Lecture #3

4 Bits code BCD code range from 0 to 9. {Total $= 2^{4-1} = 16$}

6 = 0110 6 = 0110

10 = 0001 0000 +10 =

6 code words in BCD are invalid. Ranging from 10 → 15.

BCD addition: 7 = 0111
+ 5 = 0101
1100 > 9

Now we will add six to level 2 Byte +6 = 0110
0001 0010 = 12

1897	=	0001	1000	1001	0111
+ 2905	=	0010	1001	0000	0101
		0100	0001	1001	1100
					+6 0110
		0100	0001	1010	0010

ASCII 7 bits. $2^7 - 1 = 128$

It contains 98 graphic printing characters and 34 non printing characters.

B_7 B_6 B_5 B_4 B_3 B_2 B_1

It represents column — Rows

Parity Bit. :- It identify the odd number of means. 0101 → If it changes to 0001 detected. ~~Beak~~

It counts the no of 1's, if no of 1 changed is odd Then it can be detected

1's complement. with sign bit

7 6 5 4 3 2 1 0
0 0 0 0 0 0 0 0
Sign bit

-10

1 0 0 0 1 0 1 0
→ 1 0 1 1 1 0 1 0 1
+ 1 2's

1 0 1 1 1 1 1 0 0

1 1

1

Lecture #4

(i) Subtraction by 1's complement.

$X - Y = 1010100 - \underbrace{1000011}$

1's complement

$$\begin{array}{r} 1010100 \\ +\ 0111100 \\ \hline 1\,0010000 \end{array}$$

9th ← 8 bits.

$$\begin{array}{r} +1 \\ \hline X - Y = 0010001 \end{array}$$

steps.

① Take 1's complement of Y then add if carry exist (>8 bit) then add that carry.

② If there's no end carry then take 1's complement of the ~~carry~~ answer and then insert neg sign.

$$\begin{array}{r} Y - X = 1000011 \\ +\ 0101011 \\ \hline 1101110 \end{array}$$

No end carry

Sum $= -0010001$ so take 1's com

Subtraction by 2's complement.

(a) $X - Y$

Take 2's complement of Y Then add.

$X = 1010100$

$Y = 1000011$

2's $Y = 0111101$

$$\begin{array}{r} X \quad 1010100 \\ Y \quad 0111101 \\ \hline 10010001 \end{array}$$

Discard end carry then answer is $X - Y = 0010001$

(b) $Y - X$

$$\begin{array}{r} Y = 1000011 \\ 2\text{'s} \to X \quad 0101100 \\ \hline -1101111 \end{array}$$

Then put -ve sign in answer.

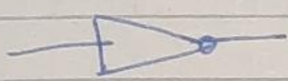
In Bolean Algebra.

$1+1=1$ 1 represent on
0 represent off.

logic gates

(i)  And $X \cdot Y$

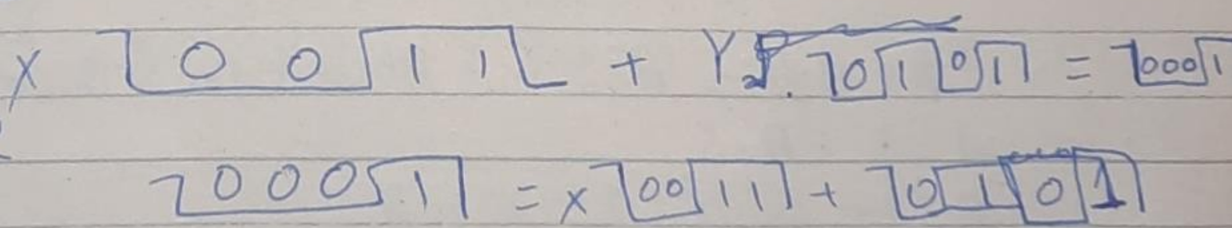
(2)  OR $X+Y$

(3)  Z $Z=\bar{X}$

wave form.

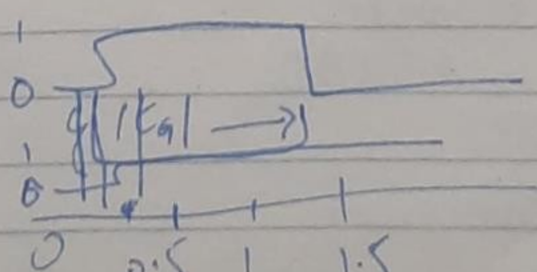
(1) And

(2) OR



Gate Delay

In actual physical gates, if one or more input changes cause the output to change it take some part of second. denoted by t_{pd}

output 

$0.5-0.2$
$=0.3$ ns

→ The order of evaluation in a Bolean algebra expression.

