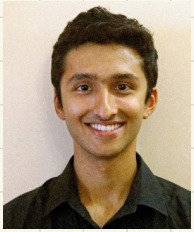
Commitment Over Unreliable Noisy Channels: When Awareness Meets Control

Manideep Mamindlapally
(TIFR, Mumbai)*

Amitalok Budkuley
(IIT Kharagpur)



Pranav Joshi
(Independent Researcher)

Anuj Kumar Yadav
(EPFL, Lausanne)

*work done while at IIT Kharagpur

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- Security Protocol
- Two parties – COMMITTER  & VERIFIER 
- Two phases

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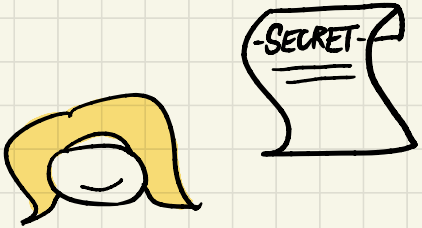

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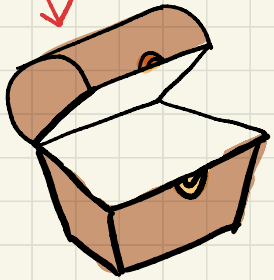

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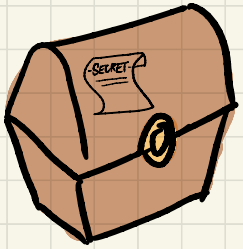

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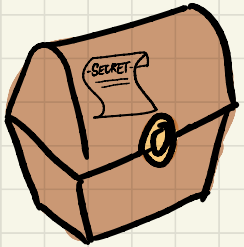

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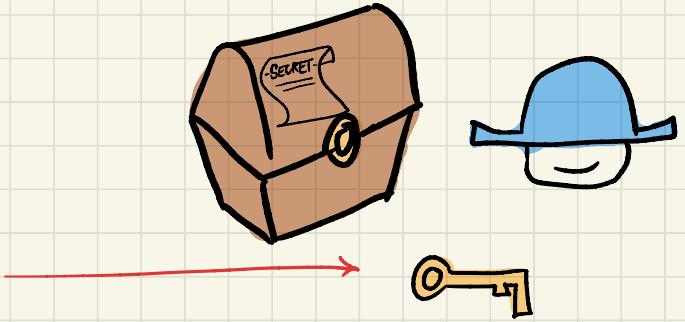

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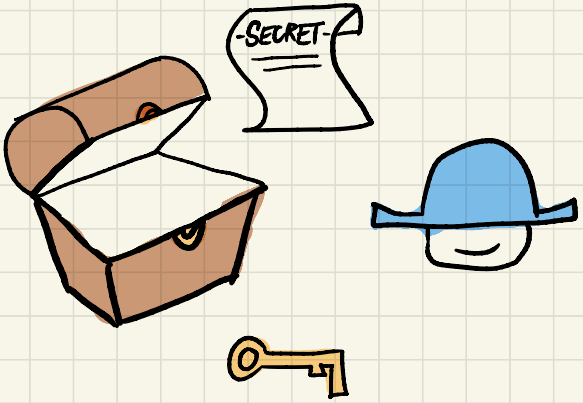

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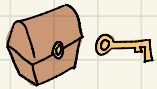
Non trivial resource: 

- Security Guarantees
  - soundness, concealment, bindingness.

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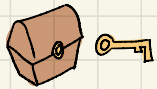
Non trivial resource: 

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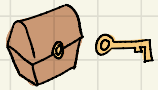
Non trivial resource: 

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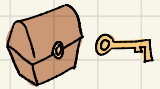
Non trivial resource: 

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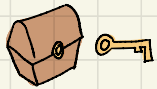
Non trivial resource: 

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– Commitment using "noisy channels" resource. [Crepeau et al STOC 1988]

↳ Unconditionally Secure

(Information theoretically secure)

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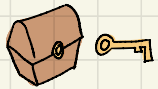
Non trivial resource: 

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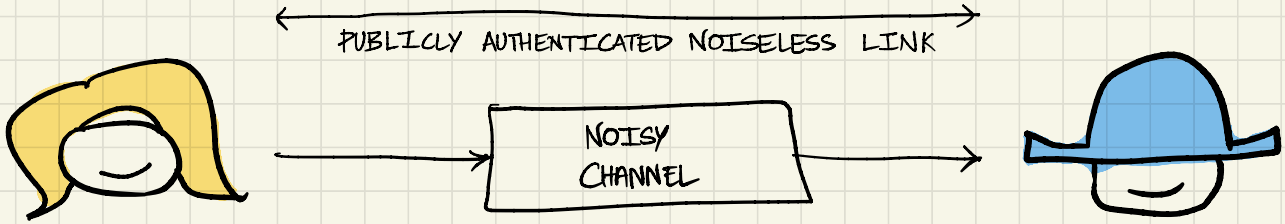
↳ Imperfectly characterised : "UNRELIABLE" NOISY CHANNELS

