HA NOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF INFORMATION AND COMMUNICATION TECHNOLOGY

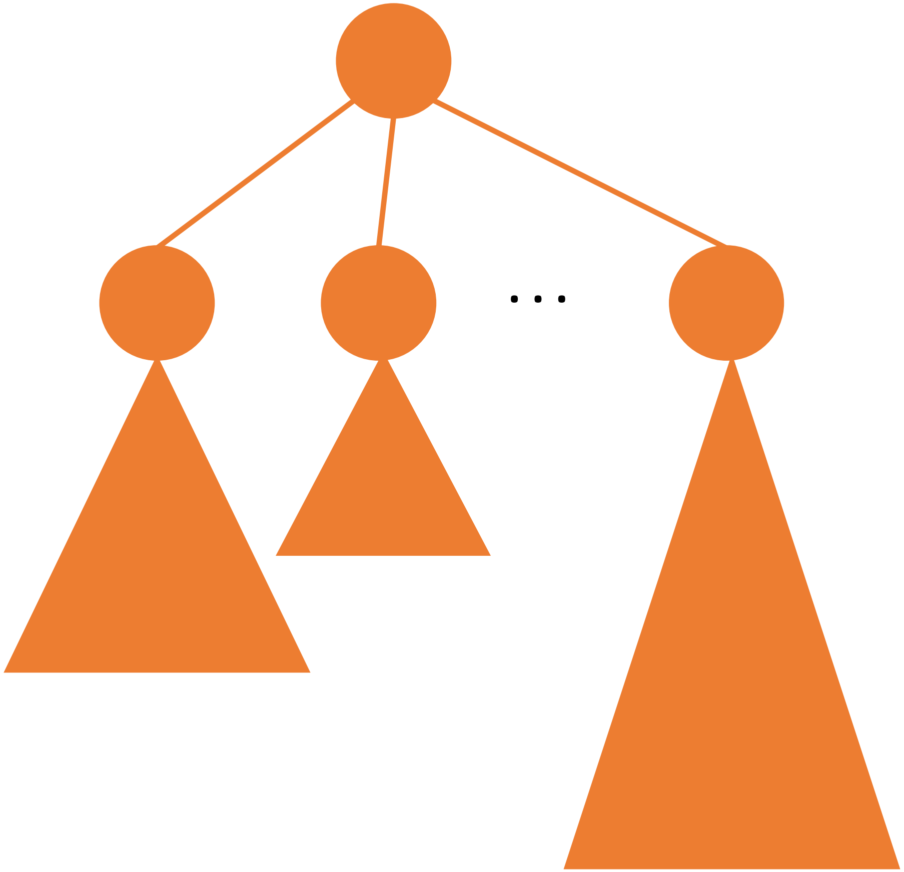
DATA STRUCTURES AND ALGORITHMS

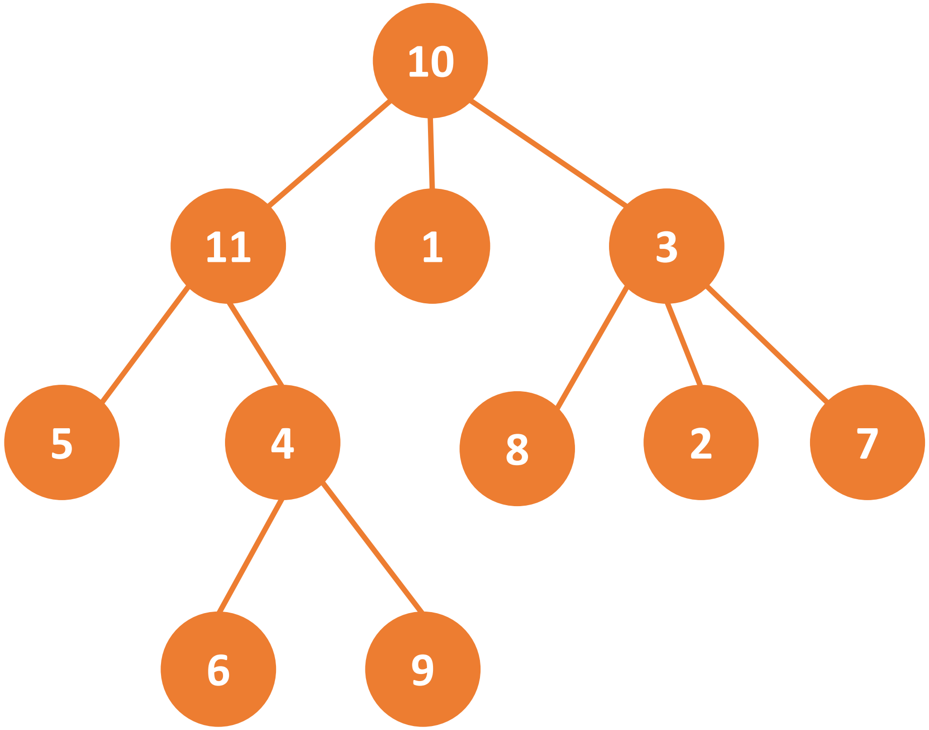
Trees

Content

- Definition
- Terminology
- Traversal
- Data structures
- Operations

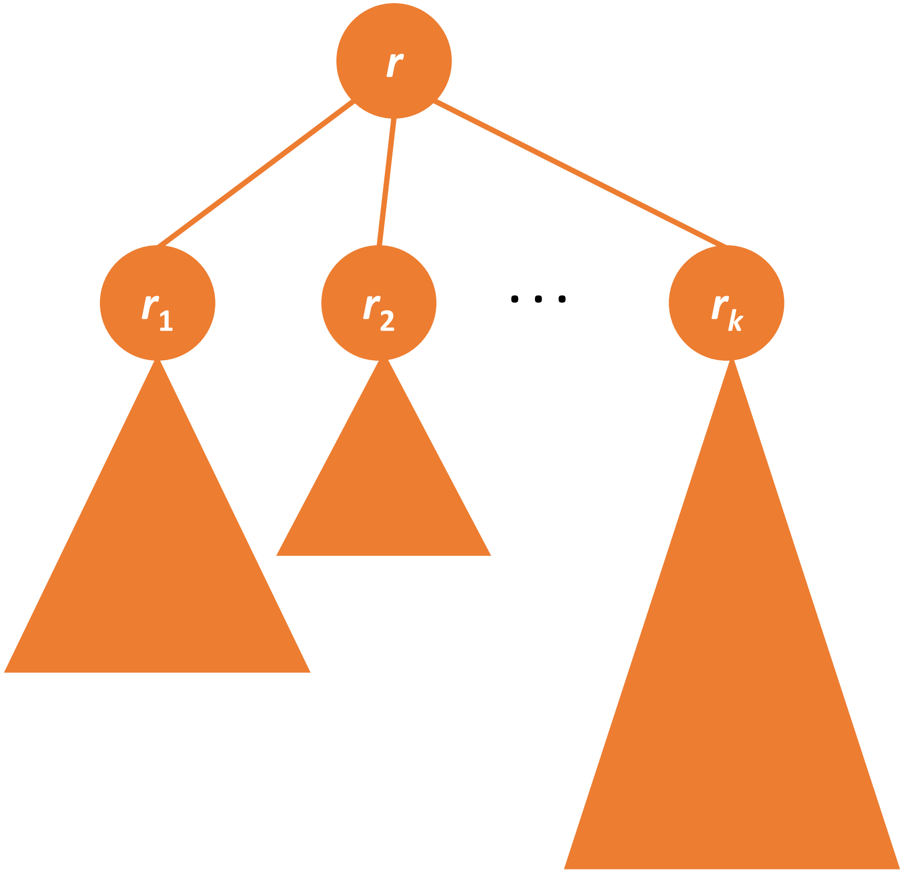
Tree: definition





