Exercises Set 6

Nov. 30 2015

Exercise 1: Lyapunov exponents

For the logistic map

$$f(x) = rx(1-x)$$

a.) numerically calculate the Lyapunov exponent as a function of r and plot it in the interval $r \in [2, 4]$.

b.) for $r = 4$ find the expression for the Lyapunov exponent using the natural density $\rho_L(x)$ (see exercise 2), and evaluate it either by using a computer algebra system (like Mathematica or Wolfram Alpha, the latter is freely available online) or numerically. (In the latter case you have to be careful at the singularities!) Compare the result to the analytically known expression $\lambda = \ln 2$.

Exercise 2: Invariant density

a.) For the logistic map with $r = 4$ confirm by insertion that the natural density

$$\rho_L(x) = \frac{1}{\pi\sqrt{x(1-x)}},$$

is a fixed point of the Frobenius-Perron equation.

b.) Find the natural density of a map with a globally attractive fixed point in the origin.

Exercise 3: Bernoulli shift

Examine the generalized Bernoulli shift

$$f(x) = bx \mod 1,$$

with $b \geq 2$ and $x \in [0, 1]$. Numerically and using the Frobenius-Perron equation confirm that the constant density is invariant for b=3. Try the same numerical confirmation for b=2. Is the constant density invariant for $b = 2.5$ as well?