Cargo Operations of Express Air Project
ORIE 5380 - Optimization Methods

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1. Executive Summary

Express Air is a cargo operator that is seeking to determine a weekly aircraft movement cycle that delivers all its cargo while minimizing total cost. As part of our analysis, we are unable to create an aircraft movement cycle with Express Air's current number of aircraft and the schedule in which cargo arrives into the system. However, feasible solutions can be found by either increasing the number of aircraft or redistributing the cargo arrivals. We ultimately recommend a solution that does the latter, with an associated cost of 15,125.

2. Problem Overview

Express Air is a cargo operator that delivers cargo between three airports (A, B, and C) using its aircraft fleet. There are two components that Express Air needs to consider when delivering cargo:

1) Cargo loads
    - Each day, a set amount of cargo loads arrives within the system that needs to be delivered between each origin-destination pair
    - Each load of cargo requires a single aircraft to carry
    - The total amount of cargo loads that needs to be delivered between each origin-destination pair on any given day is equal to the newly arrived cargo loads that day, plus any remaining cargo loads that was not delivered in the previous days. Express Air determines the portion of cargo loads that will be delivered on any given day, which is constrained by the number of aircraft available at an airport
    - There is a cost per day associated to holding and not delivering a cargo load
2) Aircraft Positioning
    - For each airport, Express Air has three options on how to handle the aircraft on any given day:
        1) The airport can use its aircraft to deliver cargo to its destinations
        2) The airport can ground the aircraft, and have it remain at the airport to be available for the next day
        3) The airport can reposition empty aircraft to other airports, so to help balance the cargo load delivery requirements
    - It takes one full day to travel between each airport
    - Any aircraft that's repositioned or used to deliver cargo will be available for use by the destination airport the following day
    - There is a cost associated to repositioning an aircraft for each origin-destination pair

Since there's a consistent weekly pattern for cargo loads that needs to be delivered, Express Air wants to find a repeatable weekly movement cycle for the aircraft. This implies that the aircraft that moves into an airport on Friday evening should be equal to the aircraft that's available for the same airport for the following Monday morning.

Furthermore, since the amount of cargo that needs to be delivered each week is fixed, we cannot optimize the revenue and costs of delivering cargo. Instead, we are tasked with minimizing the cost to reposition empty aircraft and the cost to hold cargo, while ensuring all cargo is delivered by the end of the week and that we have a repeatable weekly aircraft movement cycle.

3. Data Description

The number of aircraft that Express Air has available to distribute across its three airports is 1,200. The amount of cargo loads arriving into the system that needs to be delivered between origin destination pairs for each day is given by Table 1 below:

Origin-Destination	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	100	200	100	400	300
A-C	50	50	50	50	50
B-A	25	25	25	25	25
B-C	25	25	25	25	25
C-A	40	40	40	40	40
C-B	400	200	300	200	400

Table 1: Amount of cargo loads arriving into the system on each day that needs to be delivered between each origin-destination pair

The cost to hold one load of cargo is 10 per day, while the cost to reposition empty aircraft between airports is given by Figure 1 below:

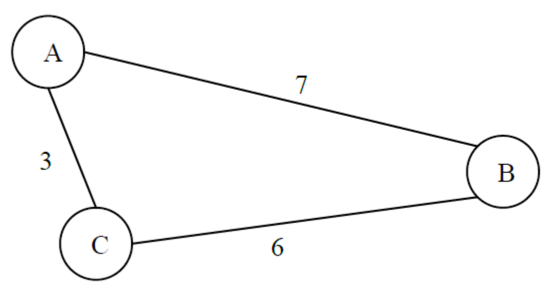

Figure 1: Repositioning costs among different airports

4. Overview of Optimization Model and Mathematical Details

To solve for Express Air's weekly movement pattern, we establish several decision variables for our optimization problem:

- Let $x_{i,j,t}$ represent the amount of cargo loads that needs to be delivered between airport i and airport j at time t
- Let $y_{i,j,t}$ represent the amount of cargo loads shipped out of airport i to airport j on day t. Since each aircraft can carry only one cargo load, this also represents the number of aircraft that is delivering cargo between airport i and airport j
- Let $z_{i,j,t}$ represent the number of aircraft repositioned from airport i to airport j on day t

- Let $s_{i,t}$ represent the number of aircraft that's grounded at airport i on day t (i.e. the aircraft does not leave the airport)

We create these decision variables $\forall i, j \in \{A, B, C\}, i \neq j, t = 1, 2, 3, 4, 5$. All decision variables are non-negative integer values.

