Exercises Set 2

Nov. 04 2015

Exercise 1: Lyapunov function

Analyse the stability of the fixed point $(0, 0)$ of the dynamical system

$$\begin{aligned}\dot{x} &= -2x - y^2 \\ \dot{y} &= -y - x^2,\end{aligned}$$

using the notion of the Lyapunov function.
(Hint: possible Lyapunov functions have the form $L(x, y) = \frac{1}{2}(\alpha x^2 + \beta y^2)$).

Exercise 2: Double well potential

Study the dynamics of a particle moving in a double well potential under the influence of friction ($\gamma > 0$). The equation of motion is

$$\ddot{x} + \gamma \dot{x} - x + x^3 = 0. \tag{1}$$

a.) Rewrite the dynamics as a system of first order differential equations and find and classify the fixed points.

b.) Write a program that solves the differential equations using a Runge-Kutta method. This could be e.g. the 4t/5th order Runge-Kutta method ("dopri5") implemented in the "ode" class from the "scipy.integrate" module.
(See docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.ode.html)

c.) Figure out a suitable numerical way to determine in which fixed point the damped dynamics will finally end.

d.) Using this, write a program that plots the basins of attractions of the two attractive fixed points (color coded) in one plot. Let it run for several values of the friction parameters.