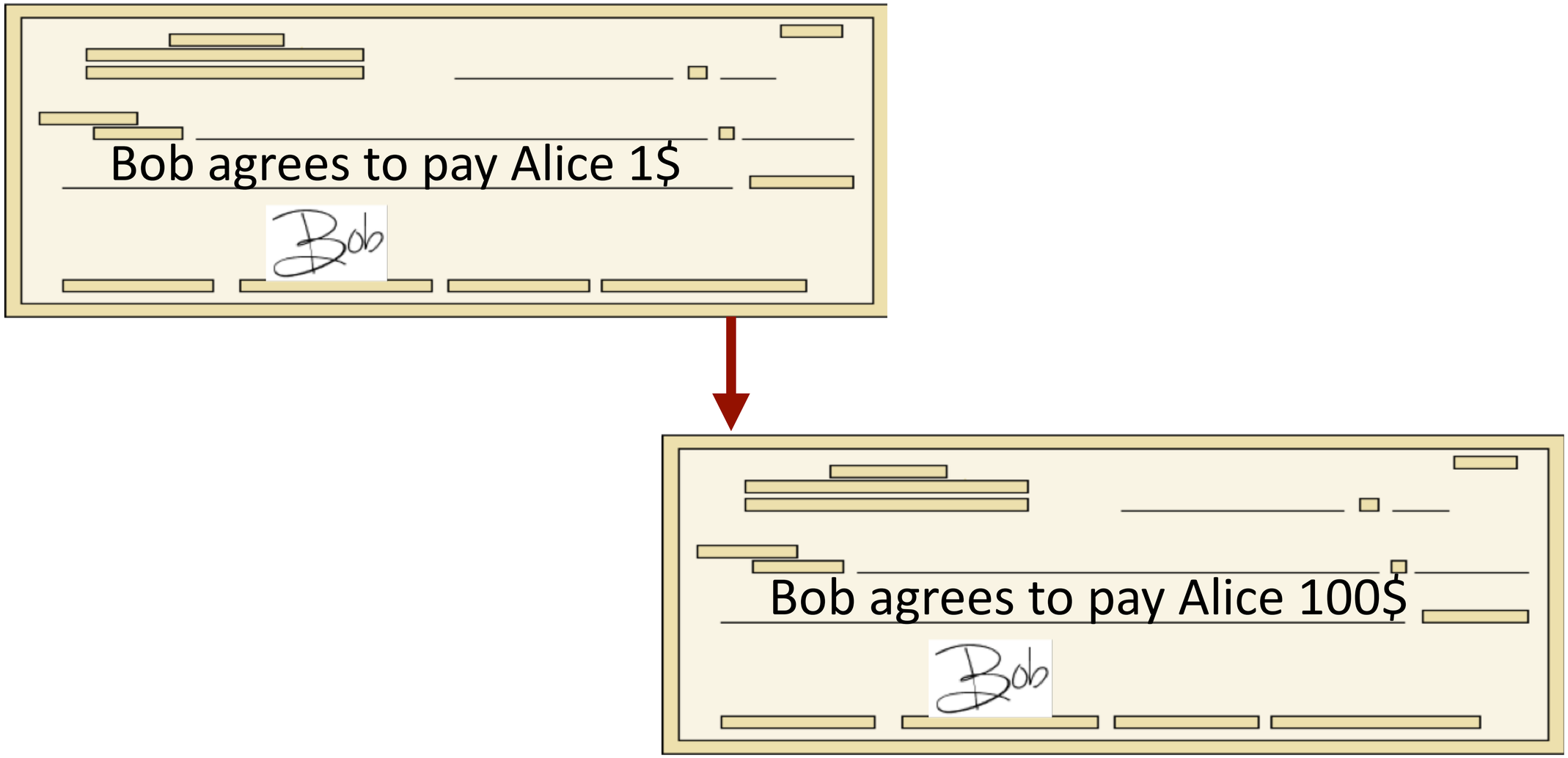
HA NOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF INFORMATION AND COMMUNICATION TECHNOLOGY

