ASSIGNMENT-1

MANHARJOT KAUR
MOHPREET SIDHU
JASWINDER SINGH

Q1

Ques 1 :-

$$Z = \sigma \times (0.a_1a_2a_3a_4)_2 \times 2^e$$

(a) '0' cannot be represented in the given floating-point, as zero is equivalent to 0 itself in binary number system

$$Z = (\sigma) \times (0.0000)_2 \times 2^e$$

(σ: sign; e: if $e=0$)

$$= (\sigma) \times (0.0000)_2 \times 2^0$$

$$= 0$$

It would be zero, as $2^0 = 1$

(b) Out of 0 to 14, 0 cannot be represented because the smallest term is $2^0 = 1$, so 0 is 0 in binary. Anything multiplied by zero will automatically become zero, which is insignificant.

$$Z = (\sigma) \times (0.0000)_2 \times 2^0$$

$$= (\sigma) \times 0 \times 1$$

$$= 0$$

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Q1 Ques 2:-

(a) Propagated error: $E_x + E_y$

$$E_x = x_T - x_A$$
$$= 0.3676 \times 10^3 - 0.368 \times 10^3$$
$$= -0.4 \times 10^{-1}$$

$$E_y = y_T - y_A$$
$$= 0.4562 \times 10^1 - 0.456 \times 10^1$$
$$= 0.3 \times 10^{-3}$$

$$E_x + E_y = -0.4000 \times 10^{-1} + 0.3000 \times 10^{-3}$$
$$= 10^{-1}(-0.4 + 0.003)$$
$$= -0.397 \times 10^{-1}$$
$$= -0.0397$$

(b) Rounding error: $x_A + y_A$

$$x_A + y_A = 0.368 \times 10^3 + 0.456 \times 10^1$$
$$= 0.37256 \times 10^3$$

Representation $= f(x_A + y_A)$
$$= 0.373 \times 10^3$$

Total Rounding error $= (x_T + y_T) - f(x_A + y_A)$
$$= 0.3721 \times 10^3 - 0.373 \times 10^3$$
$$= -0.90$$

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Q Ques 3:-

$$R_n = \frac{e^x}{(n+1)!} \times x^{n+1}$$

$$x = 5.04$$

$$e^x = e^{5.04} = 154.4700$$

$$\text{Then } R_n = \frac{e^{5.04}}{(31+1)!} \times (5.04)^{31+1}$$

$$= \frac{154.4700}{32!} \times (5.04)^{32}$$

$$R_n = 1.76380110 \times e^{-11} < 10^{-10}$$

In taylor series, we need atleast 31 terms to acquire an error less than 10^{-10}.

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Ques 4:-

$$x_2 + 3x_3 = 8$$
$$4x_1 + 6x_2 + 7x_3 = -3$$
$$2x_1 + x_2 + 6x_3 = 5$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 3 & 8 \\ 4 & 6 & 7 & -3 \\ 2 & 1 & 6 & 5 \end{array}\right]$$

$$R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 4 & 6 & 7 & -3 \\ 0 & 1 & 3 & 8 \\ 2 & 1 & 6 & 5 \end{array}\right]$$

$$R_1 \rightarrow R_1/2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 7/2 & -3/2 \\ 0 & 1 & 3 & 8 \\ 2 & 1 & 6 & 5 \end{array}\right]$$

$$R_3 \rightarrow R_3 - R_1, \quad R_1 \rightarrow R_1/2$$

$$\left[\begin{array}{ccc|c} 1 & 3/2 & 7/4 & -3/4 \\ 0 & 1 & 3 & 8 \\ 2 & -2 & 5/2 & 13/2 \end{array}\right]$$

$$R_2 \rightarrow 3/2\ R_2$$

$$\left[\begin{array}{ccc|c} 1 & 3/2 & 7/4 & -3/4 \\ 0 & 3/2 & 9/2 & 12 \\ 0 & -2 & 5/2 & 13/2 \end{array}\right]$$

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$R_1 \to R_1 - R_2$ & then $R_2 \to \frac{2}{3} R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -11/4 & -51/4 \\ 0 & +1 & +3 & 8 \\ 0 & -2 & 5/2 & 13/2 \end{array}\right]$$

$R_2 \to -2 R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -11/4 & -51/4 \\ 0 & -2 & -6 & -16 \\ 0 & -2 & 5/2 & 13/2 \end{array}\right]$$

$R_3 \to R_3 - R_2$ & then $R_2 \to \frac{R_2}{-2}$

$$\left[\begin{array}{ccc|c} 1 & 0 & -11/4 & -51/4 \\ 0 & 1 & 3 & 8 \\ 0 & 0 & 17/2 & 45/2 \end{array}\right]$$

$R_3 \to R_3 / (17/2)$

$$\left[\begin{array}{ccc|c} 1 & 0 & -11/4 & -51/4 \\ 0 & 1 & 3 & 8 \\ 0 & 0 & 1 & 45/17 \end{array}\right]$$

$R_3 \to -11/4\, R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -11/4 & -51/4 \\ 0 & 1 & 3 & 8 \\ 0 & 0 & -11/4 & -495/68 \end{array}\right]$$

$R_1 \to R_1 - R_3$ & then $R_3 \to \frac{R_3}{(-11/4)}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -93/17 \\ 0 & 1 & 3 & 8 \\ 0 & 0 & 1 & 45/17 \end{array}\right]$$