Problem Setup: Commitment Over Noisy Channels



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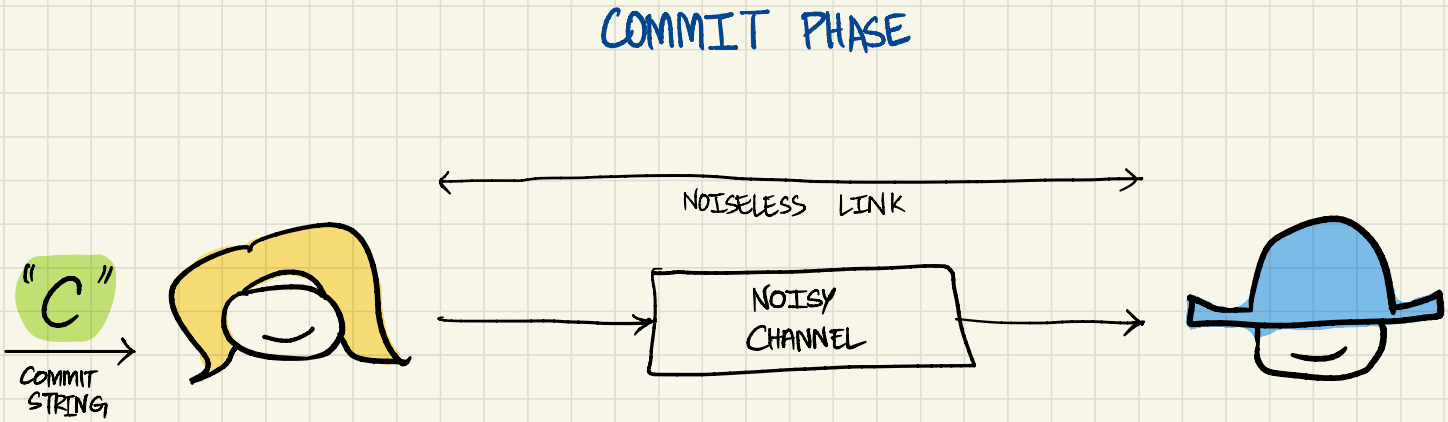

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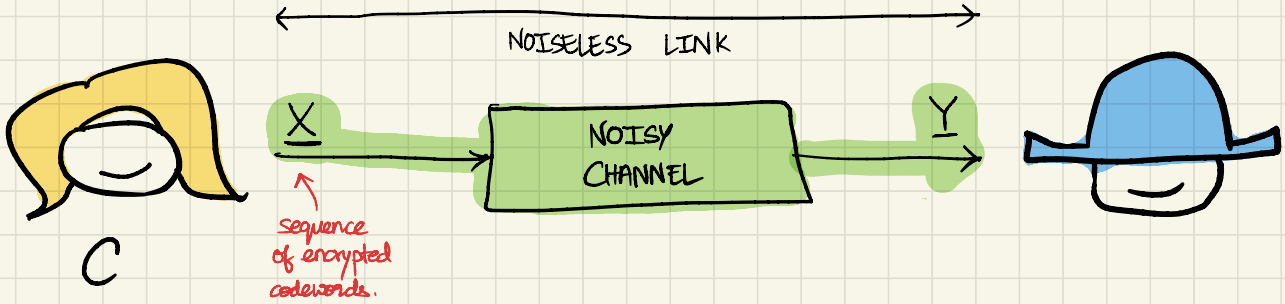
COMMIT PHASE



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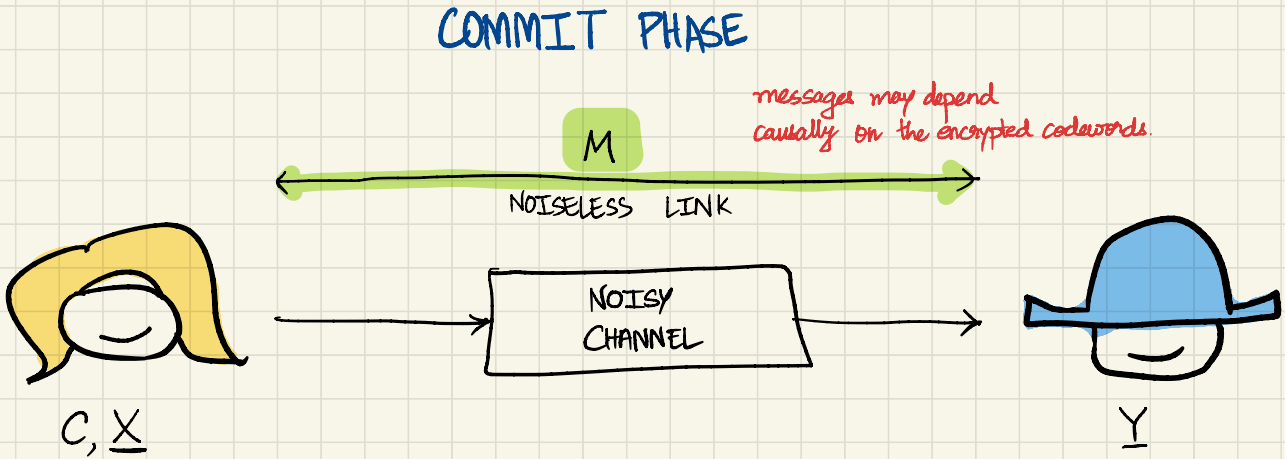

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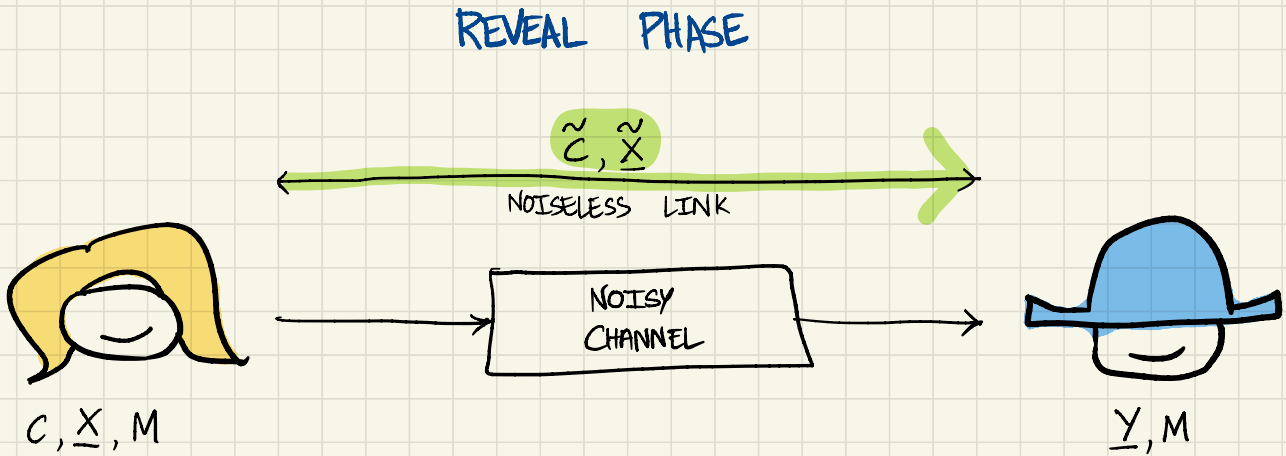

