Hash Functions and String Matching

Sriram Sankaranarayanan

Data Structures and Algorithms

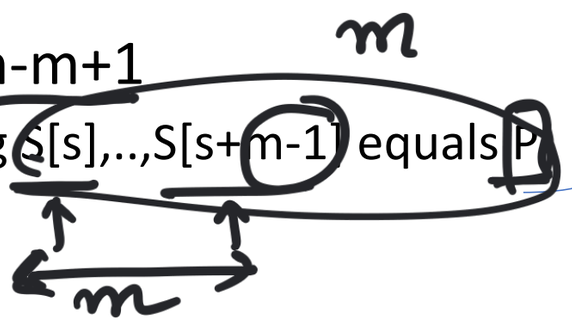
Rabin - Karp

Very important problem with lots of applications!

String Matching

- Does a given pattern 'P' of size m occur in a string 'S' of size n?
- Example:
  - P = "GATTACA" of size m = 7
  - S="GTAGATAGATTTAATTACGATTACATGATGTTGATTAGGATGATTACATATATGAATA ATAGCGCCGATATAGAT"
  - Answer: P occurs in S at the position shown in red.

- Simple algorithm:
  - For each s = 1,…, n-m+1
    - Check if substring S[s],..,S[s+m-1] equals P



$$\Theta(m \times (n - m + 1))$$

$\approx 10^9$ oper

Speeding up Matching Using Hash-Functions

Idea: Hash the pattern P using hash function: $h(P)$ ← hash of this pattern

```
compute pattern hash: r = h(P)
for each s = 1,…, n-m+1
   compute: q = h(S[s] … S[s + m − 1])   ⟵ Θ(m)   filtering
   if r = q:
         compare S[s]…S[s+m-1] with P.
```

S[1] . . . . S[s] . . S[s+m]
m.

Running time: $\Theta(m \times (n - m + 1))$

Rolling Hash Functions

$$h(s) = (p^{m-1} s_m + p^{m-2} s_{m-1} + \cdots + p^1 s_2 + s_1) \bmod M$$

$s_m \; s_{m-1} \; s_{m-2} \; \cdots \; s_1$

s_m | $s_{m-1} \; s_{m-2} \; \cdots \; s_1 \; \hat{s}$

$$h(s') = (p^{m-1} s_{m-1} + p^{m-2} s_{m-2} + \cdots + p^1 s_1 + \hat{s}) \bmod M$$

$$= (p \times (h(s) - p^{m-1} s_m) + \hat{s}) \bmod M$$

Precompute $p^{m-1} \bmod M$ and
perform two multiplications + one subtraction + one addition

Using Rolling Hash Function

```
compute pattern hash: r = h(P)
for each s = 1,…, n-m+1
   compute: q = h(S[s] … S[s + m − 1])
   if r = q:
      compare S[s]…S[s+m-1] with P.
```

$\Theta(1)$ use rolling hash function

collision

Running time: Worst case continues to be

$$\Theta(m \times (n - m + 1))$$

Assuming low probability of hash collision: we can improve the running time to

$$\Theta(m + n)$$

Problem # 2 : Check if two strings have a common substring of size m.

- Inputs: Two strings S1 of size n1, and S2 of size n2.
- Output: True if S1 and S2 have a common substring of size m, FALSE otherwise.
- Example:
  - S1 = “GATATATACAGACAATAGATAGACACACGTAGGTGCACAGT”
  - S2 = “AGGATTTAGGTGGAACCCAGAGAGTTTAGGACCAGATTAGAT”
  - m = 5
  - Answer: True

Simple Algorithm

- for i = 1 to n1-m+1
  - P = S1[i]…S1[i+m-1]
  - Use previous problem to search for pattern in S2
    - If pattern P found, then return True.
    - Else, continue.
- Return False

Assuming good hash function:

$$\Theta(n_1 \times (m + n_2))$$

Worst Case:

$$\Theta\left(n_1 \times \left((n_2 - m + 1)\, m\right)\right)$$

Idea: Use a hash table and hash functions

However, we will need extra space $\Theta(n_1)$ for the hashtable.

Improved Algorithm

For i = 1 to n1 - m+1

- Compute rolling hash $h_i = h(S1[i], \dots S1[i+m-1])$

$\Theta(n_1)$

Insert $\{(h_1, 1), (h_2, 2), \cdots, (h_{n_1-m+1}, n_1 - m + 1)\}$ into perfect hash table H.

$\Theta(n_1 - m + 1)$

For j = 1 to n2 – m + 1

- Compute rolling hash $r_j = h(S2[j] \dots S2[j+m-1])$ $\Theta(n_2)$
- Is key r_j in hashtable H?
- If yes, let k be the associated value with the key r_j
  - Compare S1[k]…, S1[k+m-1] with S2[j]…S2[j+m-1] $\Theta(m)$

If there are no spurious collisions: $\Theta(n_1 + n_2 + m)$ Otherwise: $\Theta(n_1 + (n_2 - m + 1) \times m)$