1) Parenthesis
2) NOT ~
3) AND ·
4) OR +

→ Dual and Duality Principale.

Dual → obtained by changing and to OR or OR to And

Duality If the output no change occurs then It's called duality principale.

(1) $A + A \cdot B = A$ Absortion Theoram.

L.H.S $= A \cdot 1 + A \cdot B$

$= A(1 + B) = A(1) = A$

L.H.S = R.H.S

(2) $A + BC = (A+B)(A+C)$ Proof too. Distributive law

OR distributive over And.

R.H.S

$= A \cdot A + A \cdot C + AB + BC$

$= A + AC + AB + BC$

$= A(1+C) + AB + BC$

$= A + AB + BC$

$= A(1+B) + BC$

$= A + BC$

① Consensus Theorem

$$XY + \bar{X}Z + YZ = XY + \bar{X}Z$$

Both XY and $\bar{X}Z$ have similarity in the face of x where YZ doesn't so. It can be eliminated.

$$= XY + \bar{X}Z + YZ.1$$
$$= XY + \bar{X}Z + YZ(X+\bar{X})$$
$$= XY + XYZ + \bar{X}Z + \bar{X}YZ$$
$$= XY(1+Z) + \bar{X}Z(1+Y)$$
$$= XY + \bar{X}Z$$

L.H.S = R.H.S

Dual of consensus Theorem is.

$$(X+Y)(\bar{X}+Z)(Y+Z) = (X+Y)(\bar{X}+Z)$$

Minimization.

$$X \cdot Y + \bar{X} \cdot Y = Y \qquad (x+\bar{y})(\bar{x}+y) = y$$

Demorgan's law.

$$\overline{x+y} = \bar{x} \cdot \bar{y} \qquad \overline{x \cdot y} = \bar{x} + \bar{y}$$

Boolean Algebra

- **An algebraic structure defined on a set of at least two elements, B, together with three binary operators (denoted +, · and $\overline{}$) that satisfies the following basic identities:**

1.	$X + 0 = X$	2.	$X \cdot 1 = X$	
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	
7.	$X + \overline{X} = 1$	8.	$X \cdot \overline{X} = 0$	
9.	$\overline{\overline{X}} = X$			
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$(X + Y) + Z = X + (Y + Z)$	13.	$(XY)Z = X(YZ)$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

x	y	z	$\bar{x}$	$\bar{y}$	$\bar{z}$	F_1	F_2	F_3	F_4
0	0	0	1	1	1	0	0	1	0
0	0	1	1	1	0	0	1	1	1
0	1	0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0	1	1
1	0	0	0	1	1	0	1	1	1
1	0	1	0	1	0	0	1	1	1
1	1	0	0	0	1	1	1	0	0
1	1	1	0	0	0	0	1	0	0

$F_1 = xy\bar{z}$

$F_2 = x + \bar{y}z$

$F_3 = \bar{x}\bar{y}\bar{z} + \bar{x}yz + x\bar{y}$

$F_4 = x\bar{y} + \bar{x}z$

lecture# 5

complement of a function

steps

① Take dual of the function then complement each literal

① $F = \bar{x}y\bar{z} + x\bar{y}\bar{z} \rightarrow (x+\bar{y}+z)(\bar{x}+\bar{y}+z)$

② $G = (\bar{a}+bc)\bar{d} + e \rightarrow (a\cdot(\bar{b}+\bar{c}))+d\cdot\bar{e}$

→ $F(x,y)$ then literals are $x, \bar{x}, y, \bar{y}$

→ $A\bar{B} + BC + C\bar{A}$ = 5 literals $A, \bar{A}, B, \bar{B}, C$

Bolean function

The representation of logic gate by interconnecting them is called bolean function.

① The output of one's logic gate is ~~other~~ input of other's logic gate. These logic gates ~~~~ are called building blocks of combinational circuit

Combinational logic circuits CLC

Memoryless digital logic circuits whose output at any instant in time depends upon only on combination of it's input.

