Computational Methods in Physics (PHY 365)

FA23

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Lab 27

Volume of a region

- Problem: Determine the volume of the region whose points satisfy the inequalities

$$\begin{cases} 0 \le x \le 1, \quad 0 \le y \le 1, \quad 0 \le z \le 1 \\ x^2 + \sin y \le z \\ x - z + e^y \le 1 \end{cases}$$

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- The first line defines a cube whose volume is 1.
  - The region defined by all the given inequalities is therefore a subset of this cube.

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- The first line defines a cube whose volume is 1.
  - The region defined by all the given inequalities is therefore a subset of this cube.

- If we generate 'n' random points in the cube and determine that 'm' of them satisfy the last two inequalities, then the volume of the desired region is approximately m/n.

Volume of a region

- Total random points

  total_pnts = 5000;

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  total_pnts = 5000;

- Generating random numbers

  x = rand(total_pnts , 1);

  y = rand(total_pnts , 1);

  z = rand(total_pnts , 1);

Volume of a region

- Total random points

```
total_pnts = 5000;
```

- Generating random numbers

```
x = rand(total_pnts , 1);
y = rand(total_pnts , 1);
z = rand(total_pnts , 1);
```

- Inequalities

```
cond_1 = x .^ 2 + sin(y);
cond_2 = x - z + exp(y);
```

Volume of a region

- Points satisfying the inequalities

```
points_included = 0;
for k = 1 : total_pnts
  if (cond_1(k) <= z(k) ) && (cond_2(k) <= 1)
    points_included = points_included + 1;
  end
end
```

Volume of a region

- Points satisfying the inequalities

```
points_included = 0;
for k = 1 : total_pnts
  if (cond_1(k) <= z(k) ) && (cond_2(k) <= 1)
    points_included = points_included + 1;
  end
end
```

- Volume of the region

```
reg_vol = points_included / total_pnts;
disp( ['Approximated volume of the region = ', num2str
(reg_vol) ] )
```

Numerical integration

- Problem: Use MCM to approximate the integral

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\left(x^2+y^2+z^2\right)dx\,dy\,dz.$$

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- Total number of random points

```
total_pnts = 5 * 10 ^ 4;
```

Numerical integration

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- Total number of random points

  total_pnts = 5 * 10 ^ 4;

- Initial and final limits

  x_i = −1; x_f = 1;

  y_i = −1; y_f = 1;

  z_i = −1; z_f = 1;

Numerical integration

- Multiplying factor

```
mult_fac = (x_f - x_i) * (y_f - y_i) * (z_f - z_i);
```

Numerical integration

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```
mult_fac = (x_f - x_i) * (y_f - y_i) * (z_f - z_i);
```

- Generating random numbers

```
x = x_i + (x_f - x_i) * rand(total_pnts,1);
y = y_i + (y_f - y_i) * rand(total_pnts,1);
z = z_i + (z_f - z_i) * rand(total_pnts,1);
```

Numerical integration

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mult_fac = (x_f - x_i) * (y_f - y_i) * (z_f - z_i);
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- Generating random numbers

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x = x_i + (x_f - x_i) * rand(total_pnts,1);
y = y_i + (y_f - y_i) * rand(total_pnts,1);
z = z_i + (z_f - z_i) * rand(total_pnts,1);
```

- The integrand

```
f = x .^ 2 + y .^ 2 + z .^ 2;
```

Numerical integration

- Value of the integral

  integral_val = (mult_fac / total_pnts) * sum (f);

  disp([‘Approximated value of the integral = ’, num2str (integral_val)])

References

- https://en.wikipedia.org/wiki/Monte_Carlo_integration
- https://www.scratchapixel.com/lessons/mathematics-physics-for-computer-graphics/monte-carlo-methods-in-practice/monte-carlo-integration
- https://www.youtube.com/watch?v=MKnjsqYVG4Y