Reinforcement Learning

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Outline

1. Reinforcement Learning
2. Value and Policy Iteration
3. Deep RL

Reinforcement Learning

An agent interacts with the environment to achieve its goal

+ Agent receives an observation
+ Agent makes an action.
+ Agent receives a next observation and reward
+ And repeat …

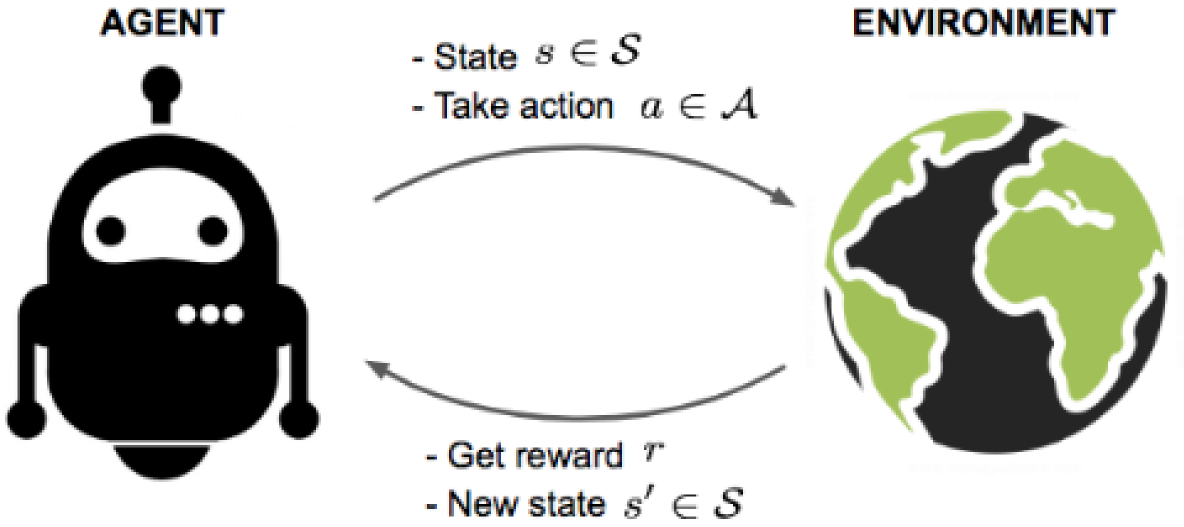


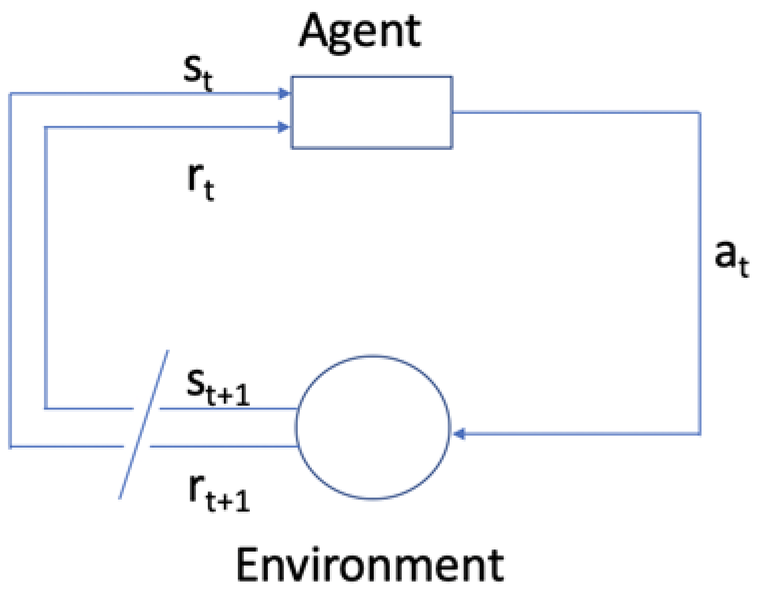
image credit to: Liliang Weng. https://lilianweng.github.io/

Reinforcement Learning: Formulation

RL uses a Markov Decision Process (MDP) definition:

$$<S, A, P, R, \gamma>$$

- S: the set of states
- A: the set of actions
- P: the transition rules, $P(s'|s, a)$
- R: the reward function, $R(s, a, s')$
- γ: Discount factor $0 < \gamma < 1$



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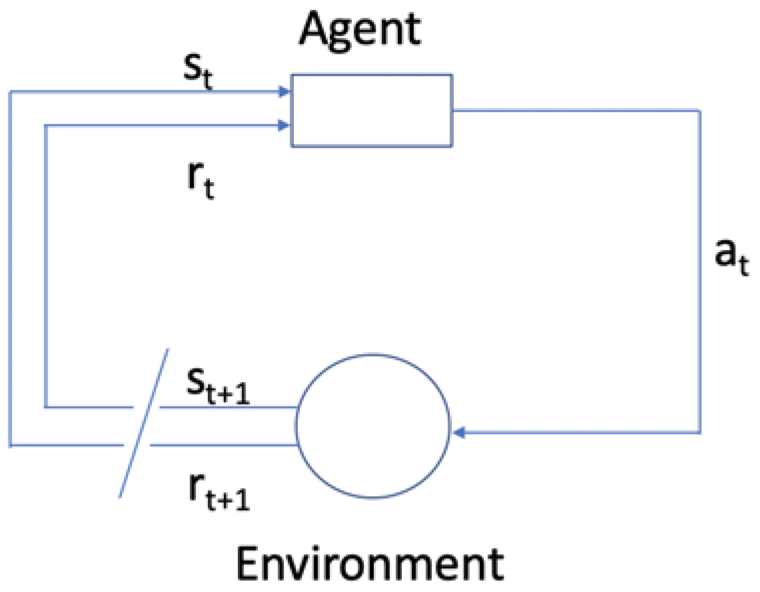
Others:

- Initial distribution: d^0
- The set of ending state: E
- Max-time horizon: H



Reinforcement Learning: Formulation

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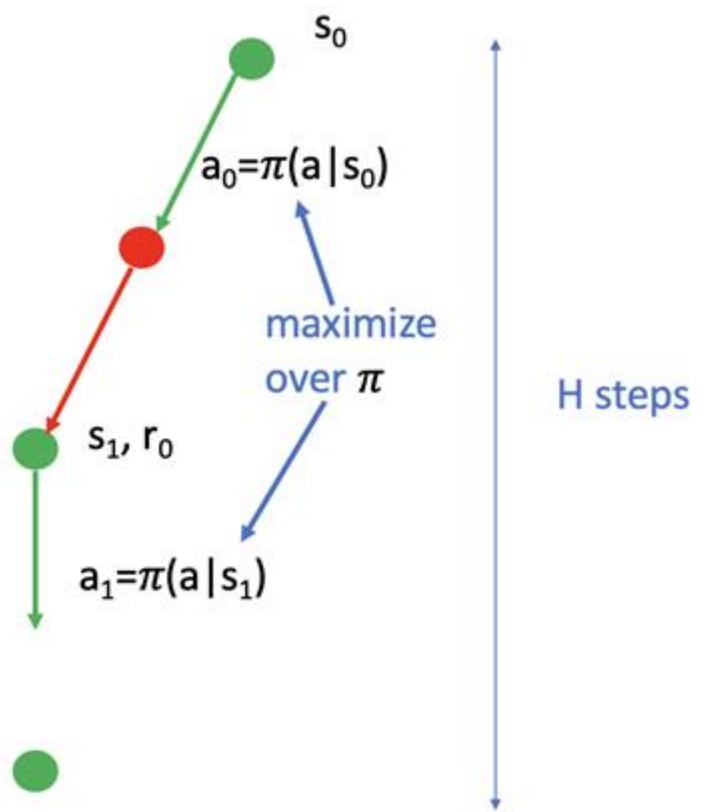
A trajectory τ is defined as with the Utility:

$$\tau = s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{H-1}, a_{H-1}, r_{H-1}, s_H(\text{terminated})$$

$$V(s_0) = U(\tau) = r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots$$

Learning a policy π: S -> A which maximizes the expected rewards:

$$\pi^* = \max_\pi E[V(s) \mid \pi]$$



Reinforcement Learning: Formulation

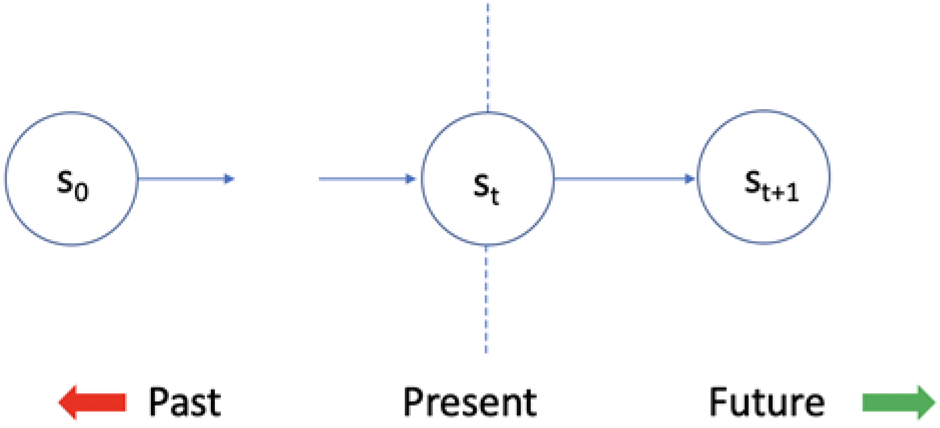
RL uses a Markov Decision Process (MDP) definition:

$$<S, A, P, R, \gamma>$$

Two assumptions:

- **MDP assumption**: $P(S_{t+1} | S_t) = P(S_{t+1} | S_t, \ldots, S_0)$

  "Future is not dependent on the Past given Present"



Reinforcement Learning: Formulation

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Two assumptions:

- **MDP assumption**: $P(S_{t+1}|S_t) = P(S_{t+1}|S_t,\ldots,S_0)$
- **Goal decomposition assumption:**

  " The goal can be achieved with a maximized reward policy"

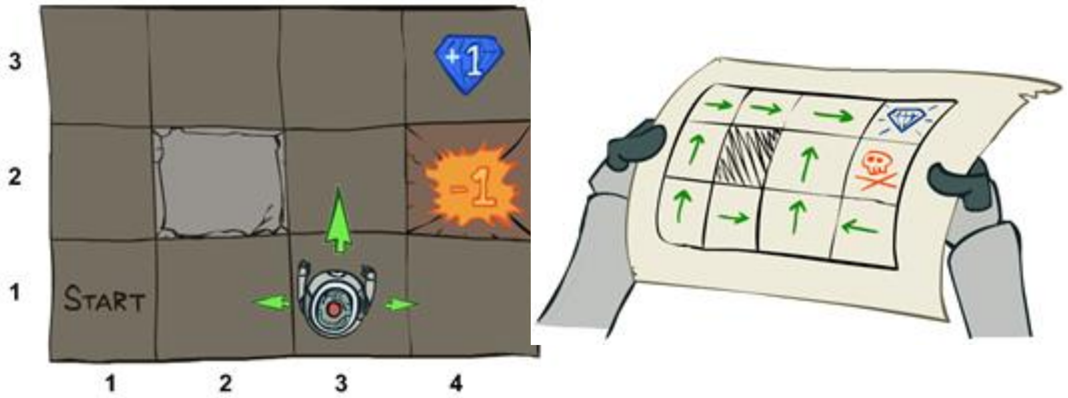

image credit to: CS188 Intro to AI at UC Berkeley (http://ai.berkeley.edu)

Reward scenario 1:
+ R(diamond) = +1
+ R(firepit) = -1
+ others = 0

Reward scenario 2:
+ R(diamond) = +1
+ R(firepit) = -1
+ others = -0.01 (Fuel Cost)

Reward scenario 3:
+ R(diamond) = +1
+ R(firepit) = -1
+ others = -0.5

Let $\gamma = 0.9$

Reinforcement Learning: Formulation

RL uses a Markov Decision Process (MDP) definition:

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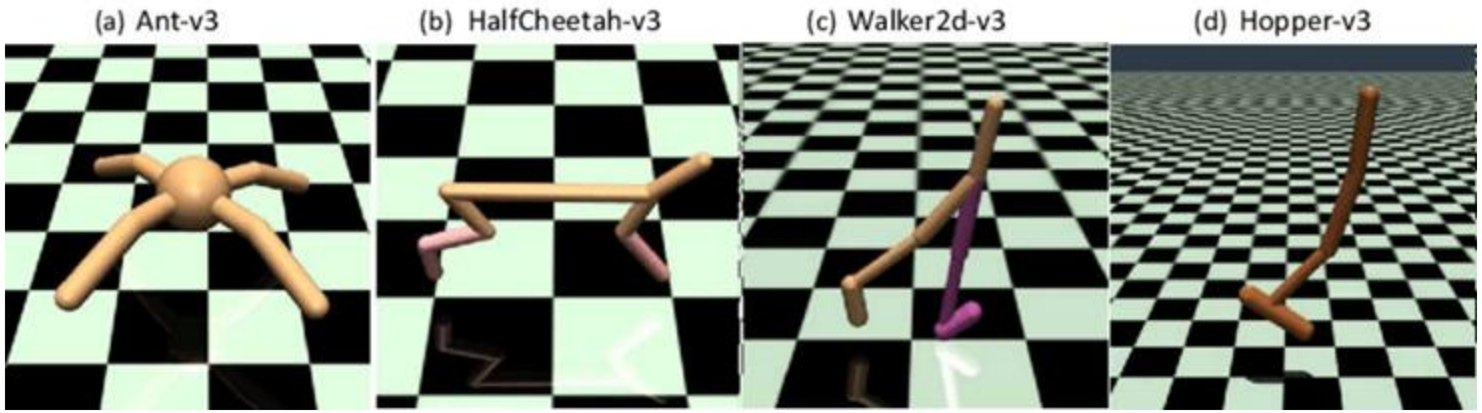
Two assumptions:

- MDP assumption
- Goal decomposition assumption:

Example problems:

- Playing games
- Robot control tasks

Atari Game - Pacman



OpenAI Gym-Mujoco

Reinforcement Learning: Formulation

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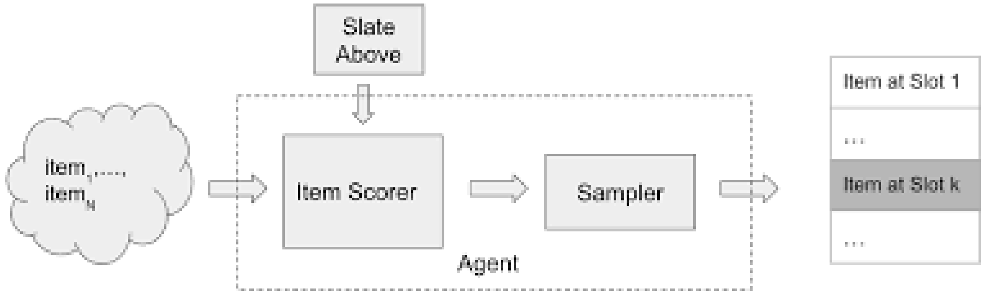
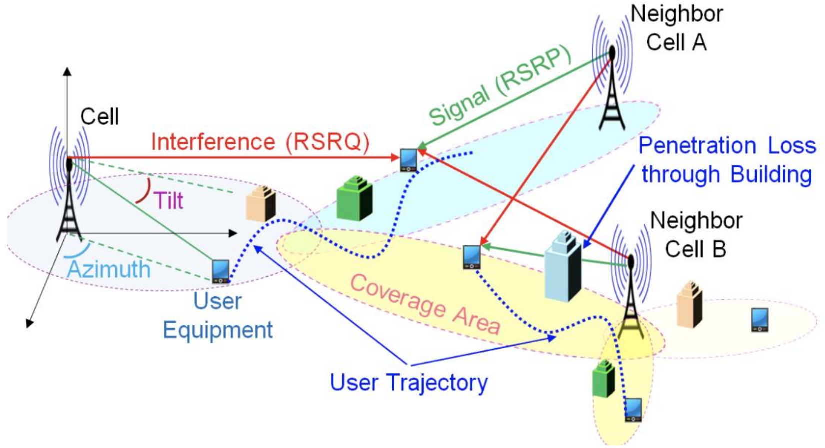
- Goal decomposition assumption:

Example problems:

- Robot control
- Playing game
- Product Recommendations
- Cell-phone coverage
- Training a “machine learning” model



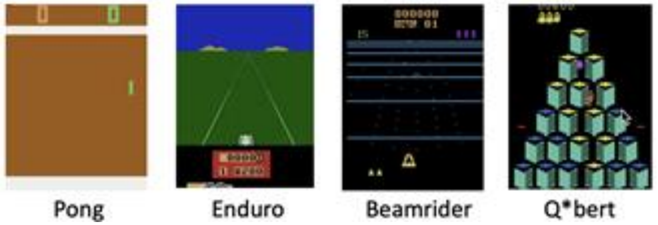
Agent for Item Ranking (image credit to: Netflix techblogs)



Cellphone control by RL
(Yongxi Tan et. al: https://arxiv.org/pdf/1802.06416.pdf)

Recent Advances in RL

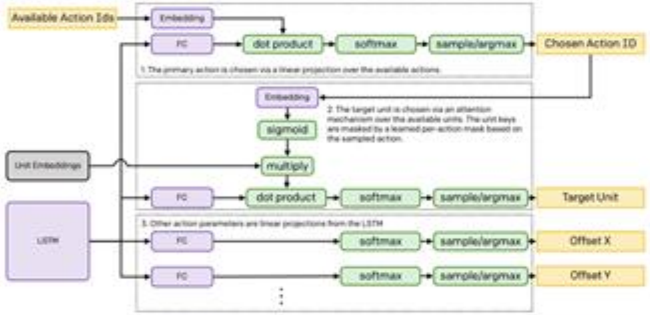
2013	Atari (DQN) [Deepmind]
2014	2D locomotion (TRPO) [Berkeley]
2015	AlphaGo [Deepmind]
2016	3D locomotion (TRPO+GAE) [Berkeley]
2016	Real Robot Manipulation (GPS) [Berkeley, Google]
2017	Dota2 (PPO) [OpenAI]
2018	DeepMimic [Berkeley]
2019	AlphaStar [Deepmind]
2019	**Rubik's Cube (PPO+DR) [OpenAI]**



Atari with Deep Q-learning (2013)

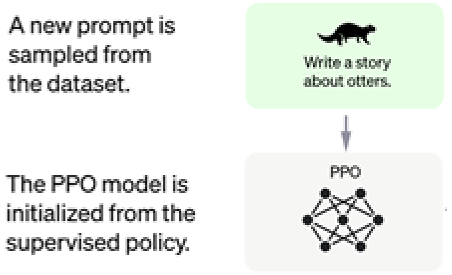
AlphaGo (2015)

OpenAI Dota2



OpenAI 5 Architecture (https://xlnwel.github.io/blog/)

(image credit to: Pieter Abeel. Foundations of Deep RL Series.)



ChatGPT=GPT3.5 +
Reinforcement Learning with Human Feedbacks

Solving RL tasks

- Exact method: Value Iteration and Policy Iteration

Solving RL: Value Iteration Methods

Exact methods: Assume |S| x |A| is tractable, and P and R is available

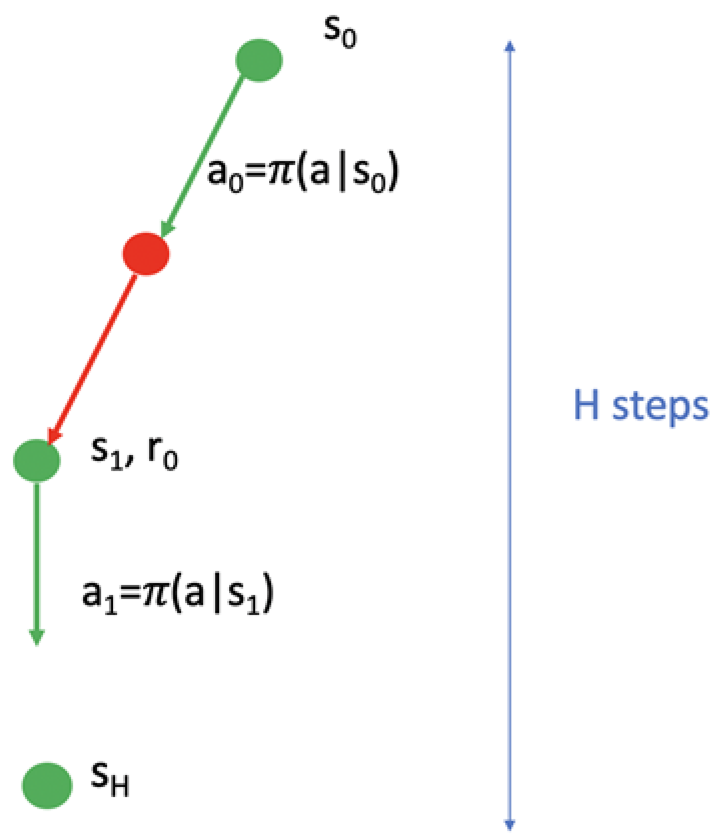
Optimal Value function is defined as:

$$V^*: S \rightarrow \mathbb{R}$$

with:

$$V^*(s) = \max_{\pi} E[U(s_0, a_0, r_0, s_1, a_1, r_1 \ldots, s_H) | \pi, s_0 = s)$$

$$U(s_0, a_0, s_1, a_1, \ldots, s_H) = r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots$$