-> No feedback
-> Imediate effect of input on output

Representation of C.L.C.

1. Bolean Algebra
2. Truth table
3. Logic Diagram.

Expression Simplification.

$$
\begin{aligned}
& AB + \bar{A}CD + \bar{A}BD + \bar{A}C\bar{D} + ABCD \\
&= AB + ABCD + \bar{A}C(D+\bar{D}) + \bar{A}BD \\
&= AB + ABCD + \bar{A}BD + \bar{A}C \\
&= AB(1+CD) + \bar{A}BD + \bar{A}C \\
&= AB[(1+C)(1+D)] + \bar{A}BD + \bar{A}C \\
&= AB + \bar{A}BD + \bar{A}C \\
&= B(A+\bar{A}D) + \bar{A}C \\
&= B(\underbrace{\cancel{A+\bar{A}}}_{1})(A+D) + \bar{A}C \\
&= AB + BD + \bar{A}C
\end{aligned}
$$

-> Absorb Theorem

5 Literals

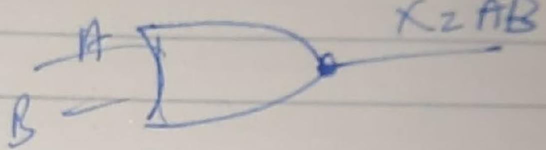
$$Z = \overline{(\bar{A}+C)\cdot(B+\bar{D})}$$

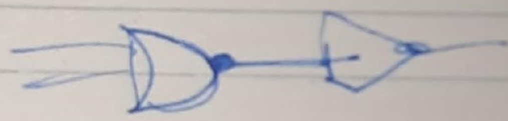
Dual

$$Z = (\bar{\bar{A}}\cdot\bar{C}) + \overline{(B\cdot\bar{D})}$$

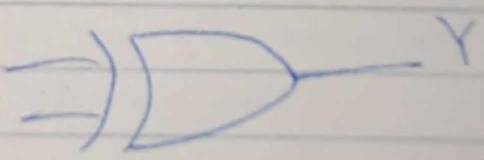
$$= A\cdot\bar{C} + \bar{B}\bar{\bar{D}} = A\cdot\bar{C} + \bar{B}D$$

NOR Gate





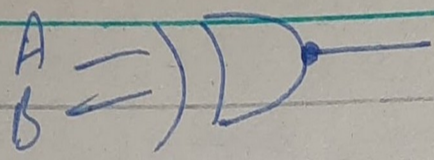
XOR



x's one if one input is one.

X	Y	Y
0	0	0
1	1	0
0	1	1
1	0	1

XNOR



The inversion of XOR

Nand or Nor are universal gates

lecture #6

1) Conanical Forms.

Bolean functions ↓ expressiod as sum of minterms or product of maxterm are called conanical forms.

2) It's useful to specify Bolean function in the form that

1) allows comparison for equality
2) has a correspondence to truth table

(3) Usage

(1) SOP aka sum of product.
(2) POS aka product of sums

(4) Minterm

And term with every variable present in orignal or complemented form called minterms.

Formula = 2^n where n is total counted literals.

Such

$X = 1$, $\bar{X} = 0$, $Y = 1$, $\bar{Y} = 0$

$XY = 1$ $\quad X\bar{Y} = (1)(0) = 0$

(5) Maxterms

OR-terms with every variable in either original or complemented way.

formula = 2^n

here = $[X = 0]$ $\bar{X} = 1$, $Y = 0$, $\bar{Y} = 1$

$X + Y = 00$

$\boxed{X + \bar{Y} = 1}$

Canonical Sum of Minterms

- Any Boolean function can be expressed as a <u>Sum of Products</u>.
  - For the function table, the <u>minterms</u> used are the terms corresponding to the 1's
  - For expressions, <u>expand</u> all terms first to explicitly list all minterms. Do this by "ANDing" any term missing a variable v with a term $(v + \bar{v})$.
- Example: Implement $f = x + \bar{x}\,\bar{y}$ as a sum of minterms.

