Computational Methods in Physics (PHY 365)

FA23

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Lab 22

Second order ODE

- $\dfrac{d^2y}{dx^2} = 0$ $\quad\quad\quad$ $y\left(0\right) = 1, y'\left(0\right) = 3$

Second order ODE

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- Trick: Write the ODE as two first-order coupled ODEs.

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- Let $y(1) = y \rightarrow y' [\equiv dy(1)] = y(2); y'' = dy(2)$.
- Initial and final x

```
x_i = 0;
x_f = 5;
x = [x_i, x_f];
```

Second order ODE

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- Initial and final x

```
x_i = 0;
x_f = 5;
x = [x_i, x_f];
```

- Initial conditions

```
IC = [1 , 3];
```

Second order ODE

- Calling the ODE solver

```
[X , Y] = ode45 (@ func_4, x , IC);
```

Second order ODE

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- Separating y and y′

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y = Y (: , 1);
y_prime = Y (: , 2);
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Second order ODE

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[X , Y] = ode45 (@ func_4, x , IC);
```

- Separating y and y'

```
y = Y (: , 1);
y_prime = Y (: , 2);
```

- Plotting the result

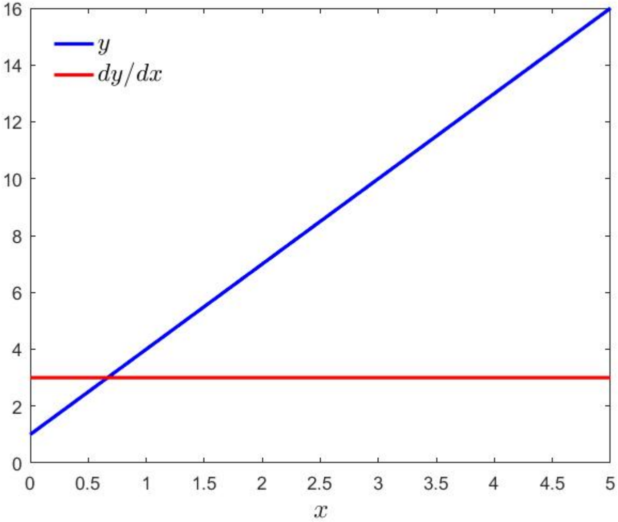
```
plot(X , y , 'b' , X , y_prime , 'r')
```

Second order ODE

- The function file

```
function dy = func_4(x , y)
dy(1) = y(2);
dy(2) = 0;
dy = dy';
```

Second order ODE



Second order ODE

- $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0; \quad y(0) = 0, y'(0) = 1$

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x_f = 5;

x = [x_i , x_f];
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Second order ODE

- $$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0; \quad y(0) = 0, y'(0) = 1$$

- Initial and final x

```
x_i = 0;

x_f = 5;

x = [x_i , x_f];
```

- Initial conditions

```
IC = [0 , 1];
```

Second order ODE

- Calling the ODE solver

  ```
  [X , Y] = ode45 (@ func_5 , x , IC);
  ```

Second order ODE

- Calling the ODE solver

  [X , Y] = ode45 (@ func_5 , x , IC);

- Extracting y from Y

  y = Y(: , 1);

Second order ODE

- Calling the ODE solver

  ```
  [X , Y] = ode45 (@ func_5 , x , IC);
  ```

- Extracting y from Y

  ```
  y = Y(: , 1);
  ```

- Plotting the result

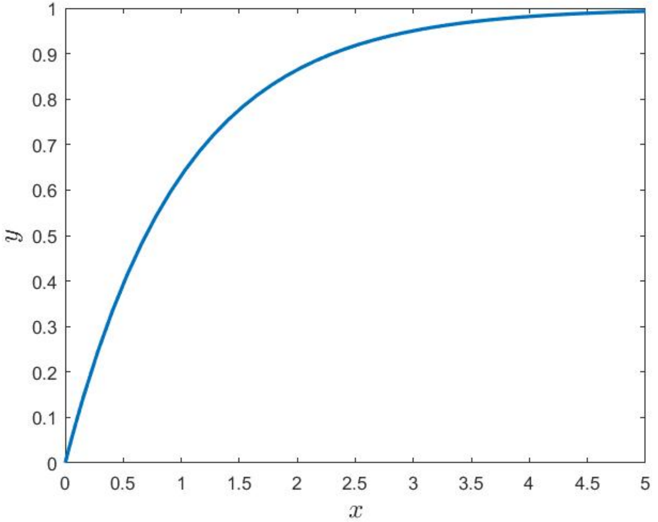
  ```
  plot(X , y)
  ```

Second order ODE

- The function file

