Computational Methods in Physics (PHY 365)

FA23

Dr. Muhammad Kamran

Department of Physics

COMSATS University Islamabad

Lab 18

Euler's method

- This numerical method does not have the utmost generality, but it is natural and capable of high precision.
- Its principle is to represent the solution of a differential equation locally by a few terms of its Taylor series.

$$x(t+h) = x(t) + hx'(t) + \frac{h^2}{2!}x''(t) + \ldots + \frac{h^m}{m!}x^{(m)}(t) + \ldots \tag{1}$$

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- For numerical purposes, the Taylor series truncated after $m+1$ terms enables us to compute $x(t+h)$ rather accurately
  - → if h is small, and
  - → if the m derivatives are known.

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$$x(t+h) \approx x(t) + hx'(t). \tag{3}$$

Euler's method

- Hence, the formula

$$x(t+h) = x(t) + hf(t, x(t)). \tag{4}$$

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- Since only two terms of the Taylor series are used in Euler's method, it is not very accurate.

- In solving an IVP, it is useful to distinguish two types of errors
  - local truncation error
  - global truncation error

Euler's method

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- For Euler's method, the local error is simply the remainder of Taylor's approximation, i.e., $O\left(h^2\right)$.
- Since at each step of Euler's method an additional truncation error is introduced, the accumulation of these errors is called the global truncation error.

Euler's method

Problem: Using Euler's method, compute an approximate value for $x(1)$ for the following ODE.

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- The interval, step size, and initial condition

  a = 0;

  b = 1;

  no_steps = 1;

  step_size = (b − a) / no_steps;

  x_0 = 1;

Euler's method

- The function

```
f = @ (t , x) x;
```

Euler's method

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```

- Applying Euler's method

```
for n = 1 : no_steps
    x = x_0 + step_size * f(a , x_0);
    x_0 = x;
    a = a + step_size;
end
```

Euler's method

- Displaying the result

```
disp ( ['The approximate value is ', num2str(x) ] )
```

References

- https://www.varsitytutors.com/differential_equations-help/initial-value-problems
- https://en.wikipedia.org/wiki/Euler_method
- https://tutorial.math.lamar.edu/classes/de/eulersmethod.aspx