Computational Methods in Physics (PHY 365)

FA23

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Lab 5

Newton-Raphson method

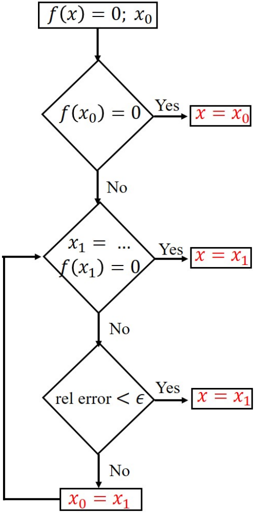

Figure: Flow chart for Newton-Raphson method

Newton-Raphson method

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- Function

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syms x
f = symfun (3 * x^2 - 5, x);
```

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- Derivative of the function

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f_der = diff(f, x);
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Newton-Raphson method

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- Derivative of the function

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- The initial guess

```
x_0 = 1;
```

Newton-Raphson method

- Minimum error

```
min_err = 10^-4;
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Newton-Raphson method

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  min_err = 10^-4;
  ```

- Calling the function

  ```
  [x_root , iterations] = new_raph_fun (f , f_der , x_0 ,
  min_err);
  ```

Newton-Raphson method

- Minimum error

  min_err = 10^–4;

- Calling the function

  [x_root , iterations] = new_raph_fun (f , f_der , x_0 , min_err);

- Displaying the result

  fprintf ('\n')

  disp (['Itertions = ' , num2str(iterations)])

  fprintf ('\n Root = %3.7f \n', x_root)

Newton-Raphson method

- The function file

```
function [x_root , iterations] = new_raph_fun (f , f_der ,
x_0 , min_err)
x_old = x_0;
rel_err = 1;
iterations = 0;
while rel_err > min_err
    f_x_old = double (subs (f , x_old));
    f_der_x_old = double (subs (f_der , x_old));
    x_new = x_old - f_x_old / f_der_x_old;
```

Newton-Raphson method

```
        rel_err = abs (x_new - x_old);
        x_old = x_new;
        iterations = iterations + 1;
    end
    x_root = x_new;
```

Newton-Raphson method

```
% The function

syms x

f = symfun(3 * x ^ 2 - 5, x);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Derivative of the function

f_der = diff(f,x);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% The initial guess

x_0 =  1.5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Minimum error

min_err = 10 ^ -4;
```

Newton-Raphson method

```
% Calling the function

[x_root,iterations] = new_raph_fun(f,f_der,x_0,min_err);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Displaying the result

fprintf('\n')

disp(['Itertions = ', num2str(iterations)])

fprintf('\n Root = %3.6f \n',x_root)
```

Newton-Raphson method

```
function [x_root,iterations] = new_raph_fun(f,f_der,x_0,min_err)

x_old = x_0;

rel_err = 1;

iterations = 0;

while rel_err > min_err

    f_x_old = double(subs(f,x_old));

    f_der_x_old = double(subs(f_der,x_old));
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Newton-Raphson method

```
    x_new = x_old - f_x_old / f_der_x_old;

    rel_err = abs(x_new - x_old);

    x_old = x_new;

    iterations = iterations + 1;

end

x_root = x_new;
```

MATLAB fzero function

- fzero calculates root of nonlinear function.
- fzero cannot find a root of a function such as x^2.
- x = fzero (fun , x0) tries to find a point x where fun (x) = 0.
- The solution is where fun (x) changes sign.

MATLAB fzero function

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- x = fzero (fun , x0) tries to find a point x where fun (x) = 0.
- The solution is where fun (x) changes sign.
- Root from one point

```
fun = @ (x)   exp ( − 3 * x) − 5 * x .^ 3 + 20; % function
x0 = 3; % initial point
x = fzero (fun ,x0);
```

MATLAB fzero function

- Root within an interval

```
fun = @ (x) x .^ 5 - 3 * x .^ 2 + 1;

x_int = [1,2]; % interval

x = fzero (fun, x_int)
```

MATLAB fsolve function

- x = fsolve (fun , x0) starts at x0 and tries to solve the equations fun (x) = 0, an array of zeros.

- x = fsolve (fun , x0 , options) solves the equations with the optimization options specified in options.

- [x , fval] = fsolve (__), for any syntax, returns the value of the objective function fun at the solution x.

MATLAB fsolve function

- x = fsolve (fun , x0) starts at x0 and tries to solve the equations fun (x) = 0, an array of zeros.
- x = fsolve (fun , x0 , options) solves the equations with the optimization options specified in options.
- [x , fval] = fsolve (__), for any syntax, returns the value of the objective function fun at the solution x.
- Example

  fun = @ (x) exp (− 3 * x) − 5 * x .^ 3 + 20; % function

  x0 = 3; % initial point

  [x , fval] = fsolve (fun , x0);

References

- https://en.wikipedia.org/wiki/Newton%27s_method
- https://web.mit.edu/10.001/Web/Course_Notes/NLAE/node6.html
- https://personal.math.ubc.ca/~anstee/math104/newtonmethod.pdf
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- https://www.mathworks.com/help/optim/ug/fsolve.html
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