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APPLIED STATISTICS AND EXPERIMENTAL DESIGN

Probability Theory

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Applied Statistics and Experimental Design

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II. Basics of Probability theory

- 2.1. Probability.
- 2.2. Random variables.
- 2.3. Law of large number
- 2.4. Normal probability distribution (Gaussian distribution).

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2.1. Probability

- Probability
  - Laplace classic definition of probability:

$$P(A) = \frac{\text{Number of outcomes favorable to } A}{\text{Total number of possible outcomes}},$$

  - Relative frequency definition of probability:

$$P(A) = \lim_{n \to \infty} \frac{n_A}{n}$$

2.1. Probability

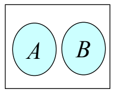
- Kolmogorov axiomatic formulation
  - Ω: sample space: set of all experimental outcomes
    $\Omega = \{ \xi_1, \xi_2, \ldots \xi_k, \ldots, \xi_n, \ldots \}$
  - Event – any subset of Ω. Number of subset of sample space : 2^n if $n < \infty$.
  - σ-field F of subsets of Ω
  - P: a probability measure on the sets in F
    - A – any event
    - 3 axiom of probability

$$\begin{aligned}
&\text{(i)} \quad P(A) \geq 0 \quad \text{(Probability is a nonnegative number)} \\
&\text{(ii)} \quad P(\Omega) = 1 \quad \text{(Probability of the whole set is unity)} \\
&\text{(iii)} \quad \text{If } A \cap B = \phi, \quad \text{then} \quad P(A \cup B) = P(A) + P(B).
\end{aligned}$$

  - $< \Omega, F, P >$: probability model

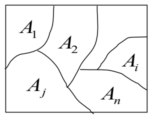
2.1. Probability

- Events: A and B
  - Mutualy exclusive events: $A \cap B = \varnothing$
  - Partition of Ω:



$A \cap B = \phi$

$$A_i \cap A_j = \phi, \quad \text{and} \quad \bigcup_{i=1}^{n} A_i = \Omega$$



  - Example: experiment of tossing two coins simultaneously
    - Elementary events:

$$\xi_1 = (H,H), \quad \xi_2 = (H,T), \quad \xi_3 = (T,H), \quad \xi_4 = (T,T)$$

    - The subset $A = \{ \xi_1, \xi_2, \xi_3 \}$

Probability

- Counting the sample points
  - Evaluate is the element of chance associated with the occurrence of certain events when an experiment is performed.
  - To solve a probability problem by counting the number of points in the sample space without actually listing each element.
  - The fundamental principle of counting - **multiplication rule**:
    - If an operation can be performed in n_1 ways, and if for each of these a second operation can be performed in n_2 ways, and for each of the first two a third operation can be performed in n_3 ways, and so forth, then the sequence of k operations can be performed in $n_1 n_2 \cdots n_k$ ways.

2.1. Probability

- Conditional probability and independence
  - *N* independent trials,
  - N_A, N_B, N_{AB} : the number of times events *A*, *B* and *AB* occur.
  - For large *N* $P(A) \approx \frac{N_A}{N}, \quad P(B) \approx \frac{N_B}{N}, \quad P(AB) \approx \frac{N_{AB}}{N}.$
  - Conditional probability: P(A|B)

$$P(A \mid B) = \frac{N_{AB}}{N_B} = \frac{N_{AB}/N}{N_B/N} = \frac{P(AB)}{P(B)}$$

2.1. Probability

- Properties of conditional probability
  - P(A|B) is nonnegative:

$$P(A \mid B) = \frac{P(AB) \geq 0}{P(B) > 0} \geq 0,$$

  - P(Ω|B) = 1

$$P(\Omega \mid B) = \frac{P(\Omega B)}{P(B)} = \frac{P(B)}{P(B)} = 1,$$

  - If A∩C = ∅ ,

$$P(A \cup C \mid B) = P(A \mid B) + P(C \mid B),$$

2.1. Probability

- If B ⊂ A then P(A|B) = 1
- If A ⊂ B then P(A|B) > P(A)
- Let, A_1, A_2, …, A_n are pair wise disjoint and their union is Ω:

$$A_i \cap A_j = \varnothing, \qquad \bigcup_{i=1}^{n} A_i = \Omega.$$

  - B is an event

$$P(B) = \sum_{i=1}^{n} P(BA_i) = \sum_{i=1}^{n} P(B \mid A_i) P(A_i).$$

2.1. Probability

- Independence: *A* and *B* events
  P(AB) = P(A) P(B)
  - If A and B are independent events:

$$P(A \mid B) = P(A)$$

- Bayes theorem

$$P(A \mid B) = \frac{P(B \mid A)}{P(B)} \cdot P(A)$$

- Generalized Bayes theorem:

$$P(A_i \mid B) = \frac{P(B \mid A_i) P(A_i)}{P(B)} = \frac{P(B \mid A_i) P(A_i)}{\sum_{i=1}^{n} P(B \mid A_i) P(A_i)},$$

