



HA NOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF INFORMATION AND COMMUNICATION TECHNOLOGY

DATA STRUCTURES AND ALGORITHMS

Lists

Content

- Definition
- Abstract Data Type
- Array
- Linked lists
- Stacks
- Queues

Definition

- Data structures store objects in a linear structure
- Specify the first object of the list
- Each object: has a unique successor



Abstract Data Type

- Notations:
 - L : a list
 - x : an object (element) of the list
 - p : position type
 - $\text{END}(L)$: return the position after the position of the last element of L

Abstract Data Type

- Operations
 - $\text{Insert}(x, p, L)$: insert an element x at the position p in L
 - $\text{Locate}(x, L)$: return the position of x in L
 - $\text{Retrieve}(p, L)$: return the element at the position p in L
 - $\text{Delete}(p, L)$: remove the element at the position p in L
 - $\text{Next}(p, L)$: return the next position of p in L
 - $\text{Prev}(p, L)$: return the previous position of p in L
 - $\text{MakeNull}(L)$: make L null
 - $\text{First}(L)$: return the first position of L

Array

- Objects are allocated continuously
- Access objects via indices
- Insertions and removals need to consolidate elements

Array

- Declaration
 - **int a[100000];** // array
 - **int n;** // number of active
// elements (started from 0)

```
void insert(int x, int p) {  
    for(int j = n-1; j >= p; j--)  
        a[j+1] = a[j];  
    a[p] = x; n = n + 1;  
}  
  
void delete(int p) {  
    for(int j = p+1; j <= n-1; j++)  
        a[j-1] = a[j];  
    n = n - 1;  
}  
  
int retrieve(int p) {  
    return a[p];  
}
```

Array

- Declaration
 - **int a[100000];** // array
 - **int n;** // number of active
//elements (started from 0)

```
int locate(int x) { // vị trí đầu tiên của  
x trong danh sách  
    for(int j= 0; j <= n-1; j++)  
        if(a[j] == x) return j;  
    return -1;  
}  
void makeNull() {  
    n = 0;  
}  
int next(int p) {  
    if(0 <= p && p < n-1) return p+1;  
    return -1;  
}  
int prev(int p) {  
    if(p > 0 && p <= n-1) return p-1;  
    return -1;  
}
```


Pointers and linked lists

- Pointers: address of variables, objects in the memory
- **int* p**: **p** is a pointer to an **int** variable. Value of **p** specifies the address of the variable it points
- **int a; int* p = &a;**// p points to a
- Structure

```
typedef struct  TNode{  
    int a;  
    double b;  
    char* s;  
}TNode;
```

- **TNode* q**: **q** is a pointer to a variable of type **TNode**

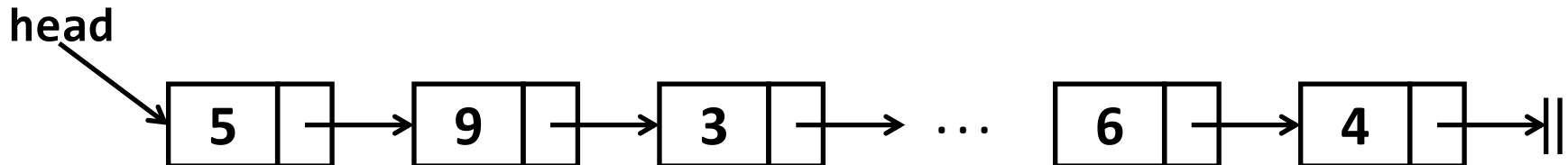
Pointers and linked lists

- `q->a`: access the the field `a` of the object
- `q = (TNode*)malloc(sizeof(TNode))`:
allocate an object pointed by `q`

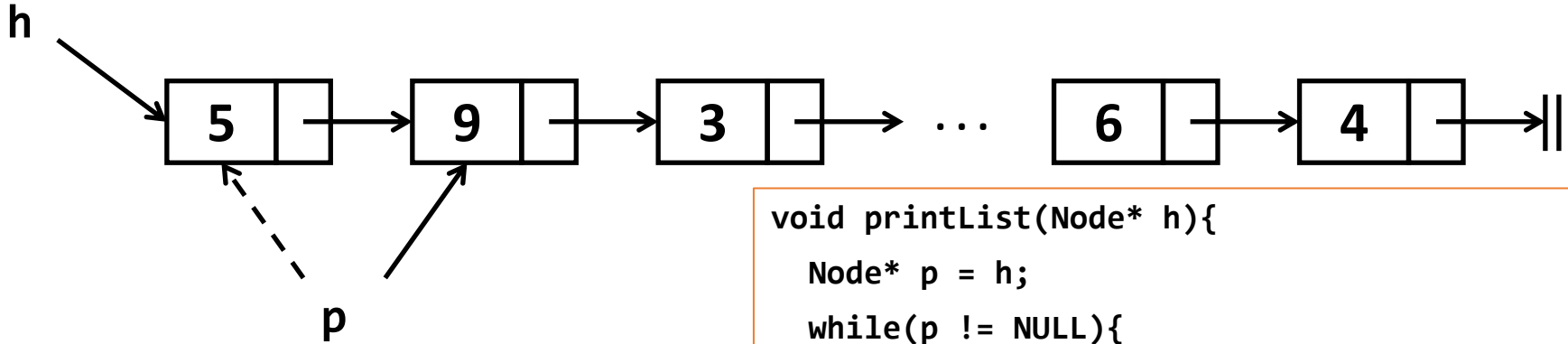
```
int main() {  
    int a = 123;  
    int* p;  
    p = &a;  
    *p = 456;  
    printf("a = %d\n",a);  
}
```

Single linked lists

```
struct Node{  
    int value;  
    Node* next; // pointer to the successor  
};  
Node* head; // pointer to the first element of the list
```