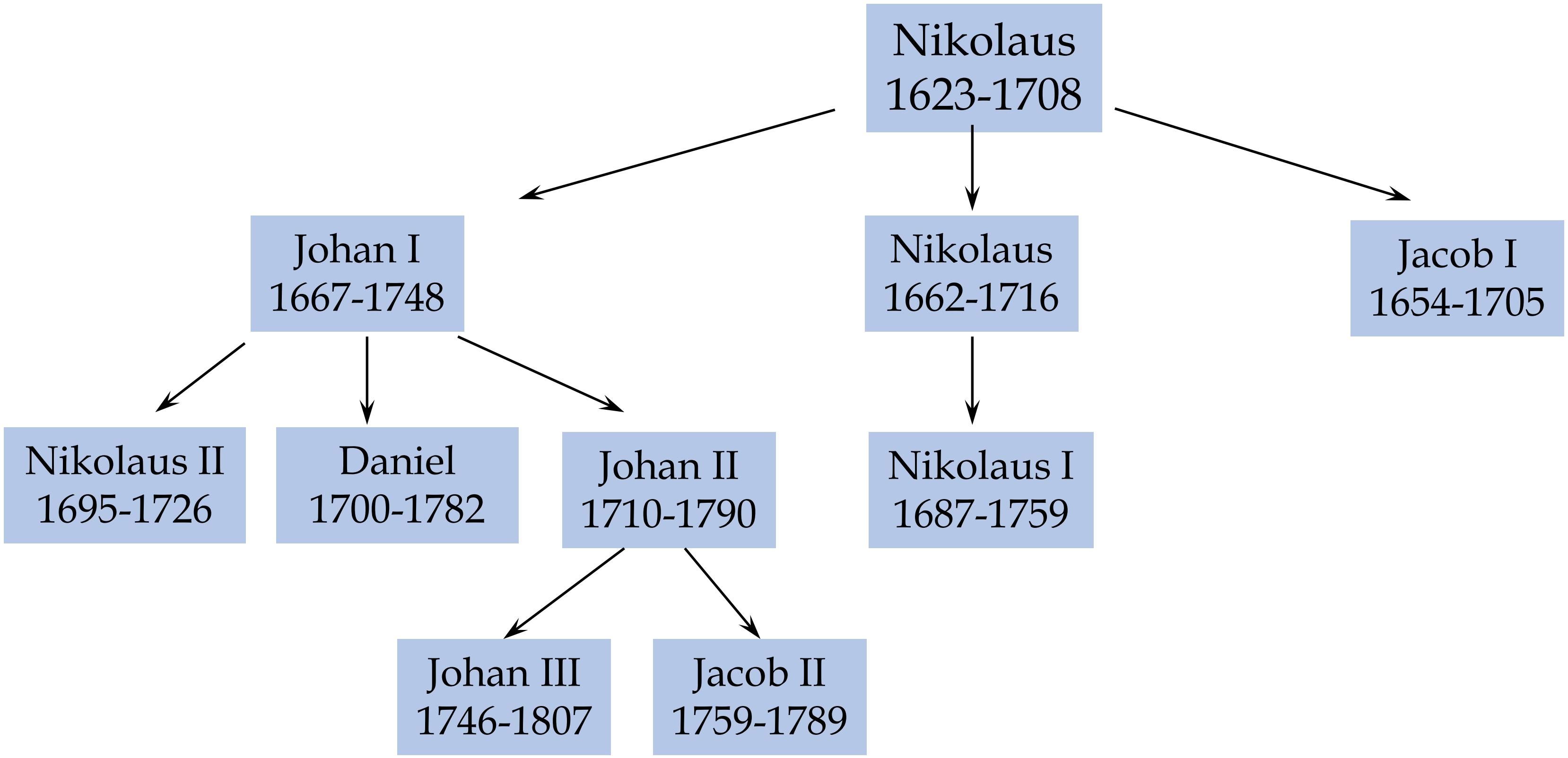
Tree: definition

- Store objects in a hierarchical structure
- Recursive definition
  - Base case: r is a node, T is a tree containing only one node r which is also the root of T
  - Recursion:
    - Suppose T_1, T_2, …, T_k are trees rooted at r_1, r_2, …, r_k
    - Given a node r
    - Make r_1, r_2, .., r_k children of r creating a new tree T

