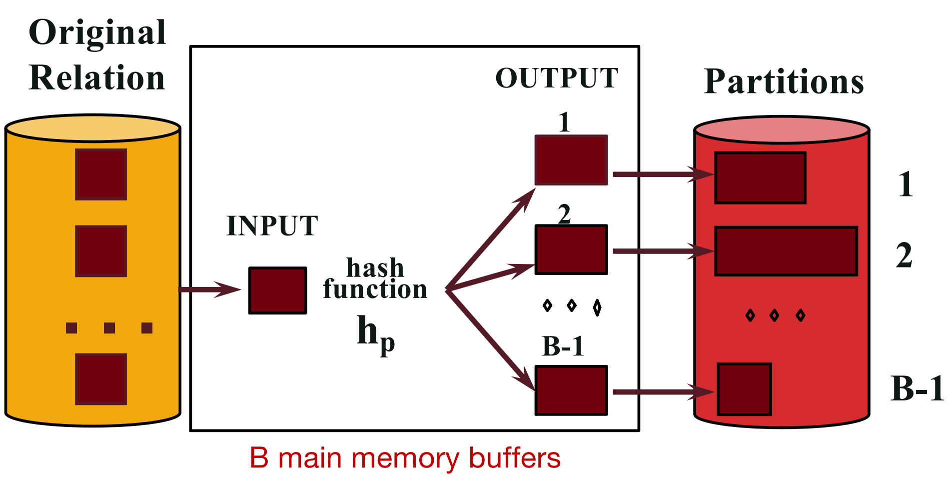
- Many times we don't require order
  - E.g., remove duplicates, form groups
- Often just need to group matches together
- Hashing does this
  - But how to do it out-of-core??

## Streaming Partition (Divide)

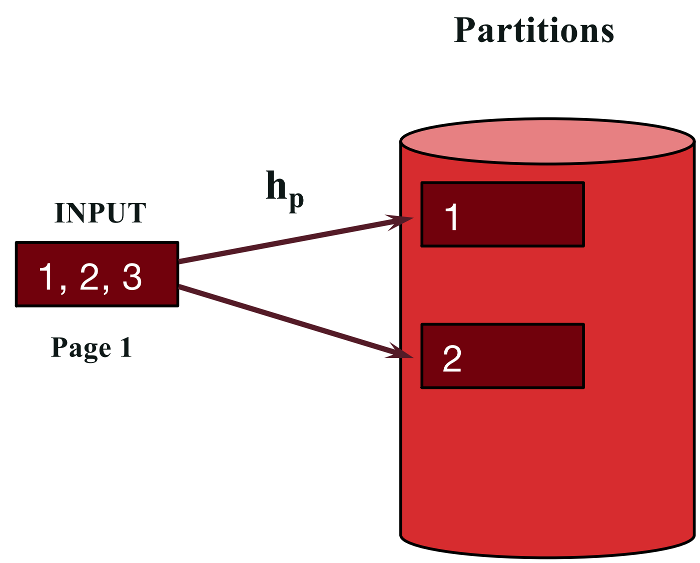
- Use a hash function $\mathbf{h_p}$ to stream records and create partitions
  - All matches grouped together in the same partition
  - Each partition can have a mix of values based on $h_p$
  - Partitions are written to disk
    - Each partition can take up multiple pages

# Two Phases: Divide

- Partition:
  (Divide)