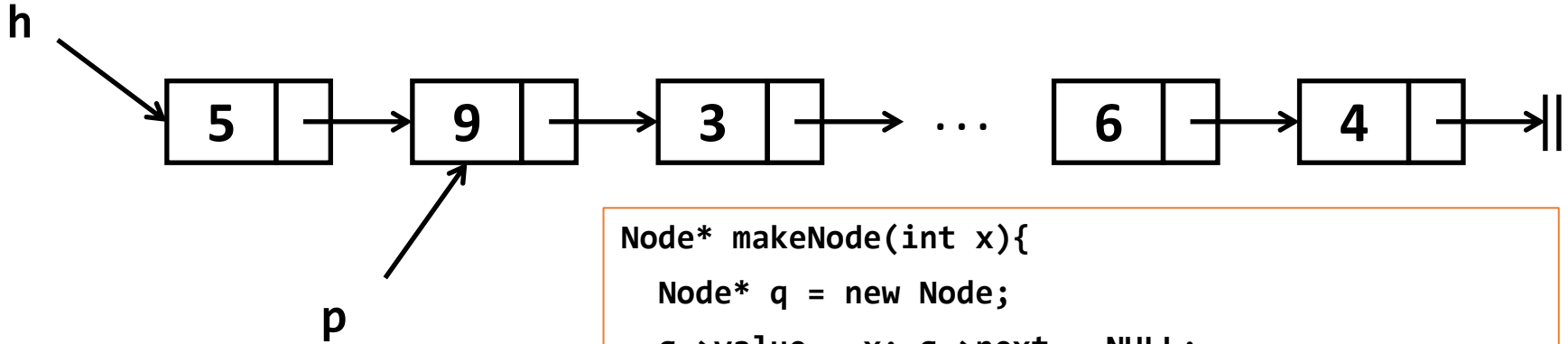
Single linked lists



```
void printList(Node* h){
    Node* p = h;
    while(p != NULL){
        printf("%d ",p->value);
        p = p->next;
    }
}

Node* findLast(Node* h){
    Node* p = h;
    while(p != NULL){
        if(p->next == NULL) return p;
        p = p->next;
    }
    return NULL;
}
```

Single linked lists



```
Node* makeNode(int x){
```

```
    Node* q = new Node;
```

```
    q->value = x; q->next = NULL;
```

```
    return q;
```

```
}
```

```
Node* insertAfter(Node* h, Node* p, int x){
```

```
    if(p == NULL) return h;
```

```
    Node* q = makeNode(x);
```

```
    if(h == NULL) return q;
```

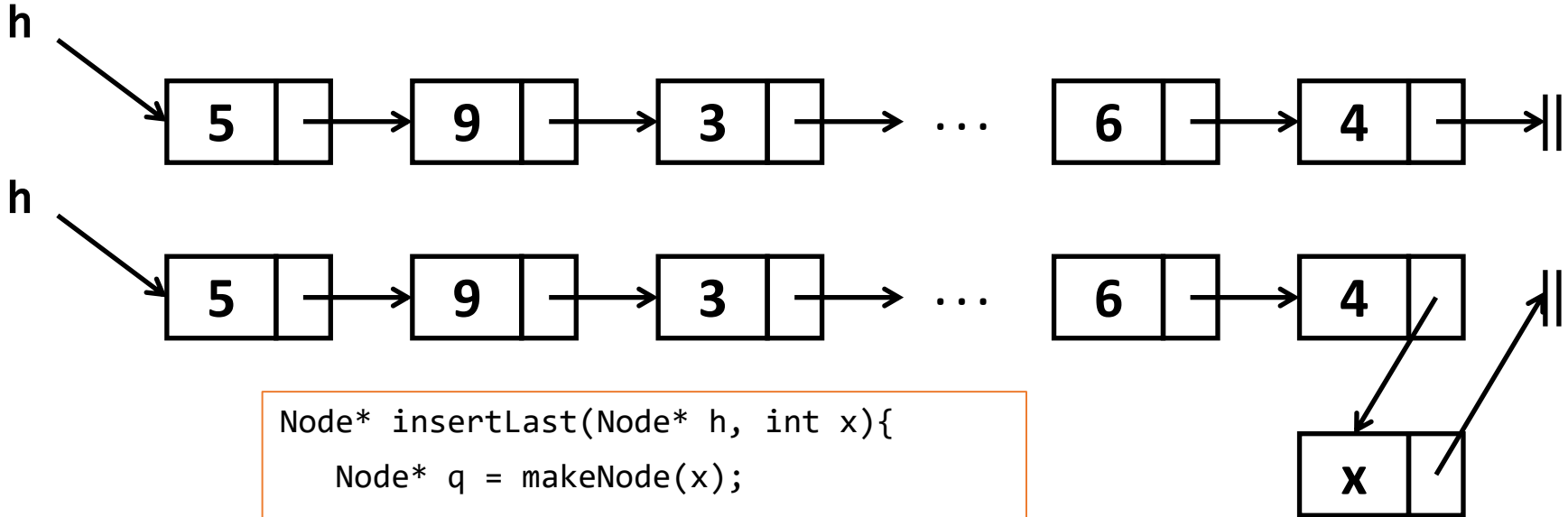
```
    q->next = p->next;
```

```
    p->next = q;
```

```
    return h;
```