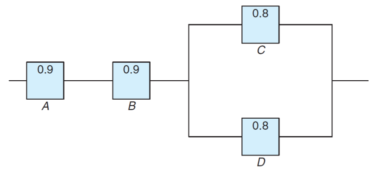
2.1. Probability

- Bayes' theorem interpretation:
  - *P*(*A*) represents the a-priori probability of the event *A*.
  - The event *B* is new knowledge obtained from an experiment.
  - Conditional probability *P(A|B)* of A given B –

  a-posteriori probability
  - The new information should be used to improve knowledge of *A*.

Probability

- Examples
  - An electrical system consists of four components. The system works if components *A* and *B* work and either of the components *C* or *D* works. The reliability (probability of working) of each component is also shown in the Figure. Find the probability that:
    - (a) the entire system works and
    - (b) the component *C* does not work, given that the entire system works. Assume that the four components work independently.



2.1. Probability

- Example:
  - In a box: 6 white and 4 black balls.
  - Remove two balls randomly without replacement.
  - P{the first one is white and the second one is black} = ?
  - Question: are events W_1 and B_2 independent ?
    - W1 = "first ball removed is white"
    - *B*2 = "second ball removed is black"

2.1. Probability

- If, in an experiment, the events $A_1, A_2, \ldots, A_k$ can occur, then

$P(A_1 \cap A_2 \cap \ldots \cap A_k) = P(A1)P(A2|A1)P(A3|A1 \cap A2) \ldots P(A_k|A_1 \cap A_2 \cap \ldots \cap A_{k-1})$.

- If the events $A_1, A_2, \ldots, A_k$ are independent, then

$P(A_1 \cap A_2 \cap \ldots \cap A_k) = P(A_1)P(A_2) \ldots P(A_k)$.

- Example
  - Three cards are drawn in succession, without replacement, from an ordinary deck of playing cards. Find the probability that the event A1 ∩ A2 ∩ A3 occurs, where A1 is the event that the first card is a red ace, A2 is the event that the second card is a 10 or a jack, and A3 is the event that the third card is greater than 3 but less than 7.

2.1. Probability

- Example: Two boxes $B1$ and $B2$ contain 100 and 200 light bulbs respectively. The first box ($B1$) has 15 defective bulbs and the second - 5. Suppose a box is selected at random and one bulb is picked out.
  - (a) What is the probability that it is defective?
  - (b) Suppose we test the bulb and it is found to be defective. What is the probability that it came from box 1?

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2.1. Probability

- Repeated trials, Bernoulli trials
  - Consider n independent experiments with models (Ω_1, F_1, P_1), (Ω_2, F_2, P_2), …, (Ω_n, F_n, P_n).
    - Let $\xi_1 \in \Omega_1$, $\xi_2 \in \Omega_2$,…, $\xi_n \in \Omega_2$: elementary events.
    - A joint performance of the n experiments produces an elementary events $\omega = (\xi_1, \xi_2, \ldots, \xi_n)$.
    - Consider space $\Omega = \Omega_1 \times \Omega_2 \times \ldots \times \Omega_n$: $\xi_1 \in \Omega_1, \ldots, \xi_n \in \Omega_n$,
    - Events in combined space Ω are of the form $A_1 \times A_2 \times \ldots \times A_n$.
    - If n experiments are independent, then $P(A_1 \times A_2 \times \ldots \times A_n) = P(A_1) \times \ldots \times P(A_n)$

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2.1. Probability

- An event A has probability p of occurring in a single trial. Find the probability that A occurs exactly k times in determined location, $k \le n$ in n trials.

$$P_0(\omega) = P(\{\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_k}, \cdots, \xi_{i_n}\}) =$$
$$= P(\{\xi_{i_1}\})P(\{\xi_{i_2}\}) \cdots P(\{\xi_{i_k}\}) \cdots P(\{\xi_{i_n}\}) =$$
$$= \underbrace{P(A)P(A)\cdots P(A)}_{k} \; \underbrace{P(\overline{A})P(\overline{A})\cdots P(\overline{A})}_{n-k} = p^k q^{n-k}.$$

- P{ A occurs exactly k time in n trials } = $C^k_n p^k q^{n-k}$
  - Bernoulli formula.

