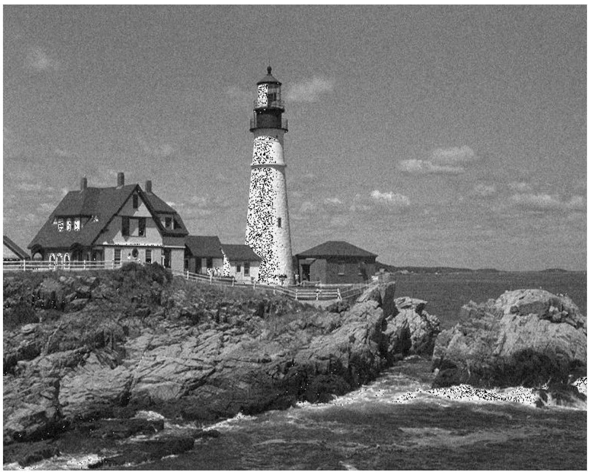
Block Matching and 3D Filtering

- From the original lighthouse.bmp, a noisy image is created by adding Gaussian noise with a variance 100.

Original Image

Noisy image



1)

- BM3D (Block-Matching and 3D filtering) is a popular denoising algorithm for image and video processing. The two main steps in BM3D algorithm are:

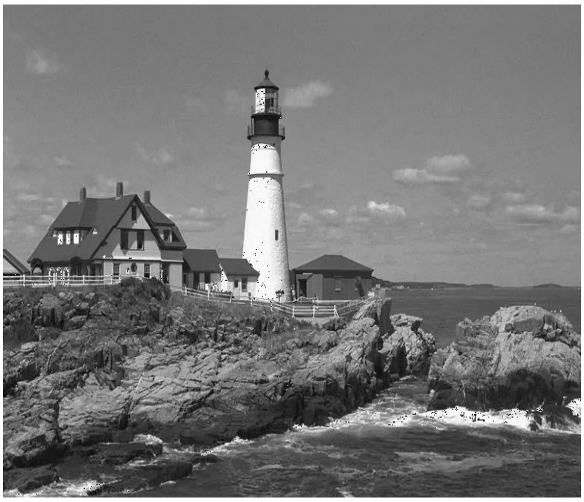
  - **Block matching and hard-thresholding**: In the first step of BM3D, similar blocks are grouped together using a block-matching algorithm, and then a hard-thresholding operation is applied to each group of blocks to reduce the noise. The hard-thresholding operation sets small coefficients in the transform domain to zero, effectively removing them from the signal.

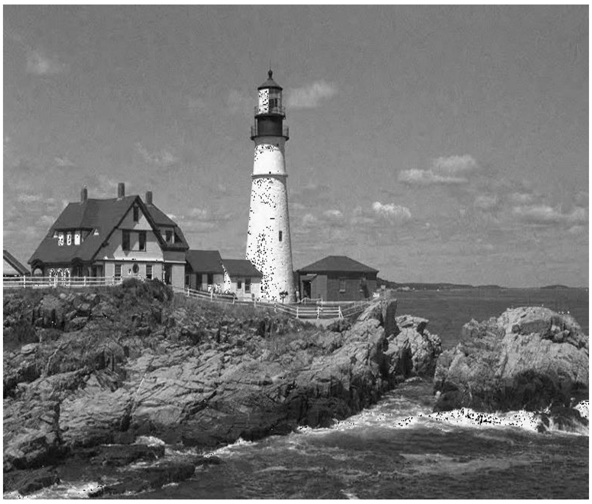
  - **Aggregation and Wiener filtering**: In the second step of BM3D, the denoised blocks are aggregated to form the final denoised image, and then a Wiener filter is applied to further reduce the remaining noise. The Wiener filter is a linear filter that estimates the signal from the noisy observation by minimizing the mean-square error between the original signal and its estimate.

Result of applying BM3D algorithm on noisy image by setting Sigma_psd = 7

First_stage

Second_Stage





```
MSE between original image and first stage: 34.9051340960
MSE between original image and second stage: 42.6120179998
```

