Computational Methods in Physics (PHY 365)

FA23

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Lab 26

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- In its simplest idealized form, the problem can be formulated in the following traditional way.
- A drunk starts out from a lamppost located on a street.
- Each step he takes is of equal length L.
- The man is, however, so drunk that the direction of each step, whether it is to the right or to the left, is completely independent of the preceding step.

Random walk problem

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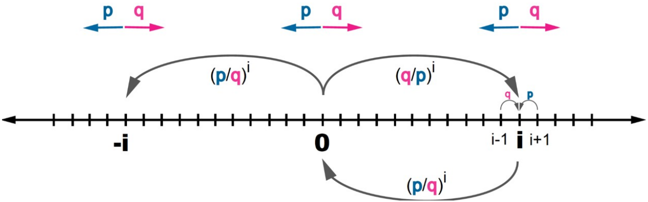
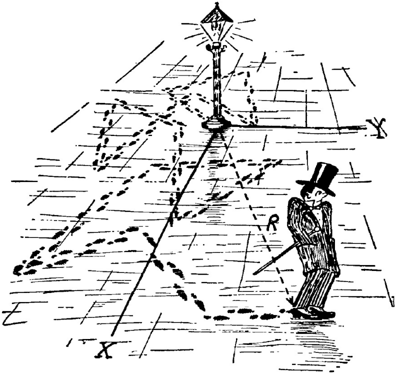


Figure: Random walk in 1 D

Random walk problem



Volume of a region

- Problem: Determine the volume of the region whose points satisfy the inequalities

$$\begin{cases} 0 \le x \le 1, \quad 0 \le y \le 1, \quad 0 \le z \le 1 \\ x^2 + \sin y \le z \\ x - z + e^y \le 1 \end{cases}$$

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- The first line defines a cube whose volume is 1.
    - The region defined by all the given inequalities is therefore a subset of this cube.

- If we generate 'n' random points in the cube and determine that 'm' of them satisfy the last two inequalities, then the volume of the desired region is approximately m/n.

Volume of a region

- Total random points

```
total_pnts = 5000;
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- Generating random numbers

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y = rand(total_pnts , 1);
z = rand(total_pnts , 1);
```

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x = rand(total_pnts , 1);
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- Inequalities

```
cond_1 = x .^ 2 + sin(y);
cond_2 = x - z + exp(y);
```

Volume of a region

- Points satisfying the inequalities

```
points_included = 0;
for k = 1 : total_pnts
  if (cond_1(k) <= z(k) ) && (cond_2(k) <= 1)
     points_included = points_included + 1;
  end
end
```

Volume of a region

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  end
end
```

- Volume of the region

```
reg_vol = points_included / total_pnts;
disp( ['Approximated volume of the region = ', num2str
(reg_vol) ] )
```

Numerical integration

- Problem: Use MCM to approximate the integral

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\left(x^2+y^2+z^2\right)dx\,dy\,dz.$$

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```
total_pnts = 5 * 10 ^ 4;
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- Total number of random points

total_pnts = 5 * 10 ^ 4;

- Initial and final limits

x_i = -1;   x_f = 1;

y_i = -1;   y_f = 1;

z_i = -1;   z_f = 1;

Numerical integration

- Multiplying factor

```
mult_fac = (x_f - x_i) * (y_f - y_i) * (z_f - z_i);
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- Generating random numbers

```
x = x_i + (x_f - x_i) * rand(total_pnts,1);
y = y_i + (y_f - y_i) * rand(total_pnts,1);
z = z_i + (z_f - z_i) * rand(total_pnts,1);
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mult_fac = (x_f - x_i) * (y_f - y_i) * (z_f - z_i);
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x = x_i + (x_f - x_i) * rand(total_pnts,1);
y = y_i + (y_f - y_i) * rand(total_pnts,1);
z = z_i + (z_f - z_i) * rand(total_pnts,1);
```

- The integrand

```
f = x .^ 2 + y .^ 2 + z .^ 2;
```

Numerical integration

- Value of the integral

  integral_val = (mult_fac / total_pnts) * sum (f);

  disp([‘Approximated value of the integral = ’, num2str (integral_val)])

References

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