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2.1. Probability

- De Moivre – Laplace Theorem
  - Guess that: $n \to \infty$ with fixed p.
  - k is in the neighborhood $\sqrt{npq}$ of np.
  - Bernoulli probability estimation:

$$C_n^k p^k q^{n-k} \approx \frac{1}{\sqrt{2\pi npq}} e^{-(k-np)^2/2npq}.$$

- Stirling formula for n! approximation:

$$n! \sim \sqrt{2\pi}\, n^{n+\frac{1}{2}} e^{-n}$$

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2.1. Probability

- Estimation of Bernoulli formula

$$\binom{n}{k} p^k q^{n-k} = \frac{n!}{(n-k)!k!} p^k q^{n-k},$$

$$\binom{n}{k} p^k q^{n-k} > c_1 \sqrt{\frac{n}{2\pi(n-k)k}} \left(\frac{np}{k}\right)^k \left(\frac{nq}{n-k}\right)^{n-k}$$

$$\binom{n}{k} p^k q^{n-k} < c_2 \sqrt{\frac{n}{2\pi(n-k)k}} \left(\frac{np}{k}\right)^k \left(\frac{nq}{n-k}\right)^{n-k}$$

- Các hằng số c_1 và c_2 khá gần nhau.

$$c_1 = e^{\left\{\frac{1}{12n+1} - \frac{1}{12(n-k)} - \frac{1}{12k}\right\}} \qquad c_2 = e^{\left\{\frac{1}{12n} - \frac{1}{12(n-k)+1} - \frac{1}{12k+1}\right\}}.$$

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2.1. Probability

- Example:
  - Toss a coin *n* times. Probability of getting *k* heads in *n* trials = ?
  - Consider rolling a fair dice eight times. Find the probability that either 3 or 4 shows up five times.
  - Suppose 5,000 components are ordered. The probability that a part is defective equals 0.1. What is the probability that the total number of defective parts does not exceed 400 ?

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2.2. Random variables

- Definition
  - (Ω, F, P) - probability model for an experiment,
  - X - a function that maps every $\xi \in \Omega$ to a unique point $x \in R$
  - **Random Variable (r.v)**: a finite single valued function that maps the set of all experimental outcomes Ω into the set of real numbers R is said to be a r.v, if the set $A=\{\xi | X(\xi) \le x\}$ is an event $A \subset F$ for every x in R.

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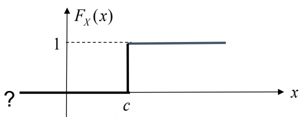
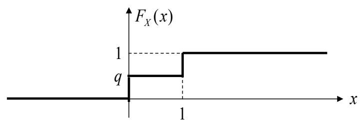
2.2. Random variables

- Probability distribution function
  - Probability distribution function of random variable X: (pdf)
  - $F_X(x)=P\{\xi | X(\xi) \le x\} \ge 0$
  - Examples
    - X – r.v: $X(\xi) = c$, $\xi \in \Omega$, $F_X(x)$ = ?
    - X – r.v: coin tossing, Ω = {H, T}. X(T)=0, X(H)=1. $F_X(x)$ = ?





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2.2. Random variables

- Properties of probability distribution function
  - $F_X(x)$ is a distribution function:
    - $F_X(-\infty) = 0$; $F_X(+\infty) = 1$
    - Pdf is nondecreasing function:
      if $x_1 \le x_2$ then $F_X(x_1) \le F_X(x_2)$
    - Pdf is right continuous: $F_X(x^+) = F_X(x)$ $\forall x$

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2.2. Random variables

- If $F_X(x_0) = 0$ for some x_0,
  then $F_X(x) = 0$ $\forall x \le x_0$
- $P\{ X(\xi) > x \} = 1 – F_X(x)$
- If $x_2 > x_1$, then
  $P\{ x_1 < X(\xi) \le x_2 \} = F_X(x_2) – F_X(x_1)$
- $P\{ X(\xi) = x \} = F_X(x) – F_X(x^-)$

