Machine Learning

(Học máy – IT3190E)

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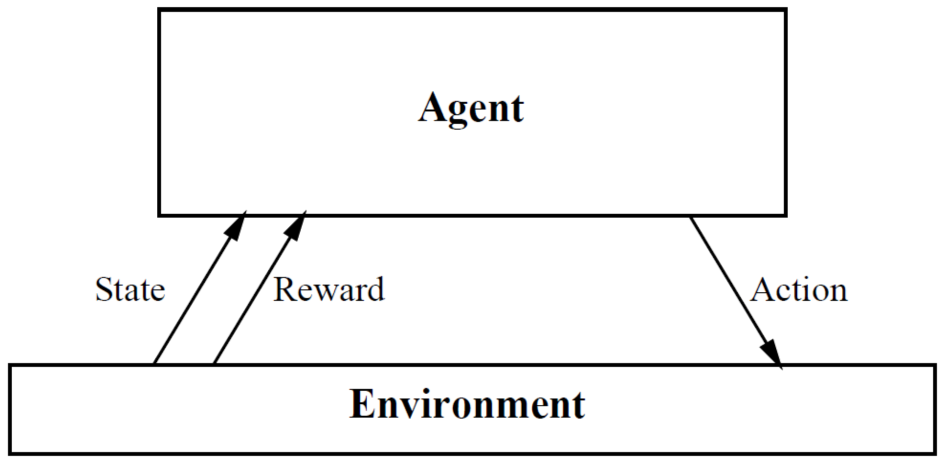
Reinforcement Learning problem



$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2} \dots$$

- **Goal:** Learn to choose actions that maximize

$$r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots, \qquad where\ 0 \leq \gamma < 1$$

(γ is the discount factor for future rewards)

(Mitchell, 1997)

Characteristics of Reinforcement learning

- What makes Reinforcement Learning (RL) different from other machine learning paradigms?
  - There is no explicit supervisor, only a **reward** signal
  - Training examples are of form $((S, A), R)$
  - Feedback is often delayed
  - Time really matters (**sequential**, not independent data)
  - Agent's actions affect the subsequent data it receives
- Examples of RL
  - Play games better than humans
  - Manage an investment portfolio
  - Make a humanoid robot walk
  - …

Reward

- A **reward** R_t is a scalar feedback signal
- Indicates how well agent is doing at step t
- The agent's job is to maximize cumulative reward
- Reinforcement learning is based on the **reward hypothesis**:
  - All goals can be described by the maximization of expected cumulative reward

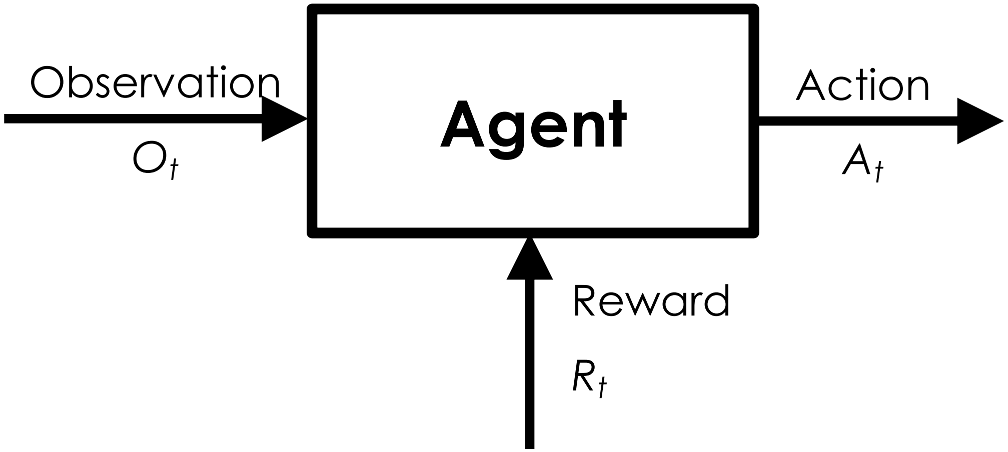
Examples of reward

- Play games better than humans
  - + reward for increasing score
  - - reward for decreasing score
- Manage an investment portfolio
  - + reward for each $ in bank
- Make a humanoid robot walk
  - + reward for forward motion
  - - reward for falling over

