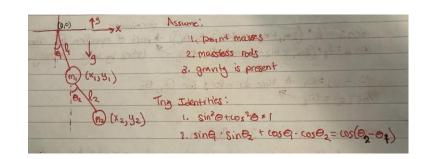
Kinematic Constants

$$\begin{split} x_1 &= l_1 \mathrm{sin}\theta_1 \\ y_1 &= -l_1 \mathrm{cos}\theta_1 \\ x_2 &= l_1 \mathrm{sin}\theta_1 + l_2 \mathrm{sin}\theta_2 \\ y_2 &= -l_1 \mathrm{cos}\theta_1 - l_2 \mathrm{cos}\theta_2 \end{split}$$



Velocities (time derivative)

$$\begin{aligned} \dot{x_1} &= \left(l_1 \cos\theta_1\right) \dot{\theta_1} \\ \dot{y_1} &= \left(l_1 \sin\theta_1\right) \dot{\theta_1} \\ \dot{x_2} &= \left(l_1 \cos\theta_1\right) \dot{\theta_1} + \left(l_2 \cos\theta_2\right) \dot{\theta_2} \\ \dot{y_2} &= \left(l_1 \sin\theta_1\right) \dot{\theta_1} + \left(l_2 \sin\theta_2\right) \dot{\theta_2} \end{aligned}$$

Potential Energy (V)

$$\begin{split} V &= \sum_{i} mgh_{i} \\ V &= m_{1}gy_{1} + m_{2}gy_{2} \\ V &= m_{1}g\left(-l_{1}\text{cos}\theta_{1}\right) + m_{2}g\left(-l_{1}\text{cos}\theta_{1} + -l_{2}\text{cos}\theta_{2}\right) \\ V &= -m_{1}g \cdot l_{1}\text{cos}\theta_{1} - m_{2}g \cdot l_{1}\text{cos}\theta_{1} - m_{2}g \cdot l_{2}\text{cos}\theta_{2} \\ V &= -\left(m_{1} + m_{2}\right)gl_{1}\text{cos}\theta_{1} - m_{2}gl_{2}\text{cos}\theta_{2} \end{split}$$

Kinetic Energy (T)

$$\begin{split} T &= \sum_{i} \left(\frac{1}{2} m v_{i}^{2}\right) \\ T &= \frac{1}{2} m v_{1}^{2} + \frac{1}{2} m v_{2}^{2} = \frac{1}{2} m_{1} \left(\dot{x}_{1}^{2} + \dot{y}_{1}^{2}\right) + \frac{1}{2} m_{2} \left(\dot{x}_{2}^{2} + \dot{y}_{2}^{2}\right) \\ T &= \frac{1}{2} m_{1} \left(l_{1}^{2} \dot{\theta}_{1}^{2} \cos^{2} \theta_{1} + l_{1}^{2} \dot{\theta}_{1}^{2} \sin^{2} \theta_{1}\right) + \frac{1}{2} m_{2} \left(\dot{x}_{2}^{2} + \dot{y}_{2}^{2}\right) \\ T &= \frac{1}{2} m_{1} \left(l_{1}^{2} \dot{\theta}_{1}^{2} \left(\cos^{2} \theta_{1} + \sin^{2} \theta_{1}\right)\right) + \frac{1}{2} m_{2} \left(\dot{x}_{2}^{2} + \dot{y}_{2}^{2}\right) \\ T &= \frac{1}{2} m_{1} l_{1}^{2} \dot{\theta}_{1}^{2} + \frac{1}{2} m_{2} \left(\dot{x}_{2}^{2} + \dot{y}_{2}^{2}\right) \end{split}$$

