consist of two uncorrelated rectangular distribution functions

$$P(\phi, \delta) = (4\sigma\gamma)^{-1} \quad \text{if } |\phi| \leq \sigma, |\delta| \leq \gamma,$$
$$= 0 \quad \text{otherwise.} \tag{1}$$

When summing the contributions from each square in order to obtain the total scattered field, each contribution will contain a phase factor relating the individual square to the origin of the coordinate system in the plane of the liquid crystal. Due to the expansions δ_{mn} this phase factor will also be a stochastic variable with a distribution function somehow related to that of δ. But the only effect of the randomness of this phase factor is to reduce the contrast of the diffraction pattern, which will not be important in what follows, and we shall therefore ignore it.

The main effect of the lattice distortions is to broaden the intensity maxima in the diffraction pattern; in particular the scale distortion δ broadens the maximum in the radial direction, whereas ϕ causes a broadening perpendicular to the radial direction. However, since the total scattered power remains the same, it is true that, at least to a first approximation, the peak intensity is decreased in the same proportion as the maximum is broadened. Thus, our procedure will be to determine the decrease of the peak intensity in a particular maximum due to distortions, and thereby obtain the broadening.

It is not difficult to show that the total scattered field at a maximum is given by

$$E = E_0 \alpha A a^{-2} f(\sigma, \gamma), \tag{2}$$

where

$$f(\sigma, \gamma) = a^2 A^{-1} \sum_{m,n} P_{\sigma\gamma}(\phi_{mn}, \delta_{mn}) \frac{1-\exp[2\pi i N(1+\delta_{mn})\sin\phi_{mn}]}{1-\exp[2\pi i(1+\delta_{mn})\sin\phi_{mn}]} \cdot \frac{1-\exp[2\pi i N(1+\delta_{mn})\cos\phi_{mn}]}{1-\exp[2\pi i(1+\delta_{mn})\cos\phi_{mn}]}, \tag{3}$$

E_0 is the incident field strength and α the scattering strength of a single scatterer. The choice of N is somewhat arbitrary, but we want N to be large enough to define a local lattice, yet small enough so that we get many squares within A. We shall tentatively set $N = 10$.

In addition to the distortions of the lattice, the scattering centers may vibrate in a random fashion around the lattice points. As is well known from x-ray scattering, this random vibration results in a loss of contrast; the intensity lost from the diffraction pattern goes into a uniform background.

We have assumed that the scattering takes place from scattering centers of negligible spatial extension. In reality the scattering is due to a more or less continuous, but periodic, change in the dielectric constant. We take the finiteness of the scatterer into account in a very approximate manner by assuming that there is a sphere of radius ρ, whose dielectric constant differs by an amount $\Delta\epsilon$ from that of the surrounding liquid, located at each scattering center. The scattering from such a sphere is given by the Mie theory, and thus α in Eq. (2) is a well known function of the scattering angle. The background scattering mentioned previously does of course also have this same angular dependence.

Finally, there is a background scattering arising from the random translations and rotations of the individual molecules. This scattering is relatively weak and of no interest in the present context.

The scattering pictures can now be evaluated in the following manner: Using a densitometer, and assuming that the film was exposed in its linear region, we obtain two intensity profiles, one in radial direction and one perpendicular to this radial direction, such that the two profiles intersect at a diffraction maximum. Comparing the width of these profiles to the ideal width (i.e., without distortions) yields two broadening factors, M_ϕ and M_δ, and the two distribution parameters σ and γ are then determined by

$$|f(\sigma, 0)| = 1/M_\phi \tag{4}$$
$$|f(0, \gamma)| = 1/M_\delta, \tag{5}$$

In Fig. 2(a) we show such a densitometric trace taken along the path indicated in Fig. 2(b), for a 6-μ-thick cell containing a nematic liquid crystal from Liquid Crystal Industries, Inc., and using a HeNe laser with a beam diameter of 0.65 mm. The lattice constant a is in this case 13.1 μ, so that the total number of scatterers within A is about 2800. Without distortions the width of a diffraction maximum, as measured between the first zero on either side of the maximum, would therefore be 3 min of arc. From Fig. 2(a) we may estimate the actual width of a diffraction maximum to be 1°15′, and we obtain $M_\delta = 23.8$. Similarly, from a second trace, taken along the dotted path in Fig. 2(b) we obtain $M_\phi = 50$. Comparing these values with a tabulation of $f(\sigma, 0)$ and $f(0, \gamma)$, we find $\sigma = 15°$, $\gamma = 0.17$, which gives us a quantitative measure of the long-range order.