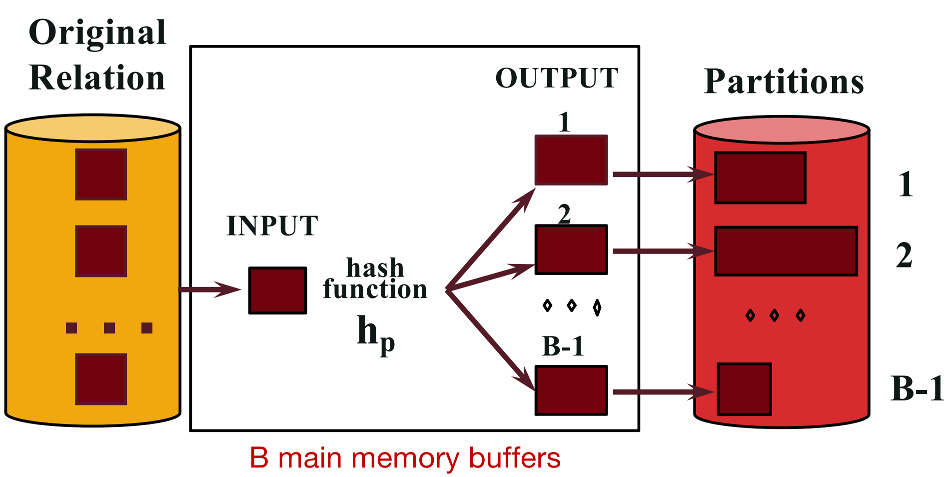



Example

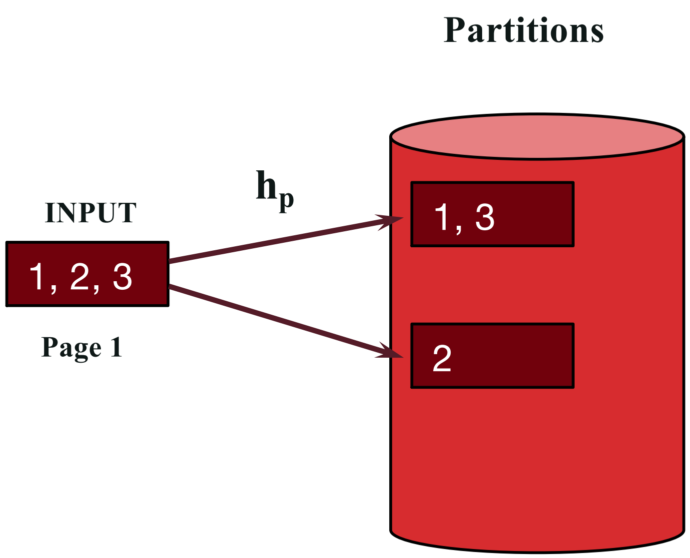


# Two Phases: Divide

- Partition:
  (Divide)

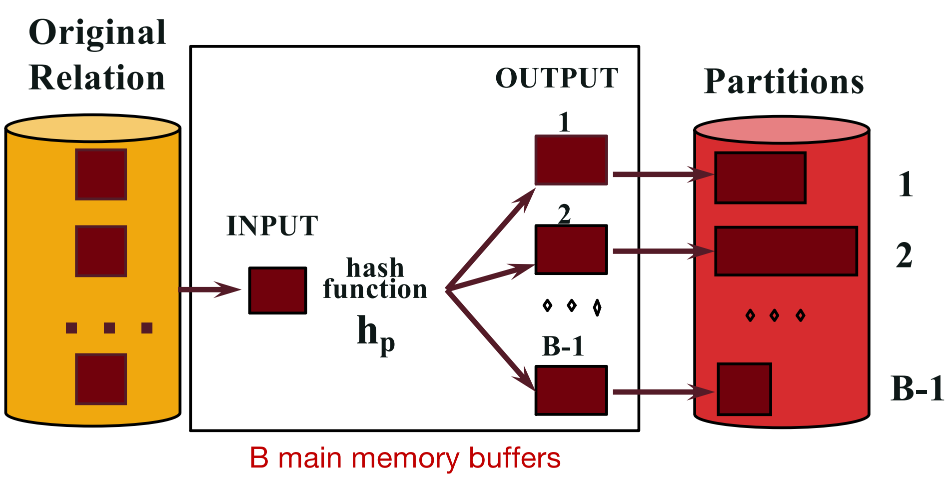



Example

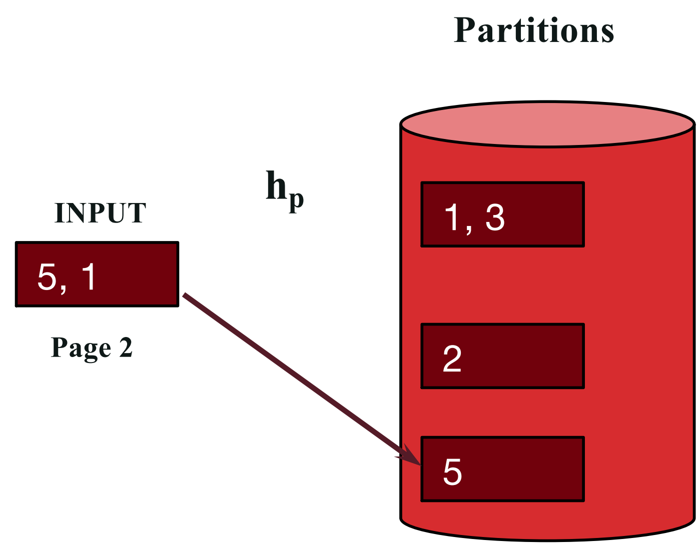


# Two Phases: Divide

- Partition:
  (Divide)