$y + \bar{y} = 1$

First expand terms: $f = x(y + \bar{y}) + \bar{x}\,\bar{y}$

Then distribute terms: $f = xy + x\bar{y} + \bar{x}\,\bar{y}$ 00

Express as sum of minterms: $f = m_3 + m_2 + m_0$

xy 11 3

$x\bar{y}$ 10 2

Another SOP Example

- Example: $F = A + \bar{B}\,C$
- There are three variables, A, B, and C which we take to be the standard order.
- Expanding the terms with missing variables:

$A + \bar{B}C = A(B + \bar{B}) + (A + \bar{A})\bar{B}C$

$= AB + A\bar{B} + A\bar{B}C + \bar{A}\bar{B}C$

$= AB(C + \bar{C}) + A\bar{B}(C + \bar{C}) + A\bar{B}C + \bar{A}\bar{B}C$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$

- Collect terms (removing all but one of duplicate terms): $ABC + AB\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$

111 110 100 101 001

- Express as SOP:

$m_7 + m_6 + m_4 + m_5 + m_1$

$F = m_1 + m_4 + m_5 + m_6 + m_7$

⑥ Standard Order.

(i) Order of minterms and maxterms matters.

② Both of terms are designated with a subscript. It's a number expressed as binary number which shows that the literal is represented in the orignal format or complemented one.

-> Minterm.

x, Y 0 = complemented

1 = Not ''

-> Max term =

X, Y 1 = complemented

0 = N.C

⑦ Question Practice

(i) Minterm of 6 Represented as Σm

Binary of 6 =
$$\begin{array}{r|l} 2 & 6 \\ \hline 2 & 3 - 0 \\ & 1 - 1 \end{array}$$

1 1 0

$= X Y \bar{Z}$

(2) Maxterm of 6 Πm

$= 1 1 0$

$= \bar{x} + \bar{y} + z$

(8) Relationship b/w Min and Max term.

By Demorgan's
- i) $\overline{X \cdot Y} = \bar{X} + \bar{y}$
(2) $\overline{X + y} = \bar{x} \cdot \bar{y}$

They are complement of each other.

steps
- i) Take dual then inverse

(9) Observations.

(i) In Minterm table. Then in a row only one 1 is present in 2^n terms. All the other terms are 0.

(2) In maxterm table. There will be only one zero in a row. All others are 1.

(3) SOP or POS are used for stating Bolean function.

(10) SOP Example.

$$F = A + \bar{B}C$$
$$= A(B+\bar{B}) + (A+\bar{A})\bar{B}C$$
$$= AB + A\bar{B} + A\bar{B}C + \bar{A}\bar{B}C$$
$$= AB(C+\bar{C}) + A\bar{B}(C+\bar{C}) + A\bar{B}C + \bar{A}\bar{B}C$$
$$= ABC + AB\bar{C} + \underbrace{A\bar{B}C}_{m_5} + \underbrace{A\bar{B}\bar{C} + A\bar{B}C}_{same} + \bar{A}\bar{B}C$$

$$\underset{m_7}{ABC} + \underset{m_6}{AB\bar{C}} + \underset{m_5}{A\bar{B}C} + \underset{m_4}{A\bar{B}\bar{C}} + \underset{m_1}{\bar{A}\bar{B}C}$$

Expressed as SOP

$$F = m_7 + m_6 + m_5 + m_4 + m_1 = m_1 + m_4 + m_5 + m_6 + m_7$$

Representation in Shorthand SOP Form

$$F(A,B,C) = \underbrace{\Sigma_m}_{\text{represents the minterm}} (\overset{m_1}{1}, \overset{m_4}{4}, \overset{m_5}{5}, \overset{m_6}{6}, \overset{m_7}{7})$$

POS

$$f(x,y,z) = x + \bar{x}\bar{y}$$
$$= (x+\bar{x})(x+\bar{y}) = 1\cdot(x+\bar{y})$$
$$= (x+\bar{y}+z\bar{z})$$
$$= (x+\bar{y}+z)(x+\bar{y}+\bar{z})$$

$$f = M_2 . M_3$$

Convert SOP to POS.

$$f(A,B,C) = A\bar{C} + BC + \bar{A}\bar{B}$$

Distributive law $\quad x + yz = (x+y).(x+z)$

Let $\quad A\bar{C} + BC = x, \quad \bar{A}\bar{B} = yz$

Then

$$= (A\bar{C} + BC + \underset{y}{\bar{A}})(A\bar{C} + BC + \underset{z}{\bar{B}})$$
$$= \cancel{[\bar{A} + A\bar{C} + BC]}\,[\bar{B} + BC + A\bar{C}]$$
$$= (\bar{C} + BC + \bar{A})(A\bar{C} + C + \bar{B})$$
$$= (\bar{C} + B + \bar{A})(A + C + \bar{B})$$