Sequential decision making

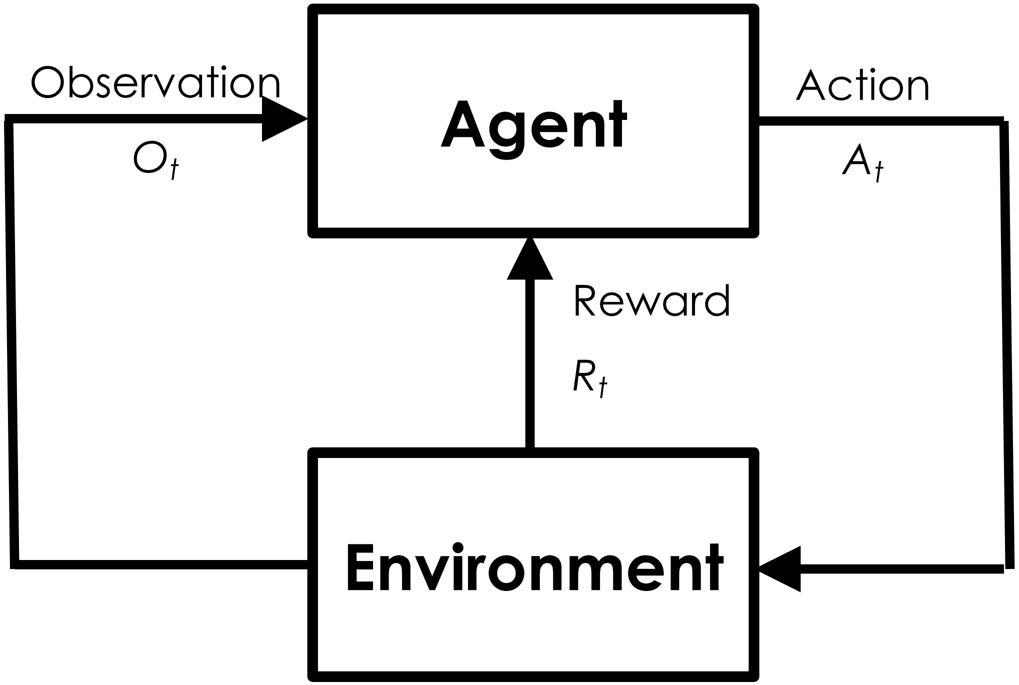
- Goal: **Select actions to maximize total future reward**
- Actions may have long term consequences
- Reward may be delayed
- It may be better to sacrifice an immediate reward to gain more long-term reward
- Examples:
  - A financial investment (may take months to mature)
  - Blocking opponent moves (might help winning chances, after many moves from now)

Agent and Environment (1)



- At each step t, the agent:
  - Executes action A_t
  - Receives observation O_t
  - Receives scalar reward R_t

Agent and Environment (2)



- At each step t, the agent:
  - Executes action A_t
  - Receives observation O_t
  - Receives scalar reward R_t
- At each step t, the environment:
  - Receives action A_t
  - Emits observation O_{t+1}
  - Emits scalar reward R_{t+1}
- t increments at environment step

History and State

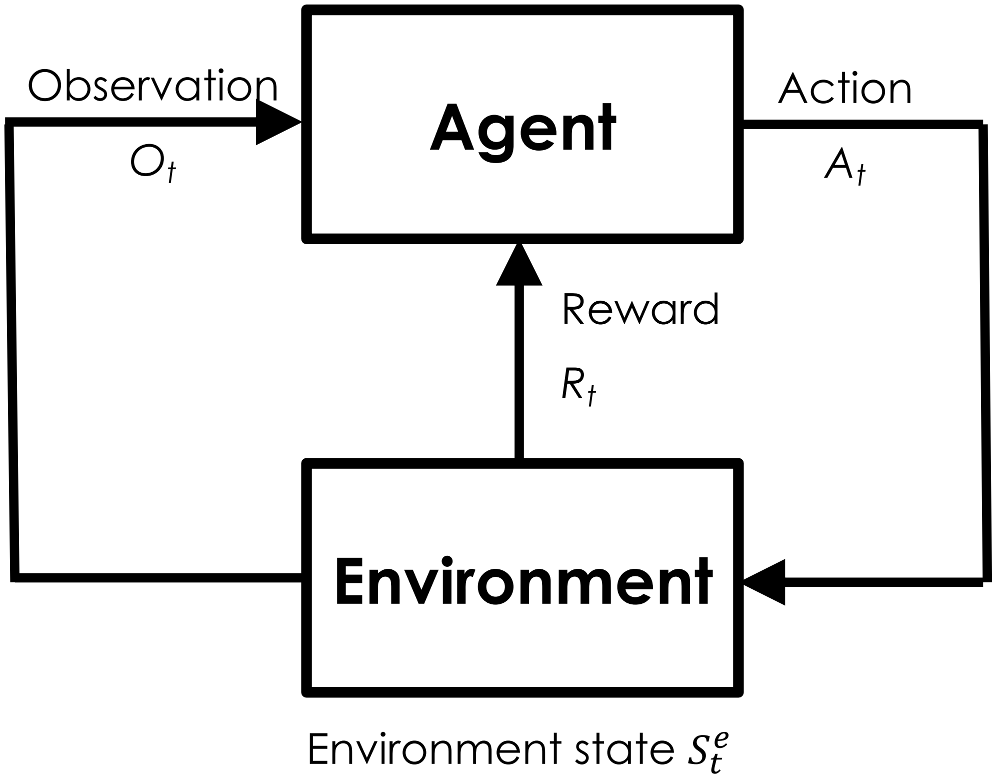
- The **history** is the sequence of observations, actions, rewards:

$$H_t = O_1, R_1, A_1, \ldots, A_{t-1}, O_t, R_t$$

  - All observable variables up to time *t*
  - The sensorimotor stream of the agent
- What happens next depends on the history:
  - The agent selects actions
  - The environment selects observations/rewards
- **State** is the information used to determine what happens next
- Formally, state is a function of the history:

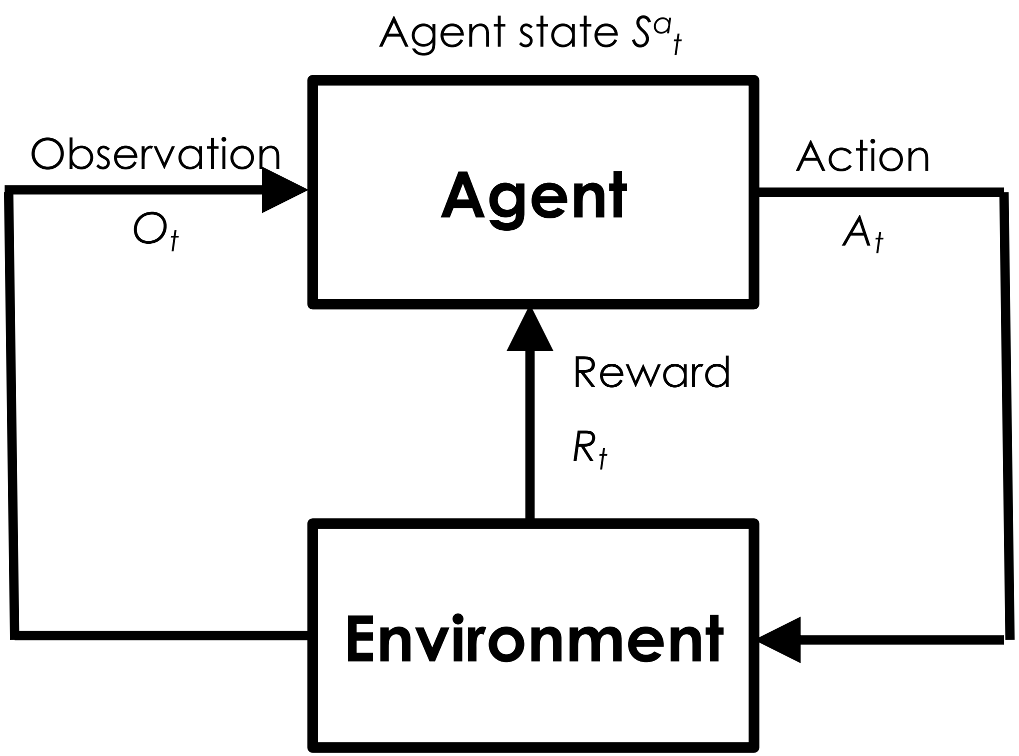
$$S_t = f(H_t)$$

Environment state



- The **environment state** S_t^e is the environment's private representation
  - The information the environment uses to pick the next observation or reward
- The environment state is not usually visible to the agent

Agent state



- The **agent state** S_t^a is the agent's internal representation
  - The information the agent uses to pick the next action
  - It is the information used by reinforcement learning algorithms
- It can be a function of history:

$$S_t^a = f(H_t)$$

Information state

- An **information state** (a.k.a. Markov state) contains all useful information from the history
- A state S_t is **Markov** if and only if:

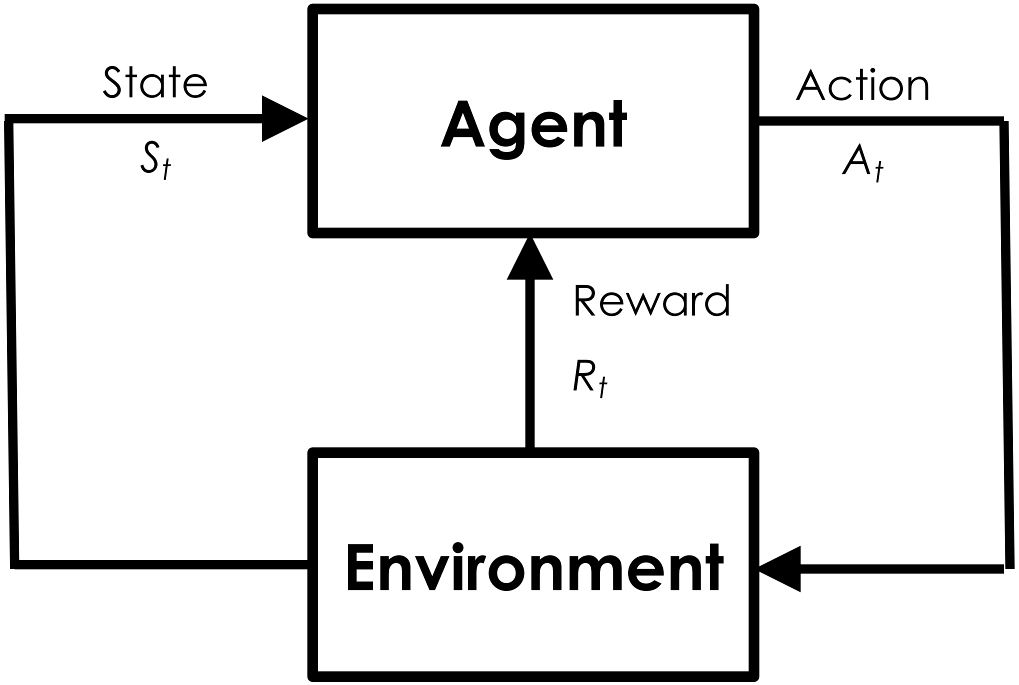
$$P(S_{t+1}|S_t) = P(S_{t+1}|\, S_1, \dots, S_t)$$

  - The future is independent of the past given the present

$$H_{1:t} \rightarrow S_t \rightarrow H_{t+1:\infty}$$

  - Once the state is known, the history may be thrown away
  - The state is a sufficient statistic of the future
  - The environment state S_t^e is Markov
  - The history H_t is Markov

Fully observable environments



- **Full observability**: Agent **directly** observes environment state

$$O_t = S_t^a = S_t^e$$

- Agent state = Environment state = Information state
- Formally, this is a Markov decision process (MDP)

Partially observable environments

- **Partial observability**: Agent **indirectly** observes environment:
  - E.g., a robot with camera vision isn't told its absolute location
  - E.g., a trading agent only observes current prices
  - E.g., a poker playing agent only observes public cards
- Now, Agent state ≠ Environment state
- Formally this is a **partially observable Markov decision process (POMDP)**
- Agent must construct its own state representation S_t^a:
  - E.g., by using complete history: $S_t^a = H_t$
  - E.g., by using a recurrent neural network: $S_t^a = \sigma(S_{t-1}^a W_s + O_t W_o)$

Major components of a RL agent

A RL agent may include one or more of these components:

- **Policy**: Agent's behavior function
- **Value function**: How good is each state and/or action
- **Model**: Agent's representation of the environment

Policy

- A **policy** is the agent's behavior
- It is a map from state to action
- *Deterministic* policy: $a = \pi(s)$
- *Stochastic* policy: $\pi(a|s) = P(A_t = a \,|S_t = s)$

Value function

- **Value function** is a prediction of future reward
- Used to evaluate the goodness/badness of states
- And therefore, to select between actions

$$v_\pi(s) = \mathbb{E}_\pi(R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots | S_t = s)$$

  where $R_{t+1}, R_{t+2}, \ldots$ are generated by following policy π starting at state *s*