Figure 2(a) also shows how the scattering is concentrated in the forward direction due to the finiteness of the scatterers. Using the simple formula for diffraction at a circular aperture ($x = 2\pi\rho/\lambda$)

$$I(\theta)/I(0) = [2J(x\theta)/x\theta]^2, \tag{6}$$

for the scattered intensity as a function of angle ($x = 2\pi\rho/\lambda$), and taking the width at $I = 1/2I_0$ in Fig. 2(a) to be about 6°, we find $\rho \approx 3$ μ, which is in reasonable accord with the value for the lattice constant.

There are, as far as further measurements go, a couple of obvious implications of this phenomenon. Firstly, by varying the laser beam diameter one can determine the actual size of the local lattice (i.e., the physically significant value of N), namely at the diameter where γ and σ fall below specified, small values. Also, the manner in which γ and σ vary with beam diameter is of interest. Secondly, there must be space charges associated with the lattice, and these should lead to observable effects, such as dispersion in the velocity of sound. We hope to return to these questions in a later publication.

It is a pleasure to thank B. Eliasson, R. Dändliker, and S. Strässler for helpful discussions, and P. Wild for supplying the liquid crystal and the scattering cell.

[1] R. Williams, J. Chem. Phys. **39**, 384 (1963).
[2] G. H. Heilmeier, L. A. Zanoni, and L. A. Barton, Proc. IEEE **56**, 1162 (1968); IEEE Trans. Electron Devices **17**, 22 (1970).
[3] S. Lu and D. Jones, Appl. Phys. Lett. **16**, 484 (1970); R. B. Meyer, Phys. Rev. Lett. **22**, 918 (1969). A recent study containing many references is F. Rondolez, thesis, Université de Paris, 1970.
[4] W. Helfrich, J. Chem. Phys. **51**, 4092 (1969).
[5] Orsay Liquid Crystal Group, Compt. Rend. **270**, 97 (1970).
[6] P. A. Penz, Phys. Rev. Lett. **24**, 1405 (1970).
[7] The existence of a diffraction pattern was mentioned by C. Deutsch and P. N. Keating, J. Appl. Phys. **40**, 4049 (1969).

Superconductor Energy Gaps from Tunneling*

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(Received 8 October 1970)

Investigations of electronic tunneling into superconductors have frequently proven to be rich sources of information concerning the superconducting energy gap. However, most of the accurate determinations of this energy gap have been confined to experiments wherein superconductors comprise both sides of the tunnel junction. This is due largely to the ease of identifying the junction bias voltage at which tunnel current sharply increases. This bias voltage is the sum of one-half the energy gap

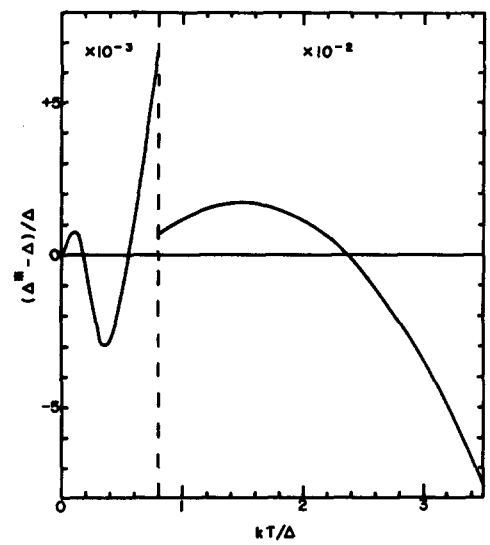

FIG. 1. The fractional difference between the BCS gap parameter, $\Delta(T)$, and the gap parameter, $\Delta^*(T)$, calculated from Eq. (3). Note the change of scale at $kT/\Delta=0.8$.

for each superconductor.[1,2] Despite this benefit it is at times neither convenient nor practically possible to produce two-superconductor junctions. But when only one side of the tunnel junction is superconducting, there is no comparable sharp increase in the current and hence the difficulty of identifying features of the junction characteristic with the superconducting energy gap. To alleviate this difficulty we have numerically computed the relation between the energy gap $2\Delta(T)$ and the junction bias voltage V_m at which the differential conductance is a maximum. The relation is well approximated by an equation which can be solved easily with a small desk-top computer or a slide rule.