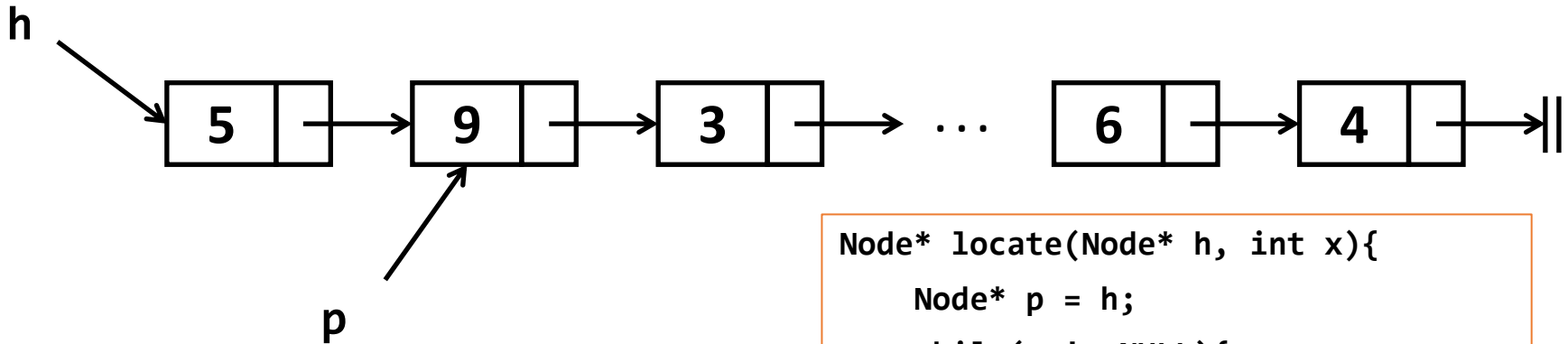
```
}
```

Single linked lists



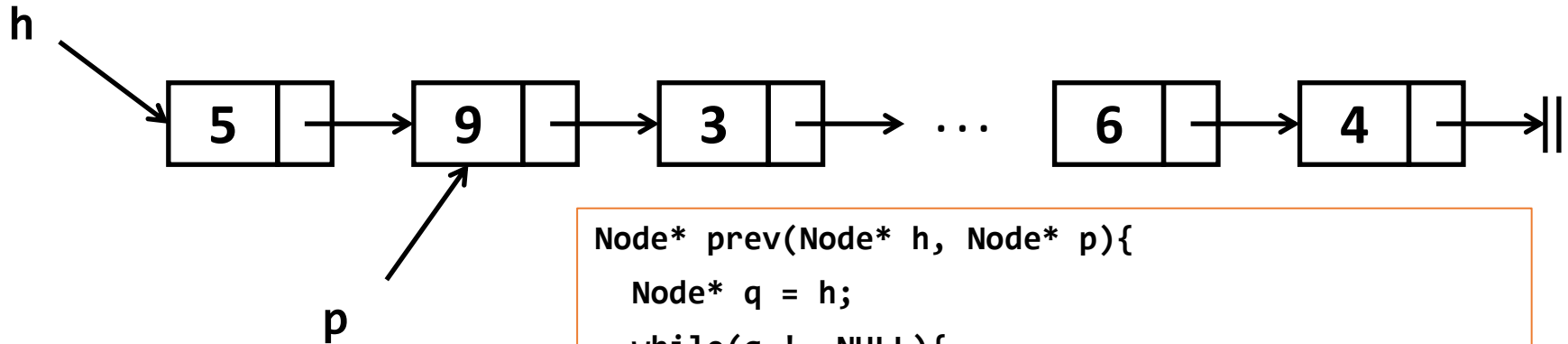
```
Node* insertLast(Node* h, int x){  
    Node* q = makeNode(x);  
    if(h == NULL)  
        return q;  
    Node* p = h;  
    while(p->next != NULL)  
        p = p->next;  
    p->next = q;  
    return h;  
}
```

Single linked lists



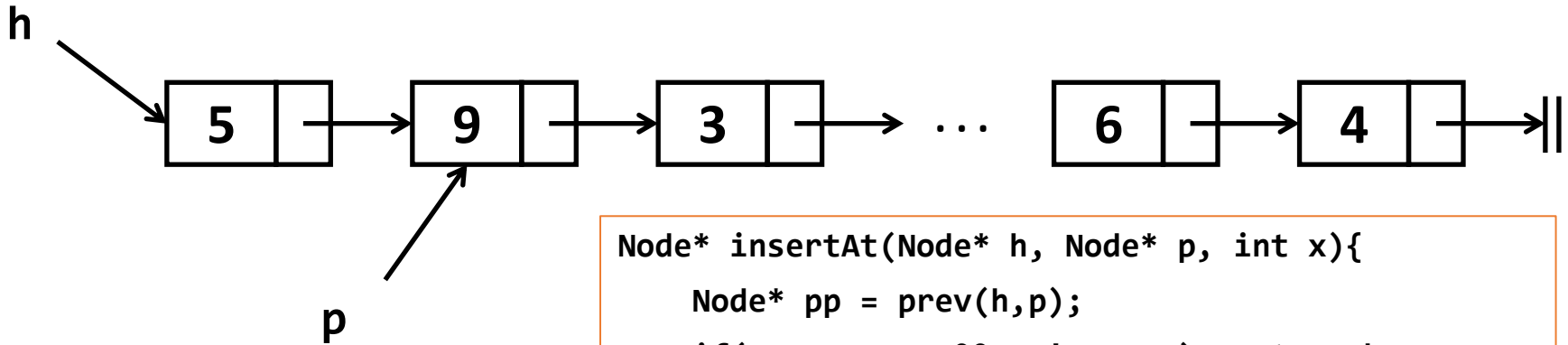
```
Node* locate(Node* h, int x){  
    Node* p = h;  
    while(p != NULL){  
        if(p->value == x) return p;  
        p = p->next;  
    }  
    return NULL;  
}
```