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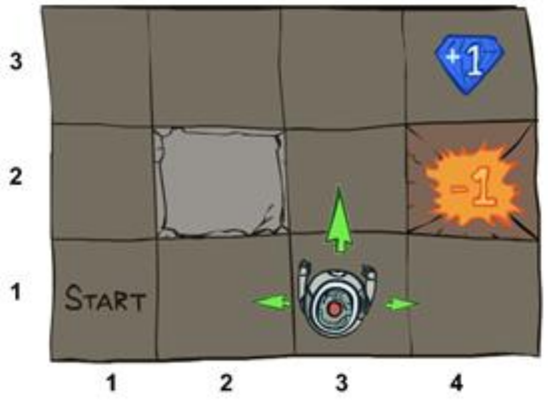
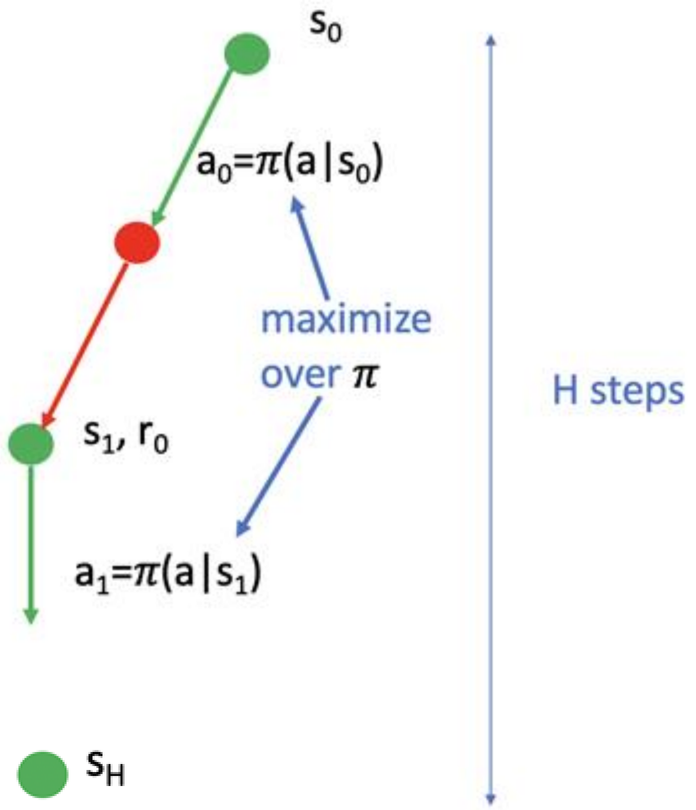


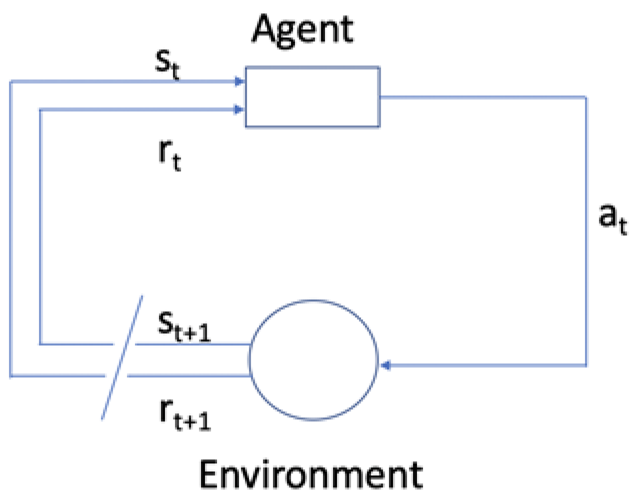


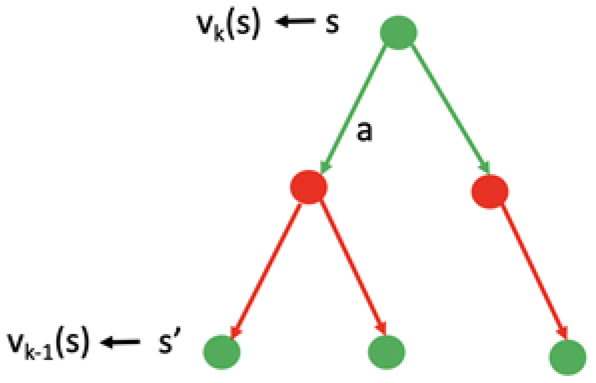
Solving RL: Value Iteration Methods

Finding V* by iterative methods:

- Start with V_0^*(s)= 0,
- Update with Bellman equation

$$V_k^*(s) \leftarrow \max_a \sum_{s'} P(s'|s,a)\left(R(s,a,s') + \gamma V_{k-1}^*(s')\right)$$





Solving RL: Value Iteration Methods

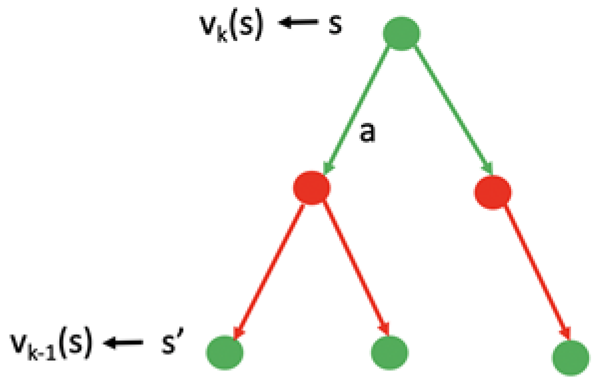
Finding V* by iterative methods:

$$V_k^*(s) \leftarrow \max_a \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma V_{k-1}^*(s')\right)$$

Convergence properties: if 0 < γ < 1, then V_k^* <- V^*

Optimal Policy: Can be extracted from V*, at any time step k

$$\pi_k^*(s) \leftarrow \arg\max_a \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma V_{k-1}^*(s')\right)$$



Solving RL problems: Policy Iteration

Policy Iteration: Similarly, we can define a value function associate with a policy

- Value Iteration steps:

$$V_k^{\pi}(s) \leftarrow \sum_{s'} P(s'|s,\pi(s))(R(s,\pi(s),s') + \gamma V_{k-1}^{\pi}(s))$$

(compare with Only Value)