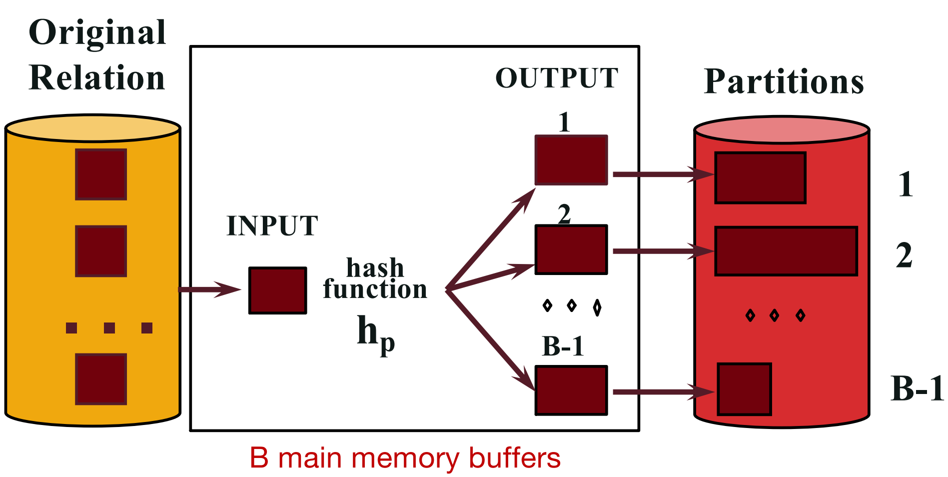



Example

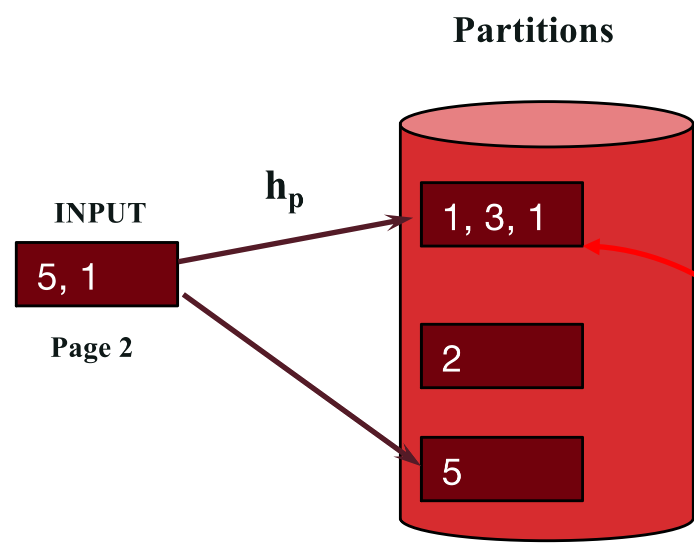


# Two Phases: Divide

- Partition:
  (Divide)




Example



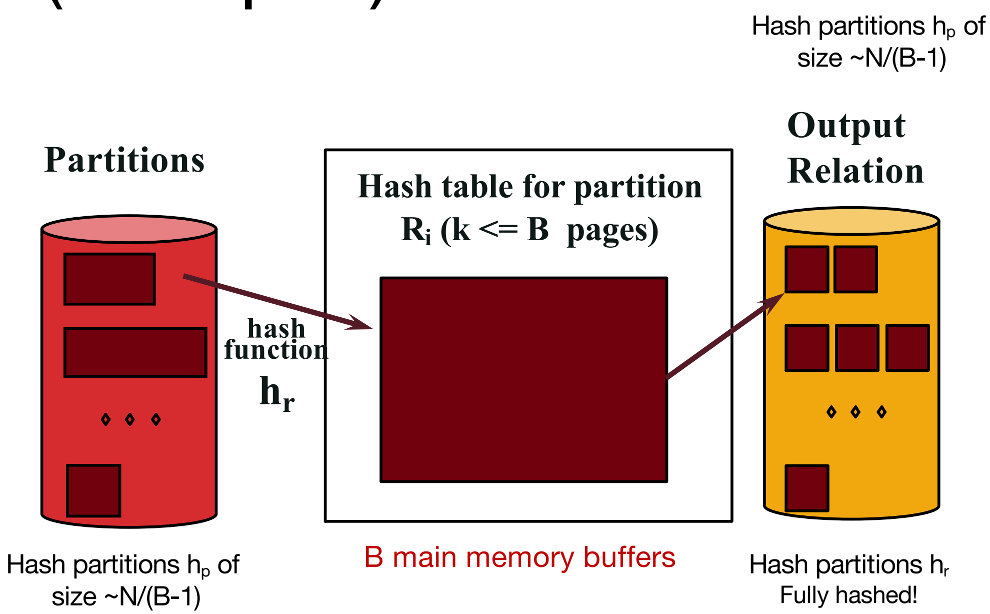

The 1's are not consecutive on disk!