The objective function is given by the sum of the holding costs and repositioning costs across all the days. The amount of cargo that we hold on a given day t per origin-destination pair can which can be calculated as the amount of cargo loads that needs to be delivered minus the amount that's actually delivered: $x_{i,j,t} - y_{i,j,t}$. The number of aircraft that's repositioned on a given day is just given by $z_{i,j,t}$. Therefore, the objective of our optimization problem is:

$$min \sum_{t=1}^{5} \sum_{i=\{A,B,C\}} \sum_{j=\{A,B,C\}} 10(x_{i,j,t} - y_{i,j,t}) + r_{i,j} z_{i,j,t}$$

where $r_{i,j}$ is the cost to reposition aircraft between airport i and airport j (given by Figure 1) and $i \neq j$.

We then establish the constraints of the problem:

C1) The number of cargo loads shipped out of each origin-destination pair cannot exceed the available cargo load to ship:

$$y_{i,j,t} \leq x_{i,j,t} \ \forall i, j \in \{A, B, C\}, i \neq j, t = 1, 2, 3, 4, 5$$

C2) The amount of cargo loads available to be shipped between each origin-destination pair on any given day is equal to the cargo that was held from the previous day (given by $x_{i,j,t} - y_{i,j,t}$) plus the arrival of new cargo loads into the system. This can be represented as:

$$x_{i,j,t+1} = x_{i,j,t} - y_{i,j,t} + D_{i,j,t+1} \ \forall i, j \in \{A, B, C\}, i \neq j, t = 1, 2, 3, 4$$

where $D_{i,j,t+1}$ is the cargo loads arriving into the system given by Table 1.

C3) Since we require that all cargo loads be delivered by the end of the week, we create a constraint that enforces that all cargo loads are delivered between each origin-destination pair by Friday:

$$x_{i,j,5} - y_{i,j,5} = 0 \ \forall i, j \in \{A, B, C\}, i \neq j$$

C4) At the beginning of the week, there should be no cargo that's held from the previous day. Therefore, we can directly assign $x_{i,j,1}$ equal to the cargo loads arriving into the system on Monday:

$$x_{i,j,1} = c_{i,j,1} \ \forall i, j \in \{A, B, C\}, i \neq j$$

C5) We also establish constraints for the flow of aircraft at each airport per day. The number of aircraft the flows into an airport at time t must equal the flow out of the airport at time $t+1$:

  - The flow into airport i at time t is equal to number of aircraft delivering to airport i, the number of aircraft that's repositioned to airport i, and the number of aircraft that's grounded at airport i at time t:

$$s_{i,t} + \sum_{j \neq i, j \in \{A,B,C\}} y_{j,i,t} + \sum_{j \neq i, j \in \{A,B,C\}} z_{j,i,t}$$

- The flow out of airport i at time $t+1$ is equal to the number of aircraft delivering out of airport i, the number of aircraft that's positioned out of airport i, and the number of aircraft that's grounded at airport i at time $t+1$:

$$s_{i,t+1} + \sum_{j \neq i, j \in \{A,B,C\}} y_{i,j,t+1} + \sum_{j \neq i, j \in \{A,B,C\}} z_{i,j,t+1}$$

Combining the two flows above, we get the constraint:

$$s_{i,t} + \sum_{j \neq i, j \in \{A,B,C\}} y_{j,i,t} + \sum_{j \neq i, j \in \{A,B,C\}} z_{j,i,t} = s_{i,t+1} + \sum_{j \neq i, j \in \{A,B,C\}} y_{i,j,t+1} + \sum_{j \neq i, j \in \{A,B,C\}} z_{i,j,t+1}$$

$$\forall i \in \{A, B, C\}, t = 1, 2, 3, 4$$

C6) In order to get a repeatable weekly movement cycle, we also need to create a constraint where the flow into airport i at the end of Friday is the same as the flow out of airport i on Monday. Following a similar logic to 5), we obtain the following constraints:

$$s_{i,5} + \sum_{j \neq i, j \in \{A,B,C\}} y_{j,i,5} + \sum_{j \neq i, j \in \{A,B,C\}} z_{j,i,5} = s_{i,1} + \sum_{j \neq i, j \in \{A,B,C\}} y_{i,j,1} + \sum_{j \neq i, j \in \{A,B,C\}} z_{i,j,1}$$

$$\forall i \in \{A, B, C\}$$

C7) Finally, we need to ensure that the number of aircraft in the system is equal to 1,200. We do this by enforcing that the number of aircraft flowing out of day 1 is equal to 1,200. The flow constraints established by C5) will ensure that all the other days will also have 1,200 aircraft in total:

$$\sum_{i \in \{A,B,C\}} s_{i,1} + \sum_{j \neq i, i, j \in \{A,B,C\}} y_{i,j,1} + \sum_{j \neq i, i, j \in \{A,B,C\}} z_{i,j,1} = 1,200$$