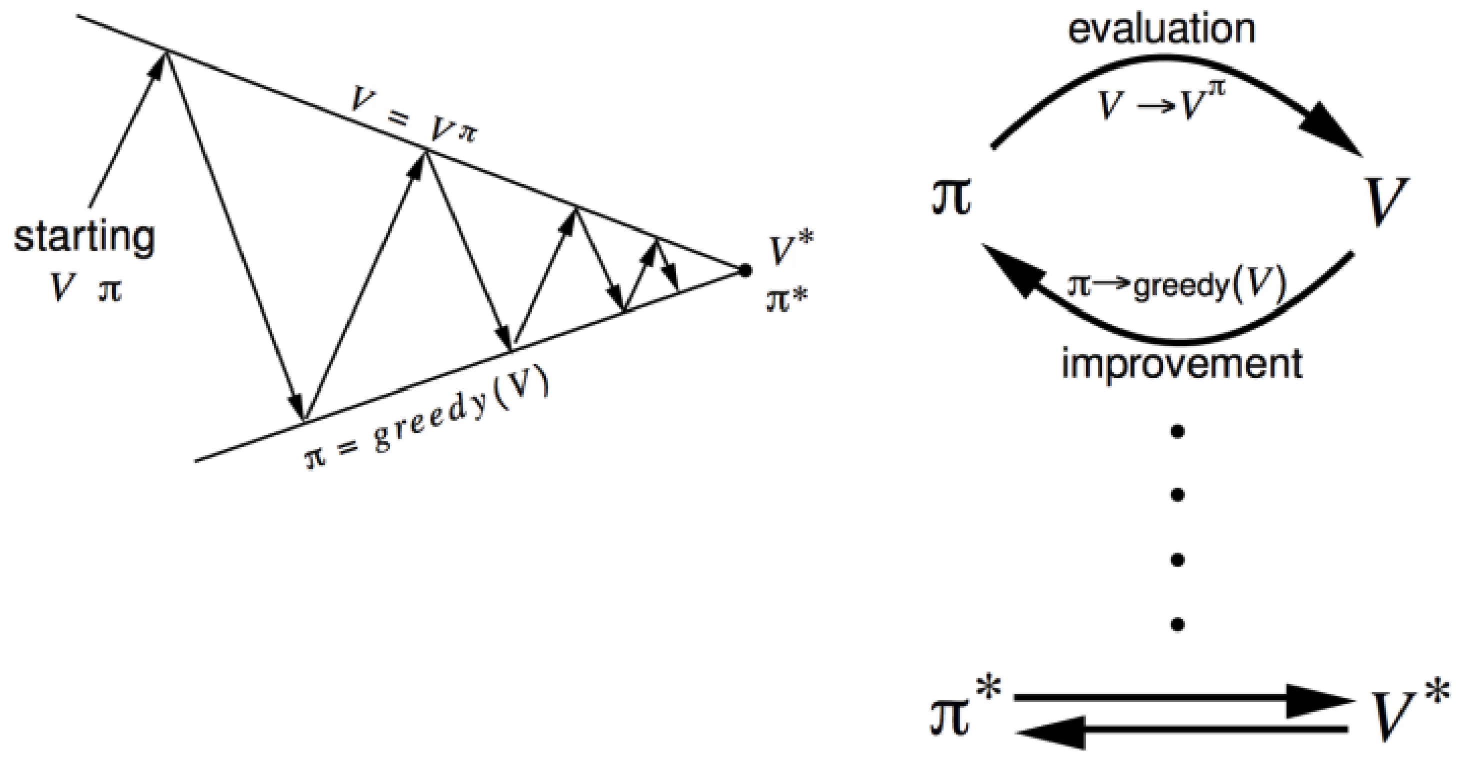
$$V_k^*(s) \leftarrow \max_a \sum_{s'} P(s'|s,a)\left(R(s,a,s') + \gamma V_{k-1}^*(s')\right)$$

- Policy Iteration steps:

$$\pi_{k+1}(s) \leftarrow \arg\max_a \sum_{s'} P(s'|s,a)\left[R(s,a,s') + \gamma V^{\pi_k}(s')\right]$$

Solving RL problems: Policy Iteration

Policy Iteration in short:



(image credit to: Deepmind. Reinforcement Learning Course Series 2021)

Deep Reinforcement Learning

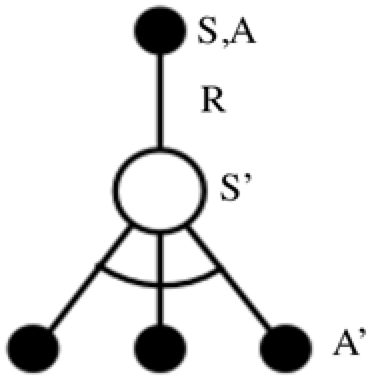
- Deep Q-Learning
- Policy Gradient
- TRPO/PPO

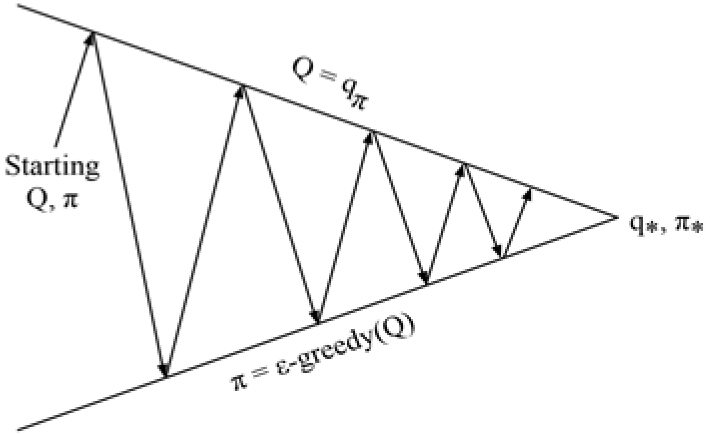
Q-learning

Q-value Iterations:

- Definitions: Q*(s, a) is the expected value starting from s do action a and acting optimally.
- Bellman equations:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*(s',a'))$$





(image credit to. Reinforcement Learning Course Series 2021)

Q-learning

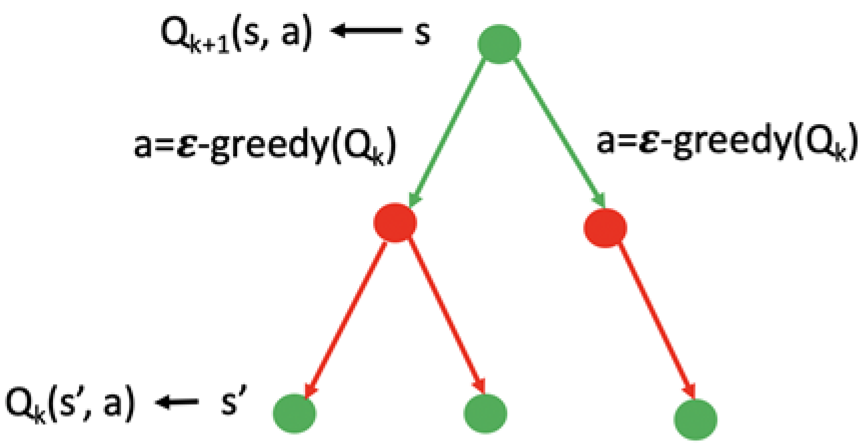
Q-value Iterations:

- Sampling based approach:

$$Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

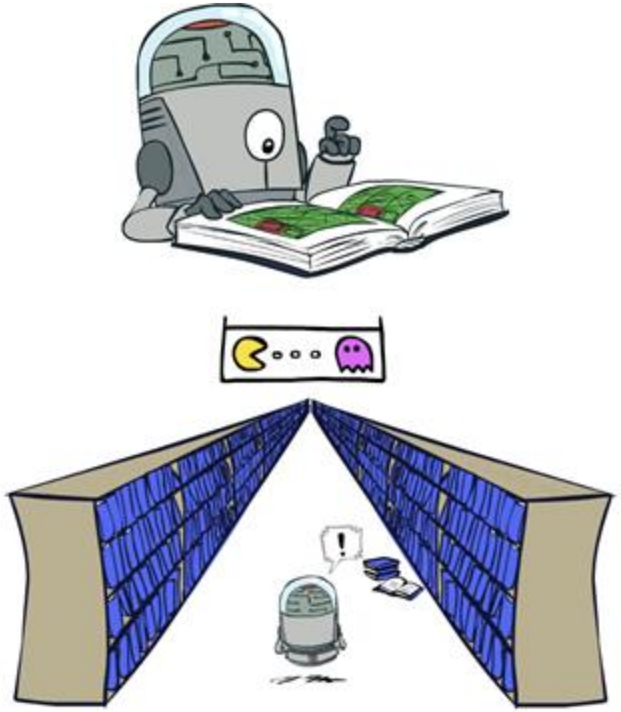
- Assume an updated at state $Q_k(s, a)$ at iteration k^{th}:
  - Sample s', R(s, a, s') from ε-greedy $Q_k(s, a)$
  - Retrieve estimate $Q_k(s', a')$
  - The target value: target(s') = R(s, a, s') + γmax $Q_k(s', a')$
  - Update with a running average value α:

$$Q_{k+1}(s,a) \leftarrow (1-\alpha)Q_k(s,a) + \alpha\,[\text{target}(s')]$$



Approximate Q-learning

- Make a fast decision
- Generalize to similar state-action pairs



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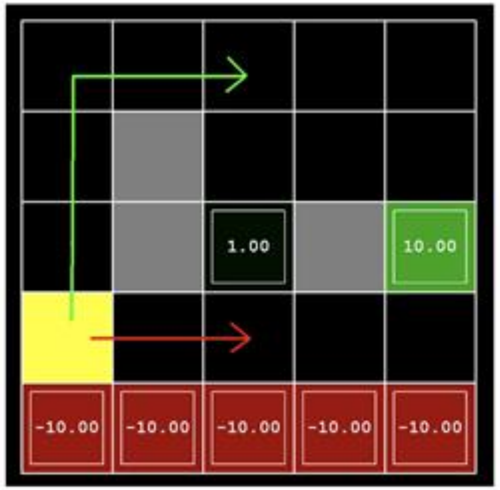
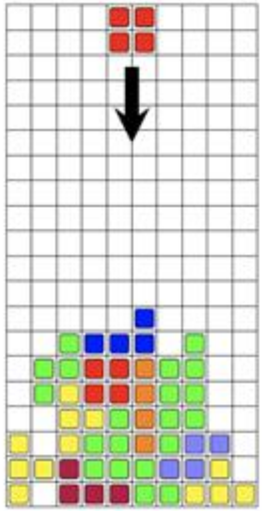


Gridworld
10^1



Tetris
10^60

Atari
10^308 (ram) 10^16992 (pixels)

(image credit to: Pieter Abeel. Foundations of Deep RL Series.)

Deep Q-learning

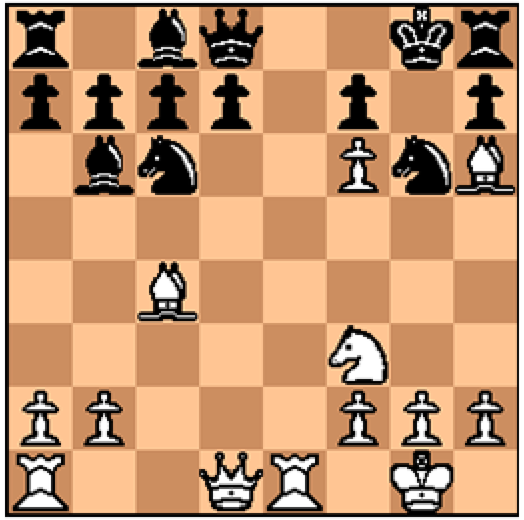
Approximate learning:

- Parameterized the Q-value by $Q_\theta(s, a)$

For examples:

- How many Queens, Knights, Rooks …
- Which are their positions …

Optimization meets Machine Learning



(image source: chessgames.com)

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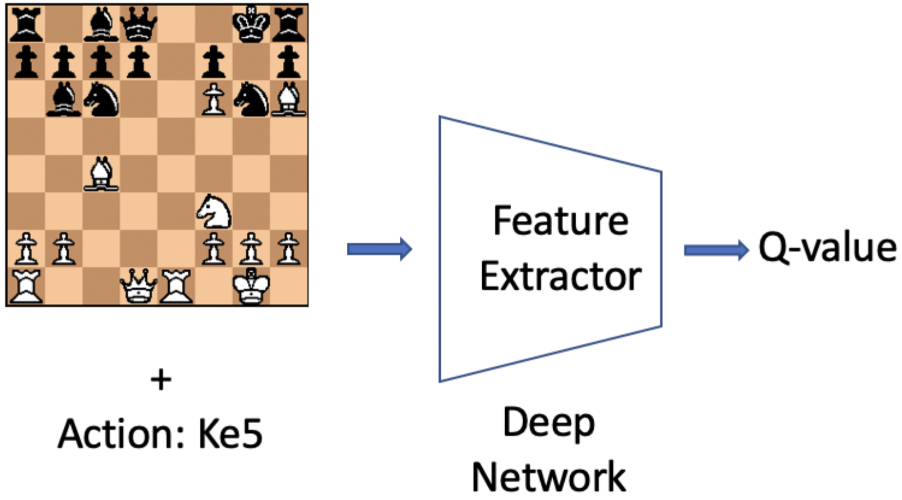
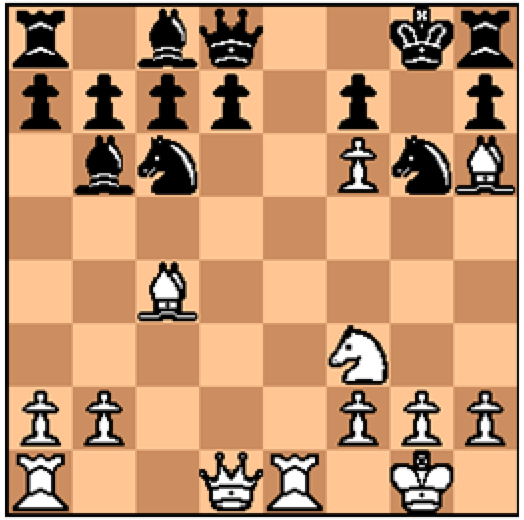




