매트랩을 활용한 양자 오류 부호의 모의 실험 연구

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A Study on Simulation of Quantum Error Correction Codes by Using MATLAB Program

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요 약

In this work, we introduce basic notations of quantum operations and quantum error correcting codes via quantum circuit model. Then, the simplest quantum error correction codes is explained and proved to correct quantum errors via MATLAB environment.

Ⅰ. Introduction

Starting with the ideas of P. Benioff (1980) and R. Feynman (1982) [1], quantum theory of information has been developed. Until now, quantum processing devices have a great deal of potential for various tasks such as factorizing large number, searching a pattern in database, cryptography. And they showed better performances than classical computers even if the best currently known algorithms are considered in classical processing devices [2].

However, to realize that potential, methods to protect fragile quantum states from unwanted evolution, errors are needed [3]. Hence, quantum error correcting codes (QECC) have been developed to protect quantum information from these errors [4].

Before implementing the error correcting codes, the necessary step is simulation in the classical computer. In this work, two basic QECC codes are first introduced. Then, the analysis and simulation is taken by MATLAB.

The rest of the paper is organized as follows. In section 2, we introduce the theory of qubits and quantum elementary operations. The quantum error correction and the solution to simulate in MATLAB are discussed in section 3. Finally, the conclusion is listed in Section 4.

Ⅱ. Qubit and quantum elementary operations

Quantum theory uses qubit to represent information, the quantum systems with two levels such as: two polarization states of photons, two energy levels of atoms, etc. A qubit, denoted as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, can be considered to have both values of $|0\rangle$ and $|1\rangle$ at the same time, the probability of value $|0\rangle$ is $|\alpha|^2$ and probability of value $|1\rangle$ is $|\beta|^2$ .This concept known as superposition is the main property of quantum computation since it allows gate operations to deal with several values in one step. Hence, the amount of information that can be represented is infinite. A qubit can be representing in vector form:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{bmatrix}1\\0\end{bmatrix} + \beta\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\alpha\\\beta\end{bmatrix}.$$

According to the probability, the condition $|\alpha|^2 + |\beta|^2 = 1$ must be satisfied. A quantum memory register is a physical system composed of n qubits, that is multiple by tensor product of some qubits. Generally, the n qubits state denote as:

$$|\psi\rangle = \sum_{i_k=\{0,1\}} \alpha_{i_1 i_2 \dots i_n} |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_n\rangle = \sum_i \alpha_i |i\rangle,$$

where $i = \sum_{k=1}^{n} 2^{n-k} i_k$. Hence, a state vector of n qubits quantum system is considered as superposition of the states that make up a base in 2^n dimensional complex Hilbert space.

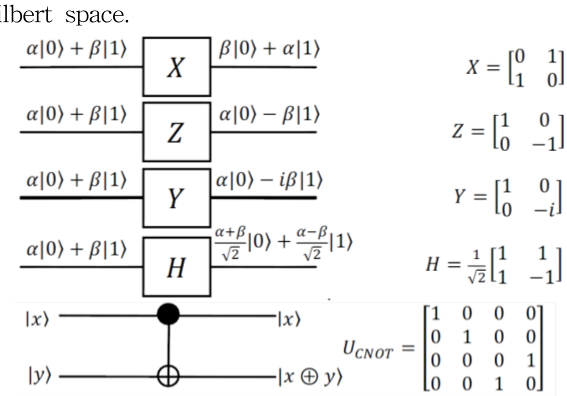


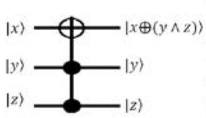
Figure 1. Summary of basic quantum gates.

In this paper, quantum circuit model is used. In this model, operations are applied to qubit by quantum gates [5]. Mathematically, we simulate the quantum gate via the matrix representation. Note that the condition for any quantum gate is revertible and the invert gate that move $U|\psi\rangle$ back to $|\psi\rangle$ satisfy $U^{-1}=U^{\dagger}$, so U is unitary matrix. Some important quantum gates are showing in **Figure** 1. In quantum depolarizing channel, $\mathbf{I}_2$ (the identity matrix), X, Z and Y form orthogonal basis of linear space of operators acting on qubit and any error acting on qubit can be represented as the combination of $\mathbf{I}_2$,X,Z and Y. Hence, there are only three types of errors that can effect on a qubit the flip error, phase error and flip-phase error. Generally, the error operators that effect to n qubits have the form: $E=e_1\otimes e_2\otimes ...\otimes e_n$ where $e_i\in\{\mathrm{I,X,Y,Z}\}$.