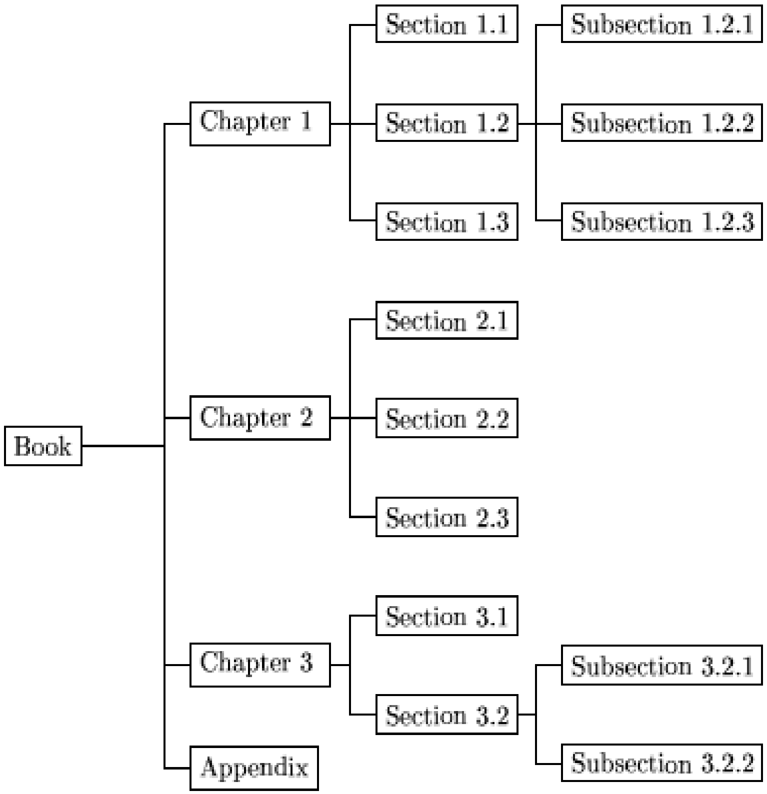


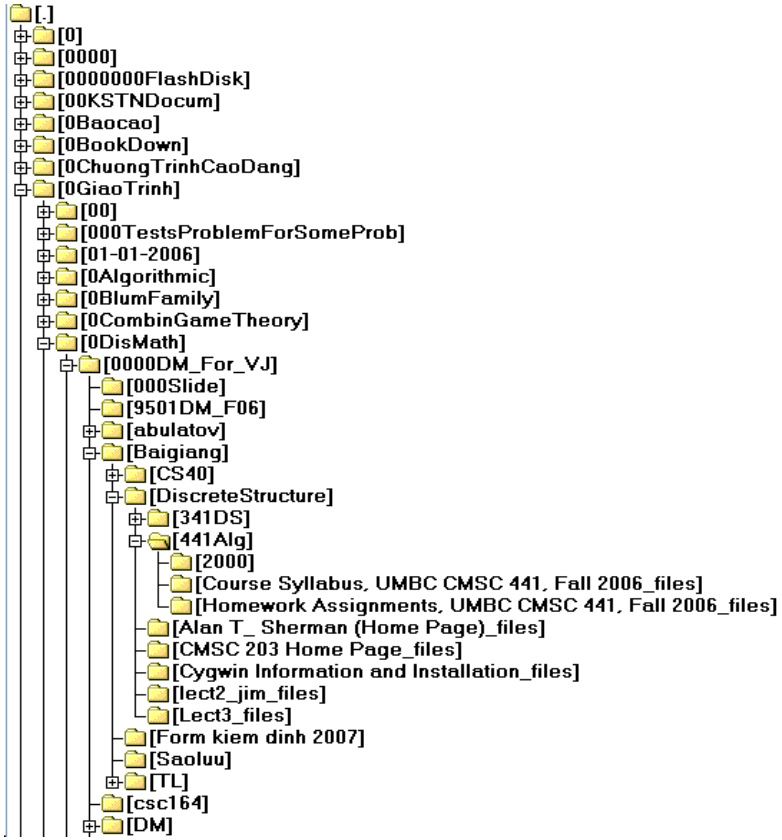
Applications

- Family tree of mathematicians of the Bernoulli family



Applications





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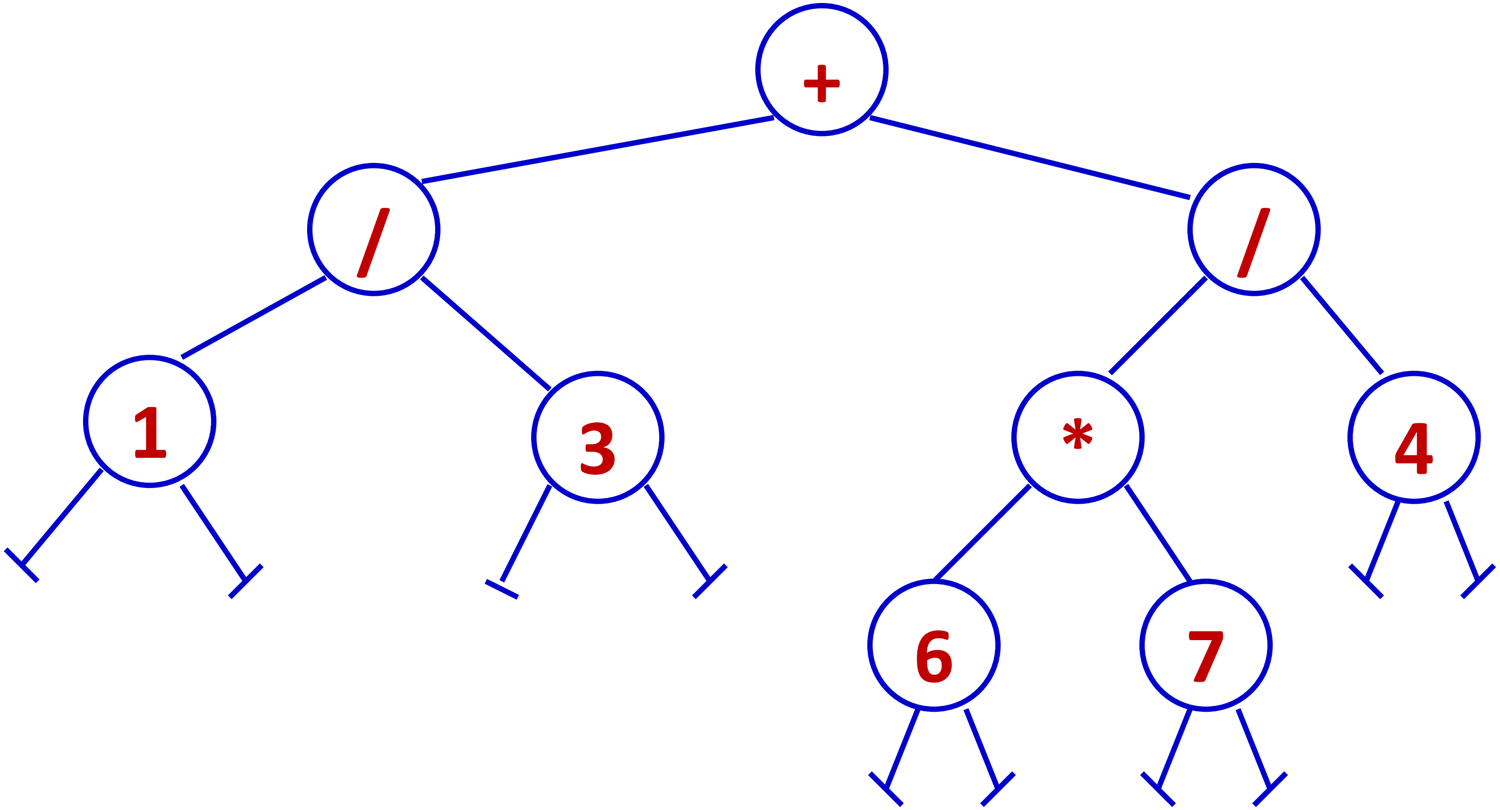
Folders organization

Applications: expression tree



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Terminology

- Path: sequence of nodes x_1, x_2, …, x_q where x_i is the parent of x_{i+1}, i = 1, 2, …, q-1. Length of the path is q-1
- Leaves: do not have children
- Internal nodes: have children
- Sibling: 2 nodes *u* and *v* are sibling if they have the same parent
- Ancestors: node *u* is an ancestor of *v* is there is a path from *u* to *v*
- Descendants: node *u* is a descendant of *v* if *v* is an ancestor of *u*
- Height: the height of a node is the length of the longest path from that node to some leaf plus 1
- Depth: the depth of a node *v* is the length of the unique path from that node to the root plus 1

Terminology

- **Root:** do not have parent (example: node A)
- B, C, D are siblings;
- **Internal nodes**: A, B, C, F
- **Leaves**: E, I, J, K, G, H, D

Con của B:

- E, F

Cha của E:

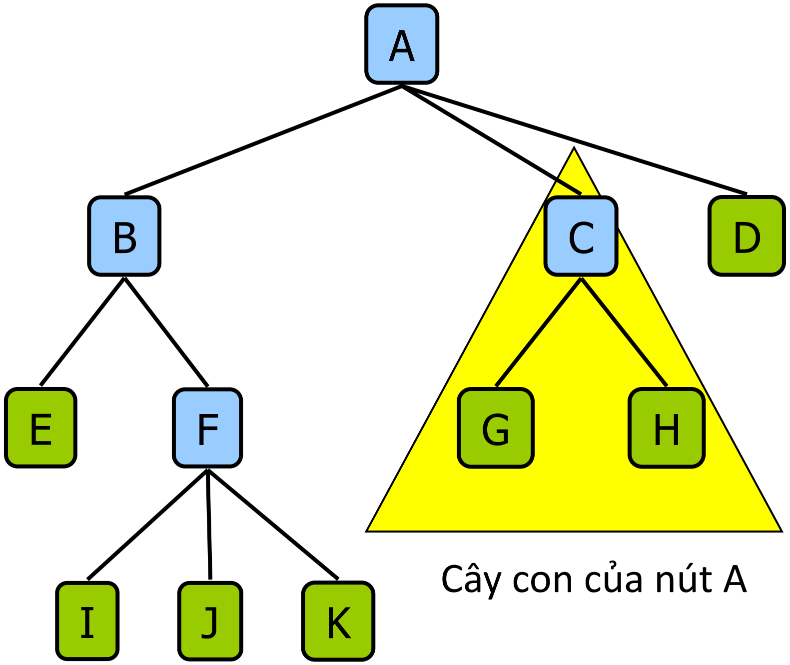
- B

Tổ tiên của F:

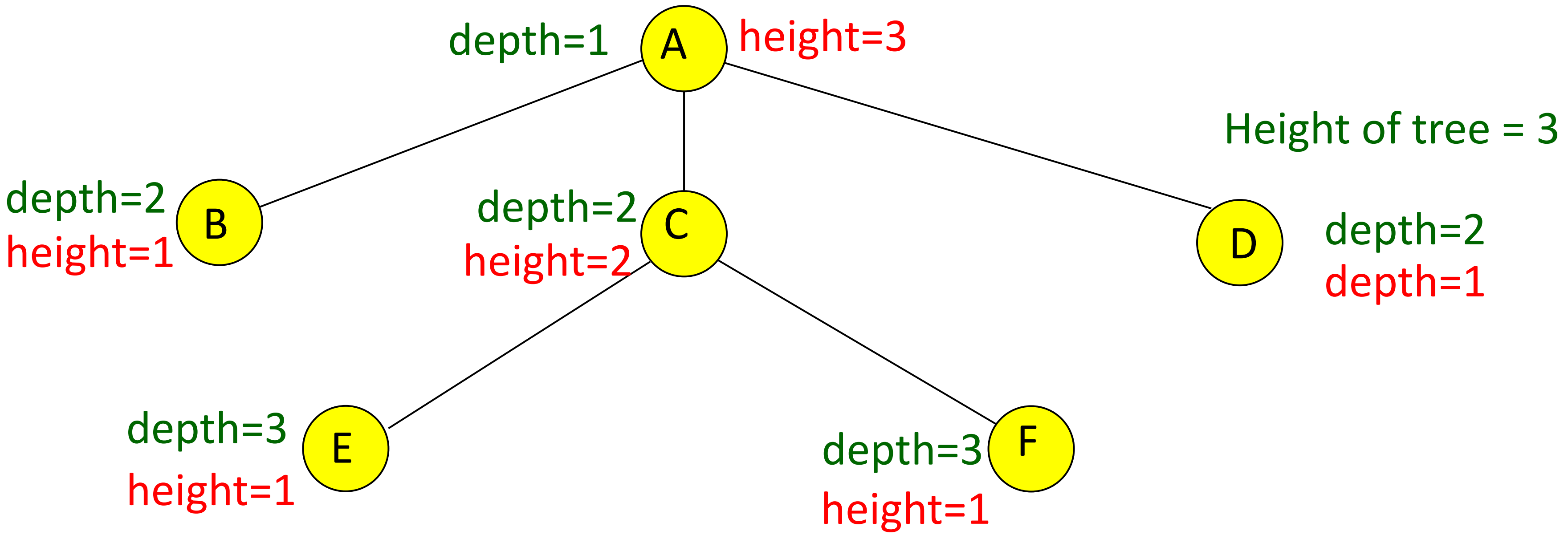
- B, A

Hậu duệ của B:

- E, F, I, J, K



Terminology



Traversal

- Visit nodes of a tree in some order
  - Pre-order traversal
  - In-order traversal
  - Post-order traversal
- Consider a tree T
  - root r
  - subtrees T_1 (root r_1), T_2 (root r_2), …, T_k (root r_k) from left to right

Traversal

- Pre-order traversal
  - Visit the root
  - Traverse T_1 in the pre-order
  - Traverse T_2 in the pre-order
  - …
  - Traverse T_k in the pre-order

```
preOrder(r){
   if(r = NULL) return;
   visit(r);
   for each p = r1, r2, .., rk {
      preOrder(p);
   }
}
```

Traversal

- In-order traversal
  - Traverse T_1 in the in-order
  - Visit the root r
  - Traverse T_2 in the in-order
  - …
  - Traverse T_k in the in-order

```
inOrder(r){
   if(r = NULL) return;
   inOrder(r1);
   visit(r);
   for each p = r₂, .., rₖ {
      inOrder(p);
   }
}
```

Traversal

- Post-order traversal
  - Traverse T_1 in the post-order
  - Traverse T_2 in the post-order
  - …
  - Traverse T_k in the post-order
  - Visit the root r

```
postOrder(r){
   if(r = NULL) return;
   for each p = r₁, r₂, .., rₖ {
      postOrder(p);
   }
   visit(r);
}
```

Data structures

- Array:
  - Suppose nodes are numbered 1, 2, …, n
  - $a[1..n]$ in which $a[i]$ is the parent of i
  - Implementation of many operations on the tree might be too complicated
- Pointer: Each node has two pointers
  - leftMostChild: a pointer to the left-most child
  - rightSibling: a pointer to the right-sibling node

Data structures

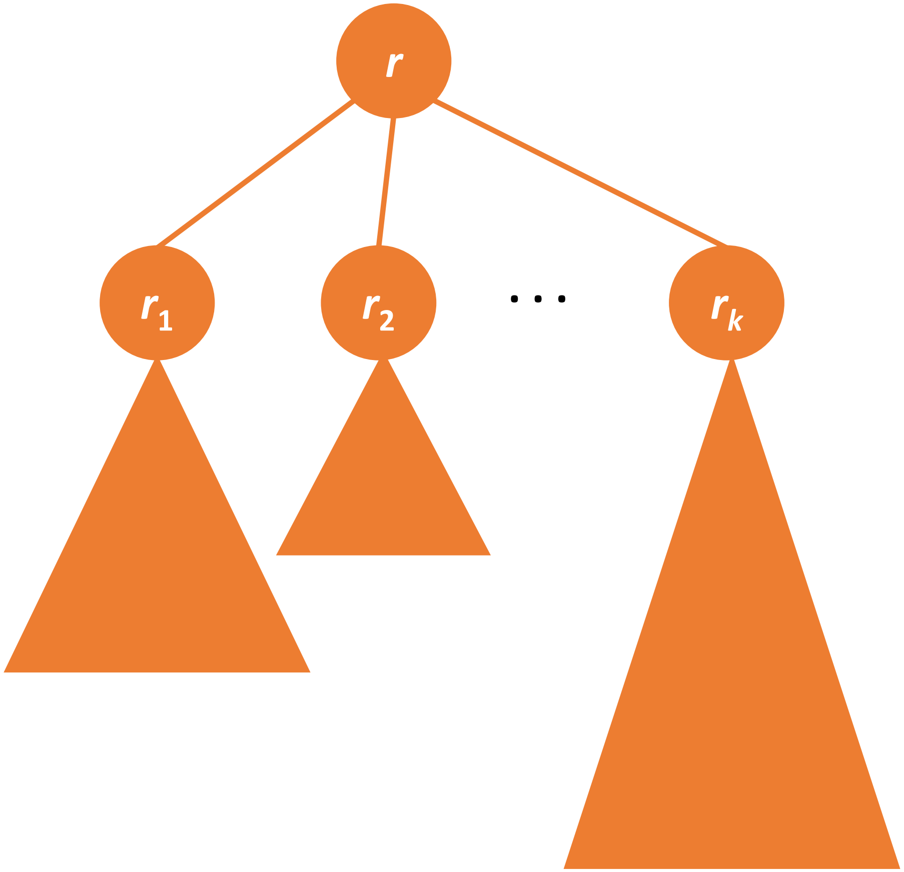
```
struct Node{
    int id; // identifier of the node
    Node* leftMostChild;// pointer to the left-most child
    Node* rightSibling;// pointer to the right-sibling
};
Node* root;// pointer to the root of the tree
```

Operations

- find(r, id): return the node having identifier id on the tree rooted at r
- insert(r, p, id): create a node having identifier id, insert that node to the end of the children list of p on the tree rooted at r
- height(r, p): return the height of node p on the tree rooted at r
- depth(r, p): return the depth of the node p on the tree rooted at r
- parent(r, p): return the parent of p on the tree rooted at r
- count(r): return the number of nodes of the tree rooted at r
- countLeaves(r): return the number of leaves of the tree rooted at r