- For each policy π, we have a value $v_\pi(s)$
- We want to find the *optimal policy* π^* such that

$$v^*(s) = \max_\pi v_\pi(s), \qquad \forall s$$

Model

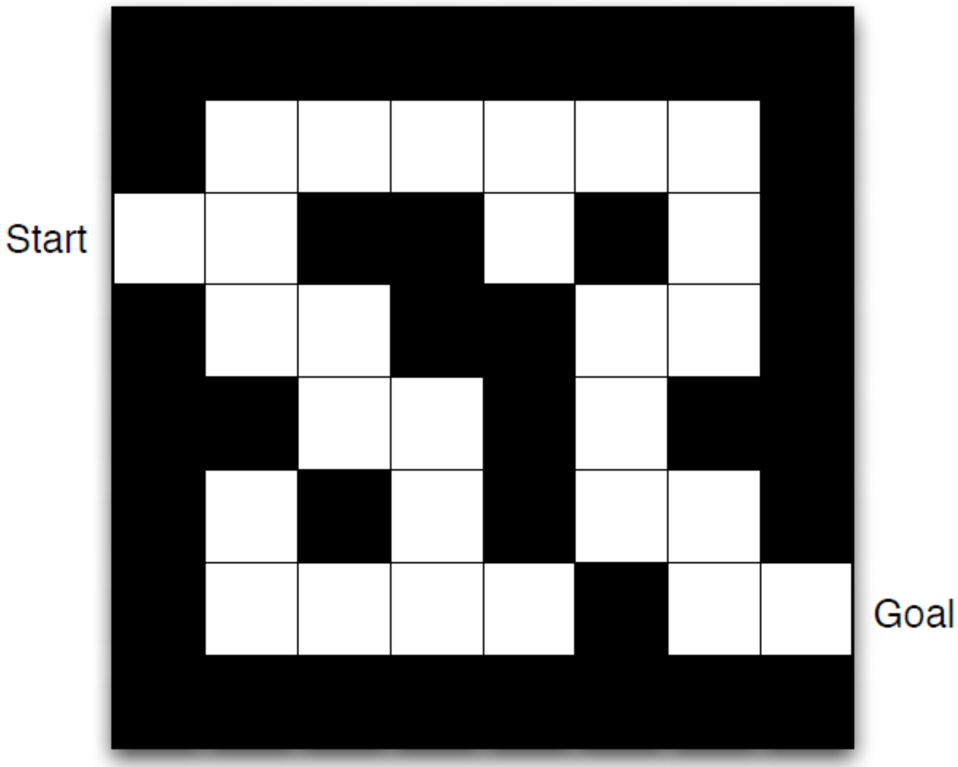
- A **model** predicts what the environment will do next
- *P* predicts the next state

