Exercises Set 3

Nov. 09 2015

Exercise 1: Topological conjugacy

Show that the flow generated by $\dot{\mathbf{x}} = \mathbf{J}\mathbf{x}$ with

$$J = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

is topologically conjugate to the flow created by $\dot{\mathbf{y}} = \mathbf{y}$.

Hint: Use the coordinate transformation that in polar coordinates (for $r \neq 0$) can be written as $\tilde{r} = r$ and $\tilde{\theta} = \theta - \ln r$.

Exercise 2: Double well potential (continuation)

Revisit the dynamics of a particle moving in a double well potential under the influence of friction ($\gamma > 0$), that you studied last week. The equation of motion is

$$\ddot{x} + \gamma\,\dot{x} - x + x^3 = 0.$$

a.) Find the stable and unstable subspaces of the saddle point.

b.) Rewrite the program you wrote last week (that solves the differential equations) so that you can follow the dynamics backwards in time.

c.) Numerically calculate the **stable** manifold of the saddle, using the hint given in the lecture (i.e. start a number of initial conditions in the stable subspace in a small vicinity of the fixed point.)

d.) Plot the manifold on top of the basins of attraction you calculated last week.