Introduction to Cryptography and Security

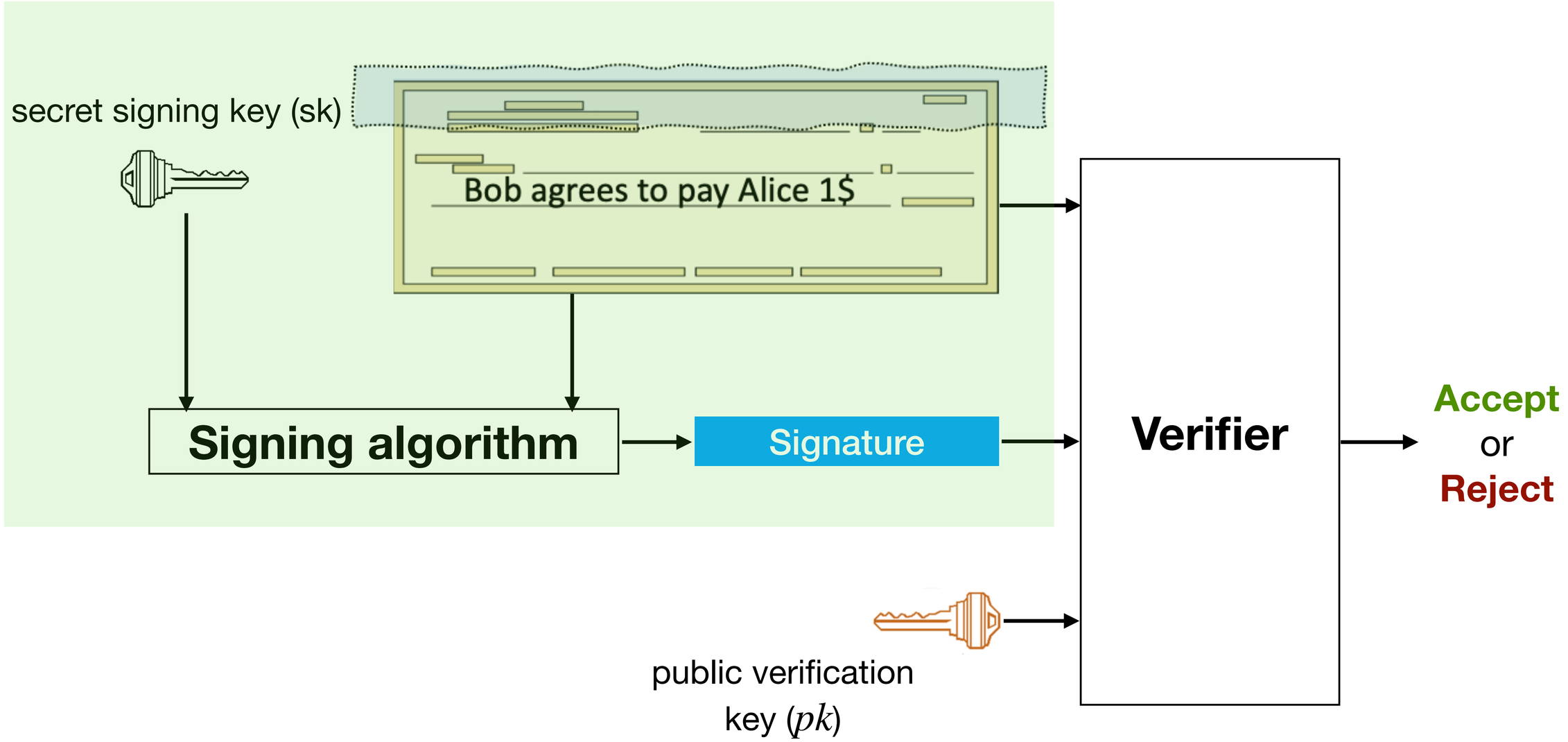
Digital Signatures

Contents

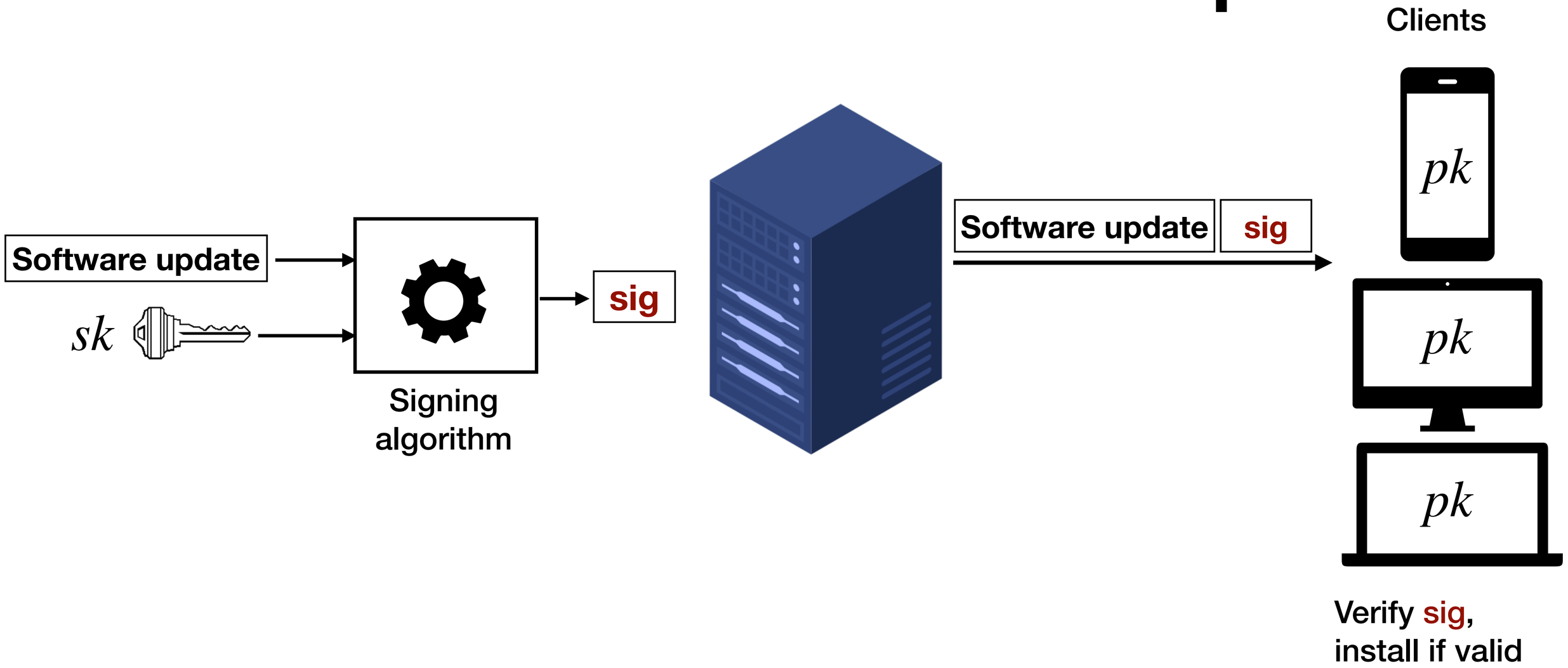
Physical signatures



Solution: make signature depend on document



A more realistic example



Digital signatures: syntax

Definition. A *signature scheme* $(\ Gen,\ S,\ V\ )$ is a triple of algorithms:

- $Gen()$ randomized algorithm outputs a key pair (pk, sk)
- $S(sk, m \in M)$ outputs signature σ
- $V(pk, m, \sigma)$ output '**accept**' or '**reject**'

