Problem Set 1

Due on March 12

Please note:

- All question is related to class materials but not every part is taught in class. In other words, some questions are a little "innovative" so to train your programming and problem-solving skill. It is normal to get stuck. Keep working and trying. Go back to the lecture code and previous exercise.
- Submit your answers (in Microsoft Word or PDF format) and your code to canvas. Your answer shall be well written. Graph and Table shall be well-formatted. Your code shall be easy for TA to run and check. Your grade will be affected if your code does not provide proper outputs, or it is confusing so that TA cannot run it.

1. In this exercise, we will write a function that can numerically solve utility maximization problem. This function feeds in a utility function u, prices p_x, p_y, and income w. It then solves the utility maximization problem numerically:

$$\max_{x,y} u(x,y), \quad \text{s.t. } p_x x + p_y y \leq w.$$

This utility maximization problem gives output of indirect utility function $v(p_x, p_y, w)$ and demand functions $x^*(p_x, p_y, w)$, $y^*(p_x, p_y, w)$.
You may follow the following instruction:
Start with a specific example: $u(x,y) = x^{\frac{1}{2}} y^{\frac{1}{3}}$, $p_x = 1$, $p_y = 2$, $w = 10$.

a. Define this utility function in R.

b. Define a grid of points representing bundles. For example $(0.1, 0.1), (0.1, 0.2), (0.1, 0.3), \cdots,$ $(0.2, 0.1), (0.2, 0.2), (0.2, 0.3) \cdots$. You may use w/p_x and w/p_y as upper bounds.

c. Based on the grid, draw a "heat map" of the utility function $u(x,y) = x^{\frac{1}{2}} y^{\frac{1}{3}}$ like the following.
[Hint: one way is the levelplot function in lattice package].

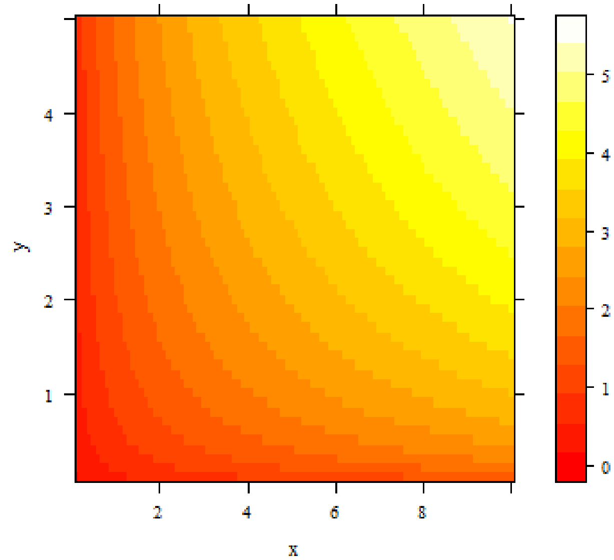
d. Find a set of bundles that satisfy the budget constraint $p_x x + p_y y \leq w$. Generate a scatter plot to show that the budget set is a triangle.

e. Find the utility maximizing bundle within the budget set. The bundle is (x^*, y^*) and the maximum is the value of indirect utility $v = u(x^*, y^*)$. Report the result.

f. Put step (a) to (d) into a big function called "UMP" (utility maximization problem). The function UMP takes $\{u(x,y), p_x, p_y, w\}$ as inputs and reports $\{x^*, y^*, v\}$ as outputs.

g. Check whether the function works by these inputs: $u(x,y) = x^{\frac{1}{2}} y^{\frac{1}{2}}$, $p_x = 4$, $p_y = 2$, $w = 20$. Report the result.

h. Following the setting in step (f), compute $x^*$for each p_x in a sequence $(1, 1.5, 2, 2.5, ..., 5)$. Show a demand curve of x in a graph.



2. Monte-Carlo study of issues in linear model specification.

Let $n = 100$, $\beta_1 = 1$, $\beta_2 = 2$, $\beta_3 = 3$. Let the number of simulation be $S = 500$. Use set.seed(1) before the simulation.

a. Generate the data by

$$\begin{pmatrix} x_{1i} \\ x_{2i} \end{pmatrix} \sim N\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & 0.2 \\ 0.2 & 1 \end{pmatrix} \right),$$

$e_i \sim N(0, 1^2)$, $y_i = \beta_1 + \beta_2 x_{1i} + \beta_3 x_{2i} + e_i$ for $i = 1, 2, ..., n$.

[Hint: you can use the mvrnorm() function in MASS package to generate multi-variate normal distribution.]

For each simulation s, generate a matrix or data frame with each row being $\{x_{1i}, x_{2i}, e_i, y_i\}$.

