EE 527: Machine Learning Laboratory

Assignment 3

Due date: 30 Jan 2023

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1. Generate a set of points around a line $y = ax + b$

   (a) Choose $a = 2$ and $b = 3$

   (b) Select the range for x as $[-10,10]$ and generate $n = 100$ values for x in that interval.

   (c) Compute the values of y for each x as $y_i = 2x_i + 3$.

   (d) Plot the line $y = 2x + 3$ in black color.

   (e) Generate a set of n points around the line using the equation

   $$y_i = 2x_i + 3 + \sigma N(0,1) \qquad (1)$$

   where σ is the standard deviation and $N(0,1)$ is the zero-mean unity variance normal distribution

   (f) Show the scatter plot of these noisy points (in red color) on the same graph generated in step (d).

2. Plot the average error surface E for different values of a and b in the interval of $[-10 : 0.1 : 10]$.

   (a) Vary both a and b in steps of 0.1 in the interval $[-10,10]$

   (b) Compute the element-wise error as $e_i = y_i - \hat{y}_i$ where $\hat{y}_i = ax_i + b$ and y_i is computed using equation (1).

   (c) Compute the average error as

   $$E = \frac{1}{n}\sum_{i=i}^{n} {e_i}^2 \qquad (2)$$

   (d) Compute the average error values for all combinations of a and b.

   (e) Plot the error surface with the values of a along x-axis, that of b along y-axis and E along z-axis.

3. Generate a set of 200 data points $\{x_i, y_i\}$ following equation (1). Split this data into train and test subsets of 100 points each, using the *train_test_split* method available in the *sklearn* library. Estimate the values of a and b using the pseudo-inverse approach on the train set. Use these estimated values to make predictions for the test set. Compute the MSE and the $R-squared$ measure. Experiment with different values of standard deviation (σ) and observe the impact on the values of MSE and R^2 .

4. Taking the points generated in Q.1 (e) as the dataset, solve for a and b using the Gradient Descent approach where the values of $\mathbf{p} = (a,b)^T$ in the $(k + 1)^{th}$ iteration is updated as

$$\mathbf{p}_{k+1} = \mathbf{p}_k - \eta \nabla_{\mathbf{p}} E|_{\mathbf{p}=\mathbf{p}_k} \quad (3)$$

Vary the update rate η and the initial values (a_0,b_0) and note the final solution after 100 iterations. Plot the trajectory of the solutions (a_k,b_k) for varying (a_0,b_0,η) on the contour plot of E on (a,b) plane.