(image source: chessgames.com)

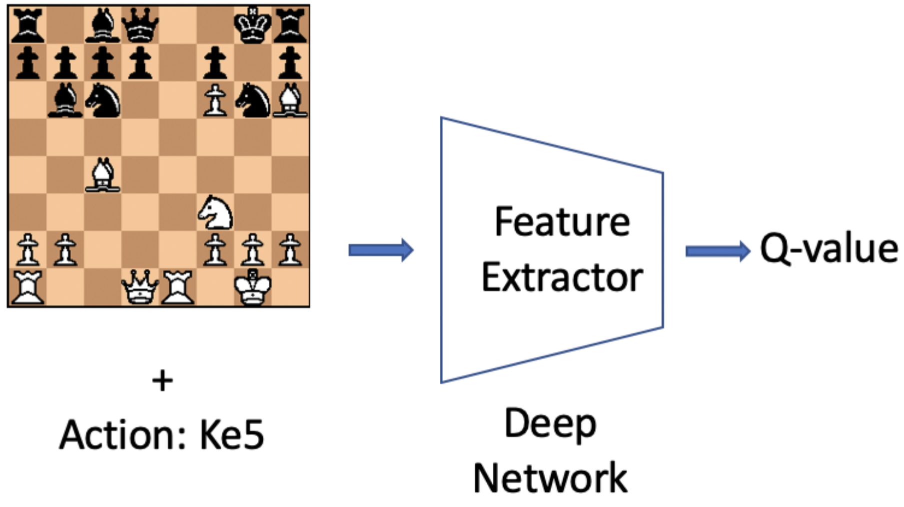
Deep Q-learning

Approximate learning:

- Parameterized the Q-value by $Q_\theta(s, a)$
- Optimization meets machine learning

What we want to achieve by θ:

- More efficient (memory, computing speeds) than store pair <s, a>
- Can make decision on key features of <s,a>
- Can have an efficient update mechanism



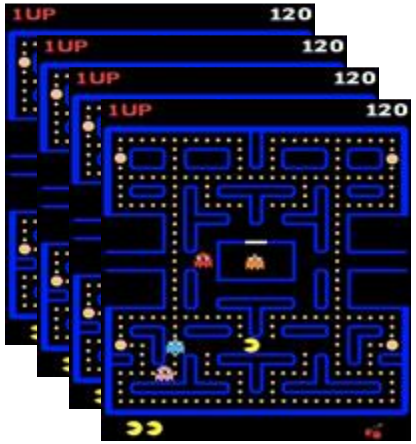
Dictionary Update: $$Q_{k+1}(s,a) \leftarrow (1-\alpha)Q_k(s,a) + \alpha\left[\text{target}(s')\right]$$

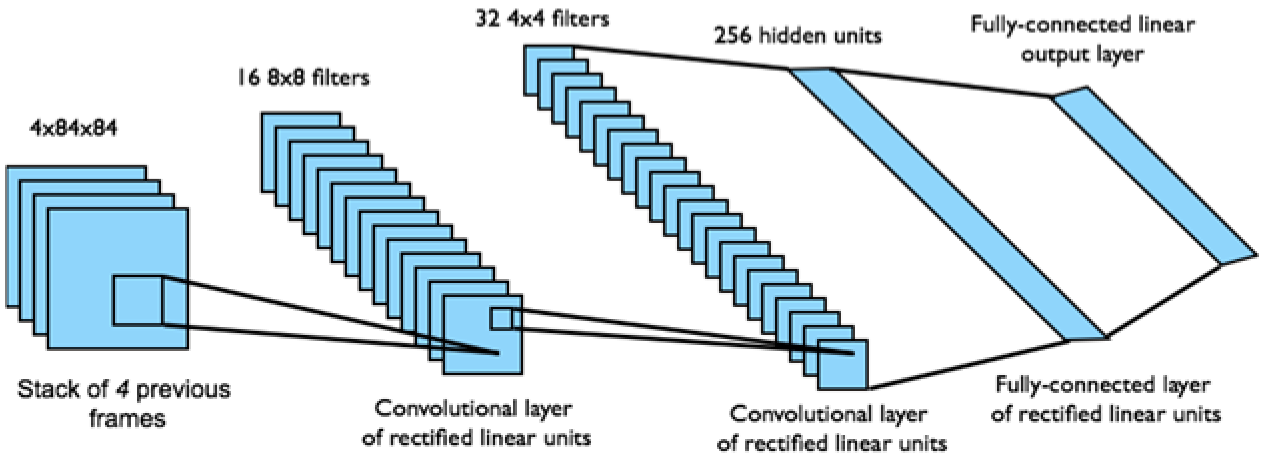
Gradient Descent Update: $$\theta_{k+1} \leftarrow \theta_k - \alpha \nabla_\theta \left[\frac{1}{2}(Q_\theta(s,a) - \text{target}(s'))^2\right]\Big|_{\theta=\theta_k}$$

Deep Q-learning

From the Nature paper [6]:

- End-to-end learning of values $Q_\theta(s, a)$ from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is $Q_\theta(s, a)$ for 18 joystick/button positions
- Reward is change in score for that step





[6] Mnih, Volodymyr, et al. "Human-level control through deep reinforcement learning." nature 518.7540 (2015): 529-533

Policy Gradient methods

Policy Gradient theorem (Sutton et al. 1999): *Given the objective function as*

$$J(\pi) = \mathbb{E}_\pi\Big[\sum_{t=0}^{\infty} \gamma^t r_t\Big]$$
$$= \mathbb{E}_\pi[Q^\pi(s_0, a_0)]$$

Then the gradient of J(π) is

$$\nabla J(\pi) = \mathbb{E}_\pi\Big[\sum_{t=0}^{\infty} \gamma^t Q^\pi(s_t, a_t) \nabla \log \pi(a_t|s_t)\Big]$$

-> The policy gradient theorem provides the closed form expression for the derivative of the objective J, which allows one to directly optimize the objective by gradient ascend

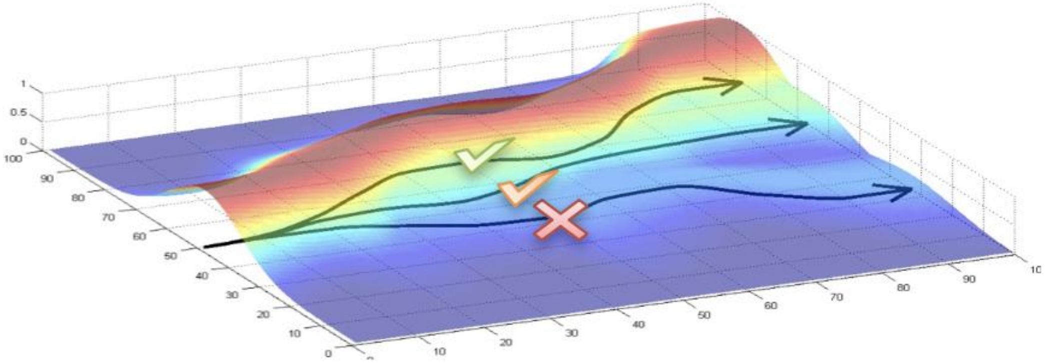
[7] Sutton, Richard S., et al. "Policy gradient methods for reinforcement learning with function approximation." *Advances in neural information processing systems* 12 (1999).