Consistency

- for all (pk, sk) output by $Gen()$,
- and for all message $m \in M$, we have

$$V(pk,\ m,\ S(sk, m)) \ = \text{‘}\textbf{accept}\text{’}$$

Security

- Attacker's power: **Chosen message attack**

  for $m_1, m_2, \ldots, m_q$, attacker is given $\sigma_i = S(sk, m_i)$

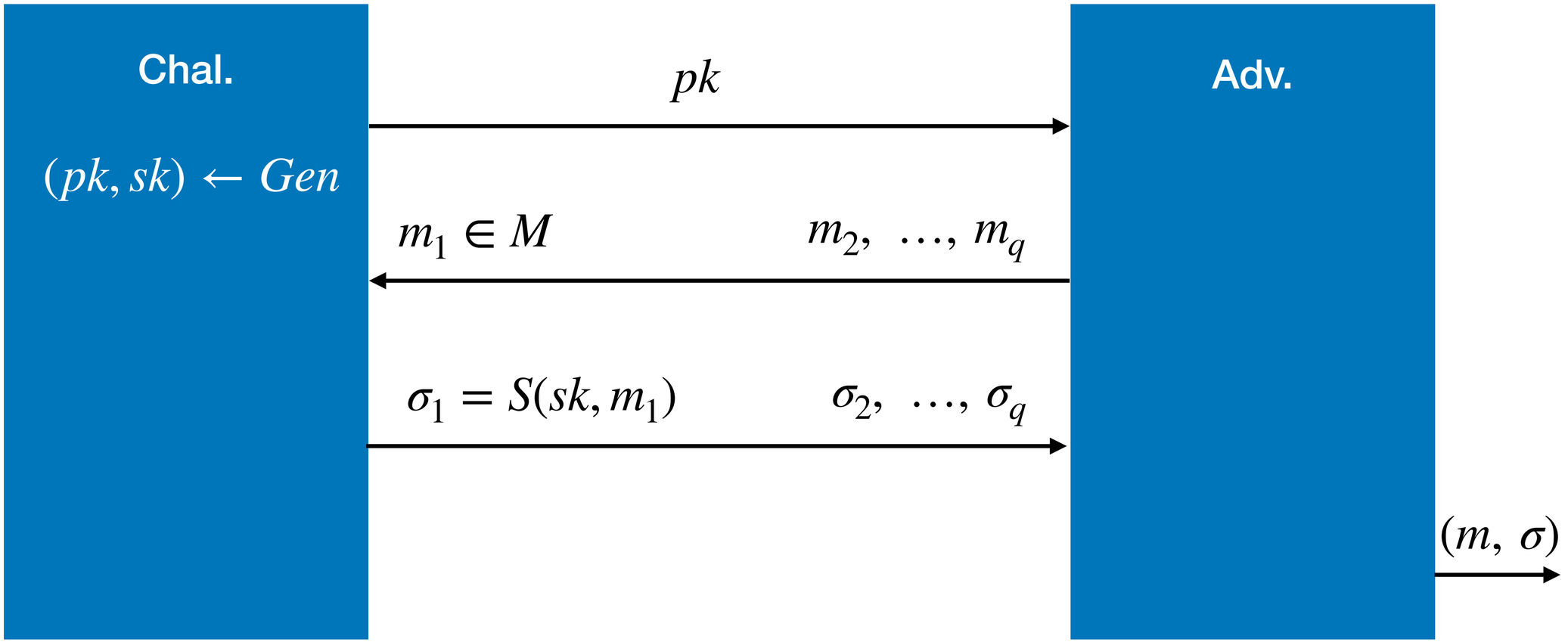
- Attacker's goal: **Existential forgery**

  produce some new valid message/sig pair (m, σ).

  $m \notin \{m_1, \ldots, m_q\}$

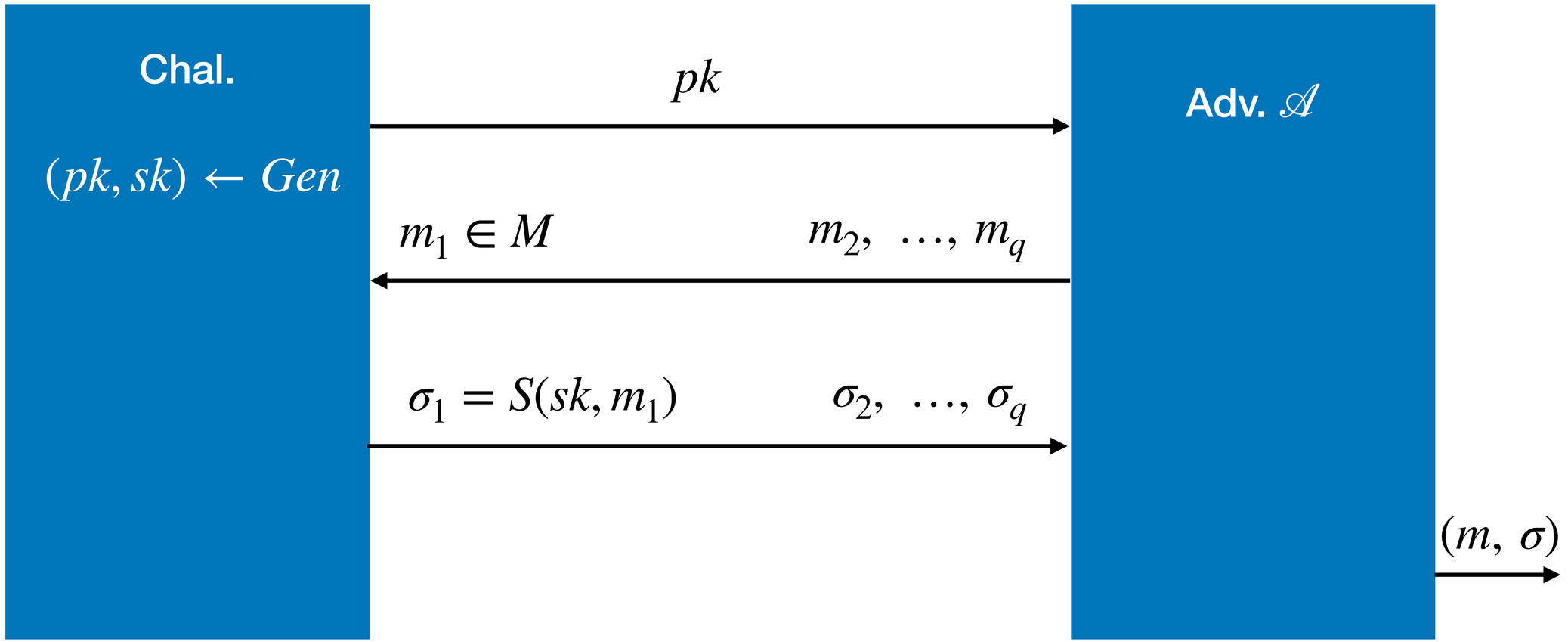
$\Rightarrow$ attacker cannot produce a valid sig. for a new message

Secure signature



Adv. **win** if $V(pk, \ m, \ \sigma) =$ '**accept**' and $m \notin \{m_1, \ldots, m_q\}$

Secure signature



Definition. A signature scheme is a secure if for all "efficient" $\mathscr{A}$:

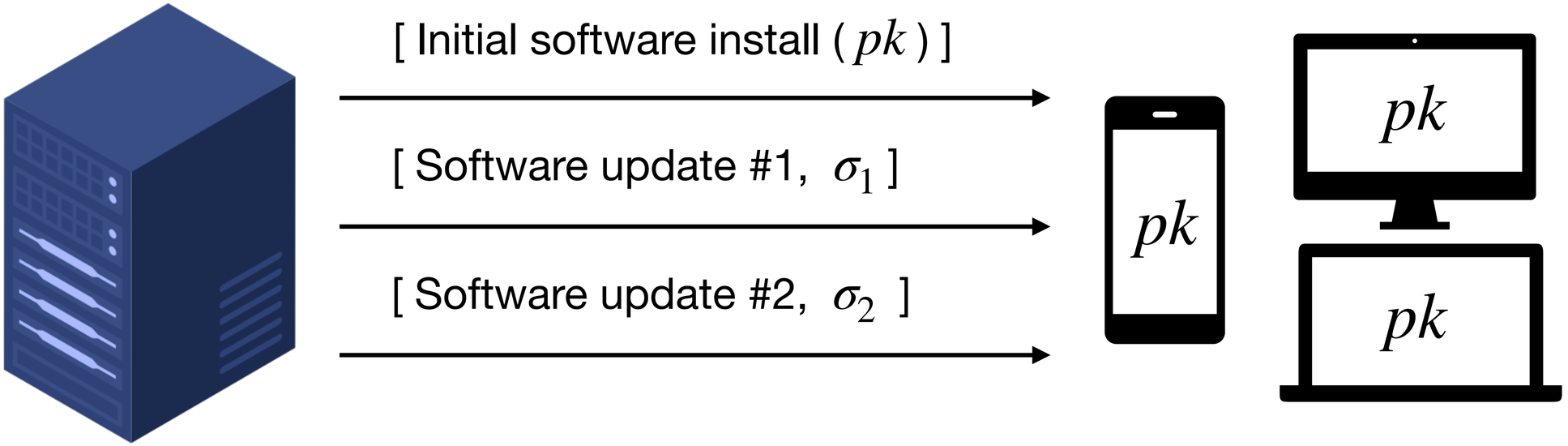
Pr[$\mathscr{A}$ **win**] is "*negligible.*"