Operations

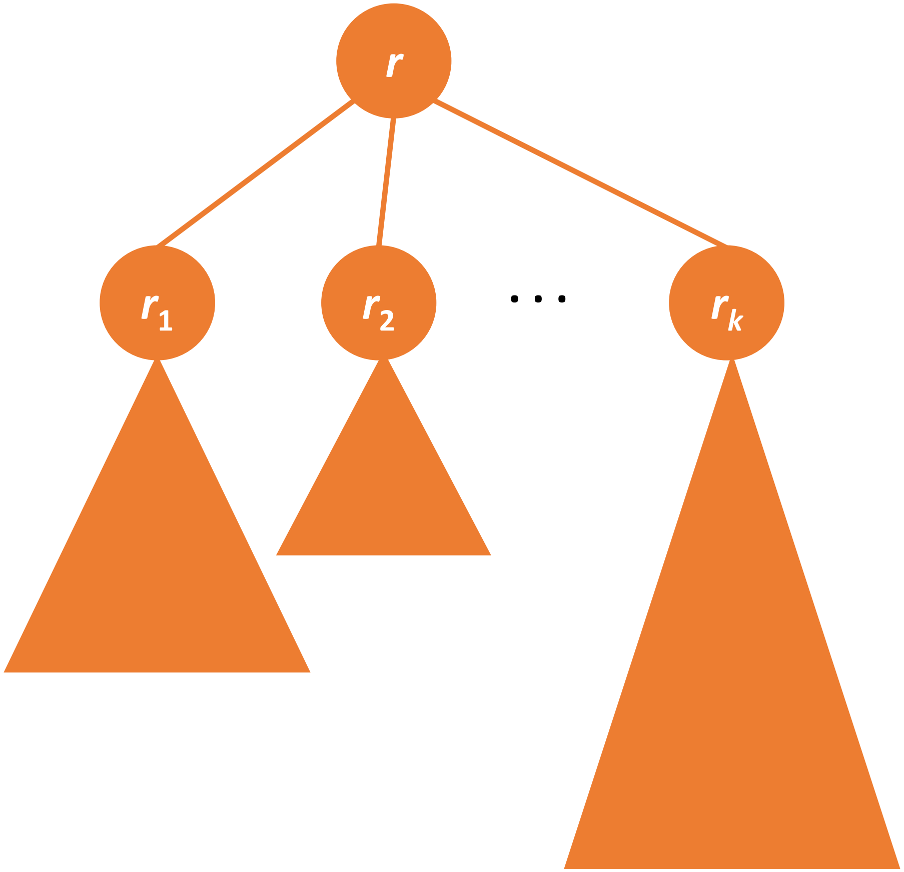
- Find a node given the identifier




```
Node* find(Node* r, int v){
    if(r == NULL) return NULL;
    if(r->id == v) return r;
    Node* p = r->leftMostChild;
    while(p != NULL){
        Node* h = find(p,v);
        if(h != NULL) return h;
        p = p->rightSibling;
    }
    return NULL;
}
```

Operations

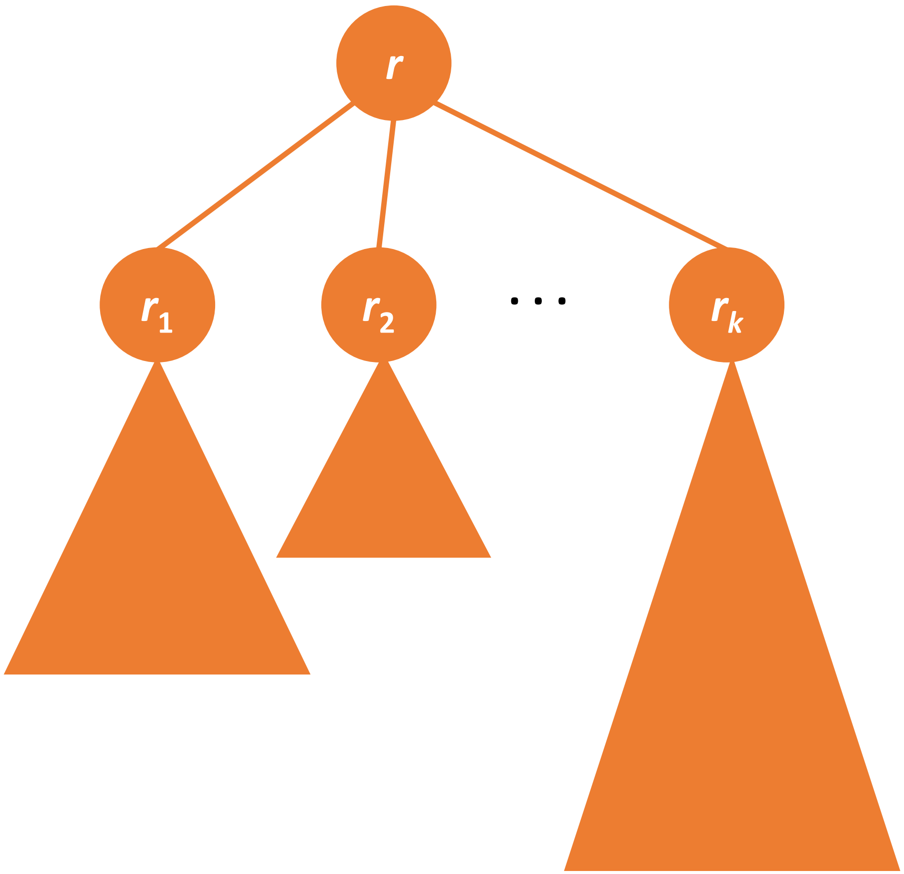
- Pre-order traversal




```
void preOrder(Node* r){
    if(r == NULL) return;
    printf("%d ",r->id);
    Node* p = r->leftMostChild;
    while(p != NULL){
        preOrder(p);
        p = p->rightSibling;
    }
}
```

Operations

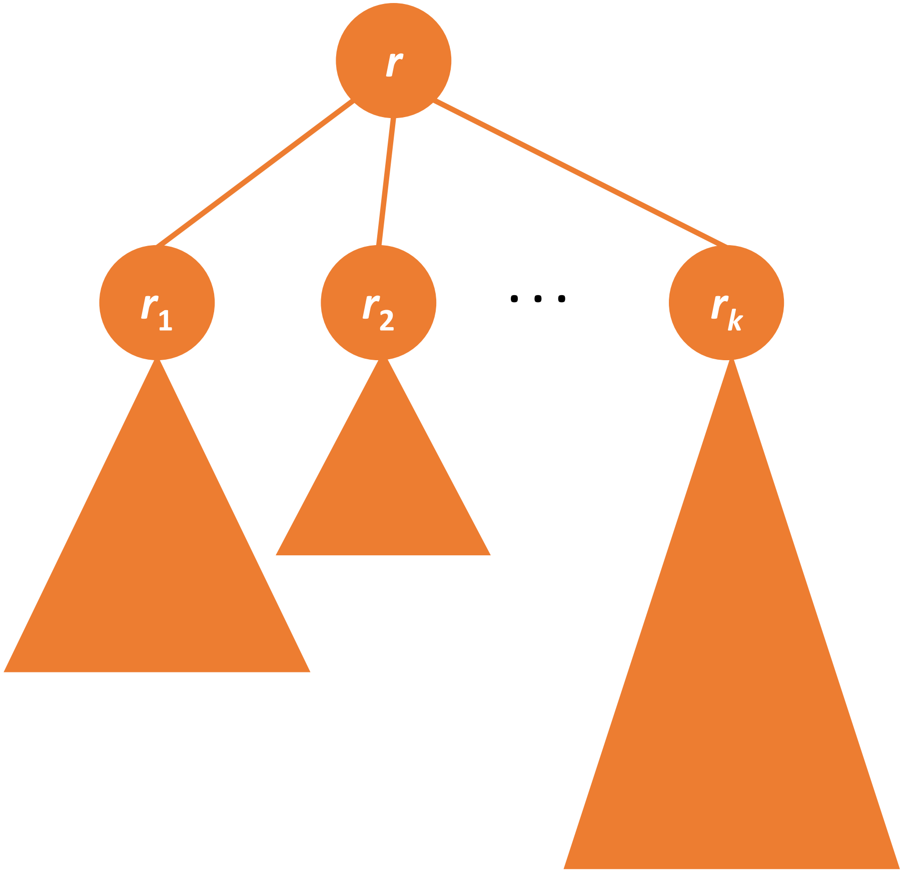
- In-order traversal




```
void inOrder(Node* r){
   if(r == NULL) return;
   Node* p = r->leftMostChild;
   inOrder(p);
   printf("%d ",r->id);
   if(p != NULL)
      p = p->rightSibling;
   while(p != NULL){
        inOrder(p);
        p = p->rightSibling;
   }
}
```