Rearrange

$f = (\bar{A}+B+\bar{C})(A+\bar{B}+C)$ to give $f = M_5 . M_2$

Minterm Function Example

- Example: Find $F_1 = m_1 + m_4 + m_7$
- $F1 = \bar{x}\ \bar{y}\ z + x\ \bar{y}\ \bar{z} + x\ y\ z$

x y z	index	m_1	+	m_4	+	m_7	$= F_1$
0 0 0	0	0	+	0	+	0	= 0
0 0 1	1	1	+	0	+	0	= 1
0 1 0	2	0	+	0	+	0	= 0
0 1 1	3	0	+	0	+	0	= 0
1 0 0	4	0	+	1	+	0	= 1
1 0 1	5	0	+	0	+	0	= 0
1 1 0	6	0	+	0	+	0	= 0
1 1 1	7	0	+	0	+	1	= 1

$\bar{x}\bar{y}z$

Sum of products +

$x\bar{y}\bar{z}$

+

xyz

Maxterm Function Example

8 4 2 1
1 1 1 0

- $F(A, B, C, D) = M_3 \cdot M_8 \cdot M_{11} \cdot M_{14}$ 4
- $F(A, B, C, D) =$

	Binary	Max.
M_3	0011	$A+B+\bar{C}+\bar{D}$
M_8	1000	$\bar{A}+B+C+D$
M_{11}	1011	$\bar{A}+B+\bar{C}+\bar{D}$
M_{14}	1110	$\bar{A}+\bar{B}+\bar{C}+D$

Maxterms and Minterms

- Examples: Two variable minterms and maxterms.

Index	Minterm	Maxterm
0	$\overline{x}\,\overline{y}$	$x + y$
1	$\overline{x}\,y$	$x + \overline{y}$
2	$x\,\overline{y}$	$\overline{x} + y$
3	$x\,y$	$\overline{x} + \overline{y}$

- The index above is important for describing which variables in the terms are true and which are complemented.

Index Examples – Four Variables

Index	Binary	Minterm	Maxterm
i	Pattern	m_i	M_i
0	0000	$\overline{a}\,\overline{b}\,\overline{c}\,\overline{d}$	$a + b + c + d$
1	0001	$\overline{a}\,\overline{b}\,\overline{c}\,d$	?
3	0011	?	$a + b + \overline{c} + \overline{d}$
5	0101	$\overline{a}\,b\,\overline{c}\,d$	$a + \overline{b} + c + \overline{d}$
7	0111	?	$a + \overline{b} + \overline{c} + \overline{d}$
10	1010	$a\,\overline{b}\,c\,\overline{d}$	$\overline{a} + b + \overline{c} + d$
13	1101	$a\,b\,\overline{c}\,d$	?
15	1111	$a\,b\,c\,d$	$\overline{a} + \overline{b} + \overline{c} + \overline{d}$

lecture # 7

Conanical terms includes all the literal and are under minterm, maxterm.

Functions Complement.

(1)

→ The complement of a function expressed as a SOP is constructed by selecting the minterms missing in the SOP, conanical form

→ Alternatively, the complement of a function expressed by a SOP is simply the POS with the same indices.

→ اگر کسی فنکشن کا میکس ٹرم نکالنی ہو تو وہ ٹرم لکھو جو اس فنکشن میں نہ ہوں، مثال

$m_i = M_i$

$M_i = \overline{m_i}$ $\quad F(x, y, z) = \Sigma_m(1, 3, 5, 7)$

$$\bar{F}(x, y, z) = \Sigma_m(0, 2, 4, 6)$$

The maxterm of " is $\Pi_m(1, 3, 5, 7)$

(2) Conversion b/w forms.