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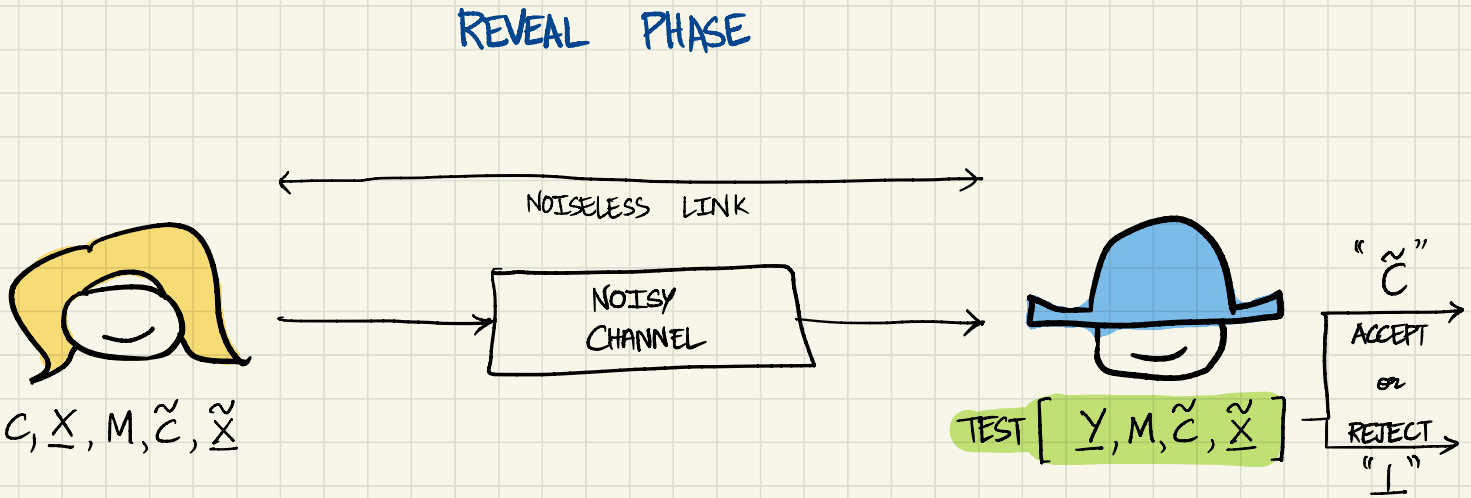

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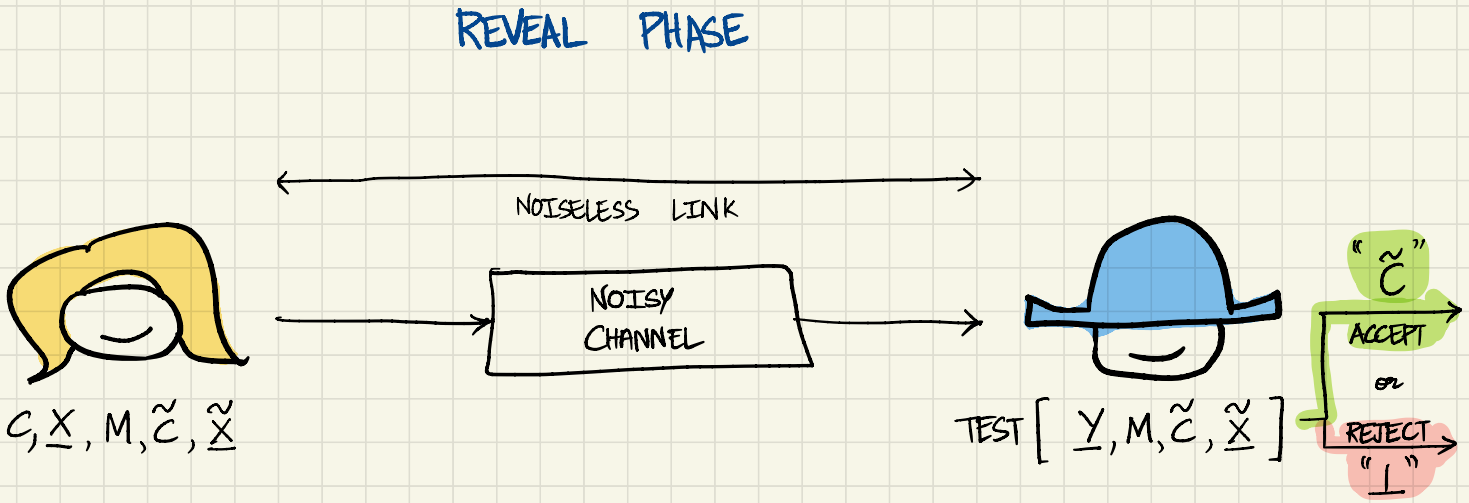