# ReHash (Conquer)

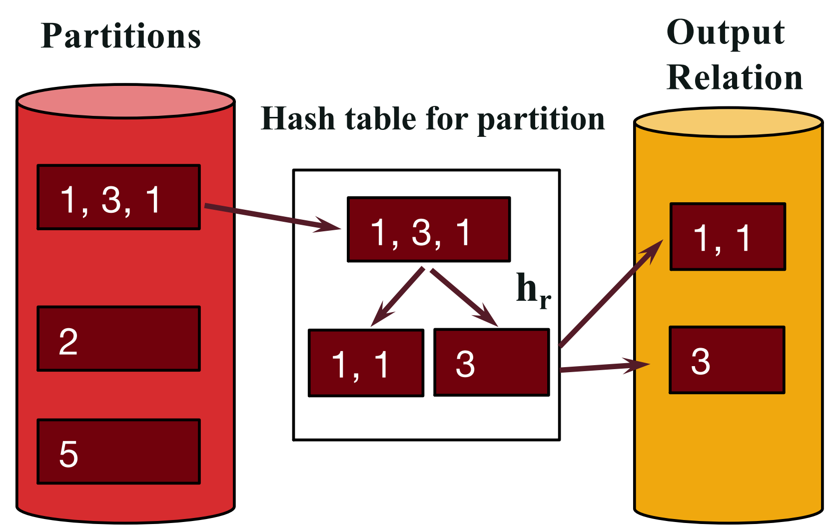
- Read each partition into RAM hash table one at a time, use *another* hash fn $\mathbf{h_r}$ to hash in RAM 
  - Each hash table bucket contains a small number of distinct values
- Read out the RAM hash table buckets and write to disk
  - Duplicate values are now contiguous on disk

# Two Phases: Conquer

- Rehash:
  (Conquer)

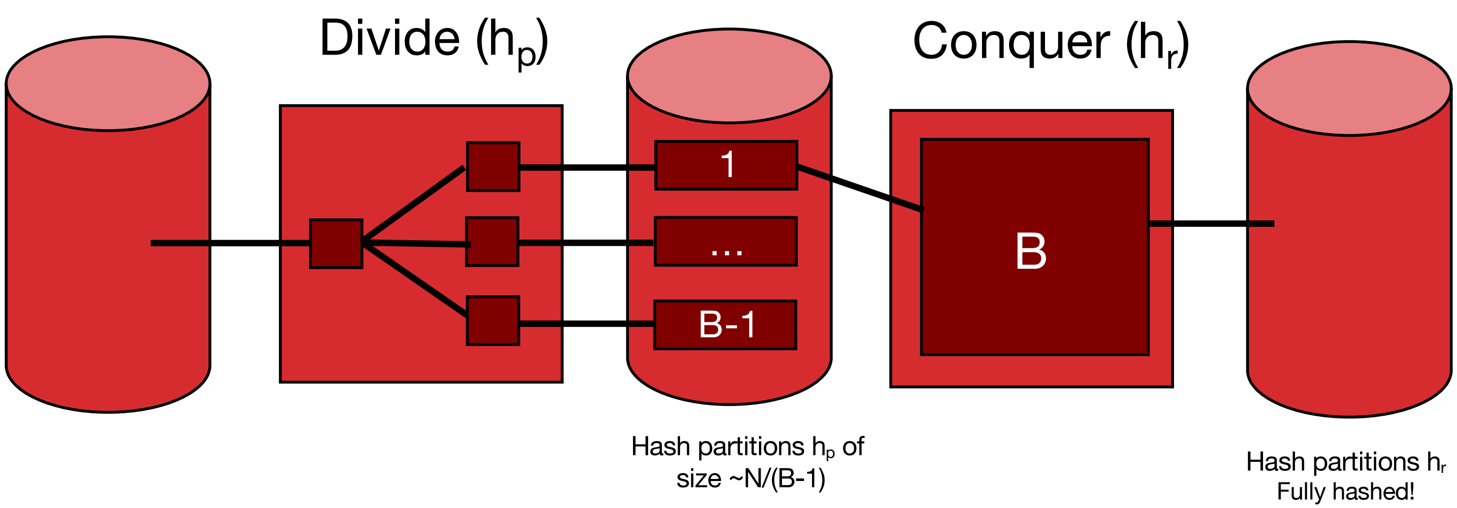



Example



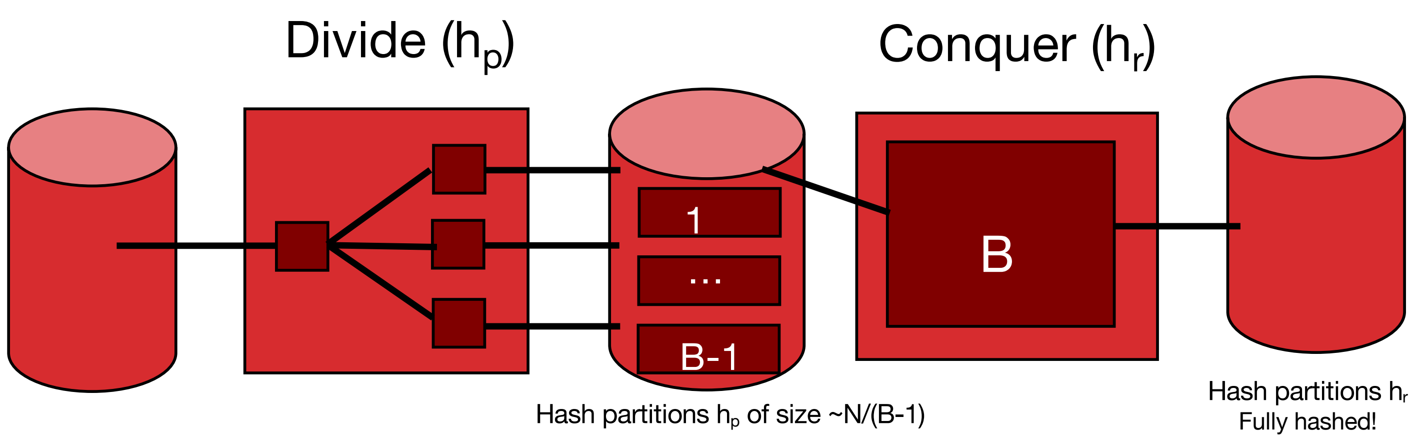
# Cost of External Hashing

Total I/Os ~ 2*N*(# passes) = 4*N
(includes initial read, final write)

