Advanced Image Processing: Assignment 3 (Due Mar 15, 2023)

Problem 1: Linear combinations of order statistics for uniformly distributed noise (10 points)
Consider the noise model $Y_i = x + Z_i$ for $i \in \{1, 2, \cdots, N\}$ for $N = 5$ where Z_i are independent and identically distributed according to a uniform distribution Unif$[-1, 1]$. Compute the order statistics filter coefficients $(\alpha_1, \alpha_2, \cdots, \alpha_N)$ that minimize the mean squared error between $\hat{X}$ and x where $\hat{X} = \sum_{i=1}^{N} \alpha_i Y_{(i)}$. (Ref: A. C. Bovik, T. S. Huang, and D. C. Munson, "A Generalization of Median Filtering Using Linear Combinations of Order Statistics," IEEE Transactions on Acoustics, Speech, and Signal Processing, vol.31, no.6, Dec. 1983).

Problem 2: Block Matching and 3D Filtering (30 points)
Take the lighthouse image provided to you, convert to greyscale and add white Gaussian noise with variance $\sigma_Z^2 = 100$ to it. Be sure to add noise in the grey scale domain where the range of pixel values is between 0 and 255. Perform the following denoising experiments on the BM3D algorithm: Read the paper "Image denoising by sparse 3D transform-domain collaborative filtering" available at `https://www.cs.tut.fi/~foi/GCF-BM3D/BM3D_TIP_2007.pdf`. Obtain the BM3D implementation available at `http://www.cs.tut.fi/~foi/GCF-BM3D/`. Based on your reading of the paper and the code, perform the following experiments:

1. Compare the MSE performance at the output of the first and second stages of the BM3D method. The BM3D algorithm has two stages in its implementation.

2. Study the performance variation of the entire algorithm with respect to the choice of the input noise variance σ_Z^2 in the algorithm. You can plot a curve between MSE and σ_Z^2 to understand this relationship. Explain why you get such a curve.

3. Replace the Wiener filter in the second stage with a hard thresholding estimate and compare the performance with the former in terms of MSE.