$$P_{ss^*}^a = P(S_{t+1} = s^* \,|S_t = s, A_t = a)$$

- *R* predicts the next (*immediate*) reward

$$R_s^a = \mathbb{E}(R_{t+1}|S_t = s;\ A_t = a)$$

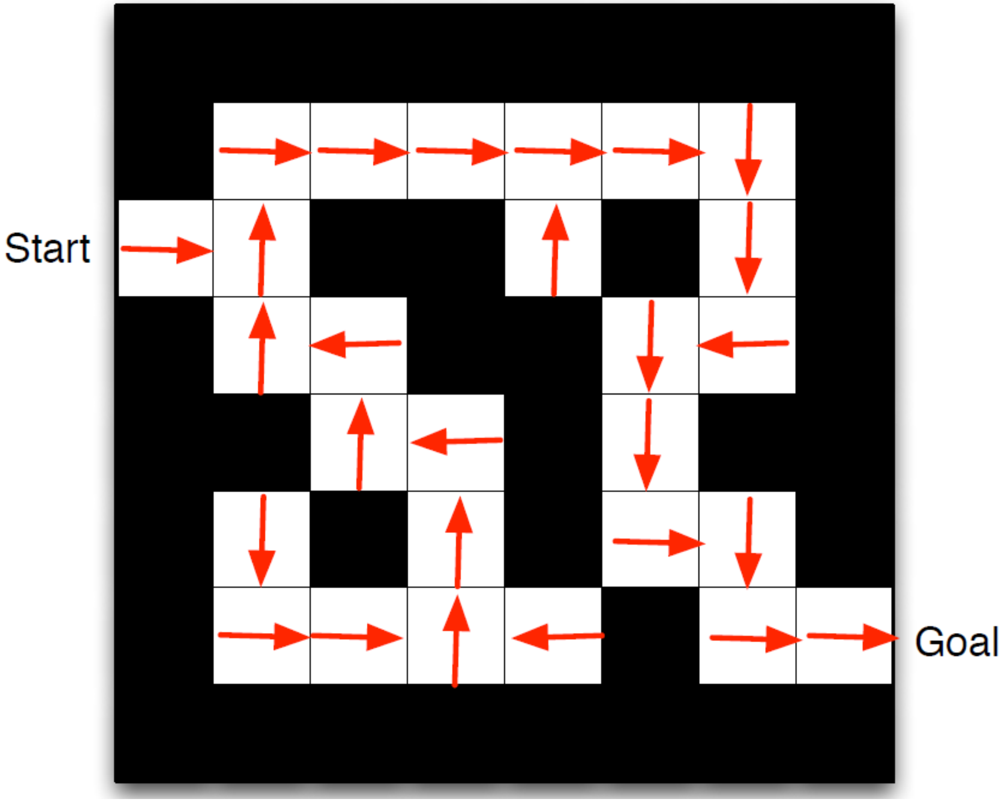
Maze example



- Rewards: -1 per time-step
- Actions: N, E, S, W
- States: Agent's location

(https://www.davidsilver.uk/wp-content/uploads/2020/03/intro_RL.pdf)

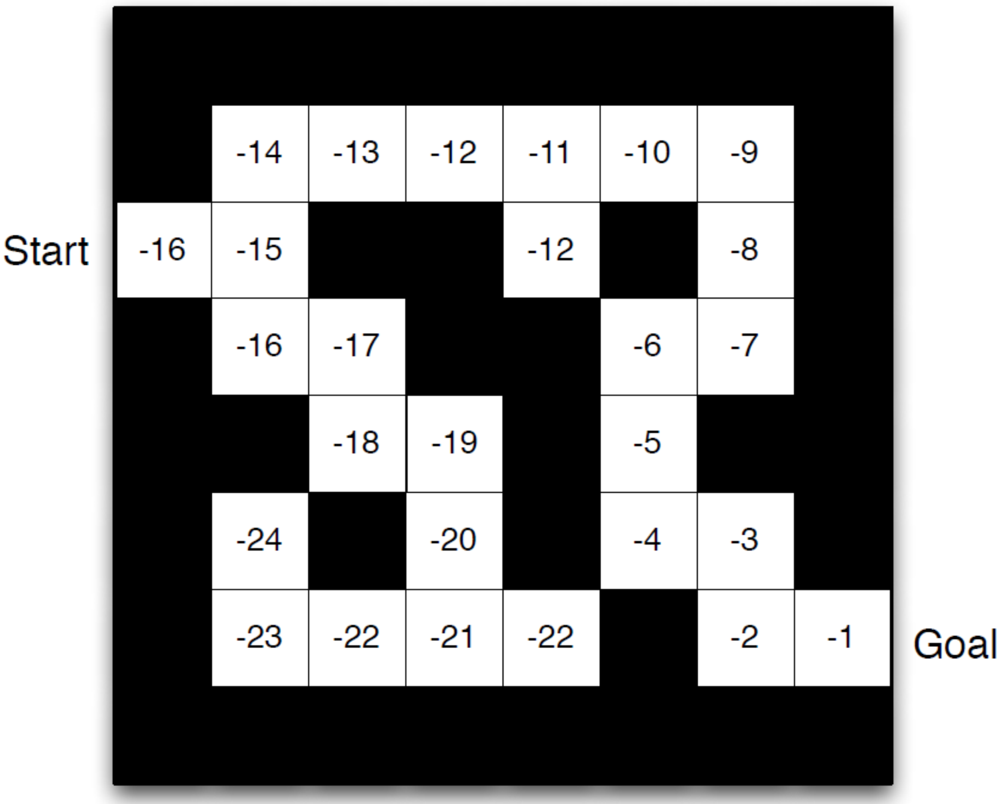
Maze example: Policy



- Arrows represent policy $\pi(s)$ for each state s

(https://www.davidsilver.uk/wp-content/uploads/2020/03/intro_RL.pdf)