So,

$x_1 \Rightarrow -93/17$
$x_2 \Rightarrow 1/17$
$x_3 \Rightarrow 45/17$

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$R_4 \rightarrow R_4 - R_2$ and th... n

Ques 5:- 6

$$A + B + 2C = 1$$
$$2A - B + D = -2$$
$$A - B - C - 2D = 4$$
$$2A - B + 2C - D = 0$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 2 & -1 & 0 & 1 & -2 \\ 1 & -1 & -1 & -2 & 4 \\ 2 & -1 & 2 & -1 & 0 \end{array}\right]$$

$R_3 \rightarrow R_3 - R_1$; $R_2 \rightarrow R_2/2$; $R_4 \rightarrow R_4/2$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 1 & -1/2 & 0 & 1/2 & -1 \\ 0 & -2 & -3 & -2 & 3 \\ 1 & -1/2 & 1 & -1/2 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 0 & -3/2 & -2 & 1/2 & -2 \\ 0 & -2 & -3 & -2 & 3 \\ 1 & -1/2 & 1 & -1/2 & 0 \end{array}\right]$$

$R_2 \rightarrow 2R_2$; $R_4 \rightarrow R_4 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 0 & -3 & -4 & 1 & -4 \\ 0 & -2 & -3 & -2 & 3 \\ 0 & -3/2 & -1 & -1/2 & -1 \end{array}\right]$$

$R_4 \rightarrow 2R_4$ and then $R_2 \rightarrow R_2/-3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 0 & 1 & 4/3 & -1/3 & 4/3 \\ 0 & -2 & -3 & -2 & 3 \\ 0 & -3 & -2 & -1 & -2 \end{array}\right]$$

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$R_4 \to R_4 - R_2$ and then $R_1 \to R_1 - R_2$ then $R_2 \to -2R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2/3 & 1/3 & -1/3 \\ 0 & -2 & -8/3 & 2/3 & -8/3 \\ 0 & -2 & -3 & -2 & 3 \\ 0 & -3 & -2 & -1 & -2 \end{array}\right]$$

$R_3 \to R_3 - R_2$ & then $R_2 \to 3R_2/2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2/3 & 1/3 & -1/3 \\ 0 & -3 & -4 & 1 & -4 \\ 0 & 0 & -1/3 & -8/3 & 17/3 \\ 0 & -3 & -2 & -1 & -2 \end{array}\right]$$

$R_4 \to R_4 - R_2$ and then $R_2 \to R_2/(-3)$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2/3 & 1/3 & -1/3 \\ 0 & 1 & 4/3 & -1/3 & 4/3 \\ 0 & 0 & -1/3 & -8/3 & 17/3 \\ 0 & 0 & 2 & -2 & 2 \end{array}\right]$$

$R_3 \to -2R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2/3 & 1/3 & -1/3 \\ 0 & 1 & 4/3 & -1/3 & 4/3 \\ 0 & 0 & 2/3 & 16/3 & -34/3 \\ 0 & 0 & 2 & -2 & 2 \end{array}\right]$$

$R_1 \to R_1 - R_3$ and then $R_3 \to 2R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -5 & 11 \\ 0 & 1 & 4/3 & -1/3 & 4/3 \\ 0 & 0 & 4/3 & 32/3 & -68/3 \\ 0 & 0 & 2 & -2 & 2 \end{array}\right].$$

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$R_2 \to R_2 - R_3$ and then $R_3 \to 2R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -5 & 11 \\ 0 & 1 & 0 & -11 & 24 \\ 0 & 0 & 2 & 16 & -34 \\ 0 & 0 & 2 & -2 & 2 \end{array}\right]$$

$R_4 \to R_4 - R_3$ and then $R_3 \to R_3/2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -5 & 11 \\ 0 & 1 & 0 & -11 & 24 \\ 0 & 0 & 1 & 8 & -17 \\ 0 & 0 & 0 & -18 & 36 \end{array}\right]$$

$R_4 \to \frac{-5R_4}{18}$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -5 & 11 \\ 0 & 1 & 0 & -11 & 24 \\ 0 & 0 & 1 & 8 & -17 \\ 0 & 0 & 0 & -5 & 10 \end{array}\right]$$

$R_1 \to R_1 - R_4$ and then $R_4 \to \frac{11}{5} R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -11 & 24 \\ 0 & 0 & 1 & 8 & -17 \\ 0 & 0 & 0 & -11 & 22 \end{array}\right]$$

$R_2 \to R_2 - R_4$ and then $R_4 \to 8R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 8 & -17 \\ 0 & 0 & 0 & 8 & -16 \end{array}\right]$$

$R_3 \to R_3 - R_4$ and then $R_4 \to R_4/8$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$x_1 = 1$
$x_2 = 2$
$x_3 = -1$
$x_4 = -2$

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Ques 6 :- Gauss Elimination Method

$$x_1 - 2x_2 - 6x_3 = 12$$
$$2x_1 + 4x_2 + 12x_3 = -17$$
$$x_1 - 4x_2 - 12x_3 = 22$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$;

$R_3 \rightarrow R_3 - R_1$

and then $R_3 \rightarrow \frac{R_3}{-2}$

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & 1 & 3 & -5 \end{array}\right]$$

$R_3 \rightarrow 8R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

z is zero in this case, so no way to find solution for these equations.

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