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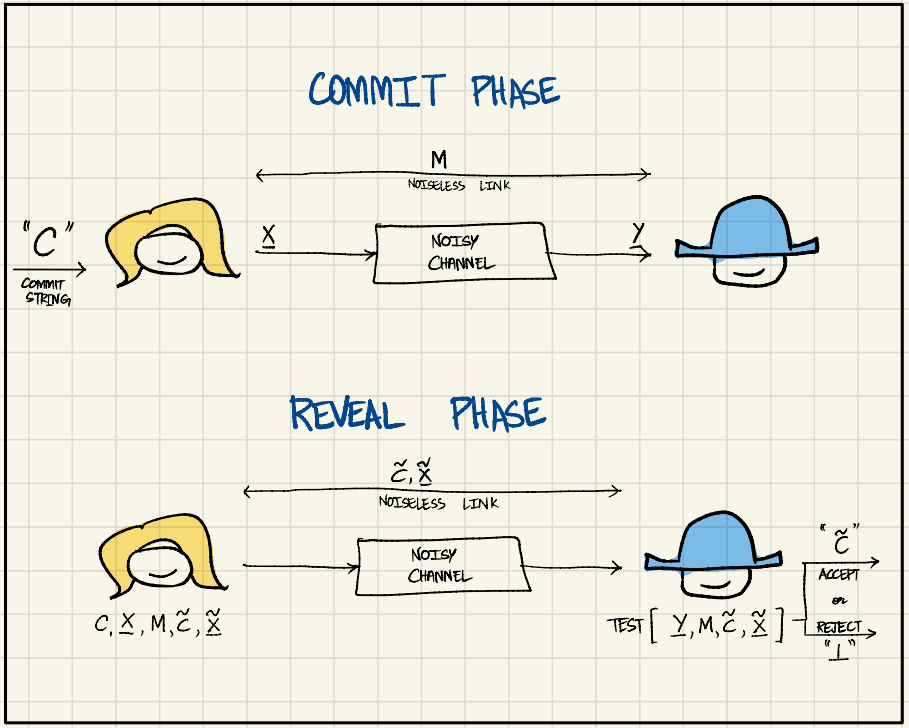

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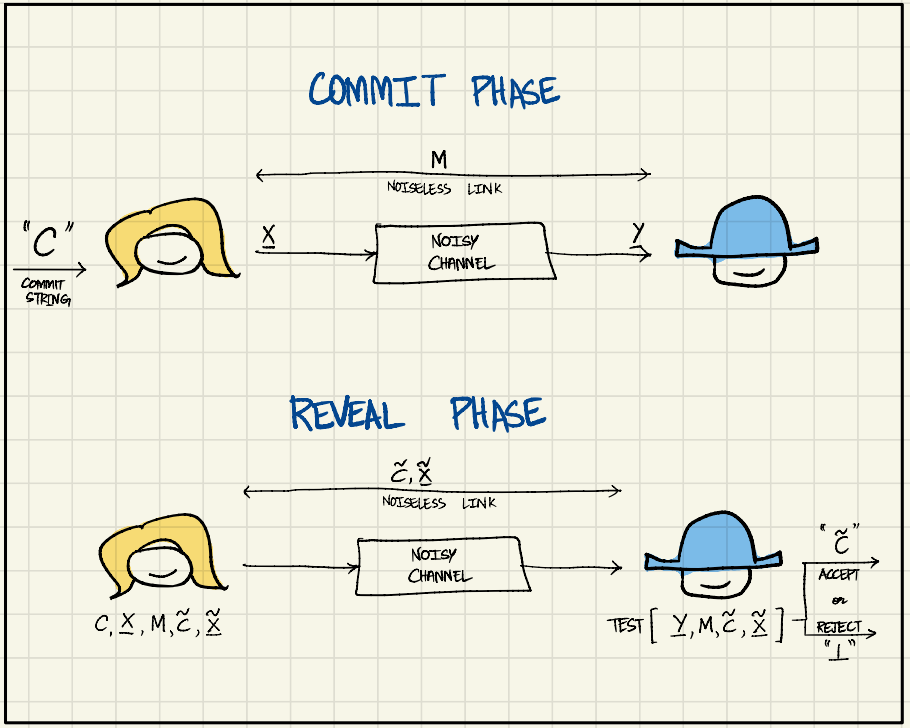
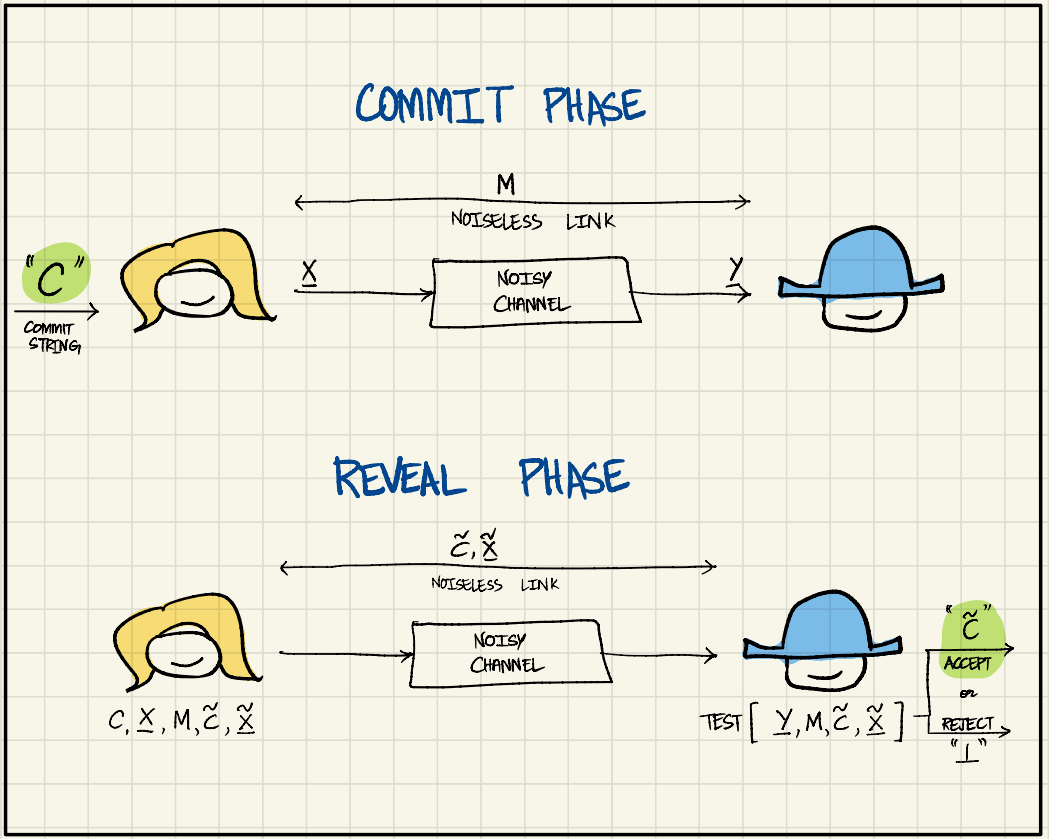


