$$K_{GL} = 0.52\,\hbar\,\chi^{1/2}/\mu_0\,e\,\xi_0\,\lambda_L^2(0) \tag{16}$$

Thus, $j_c^{2/3}(T_c)$ is predicted to be a straight line with the zero current intercept T_{c0} and the slope

$$d\,j_c^{2/3}/d\,T_c = -K_{GL}^{2/3}\,(1/T_{c0}) \tag{17}$$

The presentation of the GL theory in the present chapter is far away from being complete. However, we introduced the quantities and results which are of basic importance for a discussion of our experiments.

5.2. BCS Ground State and Quasiparticle Excitations

An important ingredient of a nonequilibrium superconducting state are nonequilibrium quasiparticles. Within the framework of the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity we will therefore briefly discuss the properties of a quasiparticle excitation of the superconducting ground state. For reviews and further information about the BCS theory see refs. 1, 2, 5, 45, and 48 - 55.

In the BCS theory the superconducting ground state consists of electron pairs ($\underline{K}\uparrow, -\underline{K}\downarrow$), so called 'Cooper pairs', where $\underline{K}$ denotes the wave number vector and $\uparrow, \downarrow$ the spin direction. There is an attractive interaction between the electrons in the pair by an exchange of virtual phonons. The physical idea is that the first electron polarizes the crystal lattice by attracting positive ions and repulsing other electrons and that the second electron is moving through the polarized region.

The graph symbolizing the contribution of effective electron-electron interaction via virtual phonons in the Hamilton operator of the problem is drawn in Fig. 7. The effective interaction potential, $V_{eff}(\underline{K}, \underline{q})$, summarizes contributions of the interaction between electrons and of electrons with 'real'

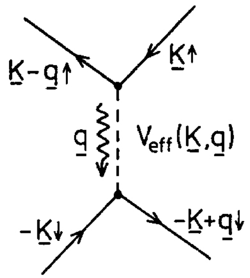

Fig. 7: *Formation of a Cooper pair ($\underline{K}\uparrow, -\underline{K}\downarrow$) by an exchange of a virtual phonon with wave number vector $\underline{q}$. Here, V_{eff} ($\underline{K}, \underline{q}$) is the effective interaction potential (see the text).*

phonons. Because the sound velocity is much smaller than the Fermi velocity of the electrons we have a time retarded electron-electron contact interaction occurring at the locus of the emission of the virtual phonon.

The BCS theory approximates the effective interaction potential by a constant as long as the wave number vectors involved in the scattering process are related to an unperturbed plane-wave energy $\eta_{\underline{K}}$ of free electrons less than $\hbar\omega_D$ above or below the chemical potential μ_F ('Fermi energy'). For all other cases the interaction potential is set to zero. Here, ω_D is the Debye frequency[26].

The many particle wave function of the superconducting ground state is approximated by BCS using the expression

$$\Psi_{BCS} = \prod_{\underline{K}} (u_{\underline{K}} + v_{\underline{K}} c^*_{\underline{K}\uparrow} c^*_{-\underline{K}\downarrow}) \, \Omega_0 \tag{18}$$

Here, $c^*_{\underline{K}\uparrow}$ and $c^*_{-\underline{K}\downarrow}$ are 'creation operators', creating an electron with wave number vector $\underline{K}$, spin up and $-\underline{K}$, spin down, respectively. Here, Ω_0 is the vacuum state with no particles present. The coefficients $u_{\underline{K}}$ and $v_{\underline{K}}$ are given by

$$|u_{\underline{K}}|^2 = (1/2)\ (1 + \varepsilon_{\underline{K}}/E_{\underline{K}}) \tag{19}$$

$$|v_{\underline{K}}|^2 = (1/2)\ (1 - \varepsilon_{\underline{K}}/E_{\underline{K}}) \tag{20}$$

where

$$\varepsilon_{\underline{K}} = \eta_{\underline{K}} - \mu_F, \quad \text{with} \quad \eta_{\underline{K}} = \hbar^2 \underline{K}^2/2\,m \tag{21}$$

$$E_{\underline{K}} = (\varepsilon_{\underline{K}}^2 + |\boldsymbol{\Delta}|^2)^{1/2} \tag{22}$$