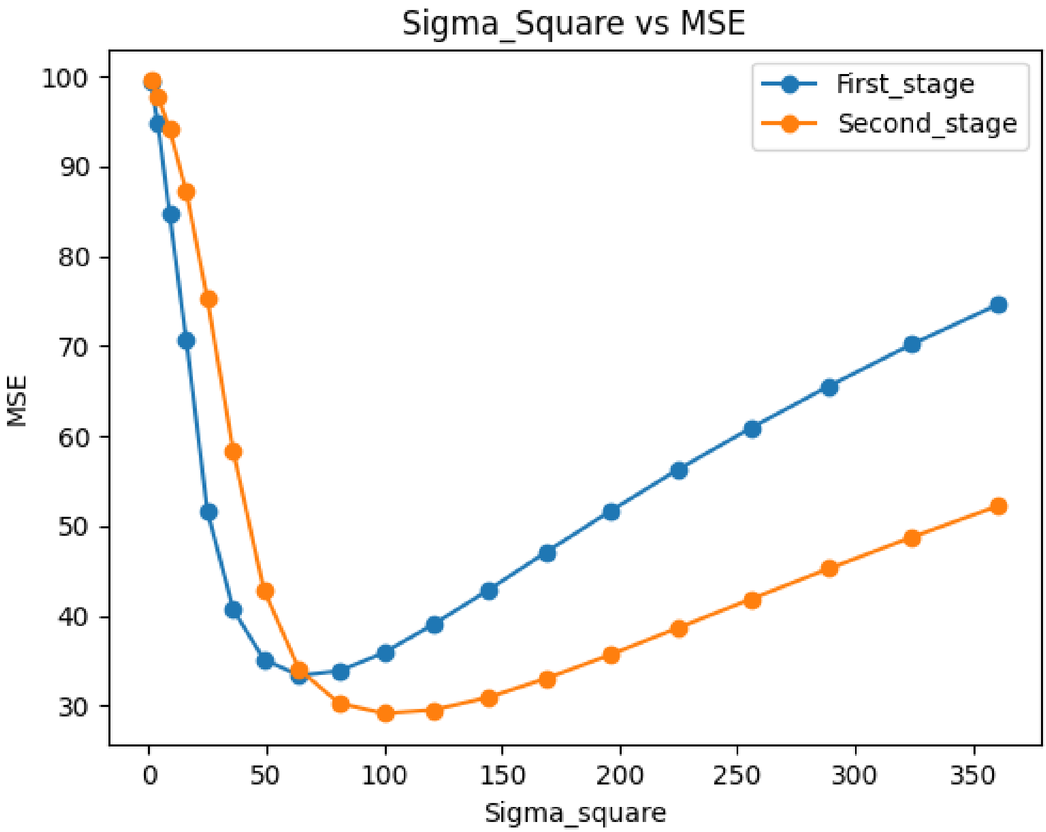
Result of applying BM3D algorithm on noisy image by setting Sigma_psd = 12

First_stage

Second_stage

```
MSE between original image and first stage: 42.5045959968
MSE between original image and second stage: 30.7064116752
```

Plot of MSE obtained between the original image and first_stage,second_stage outputs against sigma_psd set in the algorithm:



- Based on the above observations, it seems that the performance of the BM3D algorithm in denoising an image depends on the value of sigma_psd, which represents the standard deviation of the noise. When the value of sigma_psd is less than 10, the first-step output (basic estimate using hard thresholdingng) has a lower MSE compared to the second-step output (final estimate using Weiner filtering). On the other hand, when the value of sigma_psd is greater than 10, the second-step output has a lower MSE compared to the first-step output.

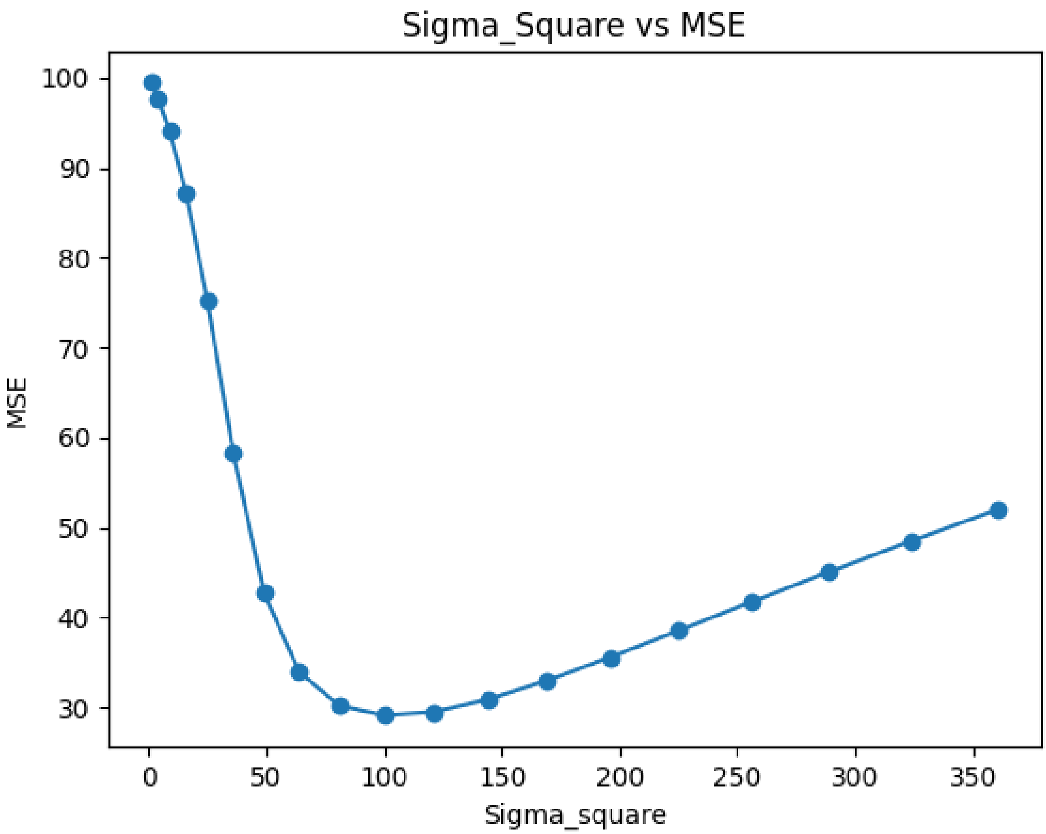
- This behavior can be attributed to the fact that BM3D algorithm is designed to perform denoising in two stages - first by grouping similar patches in the image, and then by applying collaborative filtering to the grouped patches. In the first step, hard-thresholding is applied to the patches, which effectively removes high-frequency noise. However, this can result in some loss of image details, which can contribute to a higher MSE between the first-step output and the input image.