The true model is

$$y_i = \beta_1 + \beta_2 x_{1i} + \beta_3 x_{2i} + e_i.$$

Suppose that the econometrician excludes a relevant variable x_2 and estimate the model

$$y_i = \beta_1 + \beta_2 x_{1i} + u_i.$$

What is the consequence? Show it by a histogram of $\{\hat{\beta}_2\}_{s=1}^S$.

b. Generate the data by (note that the covariance changes)

$$\begin{pmatrix} x_{1i} \\ x_{2i} \end{pmatrix} \sim N\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & \mathbf{0.8} \\ \mathbf{0.8} & 1 \end{pmatrix} \right),$$

$u_i \sim N(0, 1^2)$, $y_i = \beta_1 + \beta_2 x_{1i} + u_i$ for $i = 1, 2, ..., n$. For each simulation s, generate a matrix or data frame with each row being $\{x_{1i}, x_{2i}, u_i, y_i\}$.

The true model is

$$y_i = \beta_1 + \beta_2 x_{1i} + u_i.$$

Suppose that the econometrician includes an irrelevant variable x_2 and estimate the model

$$y_i = \beta_1 + \beta_2 x_{1i} + \beta_3 x_{2i} + e_i.$$

What is the consequence? Show it by a figure of density curves.

[Hint: you need to compare the estimates of using the true model and using the wrong model.]

c. Continue with part (a) and part (b). We are going to illustrate the properties of F test.

The restricted model is $y_i = \beta_1 + \beta_2 x_{1i} + u_i$.

The unrestricted model is $y_i = \beta_1 + \beta_2 x_{1i} + \beta_3 x_{2i} + e_i$.

What is the null hypothesis H_0 of the F test? The F test rejects the null hypothesis if the p-value is less than 0.05.

(Hint: the F test p-value of anova() can be recorded by anova(m1,m2)[[6]][2] .)

Use the simulated data from part (b) to compute the frequency of type I errors in $S = 500$ simulations.

Use the simulated data from part (a) to compute the frequency of type II errors in $S = 500$ simulations.

3. In this exercise, we will simulate rock-paper-scissor game and show how memorizing and inferring from past "data" can help a player increase winning probability.

Two player submit their action simultaneously and the payoff is shown as follow

		2		
		Rock $[r_2]$	Paper $[p_2]$	Scissor
	Rock $[r_1]$	0, 0	-1, 1	1, -1
1	Paper $[p_1]$	1, -1	0, 0	-1, 1
	Scissor	-1, 1	1, -1	0, 0

For programming convenience, denote Rock=1, Paper=2, Scissor=3.

a. Write a function that compute **player 1**'s payoff given player 1 and 2's actions as input. (e.g. $\pi_1(1,2) = -1$)

b. Think of two players that take each action with certain probability: e.g. Pr(i plays Rock) $= r_i$. Write a function that compute expected payoff of **player 1**, given r_1, p_1, r_2, p_2, that is

$$E\pi_1(r_1, p_1, r_2, p_2) = r_1\left[-p_2 + (1 - r_2 - p_2)\right] + p_1\left[r_2 - (1 - r_2 - p_2)\right] + (1 - r_1 - p_1)\left[-r_2 + p_2\right].$$

Show that a Scissor lover ($r_1 = 0.1$, $p_1 = 0.1$) has disadvantage facing Rock lover ($r_2 = 0.8$, $p_2 = 0.1$) .

c. Write a function that can simulate a naïve player that play with fixed probabilities r_i and p_i (a typical mixed strategy). Suppose player 2 that plays $r_2 = 0.4$, $p_2 = 0.3$ (a slight Rock lover), simulate his action for $S = 5000$ games.

d. Create an Artificial Intelligent (AI) player BetaGo that can beat a naïve player. Specifically, follow these instructions:

(i) At the beginning, without any information, BetaGo should play the best mixed strategy with $p_1 = 1/3$, $r_1 = 1/3$.

(ii) BetaGo can memorize all actions that it seen from its opponent. After 10 games, BetaGo starts to compute empirical probabilities $\hat{r}_2$,$\hat{p}_2$ of his opponents.

(iii) If BetaGo find out his opponent is incline to play certain action with more than 1/3 probability, then it adjusts his strategy to take advantage.

(iv) Show that BetaGo can "beat" a naïve player 2 with $r_2 = 0.4$, $p_2 = 0.3$.

(v) Show that BetaGo has more advantage as number of games (S) becomes larger.

[Note: This last exercise is difficult and takes time. There is lots of way to make even BetaGo stronger. For example, using Bayesian learning, instead of frequentiest learning. This is beyond the scope of this course.]