```
function dy = func_5(x , y)
dy(1) = y(2);
dy(2) = - y(2);
dy = dy';
```

Second order ODE



Second order ODE

- $$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + b\,y = 0; \quad y(0) = 0, y'(0) = 1$$

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- Initial and final x

```
x_i = 0;

x_f = 10 * pi;

x = [x_i , x_f];
```

Second order ODE

- $$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + b\,y = 0; \quad y\,(0) = 0, y'\,(0) = 1$$

- Initial and final x

```
x_i = 0;

x_f = 10 * pi;

x = [x_i , x_f];
```

- The parameters a and b

```
a = 10 ^ -1;

b = 1;
```

Second order ODE

- Initial conditions

  IC = [0 , 1];

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- Calling the ODE solver

  [X , Y] = ode45 (@ func_6 , x , IC);

Second order ODE

- Initial conditions

  ```
  IC = [0 , 1];
  ```

- Calling the ODE solver

  ```
  [X , Y] = ode45 (@ func_6 , x , IC);
  ```

- Extracting y from Y

  ```
  y = Y(: , 1);
  ```

Second order ODE

- Initial conditions

  IC = [0 , 1];

- Calling the ODE solver

  [X , Y] = ode45 (@ func_6 , x , IC);

- Extracting y from Y

  y = Y(: , 1);

- Plotting the result

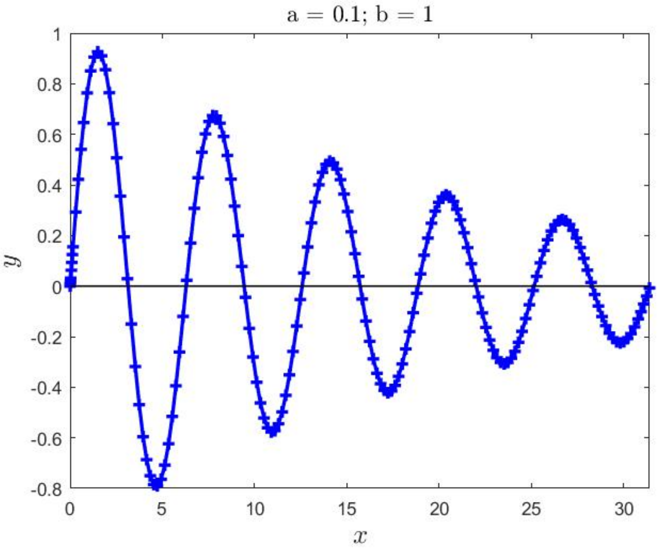
  plot(X , y)

Second order ODE

- The function file

```
function dy = func_6(x , y)
a = evalin('base' , 'a');
b = evalin('base' , 'b');
dy(1) = y(2);
dy(2) = - a * y(2) - b * y(1);
dy = dy';
```

Second order ODE



Second order ODE

- $\dfrac{d^2y}{dx^2} + a\dfrac{dy}{dx} + b\, y^2 = x; \quad y(0) = 0, y'(0) = 1$

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- Initial and final x

```
x_i = 0;

x_f = 5 * pi;

x = [x_i , x_f];
```

Second order ODE

- $$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + b\,y^2 = x; \quad y(0) = 0, y'(0) = 1$$

- Initial and final x

```
x_i = 0;
x_f = 5 * pi;
x = [x_i , x_f];
```

- The parameters a and b

```
a = 10 ^ -2;
b = 1;
```

Second order ODE

- Initial conditions

  IC = [0 , 1];

Second order ODE

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- Calling the ODE solver

  ```
  [X , Y] = ode45 (@ func_7 , x , IC);
  ```

Second order ODE

- Initial conditions

  IC = [0 , 1];

- Calling the ODE solver

  [X , Y] = ode45 (@ func_7 , x , IC);

- Extracting y from Y

  y = Y(: , 1);

Second order ODE

- Initial conditions