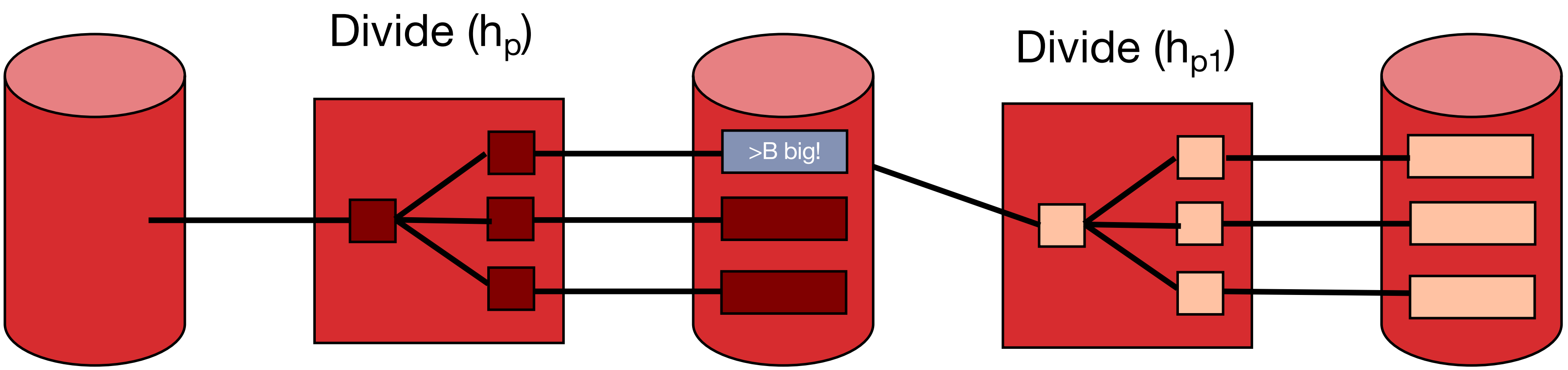


# Memory Requirement

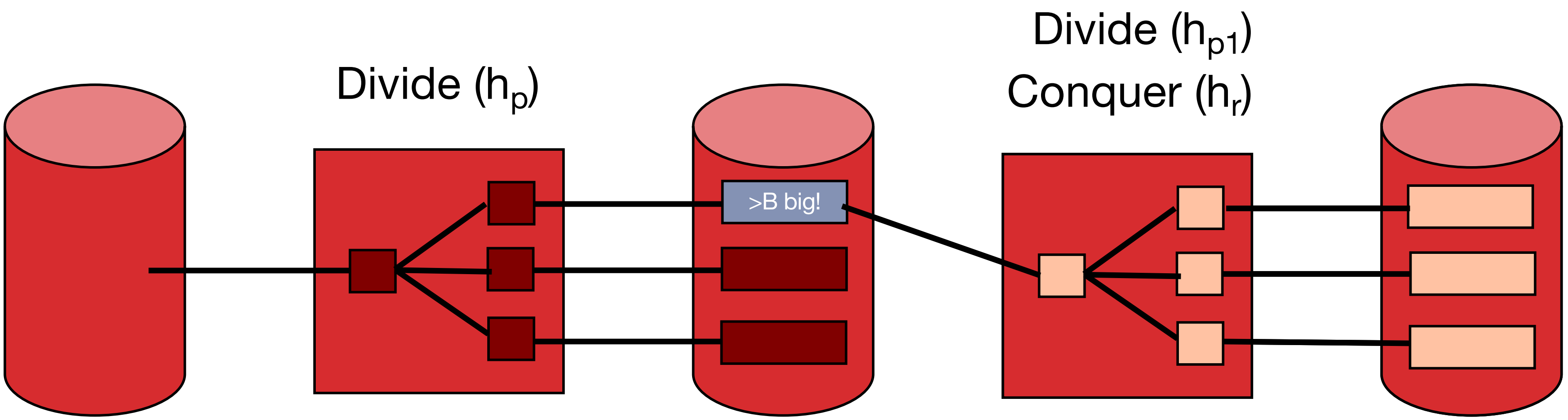
- How big of a table can we hash in exactly two passes?
  - B-1 "partitions" result from Pass 1
  - Each should be no more than B pages in size
  - Answer: B(B-1) ~ $B^2$
    - We can hash a table of size X in about $B = \sqrt{X}$ space (if we run only 2 passes)
  - Note: assumes hash function distributes records evenly!
- Have a bigger table? Recursive partitioning!