(i) $F(x, y, z) = \Sigma_m(1, 3, 5, 7)$

$$\bar{F}(x, y, z) = \Sigma_m(0, 2, 4, 6)$$

But now here double complement will result in

$$\bar{\bar{F}}(x, y, z) = \Pi_m(0, 2, 4, 6)$$

$$F(x, y, z) = \Pi_m(0, 2, 4, 6)$$

Complementing Functions

- **Find the complements of the functions by taking the duals of their equations and complementing each literal.**

1.

$$F_1 = \overline{X}Y\overline{Z} + \overline{X}\,\overline{Y}Z = (\overline{X}Y\overline{Z}) + (\overline{X}\,\overline{Y}Z)$$

The dual of F_1 is

$$(\overline{X} + Y + \overline{Z})(\overline{X} + \overline{Y} + Z)$$

Complementing each literal, we have

$$(X + \overline{Y} + Z)(X + Y + \overline{Z}) = \overline{F}_1$$

2.

$$F_2 = X(\overline{Y}\,\overline{Z} + YZ) = X((\overline{Y}\,\overline{Z}) + (YZ))$$

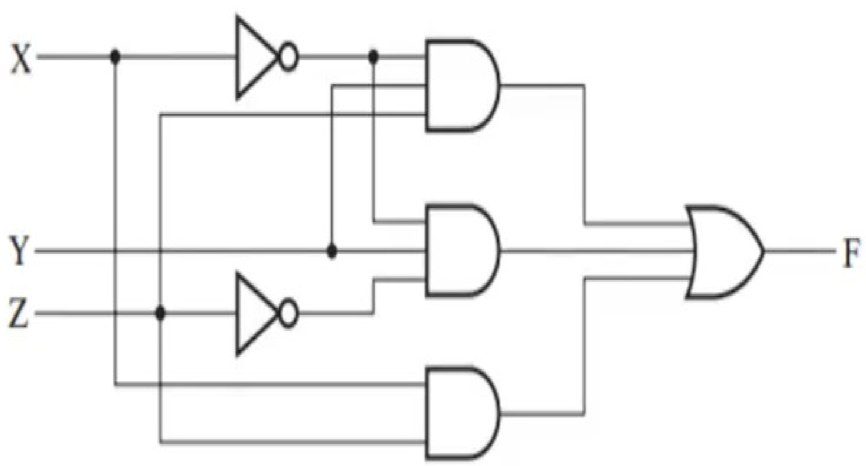
The dual of F_2 is

$$X + (\overline{Y} + \overline{Z})(Y + Z)$$

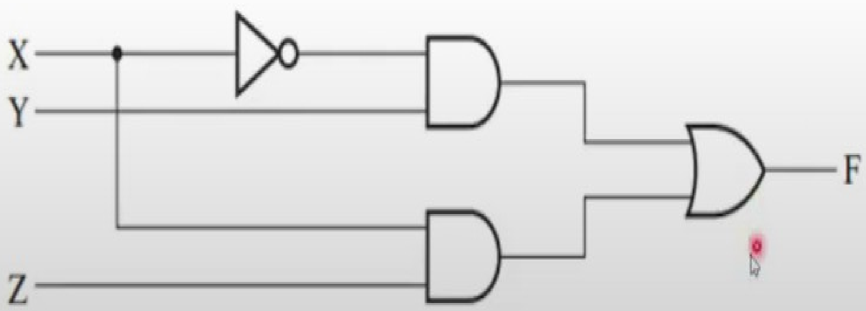
Complementing each literal yields

$$\overline{X} + (Y + Z)(\overline{Y} + \overline{Z}) = \overline{F}_2$$

Example 3: Circuit Diagram



(a) $F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$



(b) $F = \overline{X}Y + XZ$

Not obeyed by conanoical form.

Standard Forms

(i) Standard SOP form :-

Function written in the form of "OR" of "And" terms. such as $ABC + \bar{A}\bar{B}C + B$

(ii) Standard POS form.

Functions written in the form of "And" of "OR".

such as $(A+B) \cdot (A+\bar{B}+\bar{C}) \cdot C$

These mixed forms are neither SOP nor POS.

(i) $(AB+C) \cdot (A+C)$ (ii) $ABC + AC(A+B)$

Standard SOP :-

A sum of minterms form for n variables can be written down directly from a truth table.

(i) Implementation of this form is a two level network of gates such that :

→ The first level consists of n-input And gates.

→ The second level is a single OR gate (with fewer 2^n input)

$$
\begin{array}{r|l}
2 & 4 \\
2 & 2 - 0 \\
 & 1 - 0
\end{array}
$$

(ii) This form often can be simplified so that the corresponding circuit is simpler.