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2.2. Random variables

- Continuous and discrete random variables
  - X – continuous r.v if $F_X(x)$ is continuous
    - For continuous r.v, $F_X(x^-) = F_X(x)$ and $P\{X=x\} = 0$
  - If $F_X(x)$ = const, except for a finite number of jump discontinuities, then X is said to be a discrete-type r.v.
    - If x_i is such a discontinuity point, then
      $p_i = P\{ X = x_i \} = F_X(x) - F_X(x^-)$

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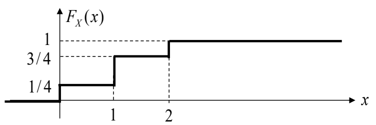
2.2. Random variables

- Examples
  - A fair coin is tossed twice, and let the r.v X represent the number of heads. Find $F_X(x)$



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2.2. Random variables

- Distribution density function
  - Derivative of distribution function $F_X(x)$ is density function $f_X(x)$ of the r.v.

$$f_X(x) = \frac{dF_X(x)}{dx}.$$

  - $F_X(x)$ is monoton nondecreasing function, so that

$$f_X(x) = \frac{dF_X(x)}{dx} = \lim_{\Delta x \to 0} \frac{F_X(x+\Delta x) - F_X(x)}{\Delta x} \geq 0,$$

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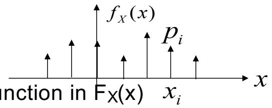
2.2. Random variables

- If r.v is continuous then $f_X(x)$ is continuous.
- For discrete r.v

$$f_X(x) = \sum_i p_i \delta(x - x_i),$$



- x_i is jump-discontinuity point function in $F_X(x)$

$$F_X(x) = \int_{-\infty}^{x} f_x(u)du.$$

- $$F_X(+\infty) = 1, \quad \int_{-\infty}^{+\infty} f_x(x)dx = 1,$$

$$P\{x_1 < X(\xi) \leq x_2\} = F_X(x_2) - F_X(x_1) = \int_{x_1}^{x_2} f_X(x)dx.$$

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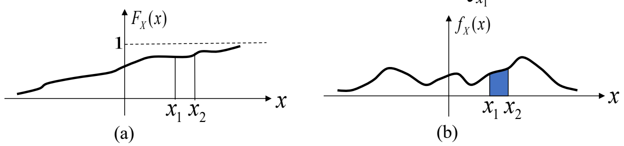
2.2. Random variables

$$P\{x_1 < X(\xi) \leq x_2\} = F_X(x_2) - F_X(x_1) = \int_{x_1}^{x_2} f_X(x)dx.$$



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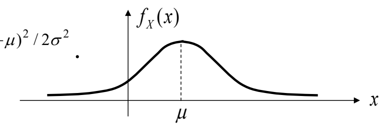
2.2. Random variables

- Some continuous random variables
  - Normal (Gaussian) random variables $X \sim N(\mu, \sigma^2)$
    - Density function

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}.$$



    - Bell shape curve
    - Distribution function

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\mu)^2/2\sigma^2} dy = G\left(\frac{x-\mu}{\sigma}\right),$$

$$G(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

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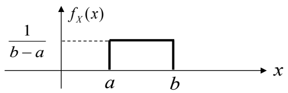
2.2. Random variables

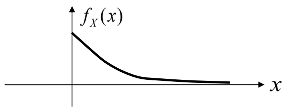
- Uniform random variables: $X \sim U(a,b),\ a<b,$
  - Density function

$$f_X(x)=\begin{cases}\dfrac{1}{b-a}, & a\le x\le b,\\ 0, & \text{otherwise.}\end{cases}$$



- Exponential random variables $X \sim \varepsilon(\lambda)$
  - Density function

$$f_X(x)=\begin{cases}\dfrac{1}{\lambda}e^{-x/\lambda}, & x\ge 0,\\ 0, & \text{otherwise.}\end{cases}$$



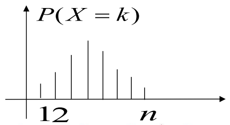
2.2. Random variables

- Some important discrete random variables
  - Bernoulli r.v: X takes values 0, 1

$$P(X=0)=q,\quad P(X=1)=p.$$

  - Binomial r.v $X \sim B(n,p),$

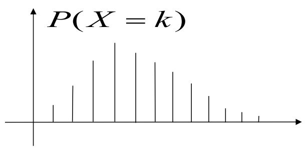
- $$P(X=k)=\binom{n}{k}p^k q^{n-k},\ k=0,1,\cdots,n.$$