  ```
  IC = [0 , 1];
  ```

- Calling the ODE solver

  ```
  [X , Y] = ode45 (@ func_7 , x , IC);
  ```

- Extracting y from Y

  ```
  y = Y(: , 1);
  ```

- Plotting the result

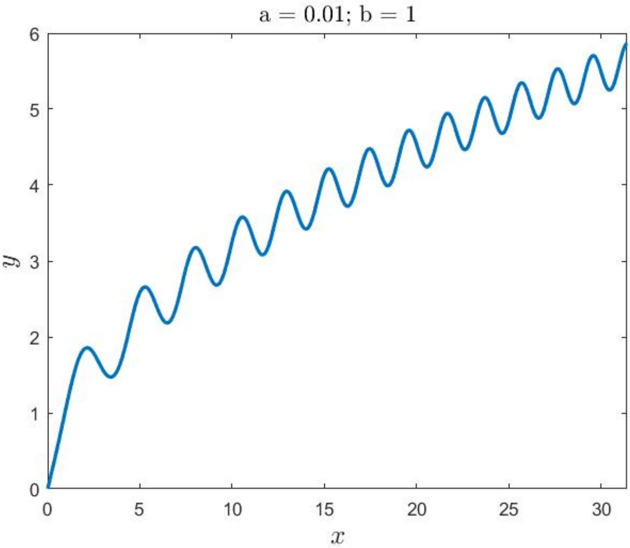
  ```
  plot(X , y)
  ```

Second order ODE

- The function file

```
function dy = func_7(x , y)
a = evalin('base' , 'a');
b = evalin('base' , 'b');
dy(1) = y(2);
dy(2) = x - a * y(2) - b * y(1) .^2;
dy = dy';
```

Second order ODE



MATLAB dsolve function

- S = dsolve(eqn) solves the differential equation “eqn”.
    - eqn is a symbolic equation.

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MATLAB dsolve function

- S = dsolve(eqn) solves the differential equation “eqn”.
  - eqn is a symbolic equation.
- S = dsolve(eqn , cond) solves eqn with the initial or boundary condition “cond”.
- S = dsolve(eqn , cond , Name , Value) uses additional options specified by one or more Name,Value pair arguments.
- [y1 , . . . , yN] = dsolve(___) assigns the solutions to the variables y1,. . . ,yN.

First order ODE

- $\frac{\mathrm{dx}}{\mathrm{dt}} = 0; \qquad \mathrm{x}(0) = 2.$

First order ODE

- $\frac{dx}{dt} = 0; \qquad x(0) = 2.$

- The differential equation

  syms x(t);

  my_eqn = diff(x , t) == 0;

First order ODE

- $\frac{dx}{dt} = 0; \qquad x(0) = 2.$

- The differential equation

```
syms x(t);
my_eqn = diff(x , t) == 0;
```

- Initial condition

```
IC = x(0) == 2;
```

First order ODE

- $\frac{dx}{dt} = 0; \qquad x\,(0) = 2.$

- The differential equation

  syms x(t);

  my_eqn = diff(x , t) == 0;

- Initial condition

  IC = x(0) == 2;

- Solving the equation

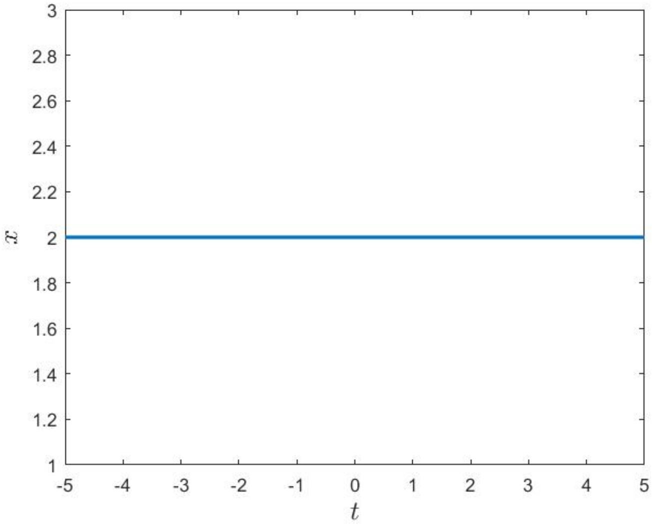
  my_sol = dsolve(my_eqn , IC , 't');

First order ODE

- Plotting the result

  fplot(my_sol)

First order ODE



First order ODE

- $\dfrac{dx}{dt} = \sin(x); \qquad x(0) = 1.$

First order ODE

- $\dfrac{dx}{dt} = \sin(x)$; $x(0) = 1$.

- The differential equation

  syms x(t);

  my_eqn = diff(x , t) == sin (x);

First order ODE

- $\dfrac{dx}{dt} = \sin(x);\qquad x(0) = 1.$

- The differential equation