Contents

- What is Digital Signature?
- **Applications**
- The RSA Digital Signature Scheme
- The Elgamal Digital Signature Scheme
- The Digital Signature Algorithm (DSA)

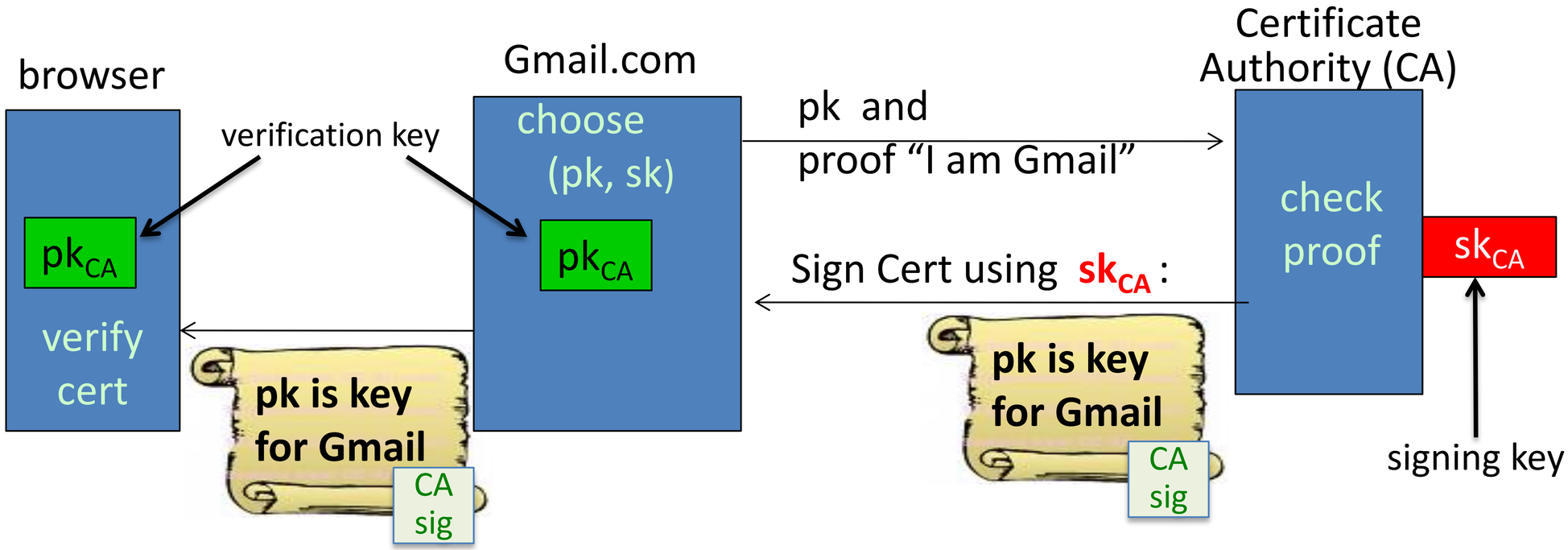
Code signing

- Software vendor signs code
- Clients have vendor's pk. Install software if signature verifies.