Operations

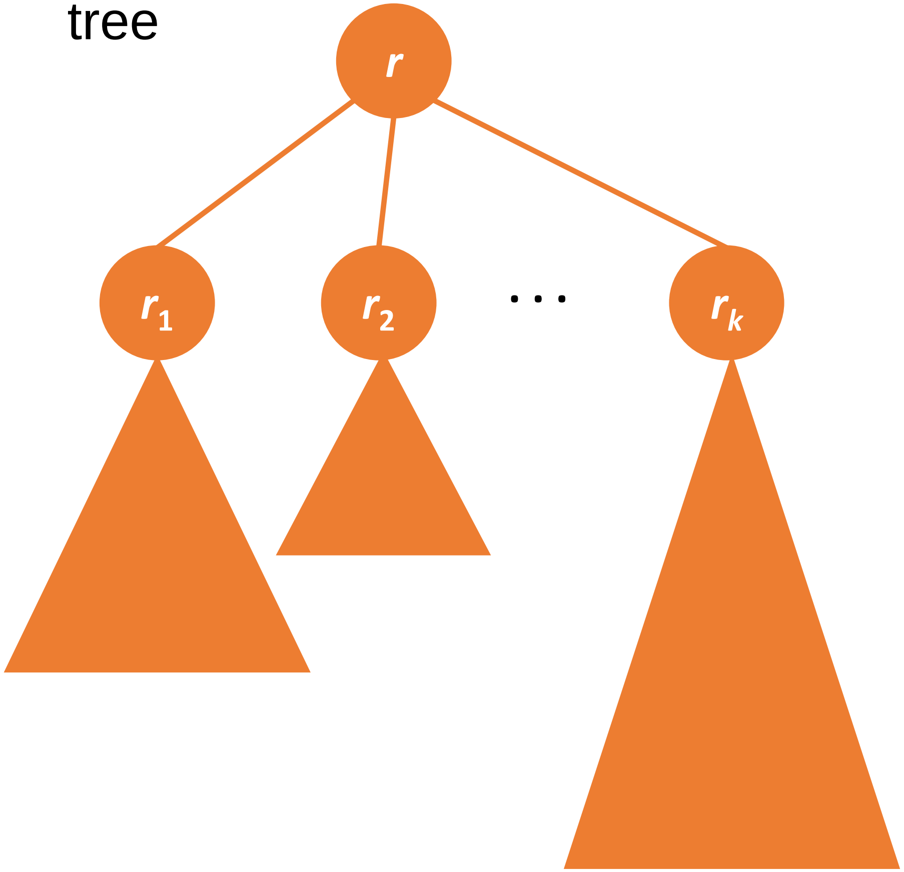
- Post-order traversal




```
void postOrder(Node* r){
    if(r == NULL) return;
    Node* p = r->leftMostChild;
    while(p != NULL){
        postOrder(p);
        p = p->rightSibling;
    }
    printf("%d ",r->id);
}
```

Operations

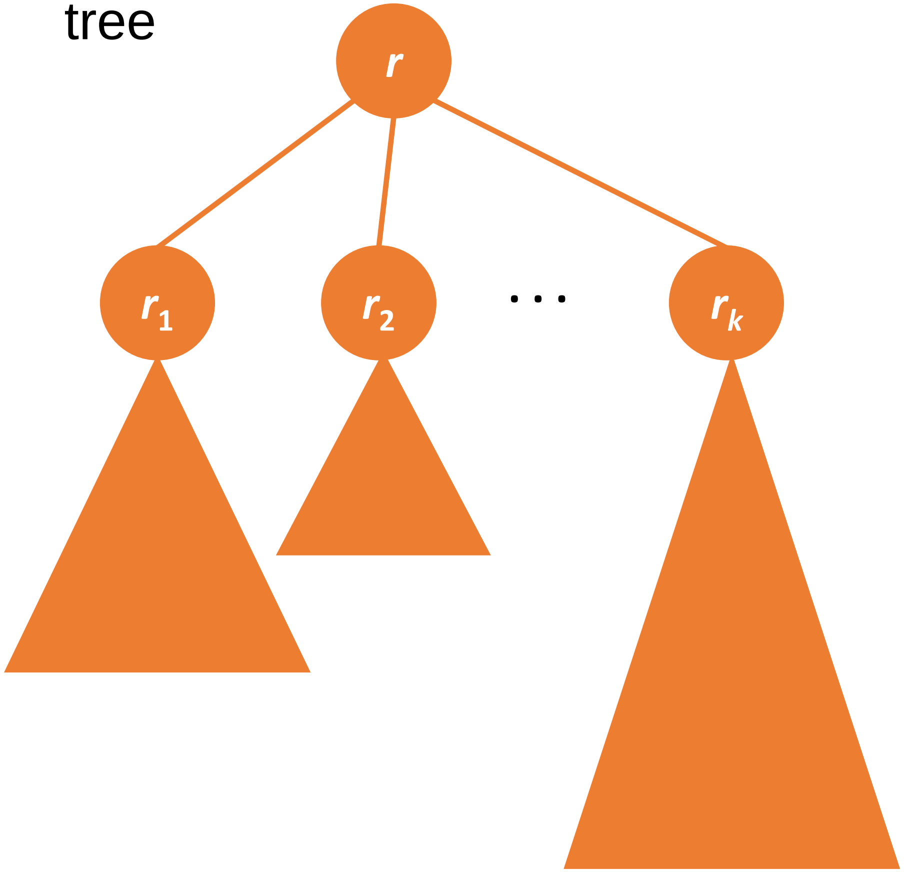
- Count the number of nodes of a tree




```
int count(Node* r){
    if(r == NULL) return 0;
    int s = 1;
    Node* p = r->leftMostChild;
    while(p != NULL){
        s += count(p);
        p = p->rightSibling;
    }
    return s;
}
```