```
syms x(t);
my_eqn = diff(x , t) == sin (x);
```

- Initial condition

```
IC = x(0) == 1;
```

First order ODE

- $\frac{dx}{dt} = \sin(x)$; $x(0) = 1$.

- The differential equation

  syms x(t);

  my_eqn = diff(x , t) == sin (x);

- Initial condition

  IC = x(0) == 1;

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First order ODE

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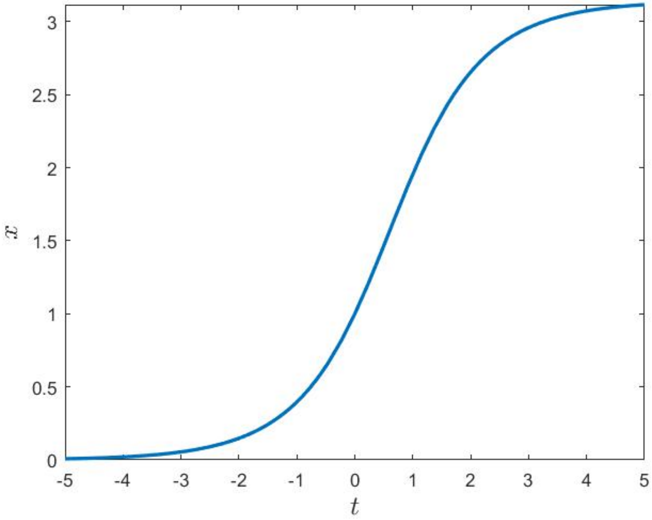
First order ODE



Second order ODE

- $\dfrac{d^2x}{dt^2} = t + \exp(1 - t)$; $\quad x(0) = 0, x'(0) = 1$.

Second order ODE

- $\frac{d^2x}{dt^2} = t + \mathrm{exp}\left(1 - t\right);\qquad x\left(0\right) = 0, x'\left(0\right) = 1.$

- The differential equation

```
syms x(t);

my_eqn = diff(x , t , 2) == t + exp(1 - t);
```

Second order ODE

- $\dfrac{d^2x}{dt^2} = t + \exp(1 - t)\,; \qquad x(0) = 0, x'(0) = 1.$

- The differential equation

```
syms x(t);
my_eqn = diff(x , t , 2) == t + exp(1 - t);
```

- Initial conditions

```
Dx = diff(x , t);
IC = [x(0) == 0, Dx(0) == 1];
```

Second order ODE

- Solving the equation

```
my_sol = dsolve(my_eqn , IC , 't');
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Second order ODE

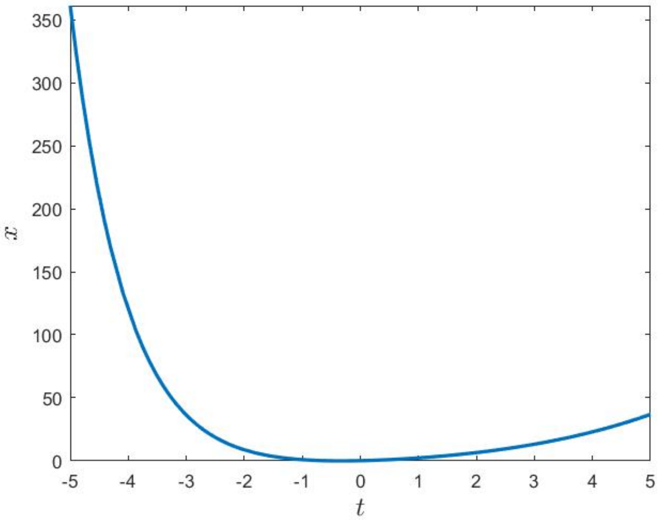
- Solving the equation

  my_sol = dsolve(my_eqn , IC , 't');

- Plotting the result

  fplot(my_sol)

Second order ODE



References

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- https://www.mathworks.com/help/symbolic/dsolve.html
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