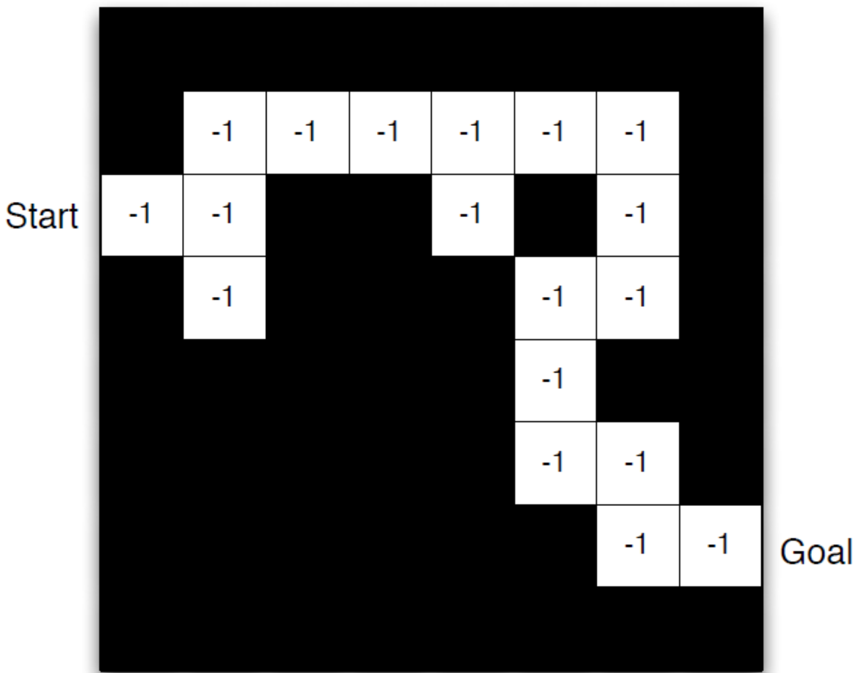
Maze example: Value function



- Numbers represent value $v_\pi(s)$ of each state s

(https://www.davidsilver.uk/wp-content/uploads/2020/03/intro_RL.pdf)

Maze example: Model



(https://www.davidsilver.uk/wp-content/uploads/2020/03/intro_RL.pdf)

- Agent may have an internal model of the environment
- Dynamics: How actions change the state
- Rewards: How much reward from each state
- Grid layout represents transition model $P_{ss'}^{a}$
- Numbers represent immediate reward R_s^a from each state *s* (same for all actions *a*)

Categorizing RL agents (1)

- Value-based
  - No policy
  - Value function
- Policy-based
  - Policy
  - No value function
- Actor critic
  - Policy
  - Value function

Categorizing RL agents (2)

- Model-free
  - Policy and/or Value function
  - No model
- Model-based
  - Policy and/or Value function
  - Model

Exploration and Exploitation (1)

- Reinforcement learning is like *trial-and-error learning*
- The agent should discover a good policy
- from its experiences of the environment
- without losing too much reward along the way

Exploration and Exploitation (2)

- **Exploration** finds more information about the environment
- **Exploitation** exploits known information to maximize reward
- It is usually important to **both** *explore* and *exploit*

Exploration and Exploitation: Examples

- Restaurant selection
  - Exploitation: Go to your favorite restaurant
  - Exploration: Try a new restaurant
- Online banner advertisements
  - Exploitation: Show the most successful advertisement
  - Exploration: Show a different advertisement
- Game playing
  - Exploitation: Play the move you believe is best
  - Exploration: Play an experimental move

Q-Learning: What to learn

- We might try to have agent learn the value function v_π
- It could then do a *lookahead* search to choose best action from any state s because

$$\pi(s) = \arg\max_a \Big(r(s,a) + \gamma v_\pi\big(\delta(s,a)\big) \Big)$$

  - δ: $S \times A \rightarrow S$ will map a given action a and state s to *the next state*
  - r: $S \times A \rightarrow R$ provides the reward of action a, from state s
- A problem:
  - This works well if agent knows functions δ and r
  - But when it doesn't, it can't choose actions by this way

Q-Function

- Define new function very similar to v:

$$Q(s,a) = r(s,a) + \gamma v_\pi(\delta(s,a))$$

  - ❖ $Q(s,a)$ shows how good it is to perform action a when in state s
  - ❖ whereas $v_\pi(s)$ shows how good it is for the agent to be in state s

- If agent learns Q, it can choose optimal action

$$\pi(s) = \arg\max_a \left(r(s,a) + \gamma v_\pi(\delta(s,a)) \right) = \arg\max_a Q(s,a)$$