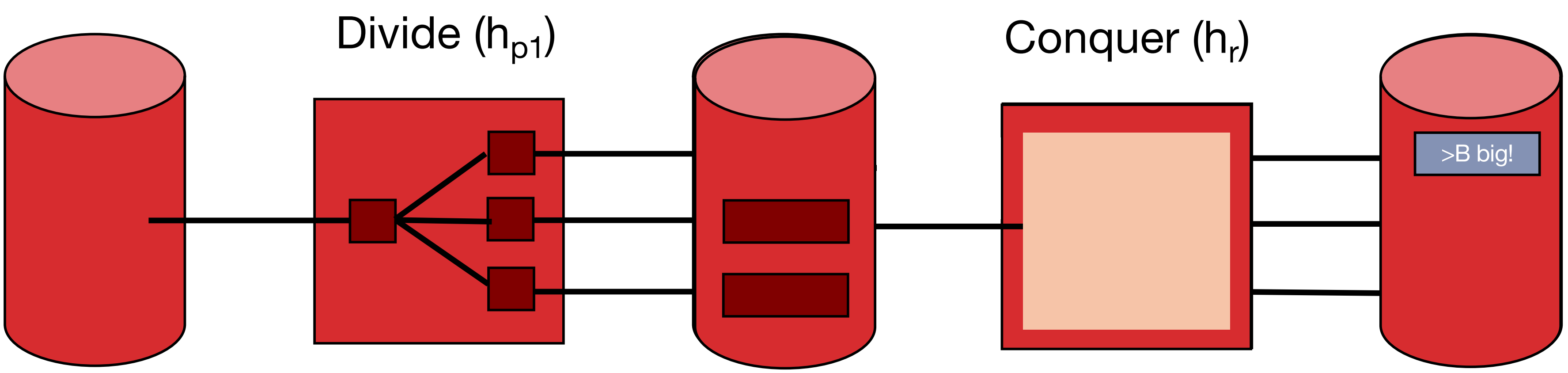
# Recursive Partitioning, Pt 1



# Recursive Partitioning, Pt 2

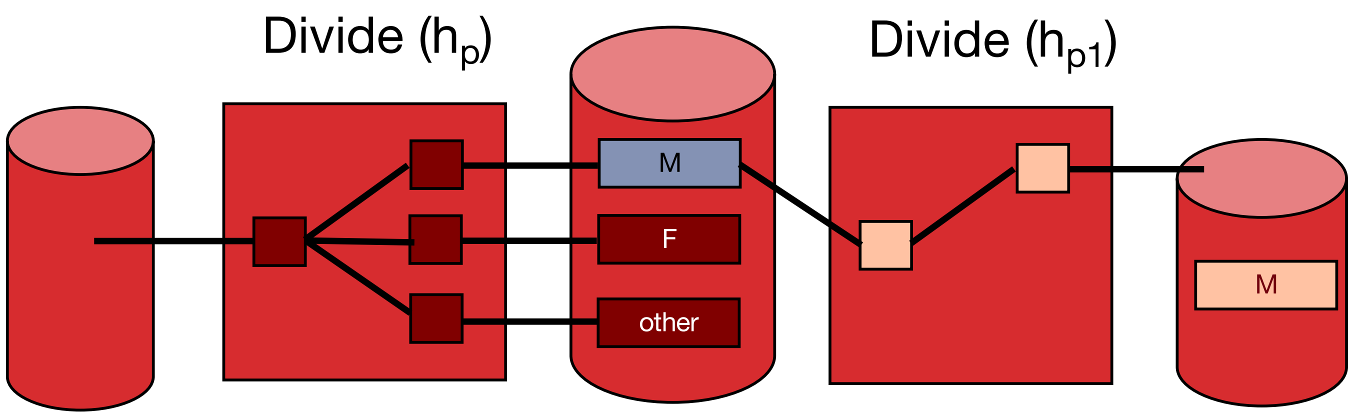


# Recursive Partitioning, Pt 3



# A Wrinkle: Duplicates

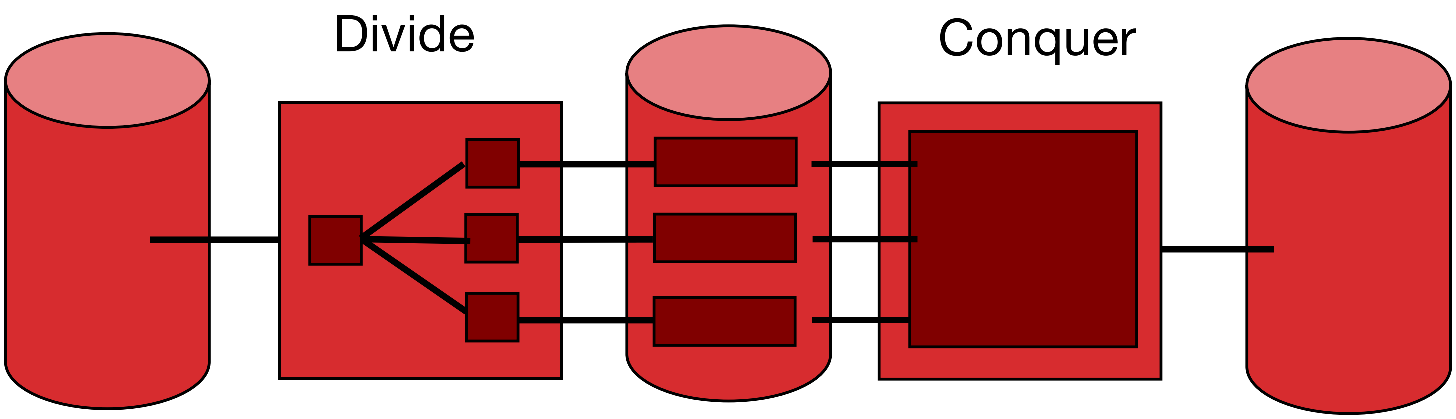
- Consider a dataset with a *very* frequent key
  - E.g. in a big table, consider the *gender* column