Policy Gradient methods

The Policy Gradient $$\nabla J(\pi) = \mathbb{E}_{\pi}\Big[\sum_{t=0}^{\infty} \gamma^t Q^{\pi}(s_t, a_t) \nabla \log \pi(a_t | s_t)\Big]$$

How good of the Action

Probability of taking a Path



(image credit to: Pieter Abeel. Foundations of Deep RL Series.)

Can be improve by changing to: How good of the Action compare with a Baseline

In practice, the Actor-Critic employs the Advantage function:

$$A^{\pi}(s_t, a_t) = Q^{\pi}(s_t, a_t) - V^{\pi}(s_t)$$

Trust Region Policy Optimization (TRPO)

The difference lemma [9] tells us that the difference in performance between two any policies can be calculated as

$$J(\tilde{\pi}) - J(\pi) = \sum_{s} \rho_{\tilde{\pi}}(s) \sum_{a} \tilde{\pi}(a|s) A_{\pi}(s, a)$$

with

$$\rho_{\pi}(s) := \sum_{t=0}^{\infty} \gamma^t \mathbb{P}_{\pi}(s_t = s)$$

is called the (improper) marginal state distribution.

=> Given π, we can maximize J($\tilde{\pi}$) by maximizing

$$J(\tilde{\pi}) = J(\pi) + \sum_{s} \rho_{\tilde{\pi}}(s) \sum_{a} \tilde{\pi}(a|s) A_{\pi}(s, a)$$

How can we optimize this?

[8]Schulman, John, et al. "Trust region policy optimization." International conference on machine learning. PMLR, 2015.
[9] Agarwal, Alekh, et al. "Reinforcement learning: Theory and algorithms." CS Dept., UW Seattle, Seattle, WA, USA, Tech. Rep 32 (2019).

Trust Region Policy Optimization (cont)

$$J(\tilde{\pi}) = J(\pi) + \sum_s \rho_{\tilde{\pi}}(s) \sum_a \tilde{\pi}(a|s) A_\pi(s,a)$$

Difficult parts, because they rely on the samples from the distribution parameterized by the variable being optimized.

just replace $\tilde{\pi}$ with pi and wish for luck

replace with importance sampling

$$J(\tilde{\pi}) \approx L(\tilde{\pi}) := J(\pi) + \sum_s \rho_\pi(s) \mathbb{E}_{a\sim\pi} \frac{\tilde{\pi}(a|s)}{\pi(a|s)} A_\pi(s,a)$$

How good is this approximation?

Trust Region Policy Optimization (cont)

Monotonic improvement guarantee [8]:

$$J(\tilde{\pi}) \geqslant L(\tilde{\pi}) - \frac{4\gamma \max |A_\pi(s,a)|}{(1-\gamma)^2} \mathrm{D}_{KL}^{\max}(\pi, \tilde{\pi})$$

=> As the *distance* between π and $\tilde{\pi}$ *decreases*, the surrogate loss L($\tilde{\pi}$) becomes an increasingly accurate estimate of the actual performance J($\tilde{\pi}$)

=> Maximize the RHS of the above inequality, and transform it into a constrained optimization problem:

$$\underset{\theta}{\text{maximize}}\, L_{\theta_{\text{old}}}(\theta)$$
$$\text{subject to } \overline{D}_{\text{KL}}^{\rho_{\theta_{\text{old}}}}(\theta_{\text{old}}, \theta) \leq \delta.$$

This is TRPO's optimization objective

[8] Schulman, John, et al. "Trust region policy optimization." International conference on machine learning. PMLR, 2015.

Proximal Policy Optimization (PPO)

Proximal Policy optimization (PPO) [10] is simpler to implement than TRPO, but also has a trust-region-like mechanism.

TRPO objective

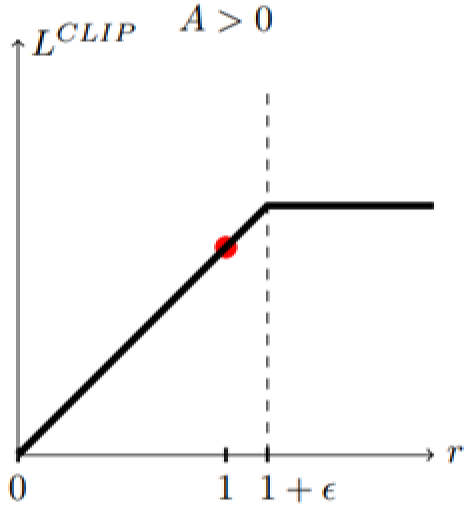
$$\underset{\theta}{\text{maximize}} \quad \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)}\hat{A}_t\right]$$

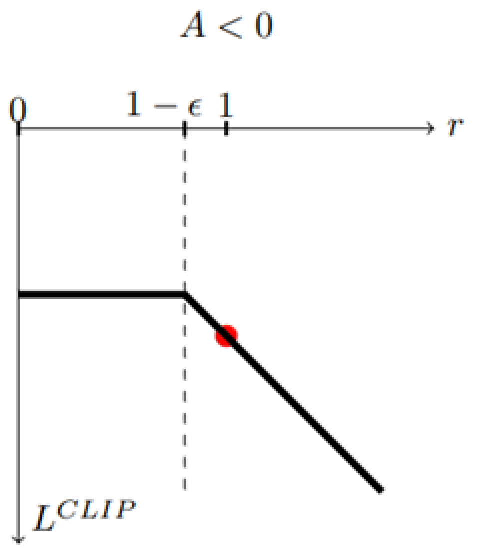
$$\text{subject to} \quad \hat{\mathbb{E}}_t[\mathrm{KL}[\pi_{\theta_{\text{old}}}(\cdot \mid s_t), \pi_\theta(\cdot \mid s_t)]] \leq \delta.$$

PPO objective

importance sampling ratio

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\left[\min(r_t(\theta)\hat{A}_t, \mathrm{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t)\right]$$





[10] John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms. 2017

PPO - Implementation that matters

To achieve the best performance of PPO, several small implementation details that might be overlooked are important to consider - without these, PPO can perform poorly. For example:

+ Value function clipping
+ Reward scaling
+ Orthogonal initialization

Many follow-up works of PPO use these choices as standard implementation.

See [11] for more information.

[11]Engstrom, Logan, et al. "Implementation matters in deep policy gradients: A case study on ppo and trpo." arXiv preprint arXiv:2005.12729 (2020).