Ⅲ. Simulation quantum error correction code

MATLAB is a computing environment that based on the operation with matrices. Hence it is useful tool in simulating the matrix formalism of quantum process. The representation of one qubit can be represented base on 2 dimensional Hilbert space $|\psi\rangle=\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$.

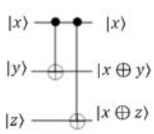
The state of quantum register n qubits can be given by the tensor product of n qubits. Any circuit is given as the series of unitary transformation applied to the quantum state. These transformations are represented by matrices form. As calculating by true table, two new matrices combined of CNOT gate is presented in **Figure** 2 and **Figure** 3.



Input	Output
000	000
001	001
010	010
011	111
100	100
101	101
110	110
111	011

$$matrix=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&1&0&0&0&0\end{bmatrix}$$

Figure 2. New CNOT-1.



Input	Output
000	000
001	001
010	010
011	011
100	111
101	110
110	101
111	100

$$matrix=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&1&0&0\\0&0&0&0&1&0&0&0\end{bmatrix}$$

Figure 3. New CNOT-2.

The simplest quantum error correction code is 3-qubit repetition code that can correct one error: bit flip or phase flip. To extend the first full quantum code, 9-qubit Shor is created by Shor [6], which can correct bit-flip, phase-flip or the combination bit, phase flip error. The quantum error correction code is simulated by quantum circuit model; they are showing in the **Figure** 4, 5 for 3-qubit repetition code as well as the 9-qubit Shor code. The quantum circuit starts with the information qubit via *kron* function in MATLAB for 3-qubits, 9-qubits, after transformation by encode step, the logical states or encoded qubits are created. For example, two logical basis states of 3-qubit are defined as: $|\mathbf{0}_L\rangle=|000\rangle;|\mathbf{1}_L\rangle=|111\rangle$ for 3-qubit repetition as well as

$$|\mathbf{0}_L\rangle=\frac{1}{\sqrt{8}}(|000\rangle+|111\rangle)\otimes(|000\rangle+|111\rangle)\otimes(|000\rangle+|111\rangle);$$
$$|\mathbf{1}_L\rangle=\frac{1}{\sqrt{8}}(|000\rangle-|111\rangle)\otimes(|000\rangle-|111\rangle)\otimes(|000\rangle-|111\rangle);$$

for 9-qubits Shor code. Using matrices transformation, the states after applying error and decoding can be found. The final states show us the correction state can be recovered the syndrome $|\mathbf{S}_e\rangle$ tell us which error has applied to logical states.

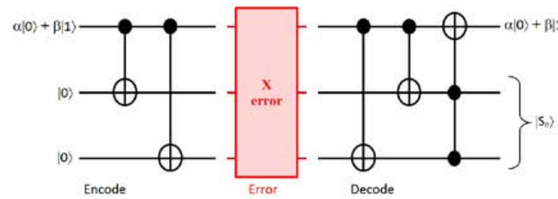


Figure 4. Circuit for X-error QECC.

Ⅳ. Conclusion

The paper presents basic information on quantum error correction, using MATLAB environment the 3-qubits repetition and 9-qubits Shor code have been proved to correct. Such simulation of simplest QECC help us better understanding of design and quantum error correction and quantum mechanism for further.

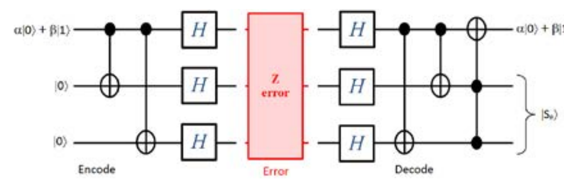


Figure 5. Circuit for Z-error QECC

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