For simplicity $|u_{\underline{K}}|^2$, $|v_{\underline{K}}|^2$, and $|\boldsymbol{\Delta}|^2$ will be denoted by $u_{\underline{K}}^2$, $v_{\underline{K}}^2$, and $\boldsymbol{\Delta}^2$ in the following. The energy $E_{\underline{K}}$ turns out to be the excitation energy for a quasiparticle excitation of the system. The parameter $\boldsymbol{\Delta}$ has to be determined selfconsistently and is the energy gap in the quasiparticle excitation spectrum. Also the coefficients $u_{\underline{K}}$ and $v_{\underline{K}}$ find a physical interpretation, because $v_{\underline{K}}^2$ and $u_{\underline{K}}^2 = 1 - v_{\underline{K}}^2$ are the occupation probability of the state $\underline{K}$ with an electron and hole, respectively. It is remarked that the expression for $\varepsilon_{\underline{K}}$ as given in eq. (21) is only valid for the equilibrium case where the chemical potential of the condensate, $\mu_{c,p}$, is equal to μ_F. In general, μ_F has to be replaced by $\mu_{c,p}$. In the presence of an electrostatic potential, Φ, $\varepsilon_{\underline{K}}$ remains unchanged, because $\eta_{\underline{K}}$ has to be replaced by $\eta_{\underline{K}} - e\,\Phi$ and at the same time μ_F and $\mu_{c,p}$, respectively, have to be replaced by electrochemical potentials. See section 5.3 for a detailed discussion.

In Fig. 8 the Cooper pair formation is illustrated. Although the effective interaction potential is only nonzero in the region $2\delta K$, there are all conduction electrons condensed into the superconducting state at $T = 0\,K$. For finite temperatures quasiparticle excitations of the ground state occur. The

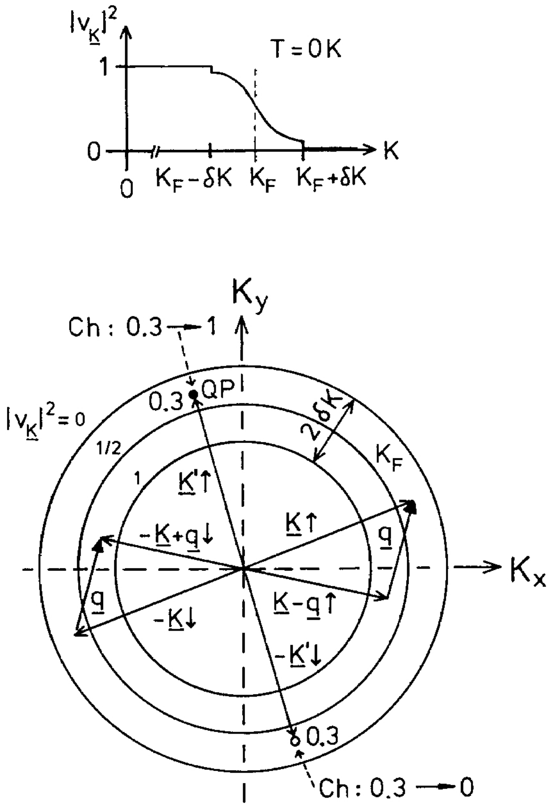

Fig. 8: *Illustration of Cooper pair formation and quasiparticle excitations in a superconductor.*

A plane cut through the $\underline{K}$ *space is drawn. The range* $2\delta K$ *for which the effective interaction potential is nonzero is indicated. The region is very small compared to the Fermi wave number* K_F *and has been magnified in the sketch. The occupation probability of a* $\underline{K}$ *state in a Cooper pair with an electron,* $|v_{\underline{K}}|^2$*, changes across the region* $2\delta K$*. It is nearly zero at the outer border and increases to about one at the inner border. The abrupt changes at the borders are an artifact of the approximation used by BCS for the effective interaction potential.*

The scattering process of Fig. 7 is redrawn into this figure. Furthermore an example for a quasiparticle excitation (QP) at $\underline{K}'\uparrow$ *is given. The value of* $|v_{\underline{K}'}|^2$ *is indicated and also the change of the occupation probability (Ch) due to the excitation of a quasiparticle.*

nature of such an excitation is also illustrated in the figure: A quasiparticle of wave number vector $\underline{K}$ and spin σ is an electron definitely occupying the state $\underline{K}, \sigma$ with its mate $-\underline{K}, -\sigma$ being definitely empty. Here, $\sigma = +1, -1$ denote the spin directions $\uparrow$ and $\downarrow$, respectively.

The excited states are described by 'Bogolubov operators' which are applied to the ground state Ψ_{BCS} and create $(\mathbb{B}^*_{\underline{K},\sigma})$ or annihilate $(\mathbb{B}_{\underline{K},\sigma})$ quasiparticle excitations. The creation operator is for instance given by

$$\mathbb{B}^*_{\underline{K},\sigma} = u^*_{\underline{K}}\ \mathbb{C}^*_{\underline{K},\sigma} - \sigma\, v_{\underline{K}}\ \mathbb{C}_{-\underline{K},-\sigma} \tag{23}$$

It creates an electron with an amplitude $u^*_{\underline{K}}$ in the state $\underline{K},\sigma$ and annihilates at the same time an electron of amplitude $v_{\underline{K}}$ in the state $-\underline{K},-\sigma$. The Bogolubov operators fulfil the exchange relations for Fermion operators. Thus, the created quasiparticles are Fermions.