Random variables

- Poisson r.v $X \sim P(\lambda),$

$$P(X=k)=e^{-\lambda}\frac{\lambda^k}{k!},\ k=0,1,\cdots,\infty.$$



2.3 Some characteristics of random variables

- Mean value

$$\eta_X=\overline{X}=E(X)=\int_{-\infty}^{+\infty}x\,f_X(x)dx.$$

  - For discrete random variable

$$\eta_X=\overline{X}=E(X)=\int x\sum_i p_i\delta(x-x_i)dx=\sum_i x_i p_i \underbrace{\int \delta(x-x_i)dx}_{1}$$

$$=\sum_i x_i p_i=\sum_i x_i\,P(X=x_i).$$

Mean and Variance

- Example: uniform random variable

$$E(X)=\int_a^b \frac{x}{b-a}dx=\frac{1}{b-a}\frac{x^2}{2}\Big|_a^b=\frac{b^2-a^2}{2(b-a)}=\frac{a+b}{2}$$

- Example: exponential random variable

$$E(X)=\int_0^\infty \frac{x}{\lambda}e^{-x/\lambda}dx=\lambda\int_0^\infty ye^{-y}dy=\lambda,$$

- Example: Poisson random variable

$$E(X)=\sum_{k=0}^{\infty}kP(X=k)=\sum_{k=0}^{\infty}ke^{-\lambda}\frac{\lambda^k}{k!}=e^{-\lambda}\sum_{k=1}^{\infty}k\frac{\lambda^k}{k!}$$

$$=e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k-1)!}=\lambda e^{-\lambda}\sum_{i=0}^{\infty}\frac{\lambda^i}{i!}=\lambda e^{-\lambda}e^{\lambda}=\lambda.$$

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2.3 Some characteristics of random variables

- Variance
  - For r.v X with mean μ
  - Variance $\sigma_X^2=E[(X-\mu)^2]>0.$
  - Or $\sigma_X^2=\int_{-\infty}^{+\infty}(x-\mu)^2 f_X(x)dx>0.$
  - Standard deviation $\sigma_X=\sqrt{E(X-\mu)^2}$
  - Variance anf means

$$Var(X)=\sigma_X^2=E[(X-\mu)^2]=E[X^2-2X\mu+\mu^2]=$$

$$E[X^2]-E[2X\mu]+E[\mu^2]=E[X^2]-2E[X]\mu+\mu^2$$

$$=E(X^2)-[E(X)]^2=\overline{X^2}-\overline{X}^2.$$

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2.3 Some characteristics of random variables

- Example:
- Example: variance of Poisson r.v.

$$\sigma_x^2=\overline{X^2}-\overline{X}^2=(\lambda^2+\lambda)-\lambda^2=\lambda.$$

- Example: variance of Gaussian r.v.

$$Var(X)=E[(X-\mu)^2]=\int_{-\infty}^{+\infty}(x-\mu)^2\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}dx.$$

$$Var(X)=\sigma^2$$

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2.3 Some characteristics of random variables

- Moments
  - X – random variables

$$m_n=\overline{X^n}=E(X^n),\quad n\geq 1$$

  - Central moments

$$\mu_n=E[(X-\mu)^n]$$

  - Relation between moments and central moments

$$\mu_n=E[(X-\mu)^n]=\sum_{k=0}^{n}C_n^k\ m_k\ (-\mu)^{n-k}.$$

  - Mean and variance:

$$\mu=m_1,\qquad \sigma^2=\mu_2.$$

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2.3 Some characteristics of random variables

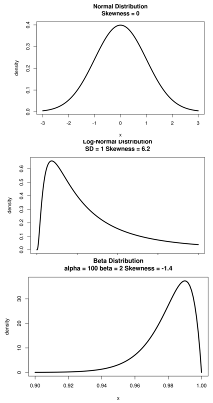
- Generalized moments of X about *a*

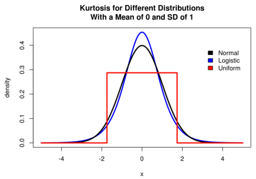
$$E[(X-a)^n]$$

- Absolute moments of X

$$E[|X|^n]$$

- Measure of skewness is $E[(X - \mu_X)^3]/\sigma^3_X$
- Measure of kurtosis is $E[(X - \mu_X)^4]/\sigma^4_X$.