Single linked lists



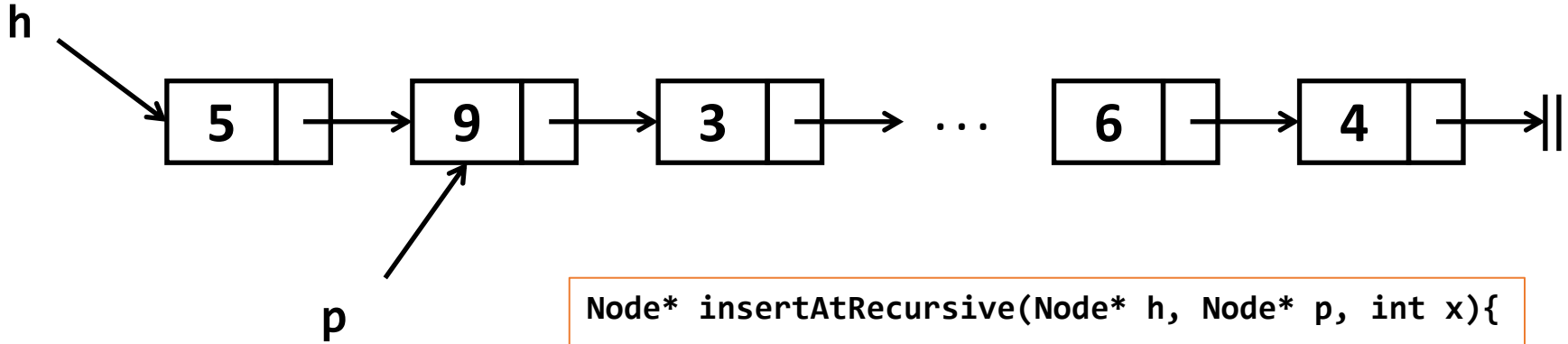
```
Node* prev(Node* h, Node* p){  
    Node* q = h;  
    while(q != NULL){  
        if(q->next == p) return q;  
        q = q->next;  
    }  
    return NULL;  
}
```


Single linked lists



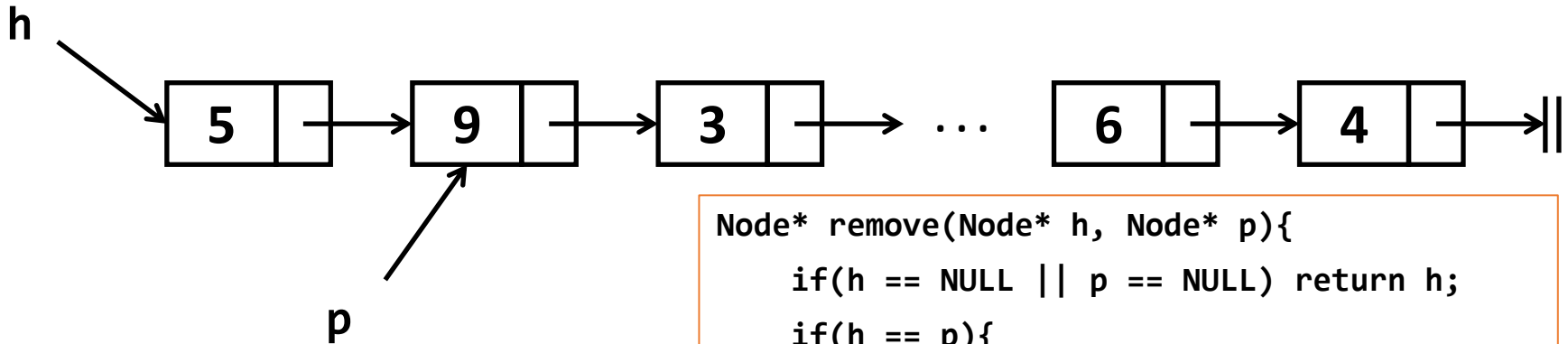
```
Node* insertAt(Node* h, Node* p, int x){  
    Node* pp = prev(h,p);  
    if(pp == NULL && p != NULL) return h;  
    Node* q = new Node;  
    q->value = x; q->next = NULL;  
    if(pp == NULL){  
        if(h == NULL)  
            return q;  
        q->next = h;  
        return q;  
    }  
    q->next = p;    pp->next = q;  
    return h;  
}
```

Single linked lists



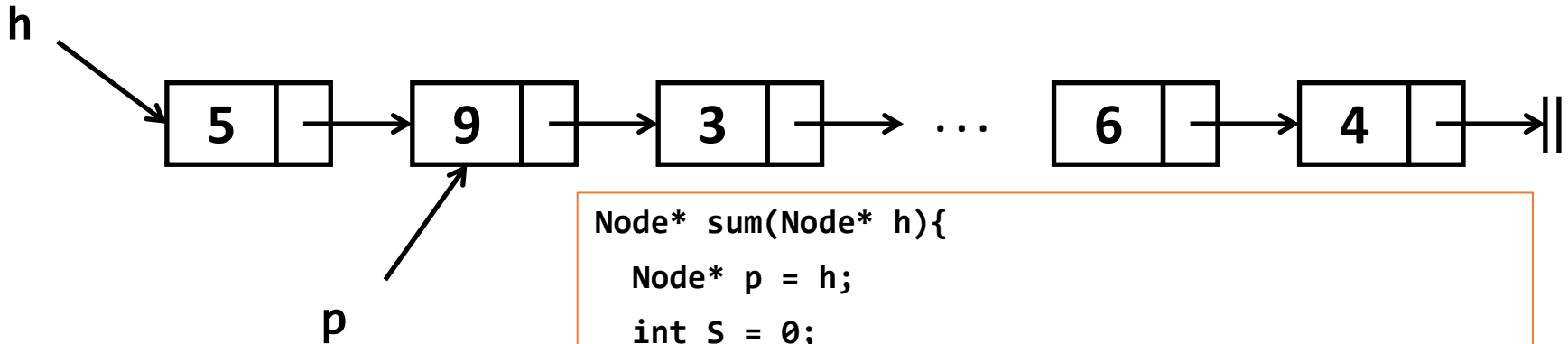
```
Node* insertAtRecursive(Node* h, Node* p, int x){  
    if(p == NULL) return h;  
    if(h == NULL || p == h){  
        return makeNode(x);  
    }else{  
        h->next = insertAtRecursive(h->next,p,x);  
        return h;  
    }  
}
```

Single linked lists



```
Node* remove(Node* h, Node* p){  
    if(h == NULL || p == NULL) return h;  
    if(h == p){  
        h = h->next;  
        delete p;  
        return h;  
    }else{  
        h->next = remove(h->next,p);  
        return h;  
    }  
}
```