The effect of a quasiparticle excitation is to block a pair state from participating in the pairing interaction. It is evident that this disturbance enhances the energy of the system above the energy of the ground state. The result of the BCS theory for the excitation energies as given in eq.(22) and illustrated in Fig.9 shows that there is a temperature dependent energy gap in the excitation spectrum.

Quasiparticle excitations may be generated by tunnel injection (or extraction) of electrons from (or into) a normal conductor. Therefore, the superconductor for instance blocks the penetration of electrons until their energy is above the pair chemical potential (which is equal to μ_F in equilibrium) by at least an amount of the energy gap. This is the reason why these experiments are a method to measure the gap.

The effective charge $Q_{\underline{K}}$ of a quasiparticle depends on the effective number of electrons $\tilde{n}$ added to the system if a quasiparticle with wave number vector $\underline{K}$ is excited. Because $\tilde{n} = u^2_{\underline{K}} - v^2_{\underline{K}}$, it follows [56]

$$Q_K = (u^2_{\underline{K}} - v^2_{\underline{K}})(-e) \tag{24}$$

Due to its effective charge the character of the quasiparticle continuously changes in the region $K_F \pm \delta K$ from 'electron-like', $Q_{\underline{K}} = -e$, at the upper border to 'hole-like', $Q_{\underline{K}} = e$, at the lower border. At K_F it is $Q_{\underline{K}_F} = 0$. For the quasiparticle at $\underline{K}'$ in Fig. 8 it is $v^2_{\underline{K}'} = 0.3$ and thus $u^2_{\underline{K}'} = 0.7$, so that $Q_{\underline{K}'} = -0.4e$ and the quasiparticle is 'more electron-like'.

The excitation spectrum in Fig.9 has two branches, a 'more electron-like', $\varepsilon_{\underline{K}} > 0$, and a 'more hole-like', $\varepsilon_{\underline{K}} < 0$, with $Q_{\underline{K}} = 0$ at $\varepsilon_{\underline{K}} = 0$. This can easiestly be seen by remembering that $Q_{\underline{K}} = (\varepsilon_{\underline{K}} / E_{\underline{K}})(-e)$.

In equilibrium the probability for the occupation of a quasiparticle state is governed by the Fermi function and the overall charge of the quasiparticles vanishes. Also in a nonequilibrium situation the branches of the excitation spectrum may be equally overpopulated so that there is no charge imbalance and the quasiparticles can simply be characterized by an effective temperature T^* which is larger than the lattice temperature. This is for instance the case for a light irradiated sample [57]. For tunnel injection experiments or in a phase-slip center, however, the two branches of the excitation spectrum are unequally populated and a charge imbalance Q^* is generated [56].

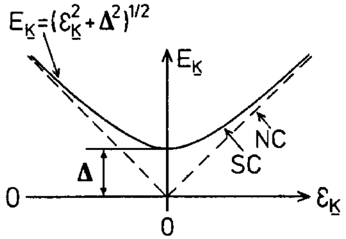

Fig. 9: *Quasiparticle excitation spectrum of a superconductor according to BCS (full line, SC) together with the corresponding spectrum of the normal conductor (dashed line, NC) with $E_K = |\varepsilon_K|$ and $E_K = \varepsilon_K$ for $\varepsilon_K < 0$ and $\varepsilon_K > 0$, respectively. Furthermore , $\Delta(T)$ is the energy gap.*

5.3. Charge Imbalance and Quasiparticle/Pair Electrochemical Potentials

In this section we discuss the electrochemical potentials in a superconductor and their relation with the charge imbalance. In a superconductor two chemical potentials and their related electrochemical potentials are introduced, assigned to the Cooper pairs and quasiparticles, respectively.

First we discuss the physical meaning of the electrochemical pair potential. Then it is shown that in equilibrium the chemical potential of the pairs is equal to the chemical potential in the normal conducting state as well for zero temperature as also for nonzero temperature.

Next, the charge imbalance of the excitation spectrum of a superconductor is defined and its relation with the chemical potential of the pairs is derived. It turns out that the pair chemical potential is different from the chemical potential (of the electrons) in the normal conducting state if there is a charge imbalance in a superconductor.

Then we turn to the problem of the chemical potential of the quasiparticles and first discuss in which situation this potential may be a meaningful quantity. Then a relation is derived between the chemical potentials of the pairs and the quasiparticles in the presence of a charge imbalance. It turns out that both potentials are equal for vanishing charge imbalance, but differ if the charge imbalance is nonzero. In equilibrium the chemical quasiparticle potential is, thus, equal to the chemical potential in the normal state. In nonequilibrium the chemical potential of the pairs, usually, is slightly larger than the normal state chemical potential, except being close to the critical temperature, where both potentials are nearly equal. Finally, we give a physical picture for the concept of a chemical potential of the quasiparticles.

From this knowledge a formula results describing the dependence of the difference between the electrochemical potentials of pairs and quasiparticles on the charge imbalance. This formula is one of the central results of this section.

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