- In the second step, Wiener filtering is applied to the patches to further reduce the noise and recover the lost image details. This results in a final estimate that has a lower MSE compared to the first-step output when the noise is high (sigma_psd > 10).

- However, when the noise is low (sigma_psd < 10), the hard-thresholding applied in the first step is already effective in removing most of the noise, and applying Wiener filtering in the second step may not have a significant effect. This can result in a final estimate that has a higher MSE compared to the first-step output.

2)

The plot of MSE obtained between the original image and output of BM3D algorithm against sigma_psd set in the algorithm:



- The fact that the MSE of the denoised image by BM3D decreases initially and then increases as the parameter sigma_psd is increased from 1 to 20, with a minimum value at sigma_psd = 10, can be explained by the trade-off between noise reduction and preservation of image details.

- When the value of sigma_psd is very small, the BM3D algorithm applies strong denoising and tries to remove as much noise as possible from the image. However, if the value of sigma_psd is too small, the algorithm may also remove some of the image details and features, resulting in a denoised image that is significantly different from the original input image, which leads to a high MSE.

- As sigma_psd is increased, the algorithm applies weaker denoising, which preserves more image details and features. However, if sigma_psd is too large, the algorithm may also preserve some of the noise in the image, leading to a denoised image that has a higher MSE.

- The optimal value of sigma_psd that leads to the minimum MSE occurs when the BM3D algorithm achieves a balance between reducing noise and preserving image details, resulting in a denoised image that is close to the original input image. In this case, the optimal value of sigma_psd is found to be 10.

Result of applying BM3D algorithm on noisy image by setting Sigma_psd = 10 gives optimal MSE.

First_stage

Second_stage

```
MSE between original image and first stage: 35.9280315940
MSE between original image and second stage: 29.0624474202
```

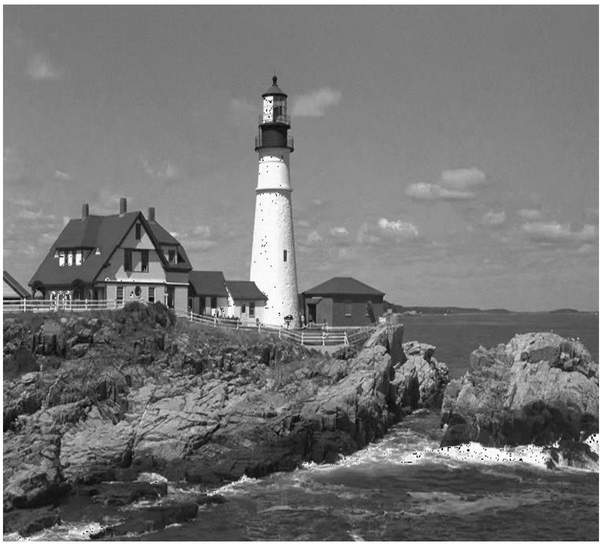
3)

- Following are the results obtained after replacing the Wiener filter in the second stage with a hard thresholding estimate in the BM3D algorithm.
- The results of the first stage are the same in both variations, as the first stage of applying hard thresholding is common.

