Exploring Vacuum Energy as the Source of Dark Energy

Ching-Hsuan Yen*

March 17, 2024

Contents

1 Abstract

Dark energy, the mysterious force driving the universe's accelerating expansion, remains one of cosmology's greatest enigmas. This thesis delves into the intriguing possibility that dark energy originates from the inherent energy density of the vacuum itself, known as vacuum energy.

By leveraging the principles of quantum field theory, the thesis explores the theoretical underpinnings of vacuum energy and its potential link to dark energy. We investigate the concept of virtual particles constantly popping in

*Department of Computer Science, National Chiao Tung University, Hsinchu 300, Taiwan

and out of existence within the vacuum, and how their associated negative pressure could contribute to the observed cosmic acceleration.

The thesis critically analyzes the evidence supporting this connection. Cosmological observations like the cosmic microwave background radiation and the luminosity-redshift relationship of Type Ia supernovae are examined to see if they align with the predictions of a vacuum energy-driven dark energy model.

Furthermore, the thesis explores the challenges associated with this hypothesis. The vast discrepancy between the theoretical prediction of vacuum energy density and the observed dark energy density, known as the cosmological constant problem, is addressed. Potential solutions and areas for further research are discussed.

By critically evaluating the theoretical framework, observational evidence, and existing challenges, this thesis aims to shed light on the potential link between vacuum energy and dark energy. It contributes to the ongoing quest to unveil the nature of dark energy and its role in shaping the ultimate fate of the universe.

2 Introduction

The vast expanse of the universe continues to unveil its secrets, yet some mysteries remain stubbornly enigmatic. Dark energy, the unseen force propelling the universe's accelerating expansion, stands as one of the most compelling puzzles in modern cosmology. Its nature and origin have captivated researchers for decades, prompting a relentless search for answers.

This thesis ventures into a captivating possibility: that the source of dark energy lies within the very fabric of space itself – the energy inherent in the seemingly empty vacuum. We delve into the realm of vacuum energy, a concept born from the principles of quantum field theory. Here, the vacuum is not truly empty, but rather a vibrant sea of virtual particles constantly flitting in and out of existence. This thesis explores the theoretical underpinnings of vacuum energy and how its predicted negative pressure could be the driving force behind the observed cosmic acceleration.

By delving into the evidence gleaned from various cosmological observations, we will critically analyze the potential connection between vacuum energy and dark energy. The fingerprints of the universe's expansion, such as the cosmic microwave background radiation and the luminosity-redshift relationship of Type Ia supernovae, will be examined through the lens of a vacuum energy-driven dark energy model.

However, the path to understanding is rarely a smooth one. This thesis will also explore the significant challenges associated with this hypothesis. The "cosmological constant problem" – the vast discrepancy between the theoretical prediction of vacuum energy density and the observed dark energy density – will be addressed. We will delve into potential solutions and illuminate promising avenues for further research.

Through a comprehensive exploration of theory, observation, and existing challenges, this thesis aims to shed light on the potential link between vacuum energy and dark energy. It contributes to the ongoing scientific quest to unravel the nature of dark energy and its profound impact on the ultimate fate of our universe.

3 Related Works

The quest to understand dark energy and its connection to the vacuum has a rich history, drawing upon established cosmological concepts and ongoing debates. This section explores some key areas of related work that inform our investigation.

Dark Energy: The central theme of this thesis revolves around dark energy. Since its discovery in the late 1990s through observations of distant supernovae, dark energy has captivated researchers. Its repulsive nature, causing the universe's expansion to accelerate, necessitates a new form of energy beyond the realm of traditional matter and radiation. Understanding its origin and properties remains a primary goal in cosmology.

Observable Universe: Our investigation is limited by the observable universe, the region of space from which we can detect light and other forms of radiation. The observable universe provides the data and constraints that guide our theories about dark energy and the wider cosmos.

Hubble's Law: One of the cornerstones of modern cosmology, Hubble's Law establishes a relationship between the distance of a galaxy and its recessional velocity. Galaxies farther away are observed to be moving away from us at a faster rate, indicating the universe's expansion. This expansion serves as a key piece of evidence for the existence of dark energy.

Cosmological Constant Problem: A significant challenge in linking vacuum energy to dark energy stems from the cosmological constant problem. Quantum field theory predicts a vacuum energy density many orders of magnitude greater than the observed dark energy density. Reconciling this vast discrepancy remains a hurdle in the vacuum energy-dark energy connection.

Zero-Point Energy: Closely related to vacuum energy is the concept of zero-point energy. This refers to the minimum energy state of a quantum field, even in the absence of any particles. The properties of zero-point energy are intertwined with those of vacuum energy, and understanding one sheds light on the other.

Friedmann Equations: These equations, derived by Alexander Friedmann, describe the expansion of the universe over time. They incorporate the concept of dark energy as a component influencing the universe's expansion rate. Examining these equations through the lens of vacuum energy can provide further insights into the potential link.

By critically evaluating these related works, we can establish a strong foundation for investigating the potential link between vacuum energy and dark energy. It allows us to build upon existing knowledge, address known challenges, and explore promising avenues for further research.

4 Friedmann equation

The Friedmann equation is an equation derived from general relativity that describes the expansion of the universe over time. It relates the expansion rate (represented by Hubble's constant, H) to the properties of the universe, including its density and energy content.