$$\begin{split} \dot{x}_{2}^{2} &= \left(l_{1}\dot{\theta}_{1}\cos\theta_{1} + l_{2}\dot{\theta}_{2}\cos\theta_{2}\right)\left(l_{1}\dot{\theta}_{1}\cos\theta_{1} + l_{2}\dot{\theta}_{2}\cos\theta_{2}\right) \\ \dot{x}_{2}^{2} &= l_{1}^{2}\dot{\theta}_{1}^{2}\cos^{2}\theta_{1} + 2l_{1}l_{2}\dot{\theta}_{1}\dot{\theta}_{2}\cos\theta_{1}\cos\theta_{2} + l_{2}^{2}\dot{\theta}_{2}^{2}\cos^{2}\theta_{2} \\ \dot{y}_{2}^{2} &= \left(l_{1}\dot{\theta}_{1}\sin\theta_{1} + l_{2}\dot{\theta}_{2}\sin\theta_{2}\right)\left(l_{1}\dot{\theta}_{1}\sin\theta_{1} + l_{2}\dot{\theta}_{2}\sin\theta_{2}\right) \\ \dot{y}_{2}^{2} &= l_{1}^{2}\dot{\theta}_{1}^{2}\sin^{2}\theta_{1} + 2l_{1}l_{2}\dot{\theta}_{1}\dot{\theta}_{2}\sin\theta_{1}\sin\theta_{2} + l_{2}^{2}\dot{\theta}_{2}^{2}\sin^{2}\theta_{2} \\ \dot{x}_{2}^{2} + \dot{y}_{2}^{2} &= l_{1}^{2}\dot{\theta}_{1}^{2}\left(\cos^{2}\theta_{1} + \sin^{2}\theta_{1}\right) + 2l_{1}l_{2}\dot{\theta}_{1}\dot{\theta}_{2}\left(\cos\theta_{1}\cos\theta_{2} + \sin\theta_{1}\sin\theta_{2}\right) + l_{2}^{2}\dot{\theta}_{2}^{2}\left(\cos^{2}\theta_{1} + \sin^{2}\theta_{1}\right) \\ \dot{x}_{2}^{2} + \dot{y}_{2}^{2} &= l_{1}^{2}\dot{\theta}_{1}^{2} + 2l_{1}l_{2}\dot{\theta}_{1}\dot{\theta}_{2}\left(\cos\left(\theta_{2} - \theta_{1}\right)\right) + l_{2}^{2}\dot{\theta}_{2}^{2} \\ T &= \frac{1}{2}m_{1}l_{1}^{2}\dot{\theta}_{1}^{2} + \frac{1}{2}m_{2}\left(l_{1}^{2}\dot{\theta}_{1}^{2} + 2l_{1}l_{2}\dot{\theta}_{1}\dot{\theta}_{2}\left(\cos\left(\theta_{2} - \theta_{1}\right)\right) + l_{2}^{2}\dot{\theta}_{2}^{2} \end{split}$$

Total Energy (L)

$$\begin{split} L &= T + V \\ L &= \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \Big(l_1^2 \dot{\theta}_1^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \Big(\cos \left(\theta_2 - \theta_1 \right) \Big) + l_2^2 \dot{\theta}_2^2 \Big) \\ &+ \Big(m_1 + m_2 \Big) g l_1 \cos \theta_1 + m_2 g l_2 \cos \theta_2 \Big) + \left(m_2 + m_2 \right) g l_1 \cos \theta_1 + m_2 g l_2 \cos \theta_2 \Big) \\ &+ \left(m_2 + m_2 \right) g l_2 \cos \theta_1 + m_2 g l_2 \cos \theta_2 \Big) + \left(m_3 + m_2 \right) g l_4 \cos \theta_1 + m_2 g l_2 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_3 \cos \theta_1 + m_3 g l_3 \cos \theta_2 \Big) + \left(m_3 + m_3 \right) g l_4 \cos \theta_1 + m_3 g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 + m_3 g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 + m_3 g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 + m_3 g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 + m_3 g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 + m_3 g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 + m_3 g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 + m_3 g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 + m_3 g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 + m_3 g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_2 \Big) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_2 \Big) g l_4 \cos \theta_1 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_2 \Big) g l_4 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_4 \cos \theta_1 \Big) g l_4 \cos \theta_2 \Big) g l_5 \cos \theta_2 \Big) g l_5 \cos \theta_1 \Big) g l_5 \cos \theta_2 \Big) \\ &+ \left(m_3 + m_3 \right) g l_5 \cos \theta_2 \Big) g l_5 \cos \theta_3 \Big) g l_5 \cos \theta_1 \Big) g l_5 \cos \theta_2 \Big) g l_5 \cos \theta_2 \Big) g l_5 \cos \theta_3 \Big) g l_5 \cos$$