Single linked lists

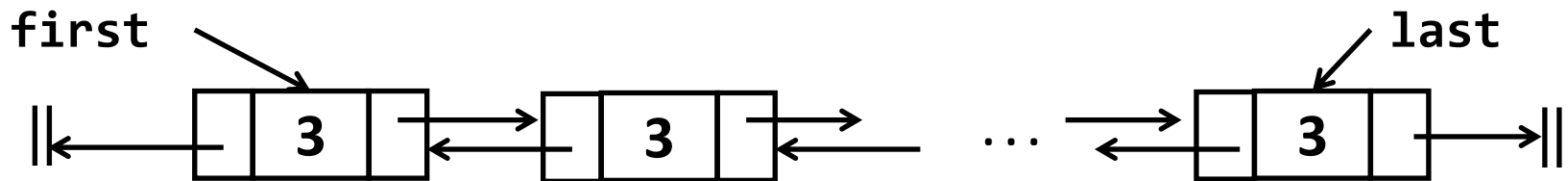
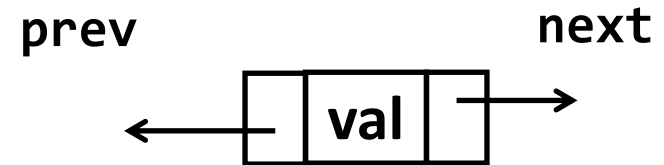


```
Node* sum(Node* h){
    Node* p = h;
    int S = 0;
    while(p != NULL) {
        S = S + p->value;
        p = p->next;
    }
    return S;
}

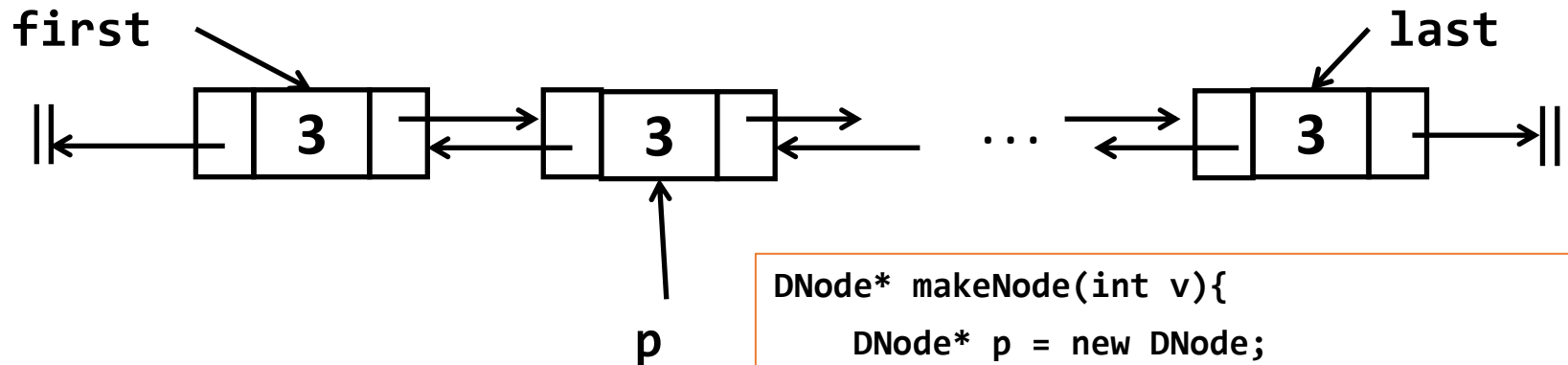
int sumRecursive(Node* h){
    if(h == NULL) return 0;
    return h->value + sumRecursive(h->next);
}
```

Doubly linked lists

```
struct DNode{  
    int val;  
    DNode* prev;// pointer to the predecessor  
    DNode* next;// pointer to the successor  
};  
  
DNode* first;// pointer to the first element  
DNode* last;// pointer to the last element
```