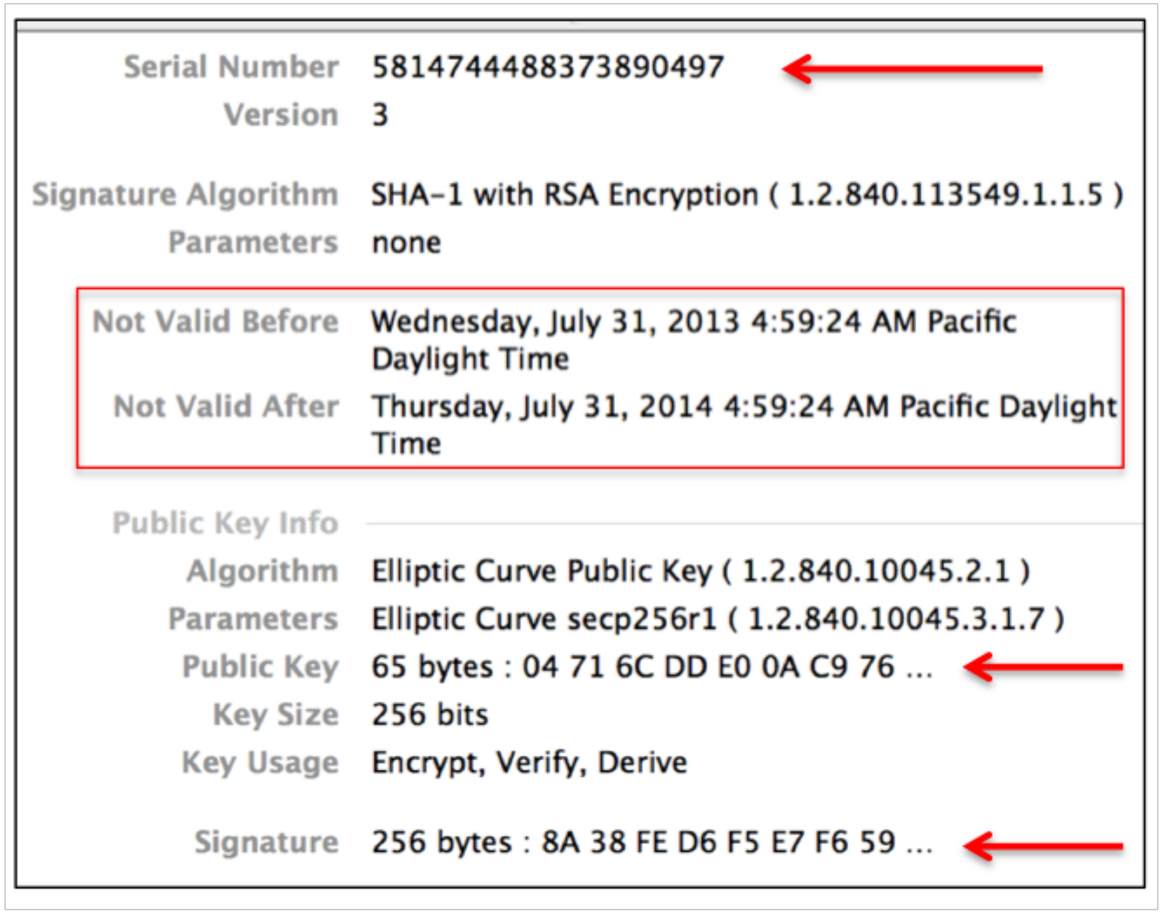
Important application: Certificates

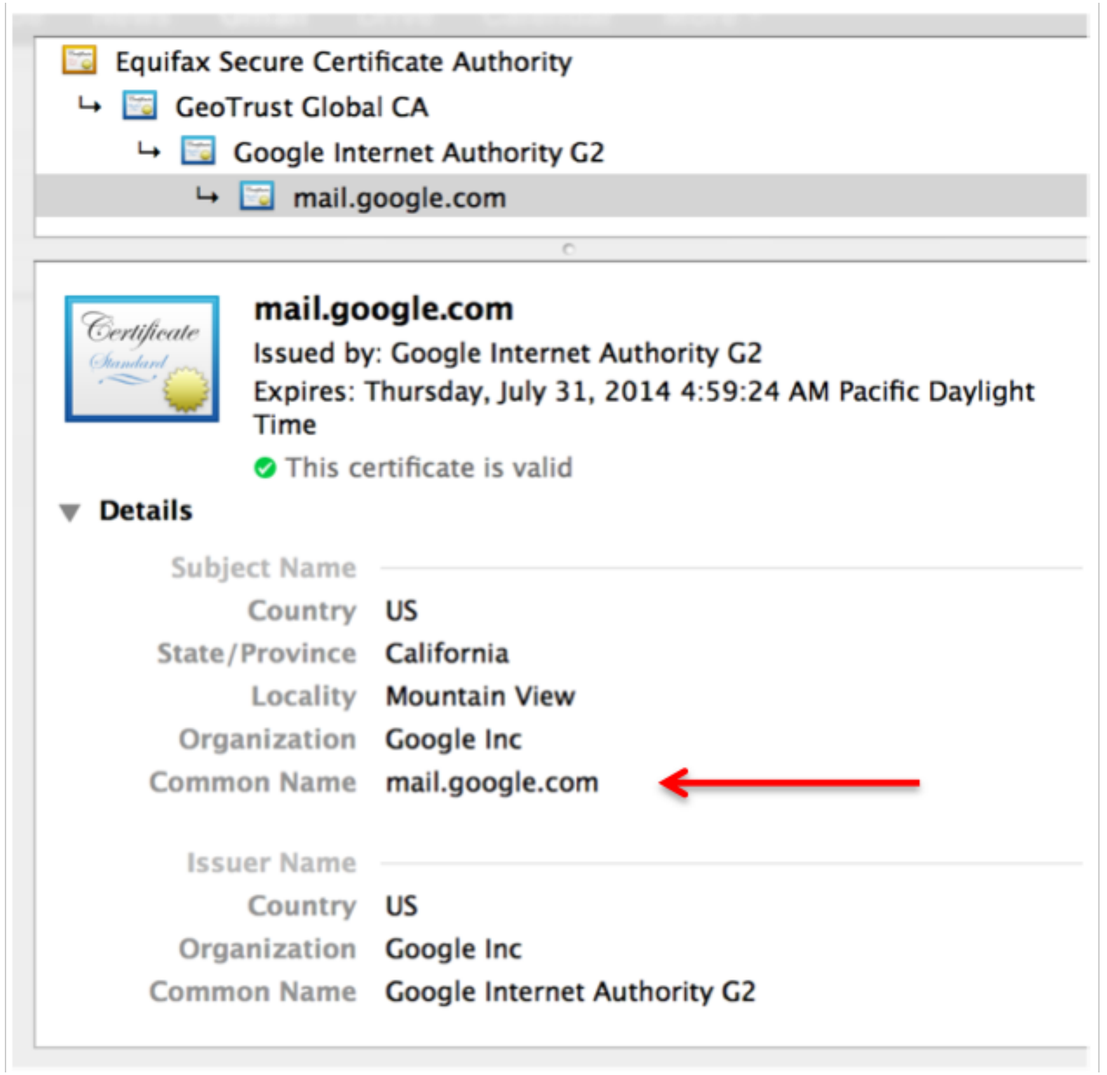
- **Problem:** browser needs server's public-key to setup a session key
- **Solution**: server asks trusted 3rd party (CA) to sign its public-key pk



Server uses Cert for an extended period (e.g. one year)

Certificates: example



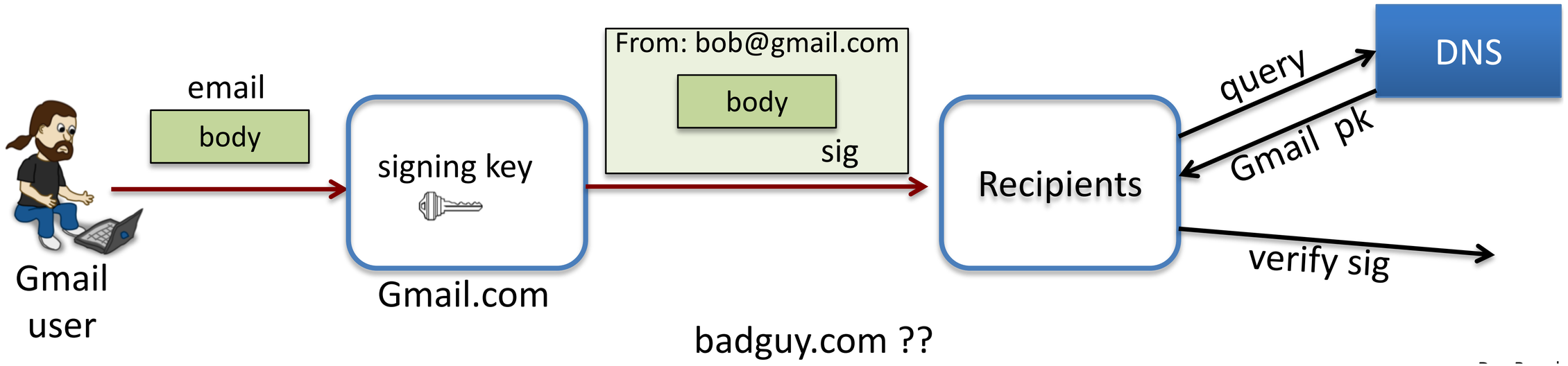


Signing email: DKIM (domain key identified mail)