Security Guarantees

1. Soundness
2. Concealment
3. Bindingness

Security Guarantee: Soundness

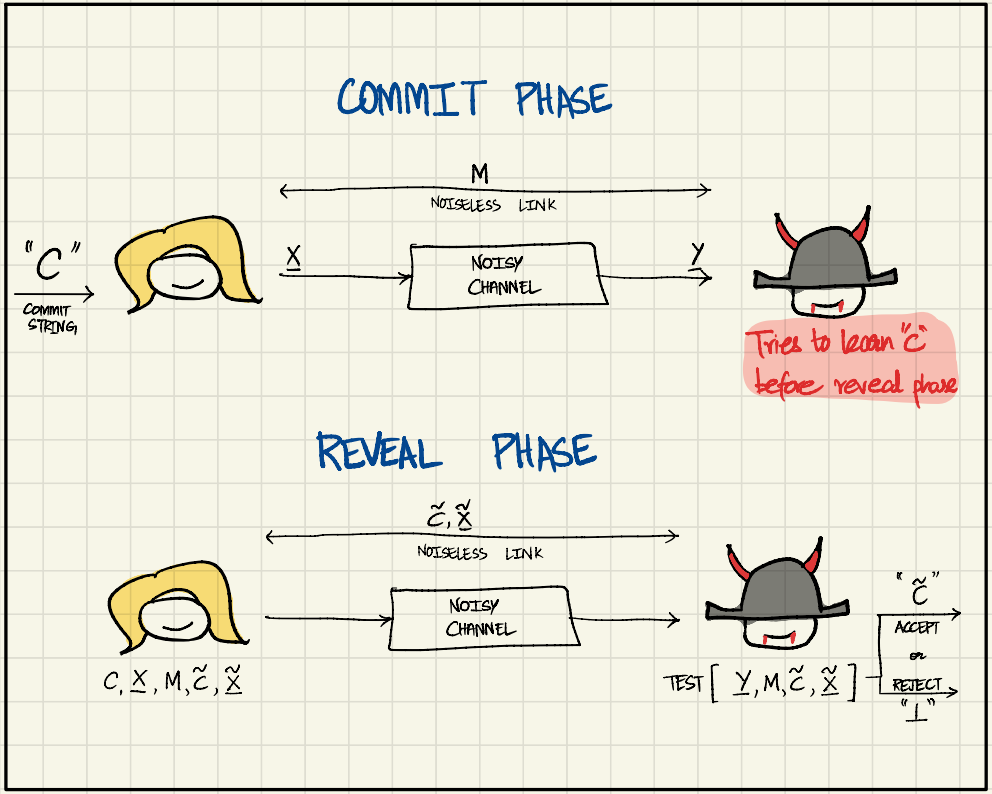


- When both Alice and Bob are honest, Bob's test T should PASS.

ϵ-Soundness

$$\mathbb{P}[T(Y, M, c, x) = \perp] \leq \epsilon \quad \forall\, c, x$$

Security Guarantee : Concealment



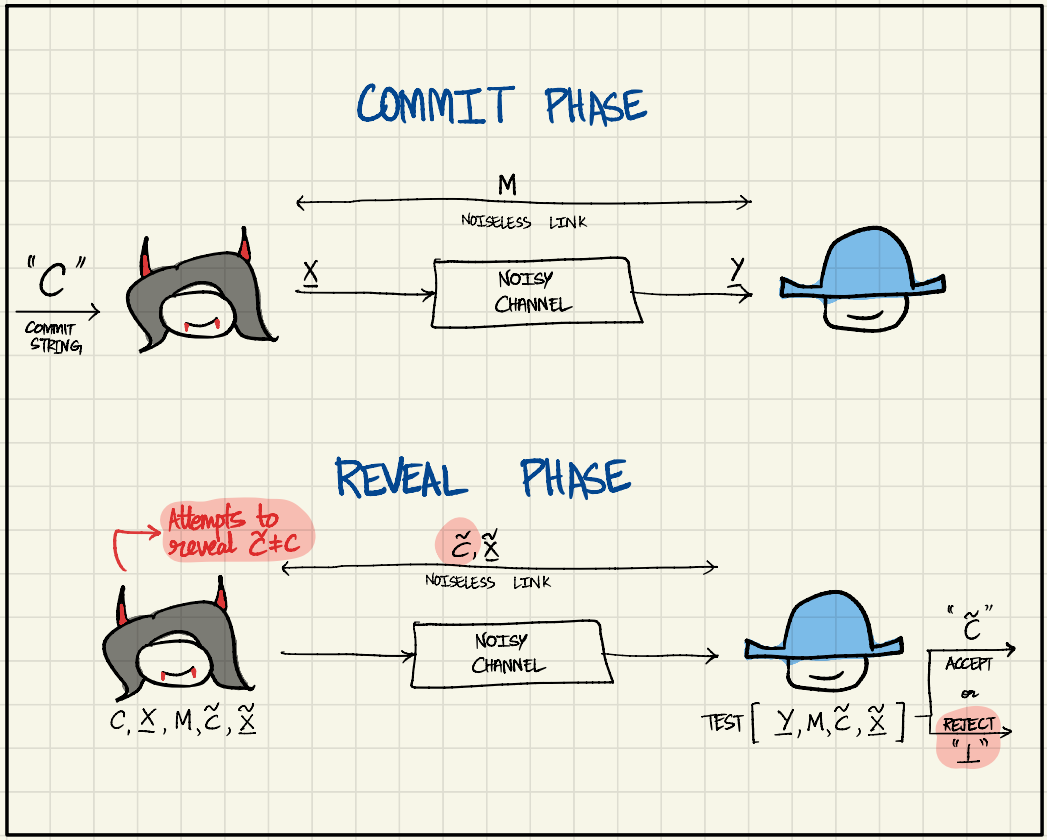
- Protection against Malicious Bob
- Ensures that Bob remains oblivious of "C" till the end of REVEAL PHASE.

