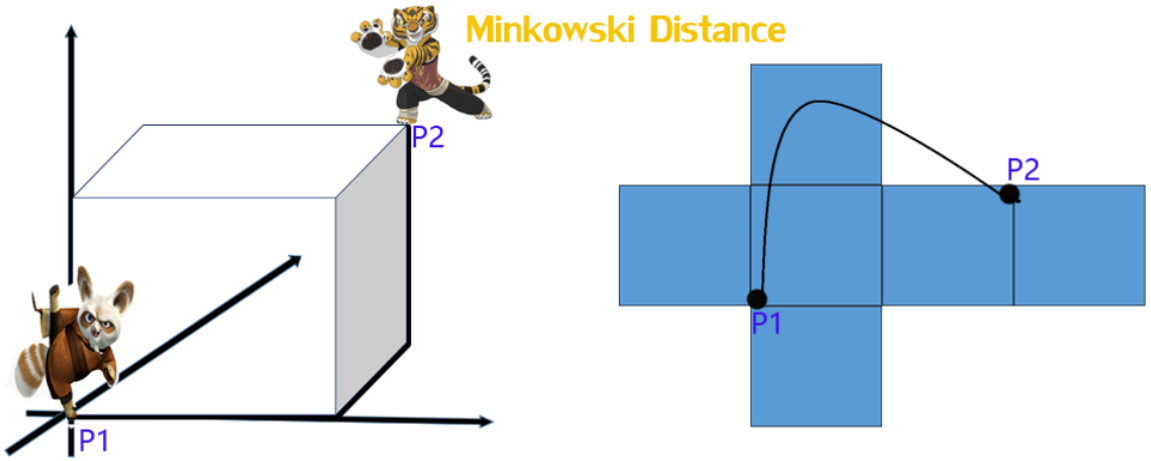
What is Minkowski Distance?

Minkowski distance is a generalized metric that serves as a flexible way to calculate the distance between two points in a multi-dimensional space. It encompasses both Euclidean and Manhattan distances as special cases, controlled by a parameter 'p'.



The Formula:

For two points $P_1 = (p_{1,1}, p_{1,2}, ..., p_{1,n})$ and $P_2 = (p_{2,1}, p_{2,2}, ..., p_{2,n})$ in an n-dimensional space, the Minkowski distance with parameter p is:

$$d = \left(\sum_{i=1}^{n} |p_{1,i} - p_{2,i}|^p\right)^{1/p}$$

- **When p = 1:** It becomes the **Manhattan distance** (sum of absolute differences).
- **When p = 2:** It becomes the **Euclidean distance** (square root of the sum of squared differences).
- **When p approaches infinity:** It becomes the **Chebyshev distance** (the maximum absolute difference between any single coordinate).

Example: Comparing Movie Ratings (with p=3)

Let's say we have two movie critics, Reviewer A and Reviewer B, and we want to compare their rating styles for three movies: "Action Movie", "Comedy Film", and "Drama Series". They rate movies on a scale of 1 to 10.

Reviewer A's Ratings:

- Action Movie: 8
- Comedy Film: 3

- Drama Series: 9
- So, Reviewer A's "point" is (8,3,9)

Reviewer B's Ratings:

- Action Movie: 6
- Comedy Film: 6
- Drama Series: 7
- So, Reviewer B's "point" is (6,6,7)

Let's calculate the Minkowski distance between Reviewer A and Reviewer B using p = 3:

1. Find absolute differences for each movie:
   - Action Movie: $|6 - 8| = |-2| = 2$
   - Comedy Film: $|6 - 3| = |3| = 3$
   - Drama Series: $|7 - 9| = |-2| = 2$
2. Raise each absolute difference to the power of p (which is 3):
   - Action Movie: $2^3 = 8$
   - Comedy Film: $3^3 = 27$
   - Drama Series: $2^3 = 8$
3. Sum these results: $8 + 27 + 8 = 43$
4. Take the p-th root (cube root) of the sum: $\sqrt[3]{43} \approx 3.503$

So, the Minkowski distance (with p=3) between Reviewer A's and Reviewer B's ratings is approximately **3.503**.

Interpretation:

The value of p in Minkowski distance influences how large differences are weighted.

- Smaller p (like p=1, Manhattan): Gives more importance to individual large differences across features.
- Larger p (like p=2, Euclidean, or p=3): Puts more emphasis on the largest differences, as squaring/cubing them makes them grow faster than smaller differences.

In this example, the Minkowski distance of 3.503 quantifies the overall dissimilarity between the two reviewers' rating styles. The choice of 'p' allows you to tune how the distance metric emphasizes or de-emphasizes differences across various dimensions.