- **Problem:** bad email claiming to be from someuser@gmail.com
  but in reality, mail is coming from domain badguy.com

  ⇒ Incorrectly makes gmail.com look like a bad source of email

- **Solution:** gmail.com sign every outgoing mail



When to use signatures

If **one** party signs and **one** party verifies: ***use a MAC***

- Often requires interaction to generate a shared key
- Recipient can modify the data and re-sign it before passing the data to a 3rd party

If **one** party signs and **many** parties verify: ***use a signature***

- Recipients cannot modify received data before passing data to a 3rd party (non-repudiation)

Review: three approaches to data integrity

1. **Collision resistant hashing:** need a read-only public space

2. **MACs:** vendor must compute a new MAC of software for every client
   ⇒ and must manage a long-term secret key
   (to generate a per-client MAC key)

3. **Digital signatures:** vendor must manage a long-term secret key

   - Vendor's signature on software is shipped with software

   - Software can be downloaded from an untrusted distribution site

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General method

- The idea was introduced by Diffie & Hellman in 1976
- Building a signature scheme from a deterministic public key crypto-system (E, D)

$$\sigma = S(sk, m) = D(sk, m)$$

$$V(pk, m, \sigma) = \begin{cases} 1 & \text{if } E(pk, \sigma) = m \\ 0 & \text{otherwise} \end{cases}$$

Textbook RSA

- Key Generation $Gen()$:
  - Choose $n = pq$ (p, q λ-bit random primes)
  - Choose e, d such that $ed = 1 \mod \varphi(n)$
  - $pk = (n, e)$ and $sk = (n, d)$
- Signature Generation: $S(sk, m) = m^d \mod n$
- Signature Verification: $V(pk, m, \sigma) = 1 \Leftrightarrow \sigma^e = m \mod n$

Example: Key generation

- Choose $p = 3$ and $q = 11$
- $n = p \cdot q = 33$
- $\varphi(n) = (3 - 1)(11 - 1) = 20$
- Choose $e = 3$
- $d = e^{-1} = 7 \mod 20$
- Output $(pk = 3,\ sk = 7)$

Example: Sign and Verify

Signature Generation

$S(sk = d = 7,\ m = 4)$:

- $\sigma = m^d \mod n$
  $= 4^7 = 16 \mod 33$

Signature Verification

$V(pk = e = 3,\ m = 4,\ \sigma = 16)$:

- $m' = \sigma^e = 16^3 = 4 \mod 33$
- Since $m = m'$ $\Rightarrow$ '**accept**'

Attack 1

- Can sign *specific* messages
  - E.g., easy to compute the e^{th} root of $m = 1$ or the cube root of $m = 8$

Attack 2

- Can sign “random” messages
  - Choose arbitrary σ ; set $m = [\sigma^e \mod n]$

Attack 3

- Can combine two signatures to obtain a third
  - Say σ_1, σ_2 are valid signatures on m_1, m_2
  - Then $\sigma = [\sigma_1 \cdot \sigma_2 \mod n]$ is a valid signature on the message $m = m_1 \cdot m_2$ because

$$(\sigma_1 \cdot \sigma_2)^e = \sigma_1^e \cdot \sigma_2^e = m_1 \cdot m_2$$

Hash & Sign

- $Gen()$: [...]
- $S(sk, m) = \mathsf{H}(m)^d \mod n$
- $V(pk, m, \sigma) = 1 \Leftrightarrow \sigma^e = \mathsf{H}(m) \mod n$

Intuition for security?

- Look at the three previous attacks…

  1. Not easy to compute the e^{th} root of $\mathrm{H}(1)$,…

  2. Choose σ but how do you find an m such that $\mathrm{H}(m) = \sigma^e \mod n$?
     $\Rightarrow$ Computing inverses of H should be hard

  3. $\mathrm{H}(m_1) \cdot \mathrm{H}(m_2) = \sigma_1^e \cdot \sigma_2^e = (\sigma_1 \cdot \sigma_2)^e \neq \mathrm{H}(m_1 \cdot m_2)$

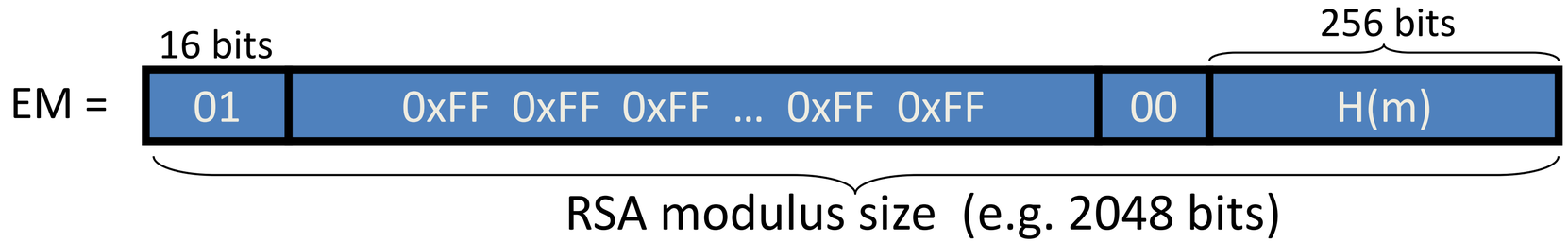
Security of RSA-FDH (Full Domain Hash)

- If the RSA assumption holds, and H is modeled as a random oracle (mapping onto $\mathbb{Z}_n^*$), then RSA-FDH is **secure**
- In practice, H is instantiated with a (modified) cryptographic hash function
  - Must ensure that the range of H is large enough!
  - A "good" hash function H :

$$\mathrm{H}(m) = n \text{ first bytes of } \mathrm{SHA256}(1\|m) \,\|\, \mathrm{SHA256}(2\|m) \,\|\, \cdots \,\|\, \mathrm{SHA256}(11\|m)$$