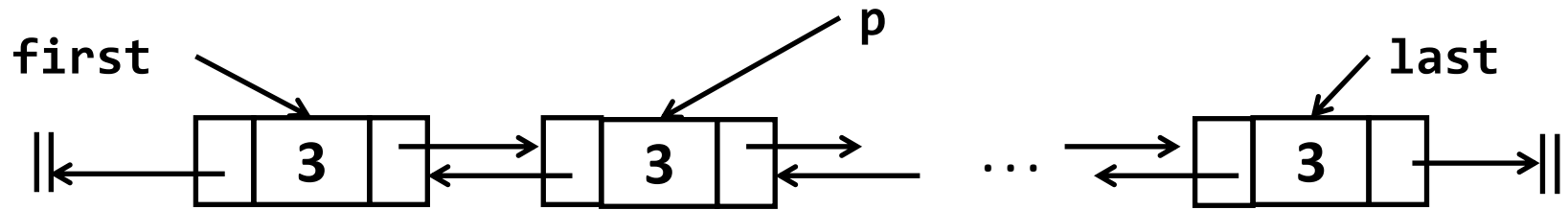


Doubly linked lists



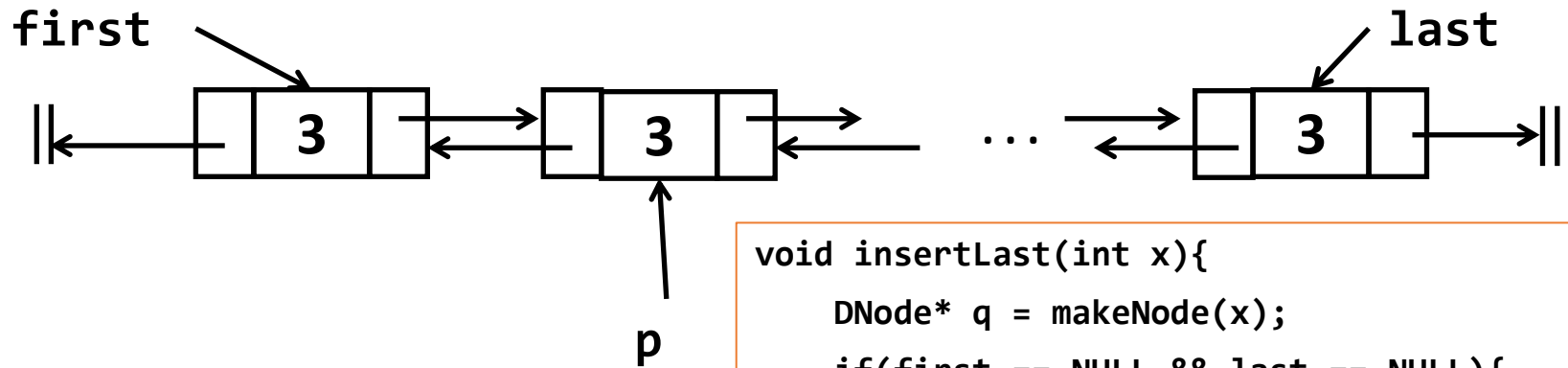
```
DNode* makeNode(int v){  
    DNode* p = new DNode;  
    p->val = v;  
    p->next = NULL;  
    p->prev = NULL;  
    return p;  
}
```

Doubly linked lists



```
void remove(DNode* p) {  
    if(p == NULL) return;  
    if(first == last && p == first){  
        first = NULL; last = NULL; delete p;  
    }  
    if(p == first){  
        first = p->next; first->prev = NULL;  
        delete p; return;  
    }  
    if(p == last){  
        last = p->prev; last->next = NULL;  
        delete p; return;  
    }  
    p->prev->next = p->next;  p->next->prev = p->prev;  delete p;  
}
```

Doubly linked lists



```
void insertLast(int x){  
    DNode* q = makeNode(x);  
    if(first == NULL && last == NULL){  
        first = q;  
        last = q;  
        return;  
    }  
    q->prev = last;  
    last->next = q;  
    last = q;  
}
```


C++ library

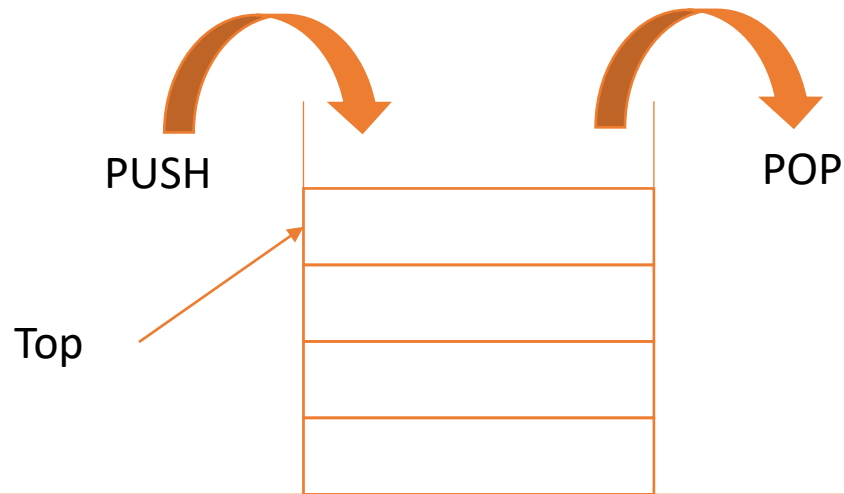
- list:
<http://www.cplusplus.com/reference/list/list/>
- operators
 - push_front()
 - push_back()
 - pop_front()
 - pop_back()
 - size()
 - clear()
 - erase()

```
#include <list>
#include <stdio.h>
using namespace std;

int main(){
    list<int> L;
    for(int i = 1; i <= 10; i++)
        L.push_back(i);
    list<int>::iterator it;
    for(it = L.begin(); it != L.end();
        it++){
        int x = *it;
        printf("%d ",x);
    }
}
```

Stacks

- Store elements in a linear structure
- Insertions and removals are performed at the top of the stack (Last In First Out – LIFO principle)
- Operations
 - Push(x, S): push an element x into the stack
 - Pop(S): remove an element out of the stack
 - Top(S): Access to the element at the top position of the stack
 - Empty(S): return true if the stack is empty



Stacks

- Application: Check the correctness of a parenthesis
 - `[({})]()`: true
 - `([] {})`: false

Stacks

- Application: Check the correctness of a parenthesis
 - $[(\{\})]()$: true
 - $([\} \{])$: false
- Algorithm:
 - Initialize a stack S
 - Scan the parenthesis from left to right
 - Meet an opening parenthesis, then push it into S
 - Meet a closing parenthesis **A**,
 - If S is empty, then return FALSE
 - Otherwise, remove an opening parenthesis **B** out of S, if **B** and **A** do not match to each other, then return FALSE
 - Termination, if S is not empty, then return FALSE, otherwise, return TRUE

Stacks

- Parenthesis checking

```
#include <stack>
#include <stdio.h>

using namespace std;

bool match(char a, char b){
    if(a == '(' && b == ')') return true;
    if(a == '{' && b == '}') return true;
    if(a == '[' && b == ']') return true;
    return false;
}
```