Operations

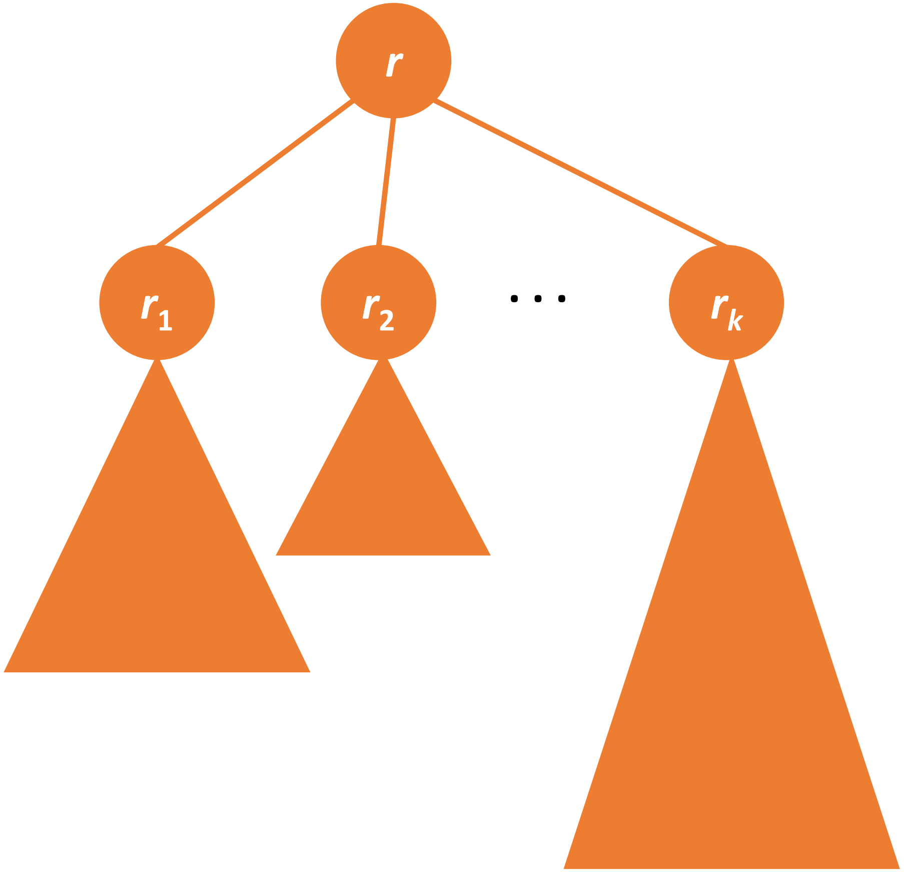
- Count the number of leaves of a tree




```
int countLeaves(Node* r){
    if(r == NULL) return 0;
    int s = 0;
    Node* p = r->leftMostChild;
    if(p == NULL) s = 1;
    while(p != NULL){
        s += countLeaves(p);
        p = p->rightSibling;
    }
    return s;
}
```

Operations

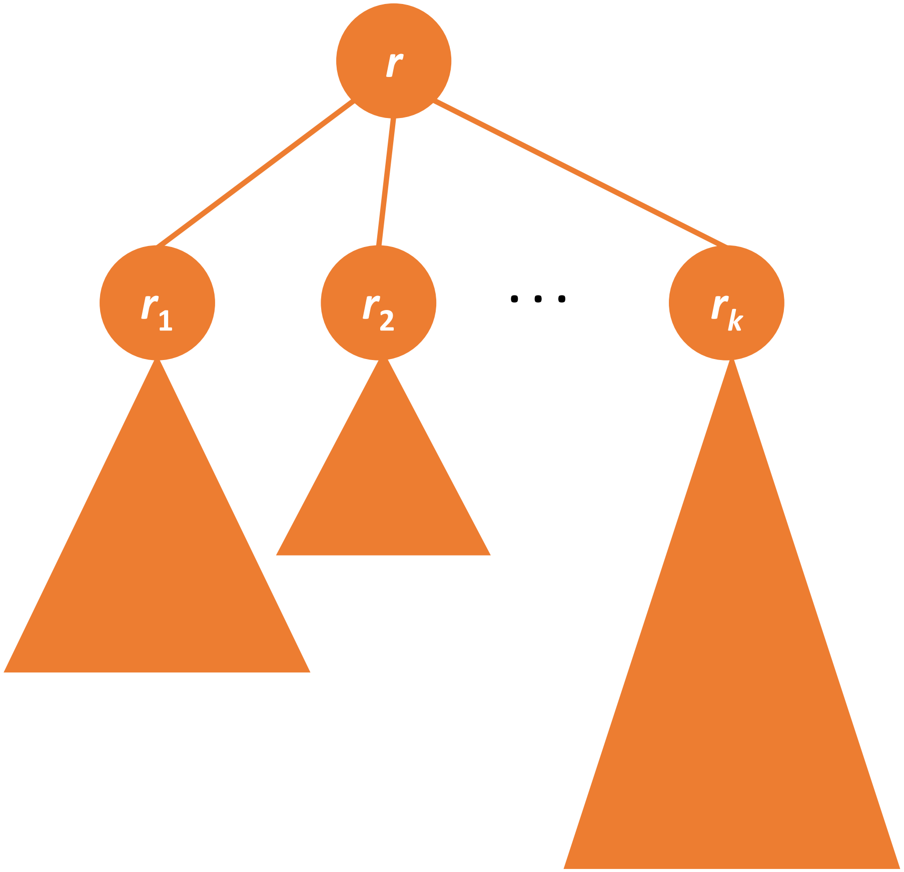
- Compute the height of a node




```
int height(Node* p){
    if(p == NULL) return 0;
    int maxh = 0;
    Node* q = p->leftMostChild;
    while(q != NULL){
        int h = height(q);
        if(h > maxh) maxh = h;
        q = q->rightSibling;
    }
    return maxh + 1;
}
```

Operations

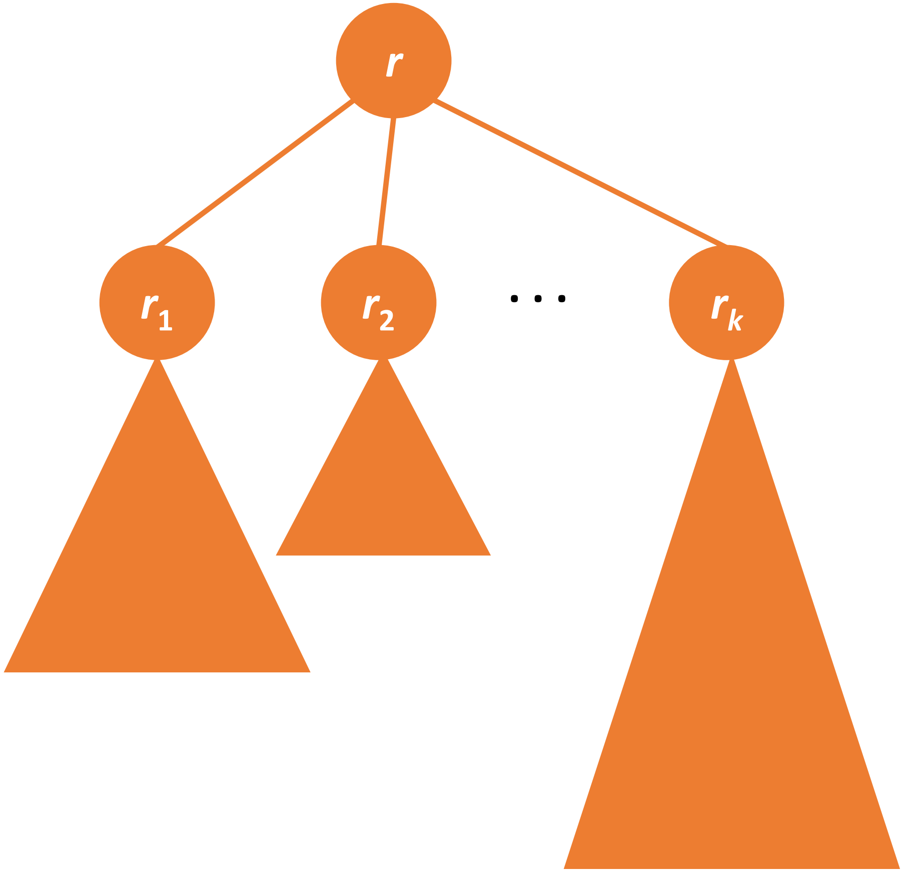
- Compute the depth of a node




```
int depth(Node* r, int v, int d){
    // d la do sau cua nut r
    if(r == NULL) return -1;
    if(r->id == v) return d;
    Node* p = r->leftMostChild;
    while(p != NULL){
        if(p->id == v) return d+1;
        int dv = depth(p,v,d+1);
        if(dv > 0) return dv;
        p = p->rightSibling;
    }
    return -1;
}
int depth(Node* r, int v){
    return depth(r,v,1);
}
```