RSA in practice - PKCS1 v1.5

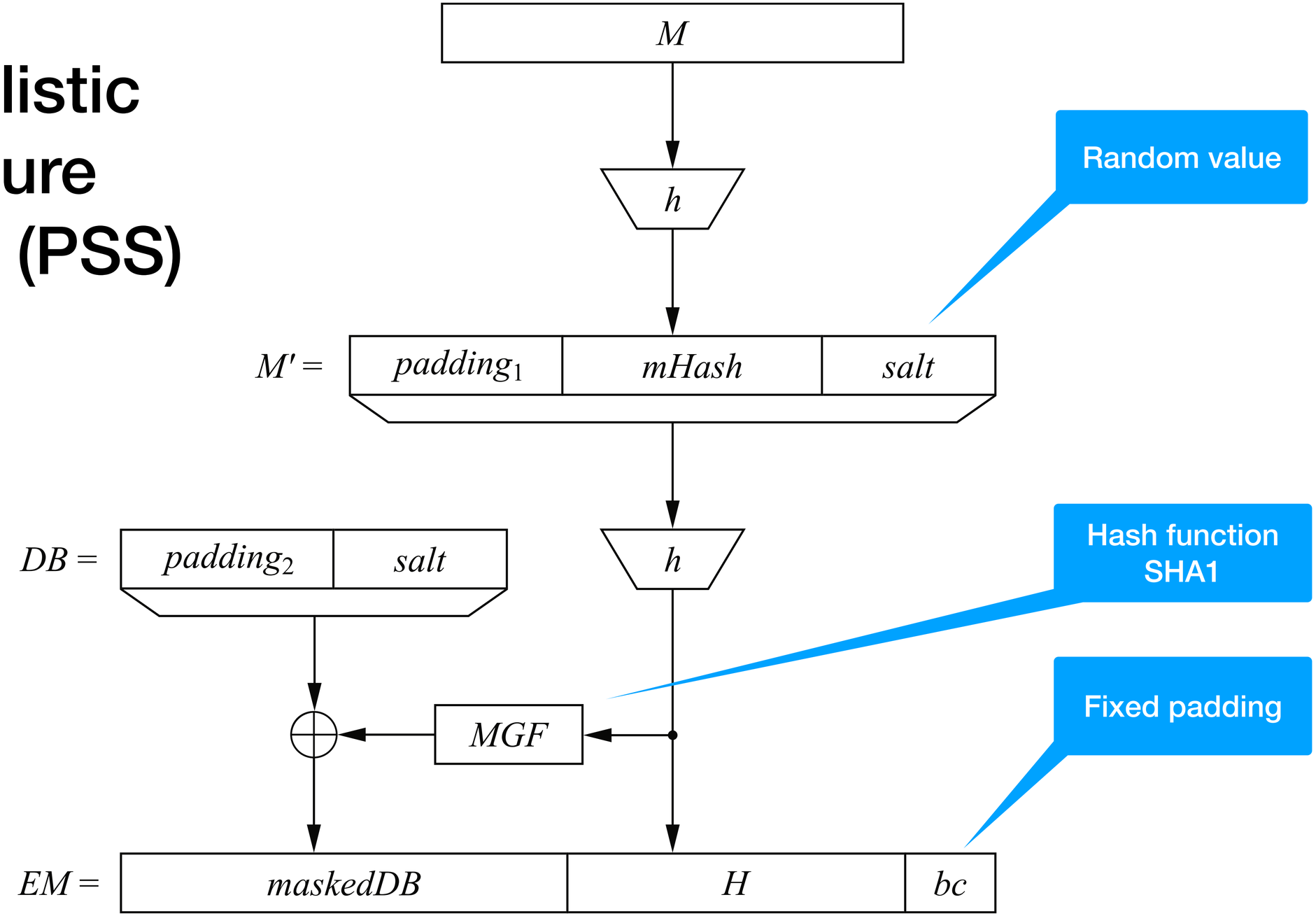
- Trapdoor permutation RSA : $pk = (n, e), sk = (n, d)$
- $S(sk, m \in M)$:



output $\sigma = (EM)^d \mod n$

- $V(pk, m \in M, \sigma)$: verify that $\sigma^e \mod n$ has correct format

Probabilistic Signature Standard (PSS)



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ElGamal Digital Signatures

- The paradigm

$$E(D(m)) = m$$

doesn't work for ElGamal, since ElGamal is not a trapdoor permutation (it is randomized.)

- The Elgamal digital signature is quite different from the Elgamal encryption scheme.

Textbook ElGamal: Key Generation

- $Gen()$:

    1. Choose a large prime p
    2. Choose a generator g of $\mathbb{Z}_p^*$ (or a subgroup of $\mathbb{Z}_p^*$)
    3. Choose a random integer $d \in \{2, 3, \ldots, p-2\}$;
    4. Output ($pk = g^d$, $sk = d$)

Textbook ElGamal: Signature Generation

- $S(sk = d, m)$:

1. Choose a random ephemeral key $k_E \in \{1, 2, \ldots, p-2\}$ such that

$$\gcd(k_E, p-1) = 1 \qquad (k_E^{-1} \mod p-1 \text{ exists})$$

2. Compute the signature parameters:

$$r = g^{k_E} \mod p,$$

$$s = \frac{(m - d \cdot r)}{k_E} \mod p-1$$

3. Output the signature $\sigma = (r, s)$

Textbook ElGamal: Signature Verification

- $V(pk = g^d,\ m,\ \sigma = (r, s))$:

    1. Compute the value

$$t = \left(g^d\right)^r \cdot r^s \mod p$$

    2. `if` $t = g^m \mod p$ `return ‘accept’ else ‘reject’`

Textbook ElGamal: Correctness

- $t = \left(g^d\right)^r \cdot r^s \mod p = \left(g^d\right)^r \cdot \left(g^{k_E}\right)^s \mod p$

$$= g^{d \cdot r + s \cdot k_E} \mod p$$

- According to Fermat's Little Theorem , the equality $g^m = t \mod p$ is equivalent to

$$m = \left(d \cdot r + s \cdot k_E\right) \mod p - 1$$

- from which the construction rule of the signature parameters s follows:

$$s = \frac{m - d \cdot r}{k_E} \mod p - 1$$

Example: Key and signature generation

Key generation $Gen()$

- choose $p = 29$
- choose $g = 2$
- Choose $sk = d = 12$
- $pk = g^d = 7 \mod 29$

Signature generation $S(sk = 12,\ m = 26)$**:**

- choose $k_E = 5$, note that $\gcd(5,28) = 1$
- $r = g^{k_E} = 2^5 = 3 \mod 29$
- $s = (m - d \cdot r) \cdot k_E^{-1} \mod p - 1$
  $= -10 \cdot 17 \mod 28$
  $= 26 \mod 28$
- Output signature $(3,\ 26)$

Example: Signature Verification

- $V(\ pk = g^d = 7,\ m = 26,\ \sigma = (3,26)\ )$:
  - $t = (g^d)^r \cdot r^s = 7^3 \cdot 3^{26} = 22 \mod 29$
  - $g^m = 2^{26} = 22 \mod 29$
  - Since $t = g^m \mod 29$ $\Rightarrow$ '**accept**'

Reuse of the Ephemeral Key k_E

- If the two messages m_1 and m_2 have the same ephemeral key k_E, then

$$s_1 = \frac{m_1 - dr}{k_E} \mod p - 1 \quad \text{and} \quad s_2 = \frac{m_2 - dr}{k_E} \mod p - 1$$

- Oscar can compute the ephemeral key

$$k_E = \frac{m_1 - m_2}{s_1 - s_2} \mod p - 1$$

- also compute the private key

$$d = \frac{m_1 - s_1 k_E}{r} \mod p - 1$$

Exercise

- Suppose that Oscar knows Bob's public key, which is given as:
  - $p = 29,\ g = 2,\ g^d = 7$
- Suppose that Bob signed two messages with the same ephemeral key k_E :
  - $[m_1,\ (r, s_1)] \ = \ [26,\ (3,26)]$
  - $[m_2,\ (r, s_2)] \ = \ [13,\ (3,1)]$
- Compute Bob's private key sk.