5. Analysis of Results and Findings

We initially attempt use Gurobi to solve the optimization problem that we've formulated above. However, the output indicates that a solution is not feasible. We can also come to the same conclusion by considering the total number of available aircraft, and the amount of cargo loads arriving into the system for Thursday and Friday for the A-B and C-B origin-destination pairs only (see Table 2 below):

Origin-Destination	Thursday	Friday
A-B	400	300
C-B	200	400

Table 2: Amount of cargo loads arriving into the system for Thursday and Friday that needs to be delivered between A-B and C-B

Assuming that we have zero cargo that we hold from previous days, the number of aircraft needed to deliver the cargo loads for A-B and C-B over Thursday and Friday is 1,300. This is because of two reasons:

1) In order for us to have a repeatable schedule, all the cargo loads must be shipped out by Friday to their destinations. No cargo loads can roll-over to the following week

2) Any aircraft that we use to deliver cargo for A-B and C-B on Thursday will only be usable by airport B on Friday. Therefore, airport A and airport C will need additional aircraft to deliver their cargo loads arriving on Friday

To illustrate this, we can take the case where airport A has 400 aircraft and airport C has 200 aircraft on Thursday, which they use to deliver all the cargo loads they received on Thursday to airport B. As a result, on Friday, airport B will have 600 aircraft. However, on Friday, airport A and airport C still needs 300 and 400 aircraft, respectively, to deliver Friday's cargo to airport B. Since 600 aircraft is "locked" at airport B, airport A and airport C will need a total of 700 aircraft to ensure that all cargo loads are shipped out by Friday. Therefore, a minimum of 1,300 aircraft is necessary to fulfill this requirement. Since, Express Air only have 1,200 aircraft available, no feasible solution exists.

Updated Formulation with Number of Aircraft as Decision Variable

One option to find a feasible solution is to simply increase the number of aircraft to handle the existing cargo arriving into the system. We do this by modifying our original formulation by:

1) Making the number of aircraft a decision variable in our optimization problem: $aircraft$

2) Updating our seventh constraint (C7) so that the aircraft flow out of day 1 is equal to $aircraft$:

$$\sum_{i \in \{A,B,C\}} s_{i,0} + \sum_{j \neq i, i,j \in \{A,B,C\}} y_{i,j,0} + \sum_{j \neq i, i,j \in \{A,B,C\}} z_{i,j,0} = aircraft$$

3) We also update the objective function to include $aircraft$. This is because obtaining aircraft generally incurs large fixed costs, so we want to minimize the total number of aircraft in our system:

$$min\ (aircraft + \sum_{t=1}^{5} \sum_{i=\{A,B,C\}} \sum_{j=\{A,B,C\}} 10(x_{i,j,t} - y_{i,j,t}) + r_{i,j} z_{i,j,t})$$

Running this new formulation through Gurobi, we determine that a minimum of 1,390 aircraft is required to have a feasible solution. The optimal objective value is 16,515. However, since the objective function contains the $aircraft$, we subtract 1,390 from 16,515 to obtain the true cost, which is 15,125. Increasing the number of aircraft beyond 1,390 does not yield any additional benefits, as the excess aircraft would just stay grounded at each airport. As mentioned before, obtaining additional aircraft is expensive, and should only be considered if the cargo arrival schedule (i.e. Table 1) is fixed.

Updated Formulation with Weekly Rollover

Another option is to allow cargo from one week to roll-over to the next week by modifying the following constraints:

1) We remove the constraint (C4) that initializes the amount of cargo at the beginning of the week

2) Instead, we introduce a new constraint that allows cargo from Friday to rollover to following week's Monday:

$$x_{i,j,0} = x_{i,j,5} - y_{i,j,5} + D_{i,j,0}\ \forall i,j \in \{A,B,C\}, i \neq j$$

This formulation yields an objective value of 17,925.