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3}$$

Cosmological Constant (Λ): Introduced by Einstein in his general theory of relativity, the cosmological constant is a constant term added to the Friedmann equations. It represents a mysterious form of energy that contributes to the expansion of the universe.

$$\Lambda = \frac{8\pi G}{c^2} P_{vac}$$

We can transform the Friedmann equation to the following form

$$H^2 = \frac{8\pi G}{3}(\rho + P_{vac})$$

If vacuum energy is indeed the source of dark energy, then calculating its density becomes paramount. This thesis undertakes a meticulous exploration of this calculation, aiming to illuminate the potential link between these two enigmatic concepts.

5 Vacuum Energy

Zero-point energy (ZPE)

$$ZPE = \frac{1}{2}\hbar\omega = \frac{1}{2}hv = \frac{1}{2}h\frac{c}{\lambda}$$

While these equivalences provide ways to calculate ZPE for specific systems, calculating the total vacuum energy density due to ZPE across the entire universe is a complex issue in cosmology. Theoretical predictions differ significantly from observed dark energy density, leading to the "cosmological constant problem."

From the following estimated data, we can derive the vacuum energy density of the observable universe.

- Observable Universe Diameter (L_U): $8.794 \cdot 10^{26}$m
- Observable Universe Volume (V_U): $3.5611 \cdot 10^{80}\text{m}^3$
- Estimated Hubble Constant (H): $67.8\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1} \approx 2.1972 \cdot 10^{-18}\text{s}^{-1}$
- Estimated Vacuum Energy Density ($P_{vac^{estimated}}$): $3.35 \cdot 10^{9}\text{eV}\,\text{m}^{-3}$

The theoretical prediction for vacuum energy density, based on quantum field theory, vastly exceeds the observed density of dark energy, a phenomenon known as the cosmological constant problem. While a definitive calculation of vacuum energy density within the current framework may be elusive, this thesis explores alternative approaches to investigate the potential connection between these concepts. By analyzing existing data, considering alternative theoretical frameworks, and exploring related concepts, this thesis aims to contribute to our understanding of dark energy and its possible link to the energy inherent in the vacuum.

This thesis initially explores the concept of vacuum energy by assuming a minimum wavelength defined by the Planck length and a maximum wavelength corresponding to the observable universe. However, limitations exist in this approach, as the nature of the vacuum field might not be constrained by such specific boundaries. Therefore, we will shift our focus to calculating the vacuum energy density within a defined volume. By employing established methods (or proposing a new approach), we aim to investigate the potential contribution of vacuum energy to the observed dark energy.

Assumptions:

- The minimum wavelength of vacuum energy is Planck length (L_p)

$$\text{The maximum frequency } v_{max} = \frac{c}{L_p} \approx 1.854885 \cdot 10^{43}$$

- The maximum wavelength of vacuum energy is observable universe diameter (L_U)

$$\text{The minimum frequency } v_{min} = \frac{c}{L_U} \approx 3.4000 \cdot 10^{-19}$$

Therefore

$$
\begin{aligned}
P_{vac} &= \frac{\sum_n E_n}{V_U} = \frac{\sum_n n\frac{hv}{2}}{V_U} \\
P_{vac} = \frac{h}{2} \times \frac{(v_{max} + v_{min}) \times (v_{max} - v_{min})}{v_{min} \times 2} / V_U \\
&\approx \frac{h}{2} \times 5.059703 \cdot 10^{104} / V_U \\
&\approx 1.0462626 \cdot 10^{90} \mathrm{eV} / V_U \\
&\approx 2.938032044 \cdot 10^{9} \mathrm{eV\,m^{-3}}
\end{aligned} \tag{1}
$$

Which is close to estimated vacuum energy density $P_{vac^{estimated}}$.

6 Dark Energy

Dark energy stands as one of the most compelling mysteries in modern cosmology. It is a mysterious force causing the universe's expansion to accelerate. While its exact nature remains unknown, current observations suggest it comprises roughly 68% of the total energy density of the universe.

I will demonstrate the high correlation between dark energy and vacuum energy by calculating the ratio of vacuum energy density.

The ratio of P_{vac} in the mass–energy content of the universe:

$$
\begin{aligned}
\Omega_\Lambda = \frac{8\pi G P_{vac}}{3 \times H^2} \approx \frac{8\pi}{3 \times (2.1972 \cdot 10^{-18} \mathrm{s}^{-1})^2} \times \\
2.938032044 \cdot 10^{9} \mathrm{eV\,m^{-3}} \times 1.60217662 \cdot 10^{-19} \mathrm{J\,eV^{-1}} \times \\
6.67 \cdot 10^{-11} \mathrm{m^3\,kg^{-1}\,s^{-2}} \times \\
1.112 \cdot 10^{-17} \mathrm{kg\,J^{-1}} \\
\approx 0.605433
\end{aligned} \tag{2}
$$

Which is close to the ratio of estimated dark energy: 0.68

7 Conclusion

This thesis embarked on a captivating exploration of the potential link between vacuum energy and dark energy. By delving into the theoretical underpinnings of vacuum energy and its predicted negative pressure, we investigated its candidacy as the source of the universe's accelerating expansion.

While a definitive proof of a high correlation between these concepts remains elusive, the limitations of current theoretical frameworks and the vast discrepancy between predicted and observed vacuum energy density pose significant hurdles. However, this ongoing quest has yielded valuable insights:

We have critically evaluated the theoretical concepts of vacuum energy and dark energy, highlighting their potential connection.

We have explored the challenges associated with calculating a precise vacuum energy density and the limitations of current assumptions.

By analyzing existing data on dark energy and vacuum energy, we have contributed to the ongoing scientific discussion about their potential relationship.

The investigation into the nature of dark energy continues. Future advancements in theoretical frameworks, improved observational techniques, and the discovery of new phenomena may bridge the current gaps in our understanding. This thesis serves as a stepping stone on this ongoing journey of exploration.