ϵ-concealment

$$I(View_B : C) \leq \epsilon$$

where $View_B = (Y, M)$

Security Guarantee: Bindingness



- Protection against Malicious Alice.

- Ensures that Bob's test FAILS when Alice reveals $\tilde{C} \neq C$.

ϵ-bindingness

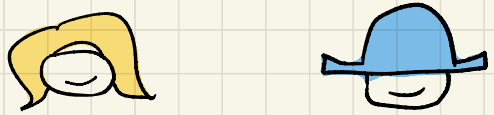
$$P\left[T(View_B, C_1, \underline{x}_1) = T(View_B, C_2, \underline{x}_2) = ACCEPT\right] \leq \epsilon$$

$$\forall\ C_1 \neq C_2, \underline{x}_1, \underline{x}_2$$

where $View_B = (\underline{Y}, M)$

Commitment Security Guarantees : Overview

SOUNDNESS



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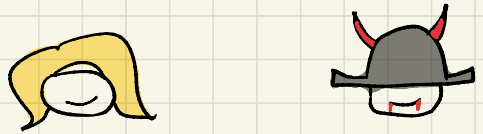

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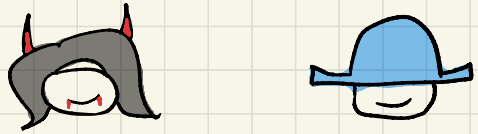


Commitment Security Guarantees : Overview

SOUNDNESS

CONCEALMENT

BINDINGNESS

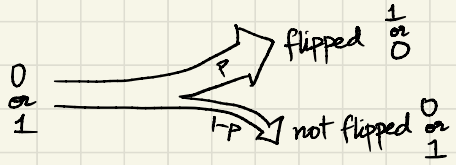
NO SECURITY GUARANTEES

Noisy Channels

"Reliable" Noisy Channels : Perfectly characterised by a Transition fxn.

Example: BSC(p)

Characterised by $\begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$



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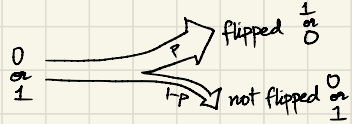

Characterised by $\begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$

Throughput : # bits that can be committed per use of noisy channel

Commitment capacity : maximum throughput achievable (satisfying security guarantees)

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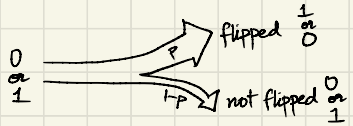

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$$C_{BSC(p)} = H(p)$$

[Winter et al ICCC 2003]

$$C_{DMC} = \max_{P_X} H(X|Y)$$

[Mamindlapally et al ISIT 2021]

$$C_{DMC,\varepsilon,\Gamma} = \max_{P_X : E_{\mathcal{R}}[\varepsilon(\mathcal{R})] \le \Gamma} H(X|Y)$$

$$= \min_{\gamma \ge 0} \max_{\mathcal{R}_y} \log\left[\sum_{x \in \mathcal{X}} \exp\{-D(W_{y|x} \| \mathcal{R}_y) + \gamma(\Gamma - \varepsilon(x))\}\right]$$

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Unreliable Noisy Channels ("compoundness")

Unreliable Noisy channels: Not perfectly characterized.

{Set of Transition fxns} ← From Alice and Bob's perspective

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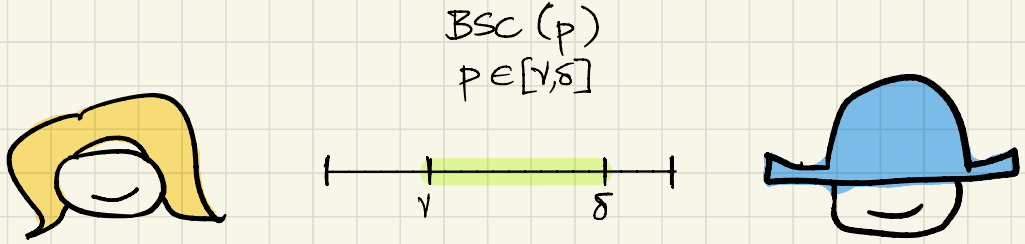

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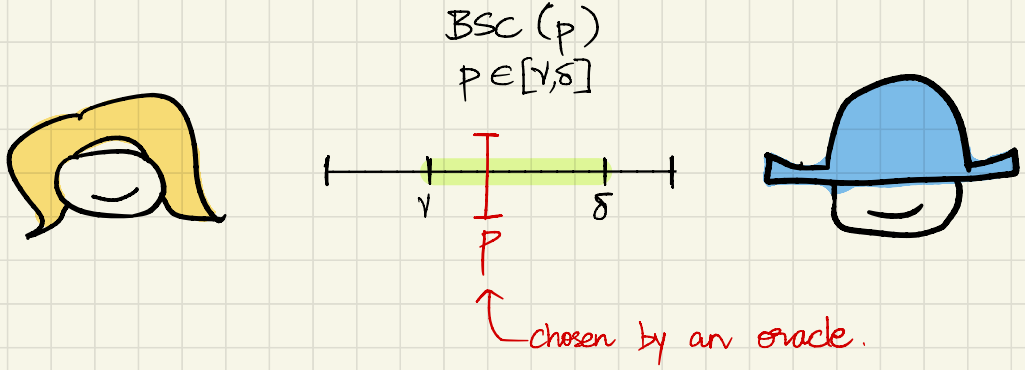