Question on Existential Forgery Attack

- In Textbook ElGamal, can you sign a “random” message? (Similar to the case of Textbook RSA)?

Existential Forgery Attack: Random message

- **Signature Generation**

  - select integers i, j where $\gcd(j, p-1) = 1$

  - compute signature:
    $r = g^i \cdot \left(g^d\right)^j \mod p$
    $s = -r \cdot j^{-1} \mod p-1$

  - compute message:
    $m = s \cdot i \mod p-1$

- **Signature Verification**

  - compute $t = (g^d)^r \cdot r^s \mod p$

  - since $t = g^m \mod p$
    $\Rightarrow$ valid signature!

Hash & Sign ElGamal

- Like RSA, the Hash and Sign scheme not only increases efficiency but also increases security.
- It helps against signature forgery attacks for “random” messages.
- The signing equation becomes:

$$s = \frac{\left(\mathrm{H}(m) \ - d \cdot r\right)}{k_E} \mod p - 1$$

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The Digital Signature Algorithm (DSA)

- It is a federal US government standard for digital signatures (DSS) and
- was proposed by the National Institute of Standards and Technology
- **Advantages :**
  - the signature is only 320 bit long
  - some of the attacks that can threaten the Elgamal scheme are not applicable.

Key Generation for DSA

- Generate a prime p with $2^{1023} < p < 2^{1024}$
- Find a prime divisor q of $p - 1$ với $2^{159} < q < 2^{160}$
- Find an element g with order $\text{ord}(g) = q$;
  i.e., g generates the subgroup with q elements
- Choose a random integer d with $0 < d < q$
- Compute $\beta = g^d$
- Output $pk = (p, q, g, \beta)$ and $sk = d$

DSA Signature Generation

- Choose an integer as random ephemeral key k_E with $0 < k_E < q$.
- Compute $r = \left(g^{k_E} \mod p\right) \mod q$
- Compute $s = \left(\mathrm{H}(m) + d \cdot r\right) \cdot k_E^{-1} \mod q$

DSA Signature Verification

- Compute auxiliary value $w = s^{-1} \mod q$
- Compute auxiliary value $u_1 = w \cdot \mathrm{H}(m) \mod q$
- Compute auxiliary value $u_2 = w \cdot r \mod q$
- Compute $v = \left(g^{u_1} \cdot \left(g^{d} \right)^{u_2} \mod p \right) \mod q$
- The verification $V(pk,\ m,\ (r, s))$ follows from:

```
if  v = r mod q  return ‘accept’  else ‘reject’
```

Standardized parameter bit lengths and security levels for DSA

p	*q*	Signature	Hash output (min)	Security levels
1024	160	320	160	80
2048	224	448	224	112
3072	256	512	256	128

Example: DSA Key and Signature Generation

Key generation

- Choose $p = 59$
- Choose $q = 29$
- Choose $g = 3$
- Choose private key
  $sk = d = 7$
- Public key
  $$pk = g^d = 4 \mod 59$$

Signature generation for H$(m) = 26$

- Choose ephemeral key $k_E = 10$
- $$r = \left(3^{10} \mod 59\right) \mod 29$$
  $$= 20 \mod 29$$
- $$s = \left(26 + 7 \cdot 20\right) \cdot 3 \mod 29$$
  $$= 5 \mod 29$$

Example: DSA Signature Verification

$V(pk = 4,\ \mathbf{H}(m) = 26,\ (r = 20,\ s = 5))$**:**

- $w = 5^{-1} = 5 \mod 29$
- $u_1 = 6 \cdot 26 = 11 \mod 29$
- $u_2 = 6 \cdot 20 = 4 \mod 29$
- $v = \left(3^{11} \cdot 4^{4} \mod 59\right) \mod 29 = 20$
- Since $v = r \mod 29$ $\Rightarrow$ the signature is '**valid**'

Computation Aspects

- **Problem:**
  - Find a cyclic group $\mathbb{Z}_p^*$ with a bit length 1024 bit, and
  - which has a prime subgroup in the range of 2^{160}
- **General approach:**
  - find the 160 bit prime q and construct the larger prime p from it.

Prime Generation for DSA

- **Output**:
  - Two primes (p, q),

    where $2^{1023} < p < 2^{1024}$ and $2^{159} < q < 2^{160}$

    such that $p - 1$ is a multiple of q

Prime Generation for DSA

1. Find prime q with $2^{159} < q < 2^{160}$ using the Miller–Rabin algorithm
2. **for** $i = 1$ to 4096
   - generate random integer M with $2^{1023} < M < 2^{1024}$
   - $M_r = M \mod 2q$
   - $p - 1 = M - M_r$ // $p - 1$ is a multiple of $2q$
   - **if** p là số nguyên tố: **return** (p, q) // use Miller–Rabin primality test
3. Goto Step 1.

Quiz 1

- Let (Gen, S, V) be a signature scheme.
- Suppose an attacker is able to find $m_0 \neq m_1$ such that
  $V(pk, m_0, \sigma) = V(pk, m_1, \sigma)$ for all σ and keys. $(pk, sk) \leftarrow Gen()$
- Can this signature be secure?

  1. Yes, the attacker cannot forge a signature for either m_0 or m_1
  2. No, signatures can be forged using a chosen message attack
  3. It depends on the details of the scheme

Quiz 2

- Alice generates a (pk, sk) and gives pk to her bank.
- Later Bob shows the bank a message m=“pay Bob 100$” properly signed by Alice, i.e., $V(pk, m, \sigma) =' yes'$
- Alice says she never signed m. Is Alice lying?

1. Alice is lying: existential unforgeability means Alice signed m and therefore the Bank should give Bob 100$ from Alice’s account
2. Bob could have stolen Alice’s signing key and therefore the bank should not honor the statement
3. What a mess: the bank will need to refer the issue to the courts

ĐẠI HỌC BÁCH KHOA

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