Stacks

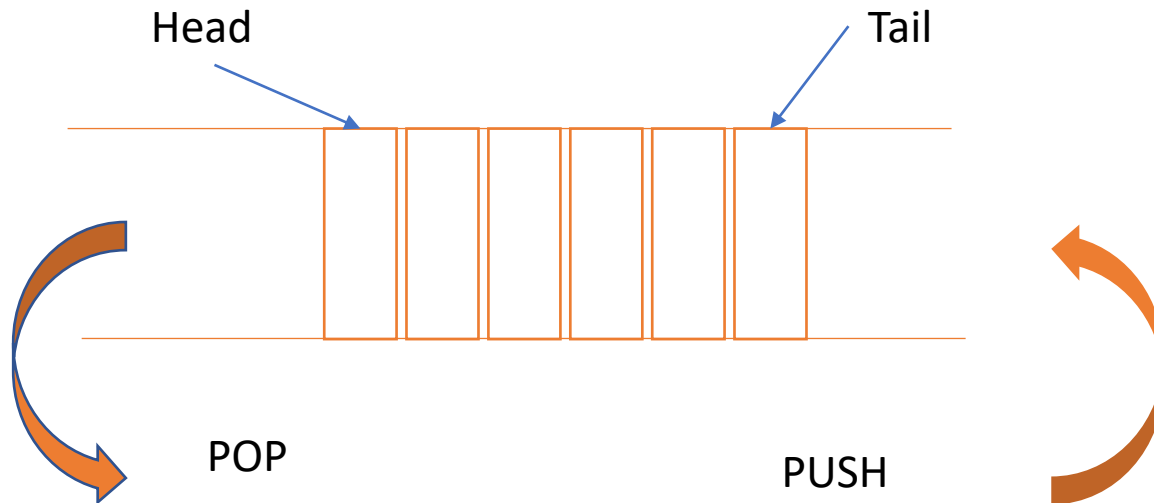
- Parenthesis checking

```
bool solve(char* x, int n){
    stack<char> S;
    for(int i = 0; i <= n-1; i++){
        if(x[i] == '[' || x[i] == '(' || x[i] == '{'){
            S.push(x[i]);
        }else{
            if(S.empty()){
                return false;
            }else{
                char c = S.top(); S.pop();
                if(!match(c,x[i])) return false;
            }
        }
    }
    return S.empty();
}

int main() {
    bool ok = solve("[({})]()",8);
}
```

Queues

- Lists with **head** and **tail**
- Insertions are performed at **tail**
- Removals are performed at **head** (First In First Out – FIFO)
- Operations
 - Enqueue(x, Q): insert an element x to the queue
 - Dequeue(Q): remove an element out of the queue
 - Empty(Q): return true if the queue is empty



Queues

- Application: Water jug
 - Given two jugs of capacities a and b (litres). Find to get c litres (a, b, c are positive integers) with following operations
 - Empty a jug
 - Fill a jug
 - Pour water from a jug to another jug

Queues

- Water jug: $a = 6$, $b = 8$, $c = 4$

Step	Operations	State
1	Fill the jug 1	(6,0)
2	Pour water from jug 1 to jug 2	(0,6)
3	Fill the jug 1	(6,6)
4	Pour water from jug 1 to jug 2	(4,8)

Queues

- Design data structures
 - State (x, y) : amount of water in the jugs 1 and 2
 - Initial state $(0, 0)$
 - Target state: $x = c$ or $y = c$ or $x + y = c$
 - State transition
 - (1) Fill the jug 1: (a, y)
 - (2) Fill the jug 2: (x, b)
 - (3) Empty the jug 1: $(0, y)$
 - (4) Empty the jug: $(x, 0)$
 - (5) Pour water from jug 1 to jug 2: $(x + y - b, b)$, if $x + y \geq b$
 - (6) Pour water from jug 1 to jug 2: $(0, x + y)$, if $x + y \leq b$
 - (7) Pour water from jug 2 to jug 1: $(a, x + y - a)$, if $x + y \geq a$
 - (8) Pour water from jug 2 to jug 1: $(x + y, 0)$, if $x + y \leq a$

Queues

- Push state (0,0) into the queue

(0,0)									
-------	--	--	--	--	--	--	--	--	--

- Pop state (0,0) out and push (6,0), (0,8) into the queue

(0,0)	(6,0)	(0,8)							
-------	-------	-------	--	--	--	--	--	--	--

- Pop state (6,0) out and push (0,6) and (6,8) into the queue

(0,0)	(6,0)	(0,8)	(0,6)	(6,8)					
-------	-------	-------	-------	-------	--	--	--	--	--

- Pop state (0,8) out and push (6,2) into the queue

(0,0)	(6,0)	(0,8)	(0,6)	(6,8)	(6,2)				
-------	-------	-------	-------	-------	-------	--	--	--	--

Queues

- Pop state (0,6) out and push (6,6) into the queue

(0,0)	(6,0)	(0,8)	(0,6)	(6,8)	(6,2)	(6,6)			
-------	-------	-------	-------	-------	-------	-------	--	--	--

- Pop state (6,8) out, do not generate new states

(0,0)	(6,0)	(0,8)	(0,6)	(6,8)	(6,2)	(6,6)			
-------	-------	-------	-------	-------	-------	-------	--	--	--

- Pop state (6,2) out and push (0,2) into the queue

(0,0)	(6,0)	(0,8)	(0,6)	(6,8)	(6,2)	(6,6)	(0,2)		
-------	-------	-------	-------	-------	-------	-------	-------	--	--

- Pop state (6,6) out and push (4,8) into the queue

(0,0)	(6,0)	(0,8)	(0,6)	(6,8)	(6,2)	(6,6)	(0,2)	(4,8)	
-------	-------	-------	-------	-------	-------	-------	-------	-------	--

Queues

- Design data structures
 - Initialize a queue Q for storing generated states
 - 2-dimensional array 2 for marking generated states
 - $\text{visited}[x][y] = \text{true}$, if the state (x, y) has been generated

Queues

- Data structures declaration

```
#include <stdio.h>
#include <stdlib.h>
#include <queue>
#include <stack>
#include <list>
using namespace std;
struct State{
    int x;
    int y;
    char* msg;// action to generate current state
    State* p;// pointer to the state generating current state
};
bool visited[10000][10000];
queue<State*> Q;
list<State*> L;
State* target;
int a,b,c;
```