To calculate the junction conductance for the case of tunneling between one metal in the normal state and one metal in the superconducting state we use the expression for the tunnel current given by Giaever and Megerle[1]:

$$I_{NS}=\frac{\sigma_{NN}}{e}\int_{-\infty}^{\infty}\frac{|E|}{(E^2-\Delta^2)^{1/2}}[f(E)-f(E+eV)]dE, \qquad (1)$$

where σ_{NN} is the conductance when both metals are normal, e is the electronic charge, f is the Fermi function, V is the voltage across the junction, E is the electronic energy measured from the Fermi energy, and $\Delta=\Delta(T)$. Note that this expression uses the BCS density of states and thus is only approximately correct for strong-coupled superconductors.[3] Setting

$$v=V/kT, \qquad c=\Delta/kT, \qquad x=E/kT, \qquad P(x)=f(x)f(-x)$$

and using the transformation, $x=u^2+c$, we find the conductance

$$\sigma(v,c)\equiv(\sigma_{NN})^{-1}(dI_{NS}/dV)$$
$$=2\int_0^{\infty}\frac{u^2+c}{(u^2+2c)^{1/2}}[P(u^2+c+v)+P(u^2+c-v)]du.$$

This expression for the conductance has been numerically integrated on a CDC 6600 computer for twenty values of the parameter c. For each choice of c the conductance is computed for thirty values of v at intervals of 0.01% about v_m, the voltage parameter which maximizes σ. Each evaluation of σ is accurate to 12 significant figures; this is checked by recalculating σ using intervals of du at least twice as large. We note also that our calculations of σ agree with earlier calculations by Bermon.[4] From the array of σ versus v we interpolate to find v_m to better than 1 part in 10^5.

We find that the relation between v_m and $\tau\equiv kT/\Delta=1/c$ is approximated to within 0.1% everywhere by the equation

$$V_m^*/\Delta=\tau v_m^*=a\tau+[1+(b\tau)^h]^{1/h}, \qquad (2)$$

with $a=1.113$, $b=2.107$, and $h=2.138$; the asterisk denotes the fitted value. The form of this fitting equation was suggested by the analytic solution of the equation $(\partial\sigma/\partial v)_T=0$, using the approximation that

$$P(x)=(4-x)/16 \qquad 4\geq x\geq 0$$
$$=(4+x)/16 \qquad 0>x\approx -4$$
$$=0 \qquad \text{elsewhere.}$$

This substitution, although crude, preserves the essential behavior of $\sigma(v)$. In this case we find

$$V_m/\Delta=\tau v_m=\tfrac{4}{3}\tau+\{1+[(8/3)\tau]^2\}^{1/2}.$$

Solving Eq. (2) for Δ yields

$$\Delta^*=[(V_m-akT)^h-(bkT)^h]^{1/h}.$$

The deviation of the gap parameter Δ^* calculated by this equation from its actual value is graphically shown in Fig. 1. This graph applies only to weak-coupled superconductors.

Equation (3) in conjunction with Fig. 1 allows one to calculate the energy gap $2\Delta(T)$ from the experimentally measured values of temperature T and of the bias voltage V_m at which the conductance is maximum.

There is another method sometimes used to determine the energy gap from the junction characteristics. This entails finding the ratio of currents I_{NS}/I_{NN} as V goes to zero, or equivalently, determining $(1/\sigma_{NN})(dI_{NS}/dV)$ at zero bias.[1,5,6] However, this method is rather sensitive to leakage (nontunneling) conductances, a fairly common problem with tunnel junctions. This procedure is also not amenable to examining the possibility of "double gaps" such as reported in lead.[7]

* This work was supported by the U. S. Atomic Energy Commission.

[1] I. Giaever and K. Megerle, Phys. Rev. **122**, 1101 (1961).

[2] J. M. Rowell, in *Tunneling Phenomena in Solids*, edited by E. Burstein and S. Lundquist (Plenum, New York, 1969), p. 281.

[3] J. R. Schrieffer, D. J. Scalapino, and J. W. Wilkins, Phys. Rev. Let. **10**, 336 (1963).

[4] S. Bermon, Tech. Rept. No. 1, National Science Foundation Grant NSF GP 1100, University of Illinois, 1964 (unpublished). This report contains tabulated values of σ correct to four decimal places for many values of the parameter c.

[5] S. Bermon and D. M. Ginsberg, Phys. Rev. **135**, A306 (1964).

[6] T. D. Clark, J. Phys. C**1**, 732 (1968).

[7] B. L. Blackford and R. H. March, Phys. Rev. **186**, 397 (1969).

Optical Waveguide Technique with Organic Dye Lasers

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(Received 23 July 1970)

Laser emission in optical waveguides may be obtained in glass fibers,[1] in p–n junctions[2,3] or in Nd-doped dielectric slabs.[3] In this letter we report a new technique of preparing optical waveguide structures and integrated lasers.

Laser dyes can be dissolved in a lot of organic solvents, the index of refraction of which may vary between 1.3 and 1.6. Optical waveguide modes exist in these liquids, if they are trans-