Operations

- Find the parent of a node




```
Node* parent(Node* p, Node* r){
    if(r == NULL) return NULL;
    Node* q = r->leftMostChild;
    while(q != NULL){
        if(p == q) return r;
        Node* pp = parent(p, q);
        if(pp != NULL) return pp;
        q = q->rightSibling;
    }
    return NULL;
}
```

Binary trees

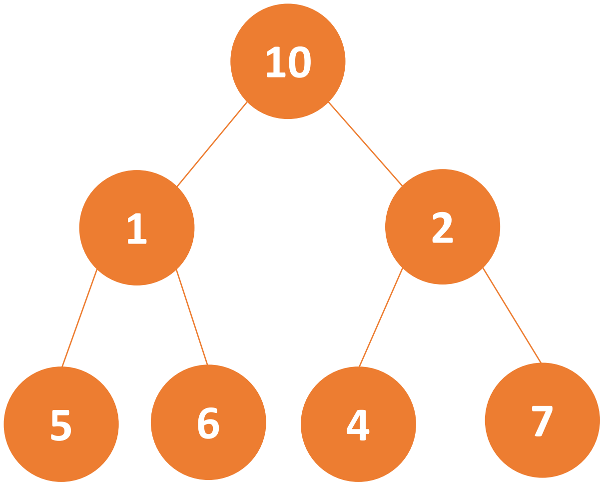
- Each node has at most two children
- Distinct between the left child and the right child

```
struct BNode{
    int id;
    BNode* leftChild; // pointer to the left child
    BNode* rightChild;// pointer fo the right child
};
```

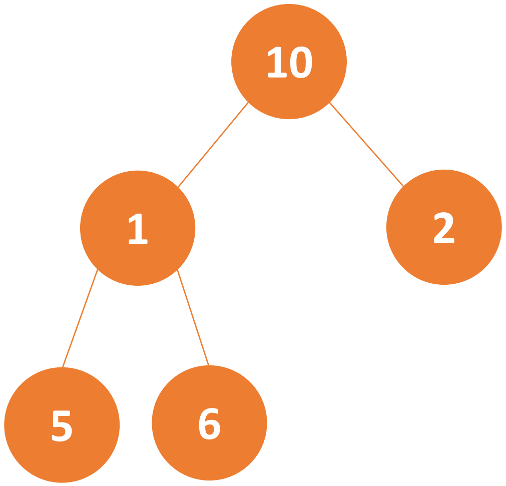
- `leftChild = NULL`: current node does not have the left child
- `rightChild = NULL`: current node does not have the right child

Binary trees

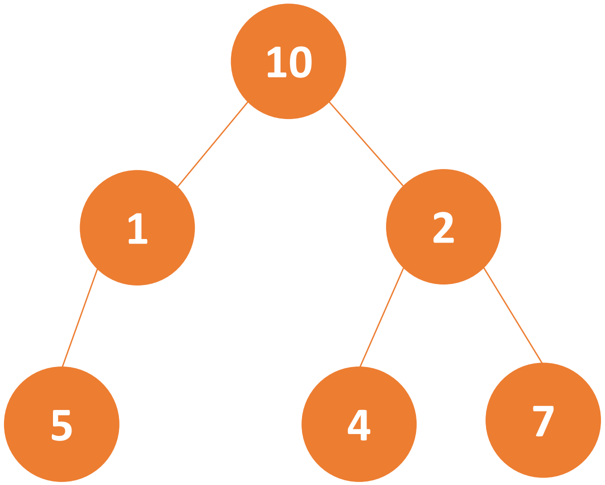
- Classification



Perfect tree



Complete tree



Balanced tree

Operations on a binary tree

```
void preOrder(BNode* r) {
  if(r == NULL) return;
  printf("%d ",r->id);
  preOrder(r->leftChild);
  preOrder(r->rightChild);
}
```

```
void inOrder(BNode* r) {
  if(r == NULL) return;
  inOrder(r->leftChild);
  printf("%d ",r->id);
  inOrder(r->rightChild);
}
```

Operations on a binary tree

```
void postOrder(BNode* r) {
  if(r == NULL) return;
  postOrder(r->leftChild);
  postOrder(r->rightChild);
  printf("%d ",r->id);

}
```

```
int count(BNode* r) {
  if(r == NULL) return 0;
  return 1 + count(r->leftChild) +
             count(r->rightChild);
}
```

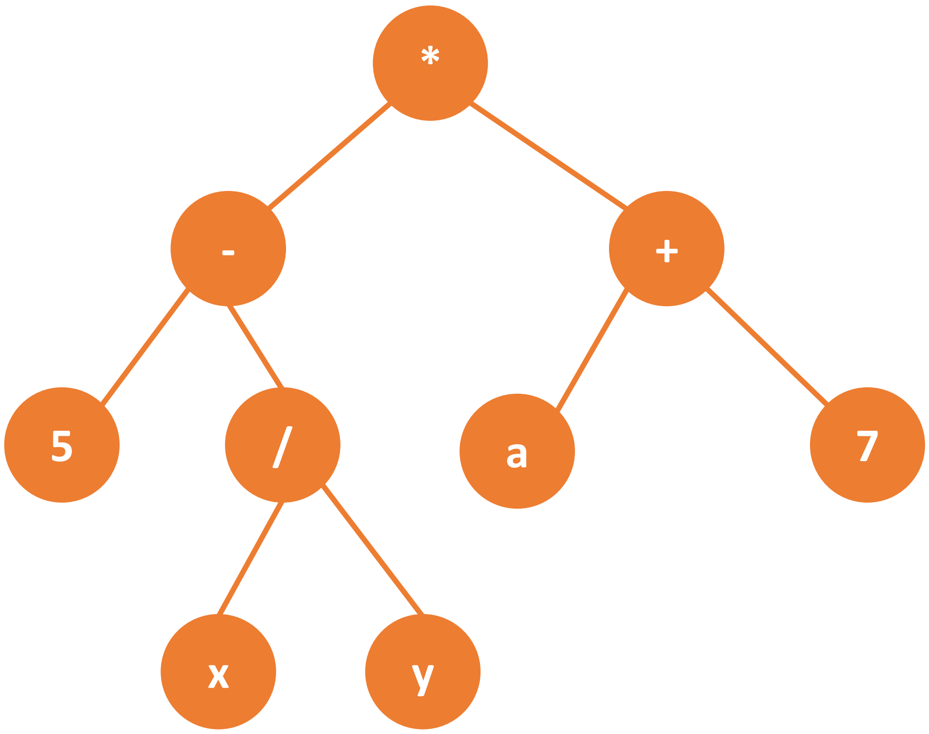
Expression trees

- Binary tree
  - Internal node are math operations
  - Leaves are operands (variables, constants)
- Infix expression: sequence of elements visited by the in-order traversal:

  (5 - x/y) * (a + 7)

- Postfix expression: sequence of elements visited by the post-order traversal:

  5 x y / - a 7 + *



Evaluation of a postfix expression

- Initialize a stack S
- Scan elements of the postfix from left to right
- If meet an operand, then push it into the stack S
- If meet an operator ***op***, then pop 2 operands A and B out of S, perform C = B ***op*** A, and then push C into S
- Termination: the element stays in the stack S is the value of the given postfix expression