Computational Methods in Physics (PHY 365)

FA23

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Lab 20

MATLAB ODE solvers

Solver	DE	Method	When to use
ode45	NDEs *	RK	Most of the time. This should be the first solver you try.
ode23	NDEs	RK	For solving moderately stiff problems.
ode113	NDEs	Adams	For solving computationally intensive problems.
ode15s	SDEs† & DAEs‡	NDF (BDF) §	If ode45 is slow because the problem is stiff

*Nonstiff DEs

†Stiff DEs

‡Diff. Algebraic Equations

§Numerical (Backward) Differentiation Formula

MATLAB ODE solvers

- An ODE problem is stiff if the solution being sought is varying slowly, but there are nearby solutions that vary rapidly.
- The numerical method must take small steps to obtain satisfactory results.
- Stiffness is an efficiency issue.
- Nonstiff methods can solve stiff problems; they just take a long time to do it.

MATLAB ODE solvers

- [T, Y] = solver(odefun,tspan,y0,options)

MATLAB ODE solvers

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solver	one of ode45, ode23, ode113, ode15s, ode23s, ode23t, or ode23tb.
odefun	A function handle that evaluates the right side of the differential equations.
tspan	A vector specifying the interval of integration.
y0	A vector of initial conditions.
options	Structure of optional parameters that change the default integration properties.

First order ODE

- $\frac{dy}{dx} = 0$ $\quad\quad y(0) = 2$

First order ODE

- $\frac{dy}{dx} = 0 \qquad y(0) = 2$

- Initial and final x

  x_i = 0;

  x_f = 10;

  x = [x_i , x_f];

First order ODE

- $\frac{dy}{dx} = 0$ $\quad\quad$ $y(0) = 2$

- Initial and final x

```
x_i = 0;
x_f = 10;
x = [x_i , x_f];
```

- Initial condition

```
IC = 2;
```

First order ODE

- $\frac{dy}{dx} = 0 \qquad y(0) = 2$

- Initial and final x

  x_i = 0;

  x_f = 10;

  x = [x_i , x_f];

- Initial condition

  IC = 2;

- Calling the ODE solver

  [X , Y] = ode45 ('func_1', x , IC);

First order ODE

- Plotting the result

```
plot(X , Y)
```

First order ODE

- Plotting the result

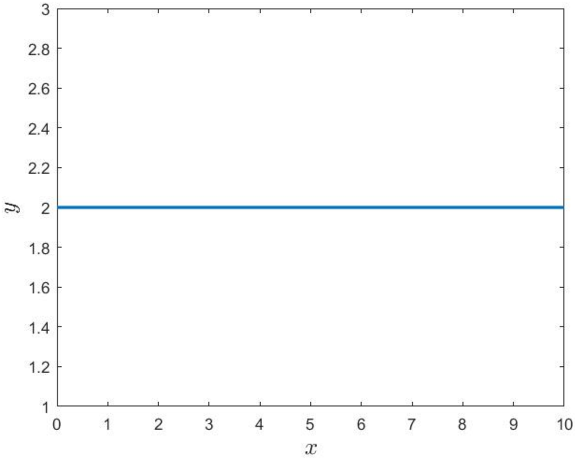
  plot(X , Y)

- The function file

  function dy = func_1(x , y)

  dy = 0;

First order ODE



First order ODE

- $\dfrac{dy}{dx} = x^3 - \exp\left(3x^2 - 2\right) + \exp\left(x^3\right)$ $\quad y(0) = 10^{-3}$

First order ODE

- $$\frac{dy}{dx} = x^3 - \exp\left(3x^2 - 2\right) + \exp\left(x^3\right) \qquad y\left(0\right) = 10^{-3}$$

- Initial and final x

  x_i = 0;

  x_f = 1.5;

  x = [x_i , x_f];

First order ODE

- $\frac{dy}{dx} = x^3 - \exp\left(3x^2 - 2\right) + \exp\left(x^3\right)$ $y(0) = 10^{-3}$

- Initial and final x

```
x_i = 0;
x_f = 1.5;
x = [x_i , x_f];
```

- Initial condition

```
IC = 10^-3;
```

First order ODE

- $\frac{dy}{dx} = x^3 - \exp\left(3x^2 - 2\right) + \exp\left(x^3\right)$ $y(0) = 10^{-3}$

- Initial and final x

  x_i = 0;

  x_f = 1.5;

  x = [x_i , x_f];

- Initial condition

  IC = 10^−3;

- Calling the ODE solver

  [X , Y] = ode45 ('func_2' , x , IC);

First order ODE

- Plotting the result

```
plot(X , Y)
```

First order ODE

- Plotting the result

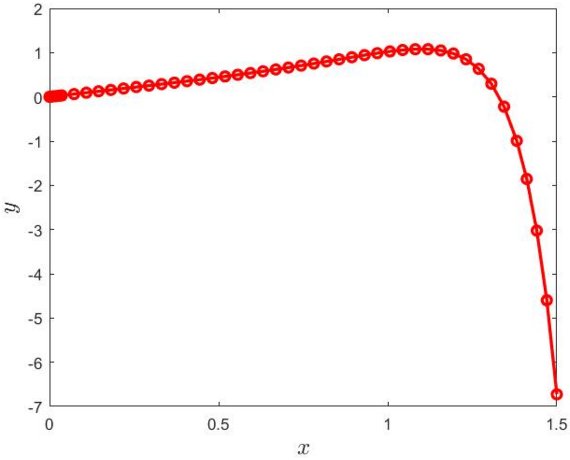
  plot(X , Y)

- The function file

  function dy = func_2(x,y)

  dy = x .^3 - exp(3 * x .^2 - 2) + exp(x .^3);

First order ODE



First order ODE

- $y^2 \frac{dy}{dx} = x^2 + \exp\left(3x^4 - 2\right)$ $\quad y(0) = 1$

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First order ODE

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  x_f = 1;

  x = [x_i , x_f];

- Initial condition

  IC = 1;

First order ODE

- $y^2 \dfrac{dy}{dx} = x^2 + \exp\left(3x^4 - 2\right)$ $y(0) = 1$

- Initial and final x

```
x_i = 0;
x_f = 1;
x = [x_i , x_f];
```

- Initial condition

```
IC = 1;
```

- Calling the ODE solver

```
[X , Y] = ode45 (@ func_3 , x , IC);
```

First order ODE

- Plotting the result

  plot (X , Y)

First order ODE

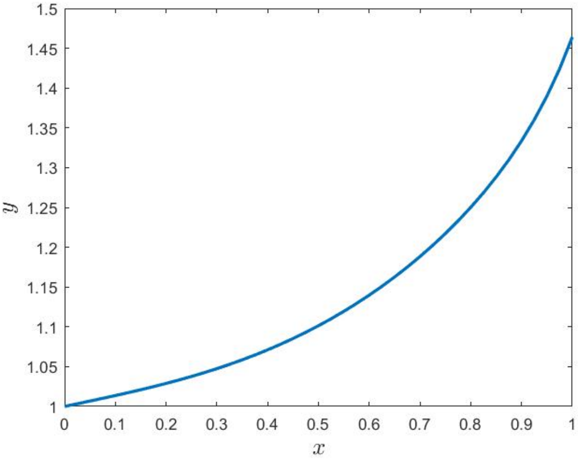
- Plotting the result

```
plot (X , Y)
```

- The function file

```
function dy = func_3(x,y)
dy = (x .^2 + exp(3 * x .^4 - 2) ) ./ y .^2;
```

First order ODE



References

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