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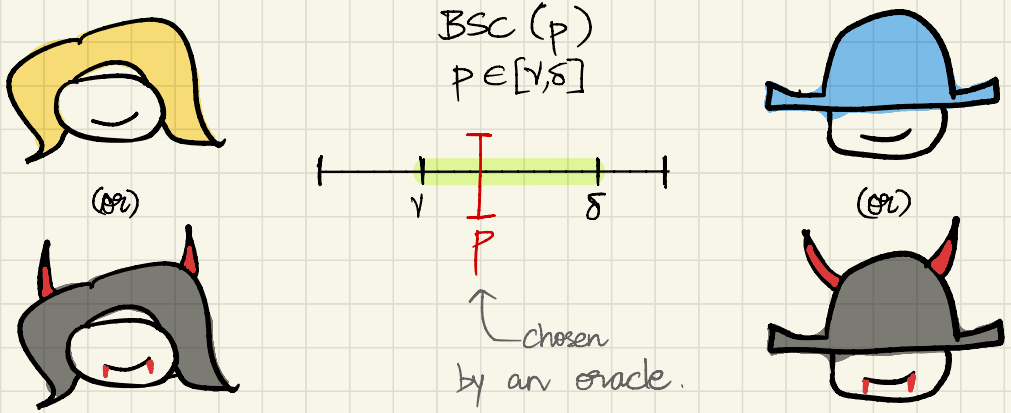

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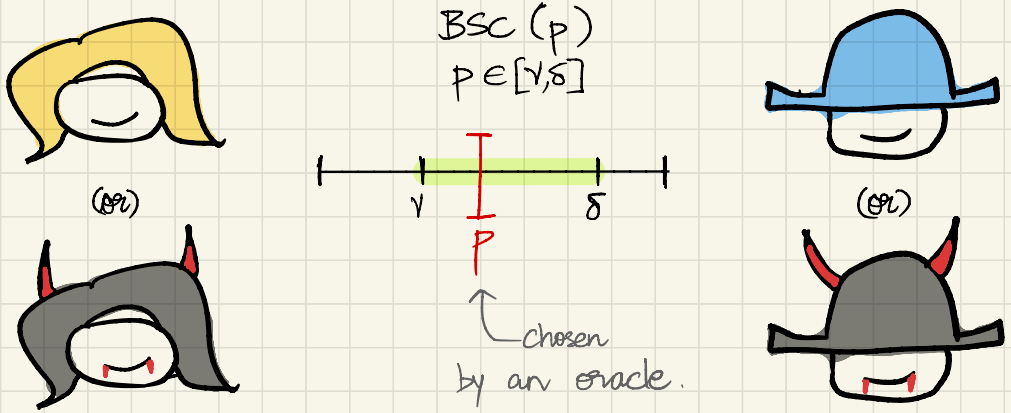

[Yadav et al NCC 2022]

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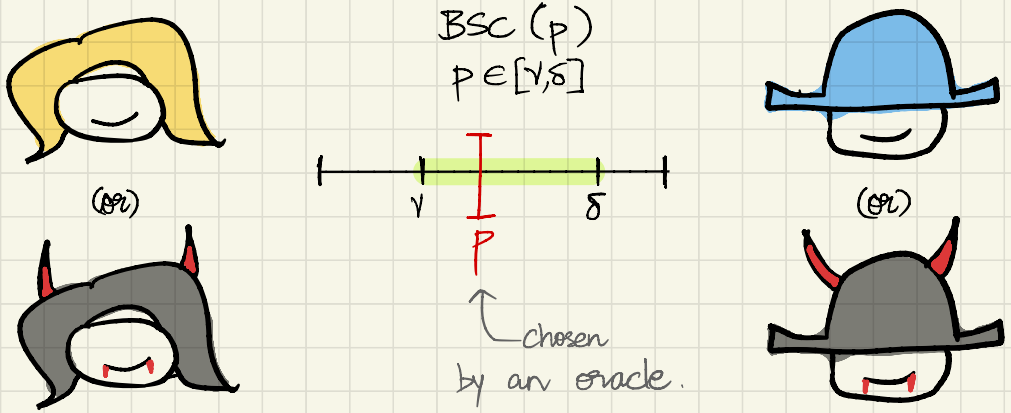

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State Unaware Compound-BSC

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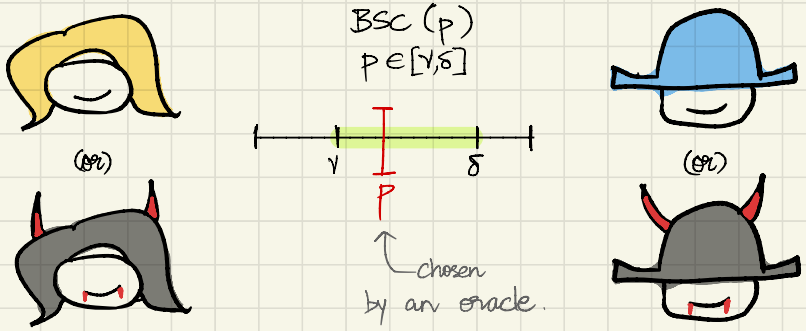


→ Further "unreliable variants"
Capabilities of Alice and Bob

① Awareness

② Control

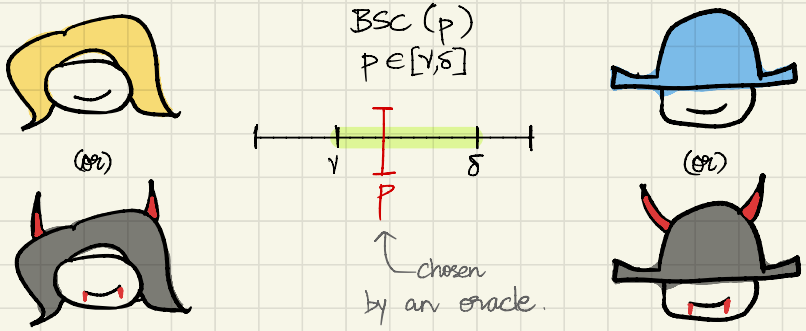
Unreliable Noisy Channels ("Compoundness")

Unreliable Noisy channels: Not perfectly characterized.

{Set of Transition fxns} ← From Alice and Bob's perspective

Example: Compound-BSC[γ,δ]

~behaves like a BSC(p) with "p" not known to the parties



→ Further "unreliable variants"
Capabilities of Alice and Bob

① Awareness]* Current Focus.