The example of the simplification :-

$$F(A, B, C) = \Sigma_m (1, 4, 5, 6, 7)$$

$$F = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

Simplified

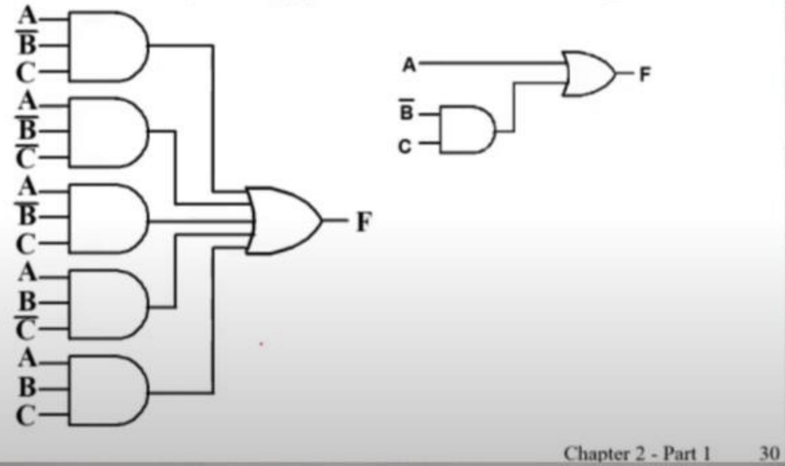
$$
\begin{aligned}
F &= \bar{A}\bar{B}C + A\bar{B}(\bar{C} + C) + AB(\bar{C} + C) \\
&= \bar{A}\bar{B}C + A\bar{B} + AB \\
&= \bar{A}\bar{B}C + A(\bar{B} + B) \\
&= A + \bar{A}\bar{B}C \\
&= (A + \bar{A})(A + \bar{B}C) \\
&= A + \bar{B}C
\end{aligned}
$$

It contains three literals as compared to 15 literals.

Ch2 Part 1 Page 30

AND/OR Two-level Implementation of SOP Expression

- The two implementations for F are shown below – it is quite apparent which is simpler!



SOP and POS observations.

(i) Canonical forms (SOPs, POS, or SSOP, SPOS) differs in complexity.
(ii) Bolean algebra can be used to manipulate into simpler form.
(iii) Simpler equations leads to simpler solution.

Properties of maxterms

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The following is a summary of the most important properties of maxterms:

1. There are 2^n maxterms for n Boolean variables. These maxterms can be generated from the binary numbers from 0 to 2^n - 1.
2. Any Boolean function can be expressed as a logical product of maxterms.
3. The complement of a function contains those maxterms not included in the original function.
4. A function that includes all the 2^n maxterms is equal to logic 1.

Properties of minterms

The following is a summary of the most important properties of minterms:

1. There are 2^n minterms for n Boolean variables. These minterms can be generated from the binary numbers from 0 to 2^n - 1.
2. Any Boolean function can be expressed as a logical sum of minterms.
3. The complement of a function contains those minterms not included in the original function.
4. A function that includes all the 2^n minterms is equal to logic 1.

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Literal Cost :

(1) Literal - a variable or it's complement.
(2) Literal cost = No of literals appeared in a ~~Boolean~~ Bolean expression - corresponding to logic Circuit Diagram.

→ **Gate Input cost.**

The number of gates inputs to the gates in the implementation corresponding exactly to the given equation (G - inverter not canted, GN - inverter counted)

⊛ For POS, or SOP - It can be found from the equations by finding the sum of
(1) All literal appearences
(2) The number of terms excluding the single literals G.
(3) Optionally. The number of distinct literal count. GN.

Example

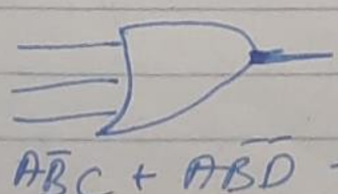
Inputs 2 — Inputs 3 — input 3 — combined literal = 3

(i) $F = BD + ABC + A\bar{C}\bar{D}$

$1 + 2 + 3 = 3$

literals = 8/8 + 3 = G(Inputs) $\underline{G = 11}$, $GN = 14$

↓



(ii) $F = BD + A\bar{B}C + A\bar{B}\bar{D} + AB\bar{C}$

Literals = 11

G = Literals + 4 = 15

GN = Literals + 3 = 18 distinct inverted complements Such as $\bar{B}, \bar{D}, \bar{C}$