- What happens during recursive partitioning?



# Question…

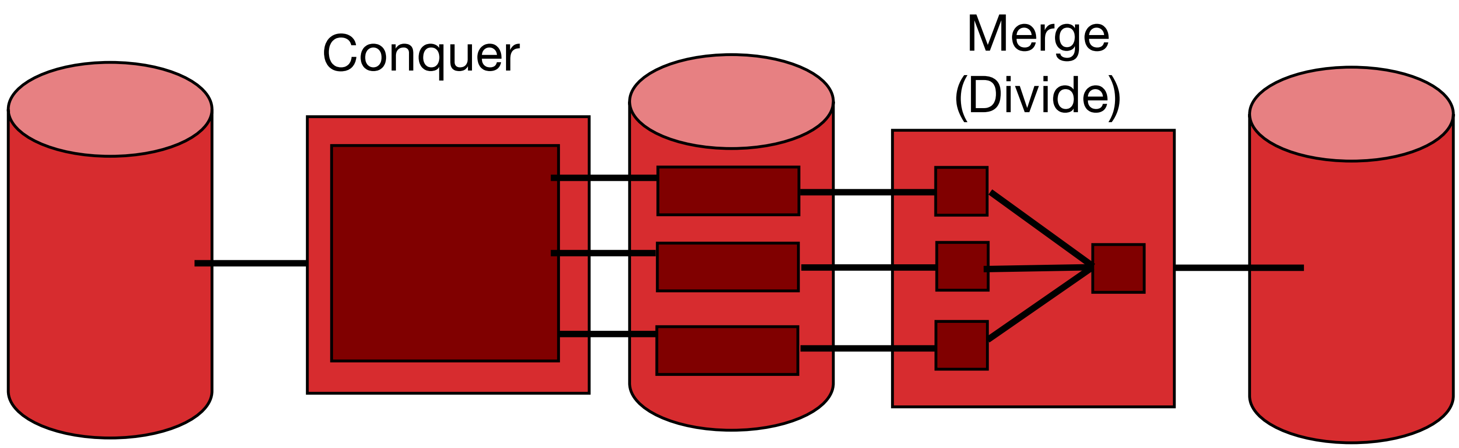
How does external hashing compare with external sorting?

# Cost of External Hashing: 1-Pass Case




**cost ~ 4*N I/Os**
**(including initial read, final write)**
**(case for 1 divide pass and 1 conquer pass only. Not the general formula!)**