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2.3 Some characteristics of random variables

- Characteristic function
  - Continuous r.v X

$$\Phi_X(\omega) \overset{\Delta}{=} E\left(e^{jX\omega}\right) = \int_{-\infty}^{+\infty} e^{jx\omega} f_X(x)dx.$$

  - We have:

$$\Phi_X(0) = 1,$$

  - And

$$|\Phi_X(\omega)| \le 1 \qquad \forall\omega$$

  - Discrete r.v X

$$\Phi_X(\omega) = \sum_k e^{jk\omega} P(X = k).$$

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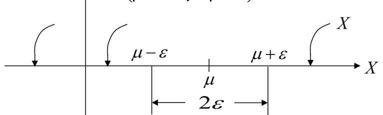
2.4 Chebychev Inequality and Law of Large numbers

- Chebychev Inequality
  - Consider an interval of width 2ε symmetrically centered around its mean μ

$$P(|X-\mu| \ge \varepsilon)\ ?$$



  - Chebychev Inequality

$$P(|X-\mu| \ge \varepsilon) \le \frac{\sigma^2}{\varepsilon^2},$$

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2.4 Chebychev Inequality and Law of Large numbers

- Weak law and strong law of large numbers
  - X_i - independent, identically distributed Bernoulli random variables:

$$P(X_i) = p, \qquad P(X_i = 0) = 1 - p = q,$$

  - k = $X_1 + X_2 + ... + X_n$ – number of successes in *n* trials
  - Weak law of large numbers:

$$P\left\{\left|\frac{k}{n} - p\right| > \varepsilon\right\} \le \frac{pq}{n\varepsilon^2}.$$

  - Strong law of large numbers:
    - The ratio *k/n* tends to *p* not only *in probability*, but *with probability* 1

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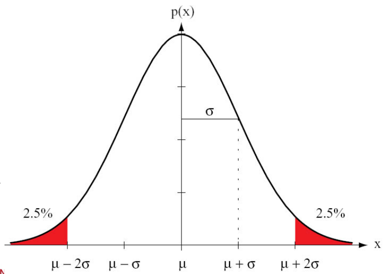
2.5. Gaussian distribution

- Gaussian distribution
  - One dimentional normal distribution

$$p(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$\mu \equiv E(x)=\int_{-\infty}^{\infty} xp(x)dx$$

$$\sigma^2 \equiv E((x-\mu)^2)=\int_{-\infty}^{\infty}(x-\mu)^2 p(x)dx$$



2.5. Gaussian distribution

- Multidimentional Gaussian distribution (*d* dimentional)

$$p(\mathbf{x})=\frac{1}{(2\pi)^{d/2}\left|\sum\right|^{1/2}}\exp\left[-\frac{1}{2}(\mathbf{x}-\mu)^T\sum{}^{-1}(\mathbf{x}-\mu)\right]$$

$$\mu \equiv E(\mathbf{x})=\int \mathbf{x}p(\mathbf{x})d\mathbf{x}$$

$$\Sigma \equiv E((\mathbf{x}-\mu)(\mathbf{x}-\mu)^T)=\int(\mathbf{x}-\mu)(\mathbf{x}-\mu)^T p(\mathbf{x})d\mathbf{x}$$

- Where, **x**, **μ** are *d*-dimentional vector
- $\sum$ covariance matrix - symmetric and semi-positive defined matrix.
- Consider cases, when $\sum$ positive defined.

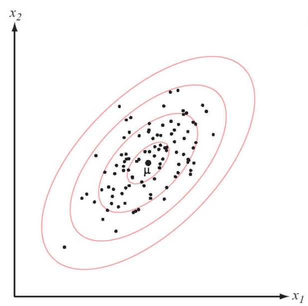
$$\mu_i = E(x_i) \qquad \sigma_{ij}=E((x_i-\mu_i)(x_j-\mu_j))$$

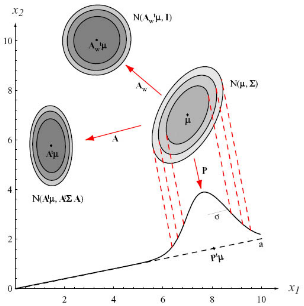
2.5. Gaussian distribution

- Covariance matrix

$$\Sigma=\begin{bmatrix}\sigma_{00} & & & \\ & \sigma_{11} & \sigma_{ij,i\neq j} & \\ & & \sigma_{22} & \\ & & & \sigma_{33}\end{bmatrix}$$

2.5. Gaussian distribution





2.5. Gaussian distribution

- Multidimentional Gaussian distribution is fully defined by d+d(d+1)/2 parameters of mean vector $\boldsymbol{\mu}$ and covariance matrix Σ.
- Mahalanobis distance:

$$r = (\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})$$

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