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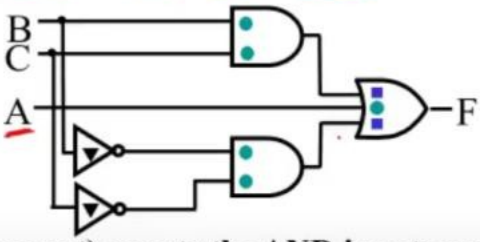
Cost Criteria (continued)

- **Example 1:** $GN = G + 2 = 9$
- $F = A + B\,C + \bar{B}\,\bar{C}$ $L = 5$ $G = L + 2 = 7$



- **L (literal count) counts the AND inputs and the single literal OR input.**
- **G (gate input count) adds the remaining OR gate inputs**
- **GN(gate input count with NOTs) adds the inverter inputs**

Cost Criteria (continued)

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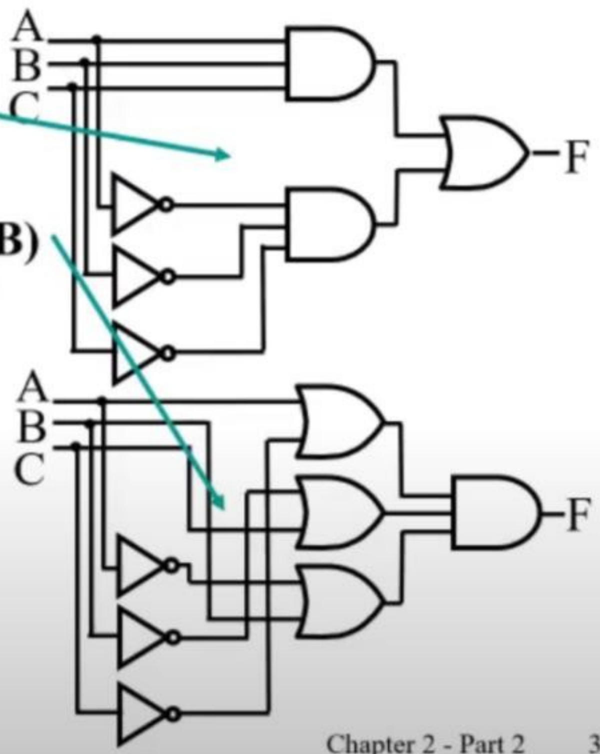
- **Example 2:**
- $F = A\,B\,C + \bar{A}\bar{B}\bar{C}$
- $L = 6 \quad G = 8 \quad GN = 11$
- $F = (A + \bar{C})(\bar{B} + C)(\bar{A} + B)$
- $L = 6 \quad G = 9 \quad GN = 12$
- **Same function and same literal cost**
- **But first circuit has better gate input count and better gate input count with NOTs**
- **Select it!**



Bolean Function Optimization.

(i) [illegible]

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Boolean Function Optimization

- **Minimizing the gate input (or literal) cost of a (a set of) Boolean equation(s) reduces circuit cost.**
- **We choose gate input cost.**
- **Boolean Algebra and graphical techniques are tools to minimize cost criteria values.**
- **Some important questions:**
  - **When do we stop trying to reduce the cost?**
  - **Do we know when we have a minimum cost?**
- **Treat optimum or near-optimum cost functions for two-level (SOP and POS) circuits first.**
- **Introduce a graphical technique using Karnaugh maps (K-maps, for short)**

$F = \Sigma m$ $\quad f = \Pi_M(1,2,3,4)$

Conversion Between Forms

① $F = \Pi_M(0,5,6,\ldots$

② $\bar{F} = \Sigma m(0,5,6,7)$

$F = \Sigma m(0,5,6,7)$

- To convert between sum-of-products and product-of-sums form (or vice-versa) we follow these steps:
  - Find the function complement by swapping terms in the list with terms not in the list.
  - Change from products to sums, or vice versa. Π_M
- Example: Given F as before: $F(x, y, z) = \Sigma_m(1,3,5,7)$
- ① Form the Complement: $\bar{F}(x, y, z) = \Sigma_m(0,2,4,6)$
- Then use the other form with the same indices – this forms the complement again, giving the other form of the original function: $F(x, y, z) = \Pi_M(0,2,4,6)$

$\bar{\bar{F}}(x,y,z) = \Pi_M(0,2,4,6)$