Lagrange Equation of Motion

$$\begin{split} \mathcal{Q}_i &= \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = 0 \\ &\frac{\partial L}{\partial \dot{\theta}_1} = m_1 l_1^2 \dot{\theta}_1 + m_2 l_1^2 \dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos \left(\theta_2 - \theta_1\right) \\ &\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) = m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos \left(\theta_2 - \theta_1\right) - m_2 l_1 l_2 \dot{\theta}_2 \sin \left(\theta_2 - \theta_1\right) \left(\dot{\theta}_2 - \dot{\theta}_1\right) \\ &\frac{\partial L}{\partial \theta_1} = m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin \left(\theta_2 - \theta_1\right) - \left(m_1 + m_2\right) g l_1 \sin \theta_1 \\ &\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos \left(\theta_2 - \theta_1\right) - m_2 l_1 l_2 \dot{\theta}_2^2 \sin \left(\theta_2 - \theta_1\right) \\ &+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin \left(\theta_2 - \theta_1\right) - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin \left(\theta_2 - \theta_1\right) + \left(m_1 + m_2\right) g l_1 \sin \theta_1 \\ &\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = \left(m_1 + m_2\right) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos \left(\theta_2 - \theta_1\right) - m_2 l_2 \dot{\theta}_2^2 \sin \left(\theta_2 - \theta_1\right) + \left(m_1 + m_2\right) g \sin \theta_1 = 0 \end{split}$$

$$\begin{split} \mathcal{Q}_i &= \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = 0 \\ &\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_2 - \theta_1) + m_2 l_2^2 \dot{\theta}_2 \\ &\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) = m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) - m_2 l_1 l_2 \dot{\theta}_1 \sin(\theta_2 - \theta_1) \left(\dot{\theta}_2 - \dot{\theta}_1 \right) \\ &\frac{\partial L}{\partial \theta_2} = -m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_2 - \theta_1) - m_2 g l_2 \sin\theta_2 \\ &\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \left(\frac{\partial L}{\partial \theta_2} \right) = m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_2 - \theta_1) + m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) \\ &+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_2 - \theta_1) + m_2 g l_2 \sin\theta_2 \\ &\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \left(\frac{\partial L}{\partial \theta_2} \right) = m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) + m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) + m_2 g l_2 \sin\theta_2 \\ &\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \left(\frac{\partial L}{\partial \theta_2} \right) = l_2^2 \ddot{\theta}_2 + l_1 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) + l_1 \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) + g \sin\theta_2 = 0 \end{split}$$

System of 4 Equations (Solved for angular accelerations in MATLAB)

$$\begin{split} &\left(m_1+m_2\right)l_1\ddot{\theta}_1+m_2l_2\ddot{\theta}_2\cos\left(\theta_2-\theta_1\right)=m_2l_2\dot{\theta}_2^2\sin\left(\theta_2-\theta_1\right)-\left(m_1+m_2\right)g\sin\theta_1\\ &l_1\ddot{\theta}_1\cos\left(\theta_2-\theta_1\right)+l_2\ddot{\theta}_2=-l_1\dot{\theta}_1^2\sin\left(\theta_2-\theta_1\right)-g\sin\theta_2 \end{split}$$

$$\dot{\theta}_1 = w_1$$

$$\dot{w}_1 = \ddot{\theta}_1$$

$$\dot{\theta}_2 = w_2$$

$$\dot{w}_2 = \ddot{\theta}_2$$