Result of applying the changed BM3D algorithm on a noisy image by setting

Sigma_psd = 7

First_stage



Second_stage

```
MSE between original image and first stage: 34.9206671420
MSE between original image and second stage: 59.4950094189
```

Result of applying the changed BM3D algorithm on noisy image by setting

Sigma_psd = 12

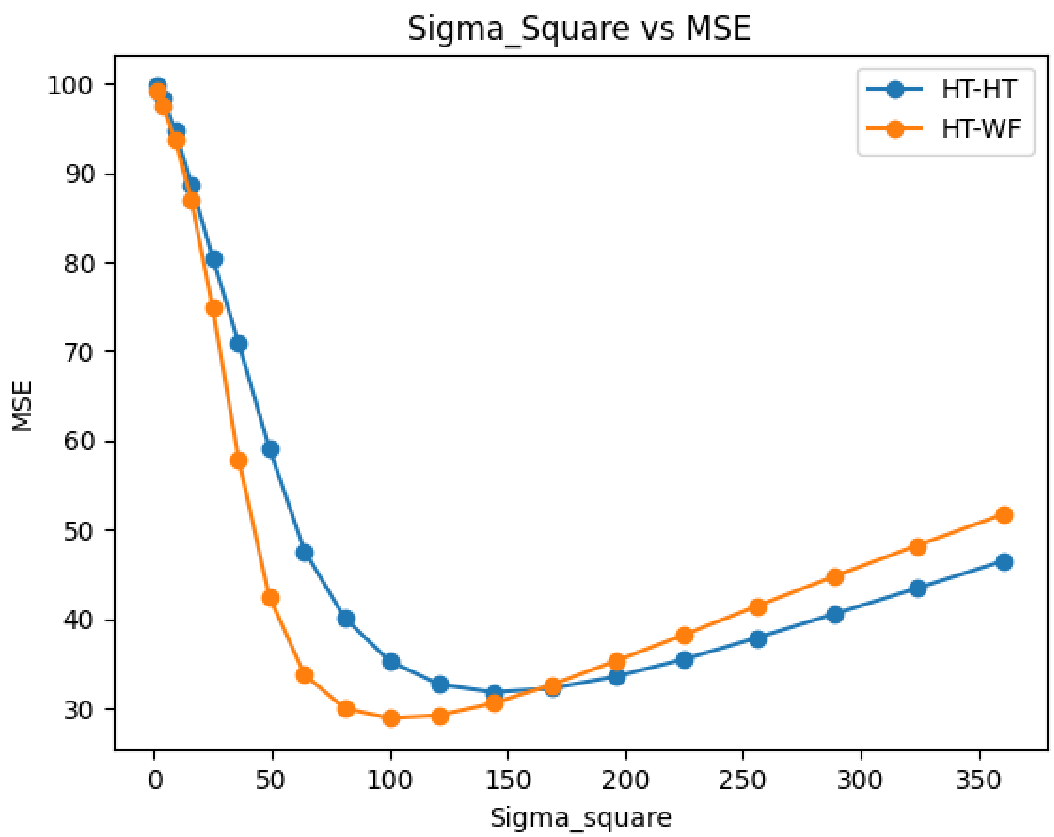
First_stage

Second_stage

```
MSE between original image and first stage: 42.6666153242
MSE between original image and second stage: 31.6824314443
```

The plot of MSE obtained between the original image and output of two variants of BM3D algorithm against sigma_psd set in the algorithm:

- HT-WF represents the algorithm of Hard Thresholding in the first stage and Weiner Filtering in the second stage.
- HT-HT represents the algorithm of Hard Thresholding in the first stage and Hard Thresholding in the second stage



- If hard thresholding is used instead of Wiener filtering in the second step of the BM3D algorithm, the denoising performance may be affected.

- In the second step of BM3D, Wiener filtering is applied to the grouped patches to further reduce the noise and recover the lost image details. Wiener filtering is a type of linear filter that estimates the clean signal by taking into account the signal-to-noise ratio (SNR) of the noisy signal. It works well when the noise is Gaussian and the SNR is known or can be estimated accurately.

- On the other hand, hard thresholding is a non-linear method that sets all coefficients below a certain threshold to zero. It works well for sparse signals, where only a small percentage of coefficients are non-zero. However, for images, most coefficients are non-zero, and applying hard thresholding may result in some loss of image details.

- Therefore, using hard thresholding instead of Wiener filtering in the second step may result in a denoised image with higher mean squared error (MSE) compared to using Wiener filtering. However, the actual denoising performance may also depend on the noise level, the image content, and other parameters of the algorithm.

- We can also observe that optimal MSE is obtained at sigma = 12(approx), which is different from the optimal MSE obtained in the original algorithm at sigma = 10(i.e. the amount of noise in the original image).

Note :

Results in this section of changing the second stage to hard thresholding are obtained by appropriately changing the __init__.py file in the bm3d module where the functionality of the BM3D algorithm is defined.