Queues

- Initialize data structures

```
void initVisited(){
    for(int x = 0; x < 10000; x++)
        for(int y = 0; y < 10000; y++)
            visited[x][y] = false;
}

bool reachTarget(State* S){
    return S->x == c || S->y == c ||
           S->x + S->y == c;
}

void markVisit(State* S){
    visited[S->x][S->y] = true;
}

void freeMemory(){
    list<State*>::iterator it;
    for(it = L.begin(); it != L.end(); it++){
        delete *it;
    }
}
```

Queues

- Generate a new state by empty jug 1

```
bool genMove1Out(State* S){
    if(visited[0][S->y]) return false;
    State* newS = new State;
    newS->x = 0;
    newS->y = S->y;
    newS->msg = "Do het nuoc o coc 1 ra ngoai";
    newS->p = S;
    Q.push(newS); markVisit(newS);
    L.push_back(newS);
    if(reachTarget(newS)){
        target = newS;
        return true;
    }
    return false;
}
```


Queues

- Generate a new state by empty jug 2

```
bool genMove2Out(State* S){
    if(visited[S->x][0]) return false;
    State* newS = new State;
    newS->x = S->x;
    newS->y = 0;
    newS->msg = "Do het nuoc o coc 2 ra ngoai";
    newS->p = S;
    Q.push(newS); markVisit(newS);
    L.push_back(newS);
    if(reachTarget(newS)){
        target = newS;
        return true;
    }
    return false;
}
```

Queues

- Generate a new state by pouring water from jug 1 to jug 2 (case 1)

```
bool genMove1Full2(State* S){
    if(S->x+S->y < b) return false;
    if(visited[S->x + S->y - b][b]) return false;
    State* newS = new State;
    newS->x = S->x + S->y - b;
    newS->y = b;
    newS->msg = "Do nuoc tu coc 1 vao day coc 2";
    newS->p = S;
    Q.push(newS); markVisit(newS);
    L.push_back(newS);
    if(reachTarget(newS)){
        target = newS;
        return true;
    }
    return false;
}
```

Queues

- Generate a new state by pouring water from jug 2 to jug 1 (case 1)

```
bool genMove2Full1(State* S){
    if(S->x+S->y < a) return false;
    if(visited[a][S->x + S->y - a]) return false;
    State* newS = new State;
    newS->x = a;
    newS->y = S->x + S->y - a;
    newS->msg = "Do nuoc tu coc 2 vao day coc 1";
    newS->p = S;
    Q.push(newS); markVisit(newS);
    L.push_back(newS);
    if(reachTarget(newS)){
        target = newS;
        return true;
    }
    return false;
}
```

Queues

- Generate a new state by pouring water from jug 1 to jug 2 (case 2)

```
bool genMoveAll12(State* S){
    if(S->x + S->y > b) return false;
    if(visited[0][S->x + S->y]) return false;
    State* newS = new State;
    newS->x = 0;
    newS->y = S->x + S->y;
    newS->msg = "Do het nuoc tu coc 1 sang coc 2";
    newS->p = S;
    Q.push(newS); markVisit(newS);
    L.push_back(newS);
    if(reachTarget(newS)){
        target = newS;
        return true;
    }
    return false;
}
```

Queues

- Generate a new state by pouring water from jug 2 to jug 1 (case 2)

```
bool genMoveAll21(State* S){
    if(S->x + S->y > a) return false;
    if(visited[S->x + S->y][0]) return false;
    State* newS = new State;
    newS->x = S->x + S->y;
    newS->y = 0;
    newS->msg = "Do het nuoc tu coc 2 sang coc 1";
    newS->p = S;
    Q.push(newS); markVisit(newS);
    L.push_back(newS);
    if(reachTarget(newS)){
        target = newS;
        return true;
    }
    return false;
}
```

Queues

- Generate a new state by filling jug 1

```
bool genMoveFill1(State* S){
    if(visited[a][S->y]) return false;
    State* newS = new State;
    newS->x = a;
    newS->y = S->y;
    newS->msg = "Do day nuoc vao coc 1";
    newS->p = S;
    Q.push(newS); markVisit(newS);
    L.push_back(newS);
    if(reachTarget(newS)){
        target = newS;
        return true;
    }
    return false;
}
```

Queues

- Generate a new state by filling jug 2

```
bool genMoveFill2(State* S){
    if(visited[S->x][b]) return false;
    State* newS = new State;
    newS->x = S->x;
    newS->y = b;
    newS->msg = "Do day nuoc vao coc 2";
    newS->p = S;
    Q.push(newS); markVisit(newS);
    L.push_back(newS);
    if(reachTarget(newS)){
        target = newS;
        return true;
    }
    return false;
}
```

Queues

- Print the sequence of operations for obtaining the objective

```
void print(State* target){
    printf("-----RESULT-----\n");
    if(target == NULL)
        printf("Khong co loi giai!!!!!!!!");
    State* currentS = target;
    stack<State*> actions;
    while(currentS != NULL){
        actions.push(currentS);
        currentS = currentS->p;
    }
    while(actions.size() > 0){
        currentS = actions.top();
        actions.pop();
        printf("%s, (%d,%d)\n",
            currentS->msg,currentS->x,
            currentS->y);
    }
}
```


Queues

- Main process for generating states and push them into the queue

```
void solve(){
    initVisited();
    // sinh ra trang thai ban dau (0,0) va dua vao Q
    State* S = new State;
    S->x = 0; S->y = 0; S->p = NULL;
    Q.push(S); markVisit(S);
    while(!Q.empty()){
        State* S = Q.front(); Q.pop();
        if(genMove1Out(S)) break;
        if(genMove2Out(S)) break;
        if(genMove1Full2(S)) break;
        if(genMoveAll12(S)) break;
        if(genMove2Full1(S)) break;
        if(genMoveAll21(S)) break;
        if(genMoveFill1(S)) break;
        if(genMoveFill2(S)) break;
    }
}
```

Queues

- Example with: $a = 4$, $b = 7$, $c = 9$

```
int main(){  
    a = 4;  
    b = 7;  
    c = 9;  
    target = NULL;  
    solve();  
    print(target);  
    freeMemory();  
}
```