Updated Formulation with Cargo Arrivals as Decision Variables

Alternatively, we can also find a feasible solution by redistributing the cargo arrivals between the days, while keeping the number of aircraft fixed at 1,200 and not allowing weekly rollovers. We do this by again modifying our original formulation by:

1) Converting the number of cargo loads arriving into the system into a decision variable: $c_{i,j,t}$

2) Since the total amount of cargo that needs to be delivered per week between each origin-destination pair is fixed, we add a new constraint that bounds the $c_{i,j,t}$:

$$\sum_{t=1}^{5} c_{i,j,t} = total_{i,j}$$

$$\forall j \neq i, i, j \in \{A, B, C\}$$

where $total_{i,j}$ is given by the Table 3.

Origin-Destination	Total
A-B	1100
A-C	250
B-A	125
B-C	125
C-A	200
C-B	1500

Table 3: The total amount of cargo that enters the system in a week that needs to be delivered between each origin-destination pair

Running this through Gurobi, we obtain an objective value of 15,125 and the following cargo arrival schedule:

Origin-Destination	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	0	0	700	400	0
A-C	0	0	0	0	250
B-A	0	0	0	125	0
B-C	0	0	0	0	125
C-A	0	0	200	0	0
C-B	650	200	100	0	550

Table 4: Feasible schedule for cargo loads arriving into system on each day for each origin-destination pair

This is just one feasible schedule. An alternative feasible schedule that yields the same objective value is shown in Table 5, where all the cargo arrivals are spread out evenly across the days:

Origin-Destination	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	220	220	220	220	220
A-C	50	50	50	50	50
B-A	25	25	25	25	25
B-C	25	25	25	25	25
C-A	40	40	40	40	40
C-B	300	300	300	300	300

Table 5: Alternate feasible schedule for cargo load arrivals, where the arrivals are spread out through the days

The choice of schedule will largely depend on factors outside the scope of this analysis, which includes the cost to have cargo arrive within the system and any limitations on the amount of cargo that can arrive at one time. If there is a cost associated with having cargo arrive into the system each day, we'd recommend the adjusting the cargo load arrival schedule to what's shown in Table 4 as there are several days where no cargo arrives into the system at all. **We also recommend this solution over all other solutions from alternative formulations (i.e. allowing rollover and increasing number of planes) as it has the lowest cost: 15,125.** The corresponding aircraft schedule to the new cargo arrival schedule shown in Table 4, and the solution to the original task is shown in Table 6, 7, and 8.

Origin-Destination	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	0	0	700	400	0
A-C	0	0	0	0	250
B-A	0	0	0	125	0
B-C	0	0	0	0	125
C-A	0	0	200	0	0
C-B	650	200	100	0	550

Table 6: Schedule for aircraft delivering cargo for each day for each origin-destination pair

Origin-Destination	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	0	0	0	0	0
A-C	0	0	0	0	0
B-A	0	700	200	125	0
B-C	200	300	0	550	275
C-A	0	0	0	0	0
C-B	0	0	0	0	0

Table 7: Schedule for repositioning aircraft for each day for each origin-destination pair

Airport	Monday	Tuesday	Wednesday	Thursday	Friday
A	0	0	0	0	0
B	350	0	0	0	0
C	0	0	0	0	0

Table 8: Schedule for holding aircraft at each airport

Intuitively, the aircraft schedules above are sensible, given that the majority of cargo is being delivered to airport B. As a result of this, airport B must constantly reposition aircraft back to airports A and C, so that A and C can continue to deliver cargo to airport B in the following days. By the same logic, it also makes sense that airport B would hold aircraft at the beginning of the week that it can reposition later.

As noted above, we notice that airport B is performing all the repositioning of aircraft. If we want to further optimize the cost, we can seek to reduce the cost to reposition aircraft from B to A and from B to C. If we reduce the repositioning cost from B to A by one and B to C by one, our total cost decreases by 1,025 and 1,325 respectively.

We also explore the marginal impact of increasing the total amount of cargo that needs to be delivered between each origin-destination pair. This is summarized in Table 9.

Origin-Destination	Marginal Cost
A-B	7
A-C	1
B-A	-7
B-C	-6
C-A	-1
C-B	6

Table 9: The marginal impact to total cost when the amount of cargo that needs to be delivered is incremented by one for each origin-destination pair

Surprisingly increasing the total cargo for B-A, B-C, and C-A reduces the total cost. This is because for each incremental cargo added for these origin-destination pairs, an aircraft is now being used to deliver the cargo for "free", instead of being repositioned, which incurs a cost. Express Air may consider adding additional cargo to these origin-destination pairs.