# Cost of External Sorting



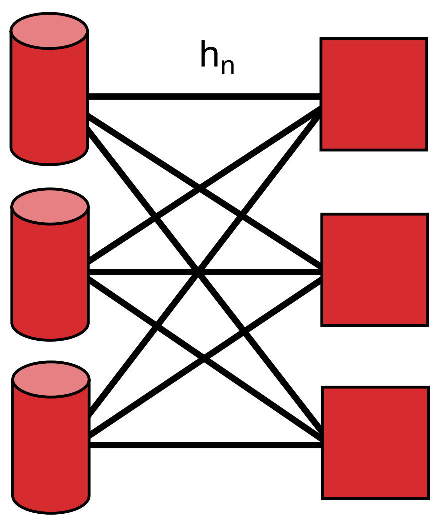

**cost = 4*N I/Os**

**(including initial read, final write)**

**(case for 1 divide pass and 1 conquer pass only. Not the general formula!)**

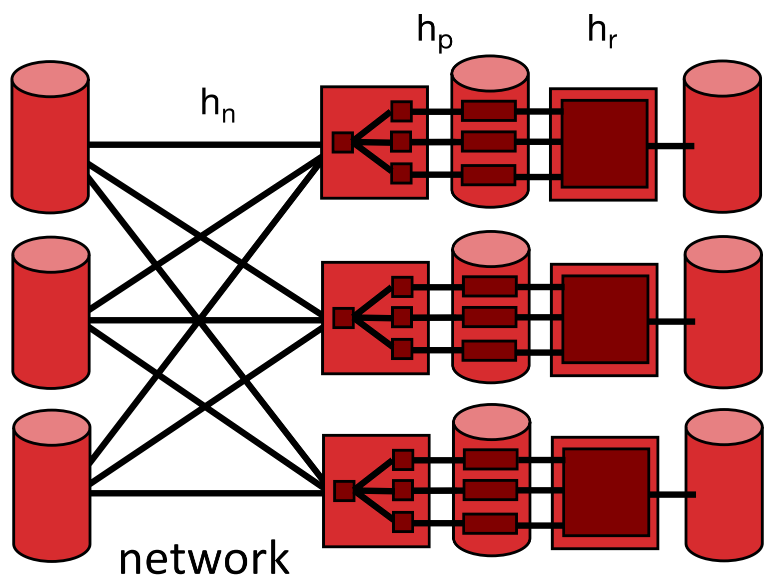
# Parallelize me! Hashing Phase 1

- Phase 1: shuffle data across machines ($h_n$)
  - stream out to network as it is scanned
  - which machine for this record?
    use (yet another) independent hash function $h_n$



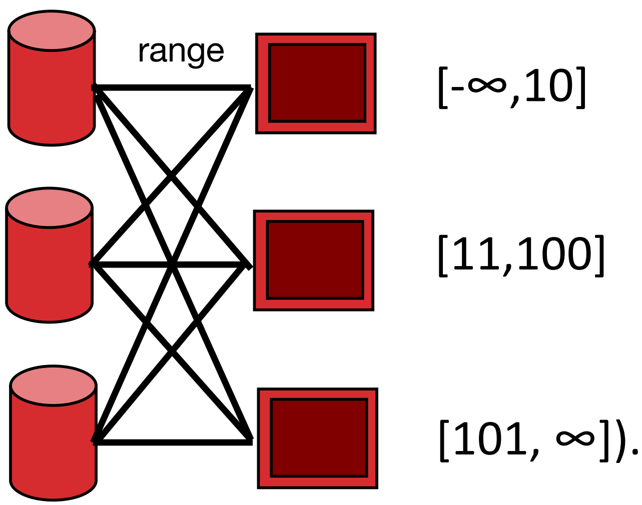
# Parallelize me! Hashing Phase 2

- Phase 1: shuffle data across machines (hn)
- Receivers proceed with phase 1 as data streams in
  - from local disk and network



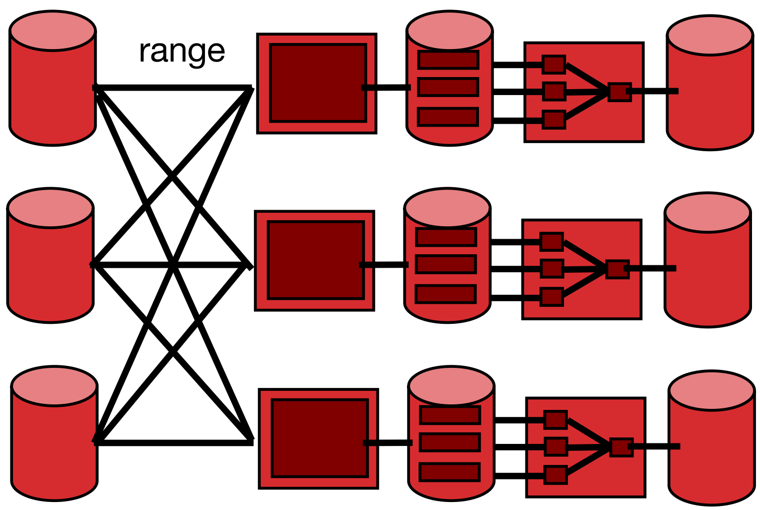
# Parallelize me!  Sorting

- Pass 1: shuffle data across machines
  - stream out to network as it is scanned
  - which machine for this record?
    Split on value range (e.g. [-∞,10], [11,100], [101, ∞]).



# Parallelize me! Sorting, cont

- Pass 1: shuffle data across machines
- Receivers proceed with pass 1 as the data streams in
- A Wrinkle: How to ensure ranges are the same #pages?!
  - i.e. avoid data skew?



# Summary

- Sort/Hash Duality
  - Hashing is Divide & Conquer
  - Sorting is Conquer & Merge
- Sorting is overkill if we just want to group tuples together
  - But sometimes a win anyhow
- Don’t forget one pass streaming and double buffering
  - Can “hide” the latency of I/O behind CPU work
- Don’t we have TB of RAM these days?
  - We still have a memory hierarchy!