- Q is the value function the agent will learn

Training rule to learn Q

- Note that Q and v_π are closely related

$$v_\pi(\mathrm{s}) = \max_{a'} Q(s, a')$$

- Which allows us to write Q recursively as

$$\begin{aligned} Q(s_t, a_t) &= r(s_t, a_t) + \gamma v_\pi(\delta(s_t, a_t)) \\ &= r(s_t, a_t) + \gamma v_\pi(s_{t+1}) \\ &= r(s_t, a_t) + \gamma \max_{a'} Q(s_{t+1}, a') \end{aligned}$$

- Let Q* denote learner (agent)'s current approximation to Q, consider the training rule

$$Q^*(s, a) \leftarrow r(s, a) + \gamma \max_{a'} Q^*(s', a')$$

  - where s' is the state resulting from applying action a in state s

Q-Learning for deterministic worlds

For each s, initialize table entry $Q^*(s,a) \leftarrow 0$

Observe current state s

Do forever:

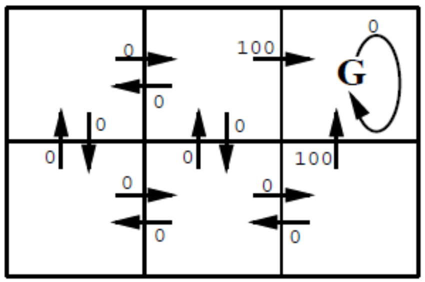
- Select an action a and execute it
- Receive immediate reward r
- Observe the new state s'
- Update the table entry for $Q^*(s,a)$ as follows:

$$Q^*(s,a) \leftarrow r + \gamma \max_{a\prime} Q^*(s',a')$$

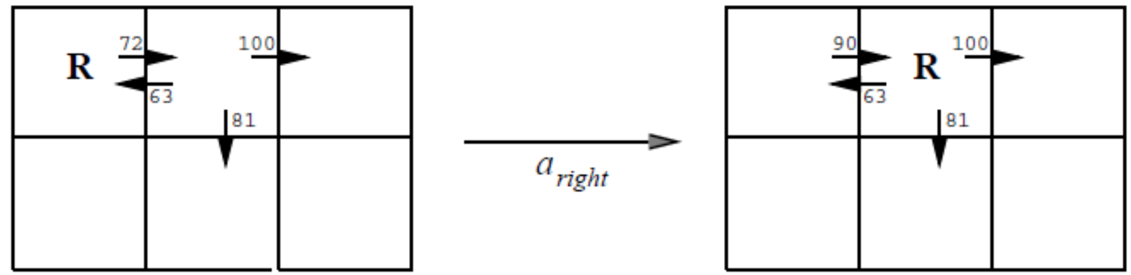
- $s \leftarrow s'$

Note:
- *Finite* action space
- *Finite* state space

Updating Q*



$r(s, a)$ (immediate reward) values



initial state: s_1 next state: s_2

- $Q^*(s_1, a_{right}) \leftarrow r + \gamma.\left(\max_{a'} Q^*(s_2, a')\right)$

$$\leftarrow 0 + 0.9 \,.\, max(63, 81, 100)$$

$$\leftarrow 90$$

- Note that if rewards are non-negative, then

$$\forall\, s, a, n\colon\ Q^*_{n+1}(s, a) \geq Q^*_n(s, a)$$

$$\forall\, s, a, n\colon\ 0 \leq Q^*_n(s, a) \leq Q(s, a)$$

  - Where Q^*_n is the value at iteration n

(Mitchell, 1997)

Q-Learning for non-deterministic worlds

- What if reward and next state are non-deterministic?
- We redefine v_π and Q by taking expected values

$$v_\pi(s) = \mathbb{E}[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots]$$

$$Q(s,a) = \mathbb{E}\left[r(s,a) + \gamma v_\pi\big(\delta(s,a)\big)\right]$$

$$= \sum_{s\prime,r} P(s',r|\, s,a)\,[r + \gamma v_\pi(s')]$$

- Q-learning generalizes to non-deterministic worlds
  - Alter the training rule at iteration n to:

$$Q_n^*(s,a) \leftarrow (1-\alpha_n).\, Q_{n-1}^*(s,a) + \ \alpha_n\left[r + \ \max_{a\prime} Q_{n-1}^*(s',a')\right]$$

  - where α_n is sometimes known as learning rate

References

- D. Silver. *Lecture 1: Introduction to Reinforcement Learning* (https://www.davidsilver.uk/wp-content/uploads/2020/03/intro_RL.pdf).
- T. M. Mitchell. *Machine Learning*. McGraw-Hill, 1997.