② Control

Unreliable Noisy Channels ("Compoundness")

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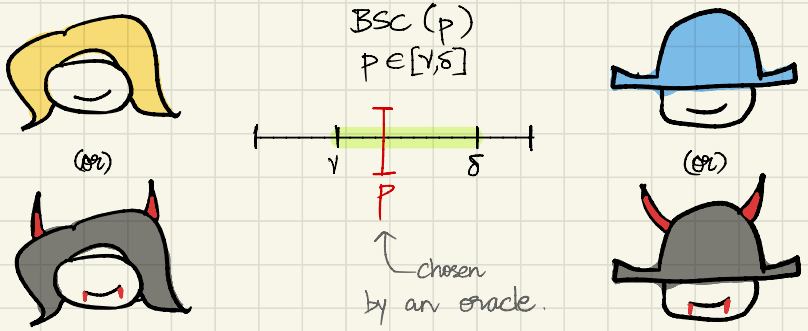

→ Further "unreliable variants"
Capabilities of Alice and Bob

① Awareness]* Current Focus.

② Control

UNCs
[Dziembowski et al EUROCRYPT 1999]

Unreliable Noisy Channels ("Compoundness")

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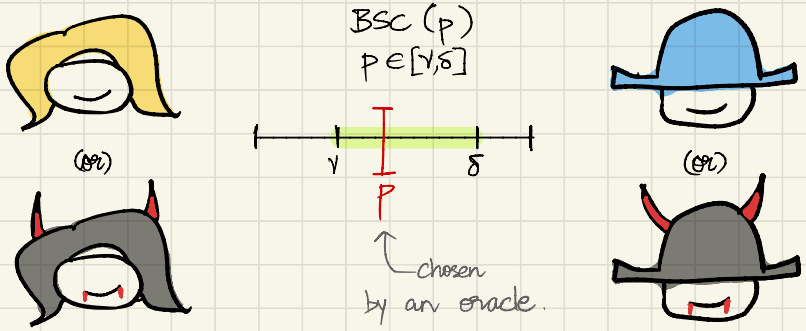


→ Further "unreliable variants" Capabilities of Alice and Bob

① Awareness]* Current Focus.

② Control — UNCs [Ahlswede et al EUROCRYPT 1999]

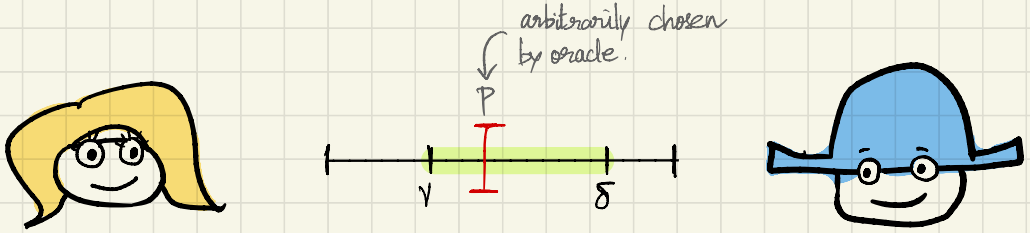
[Budkuley et al JSAC 2021] "Elasticity"

State Aware Variants of Compound BSCs

I:

State Aware Variants of Compound BSCs

I:

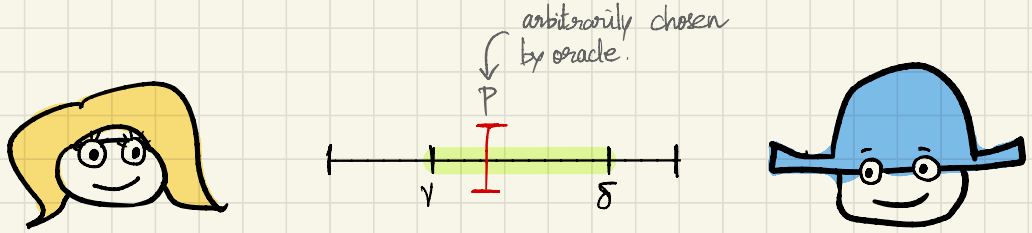


→ Channel behaves like a Compound-BSC except that Alice and Bob can now see the state 'p' that gets instantiated (because they have eyes)

→ The channel behaviour is same even for malicious Alice and Bob.

State Aware Variants of Compound BSCs

I: Alice-aware Bob-aware Variant $C\text{-}BSC_{AB}[\nu,\delta]$



→ Channel behaves like a Compound-BSC except that Alice and Bob can now see the state 'p' that gets instantiated (because they have eyes)

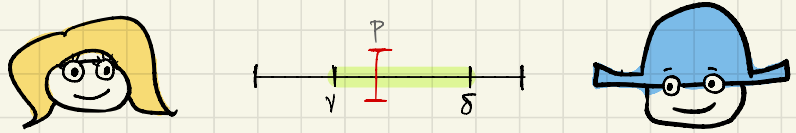
→ The channel behaviour is same even for malicious Alice and Bob.

→ We call this the Alice-aware Bob-aware variant.

State Aware Variants of Compound BSCs

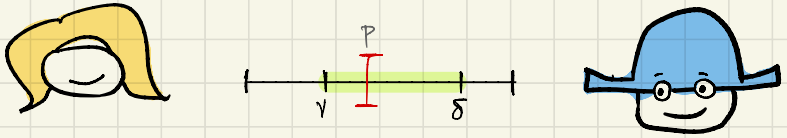
I: Alice-aware Bob-aware

$C\text{-}BSC_{A,B}[\gamma,\delta]$



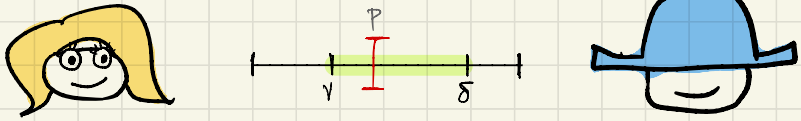
II: Alice-unaware Bob-aware

$C\text{-}BSC_{B}[\gamma,\delta]$



III: Alice-aware Bob-unaware

$C\text{-}BSC_{A}[\gamma,\delta]$



State Aware Variants of Compound BSCs

I: Alice-aware Bob-aware

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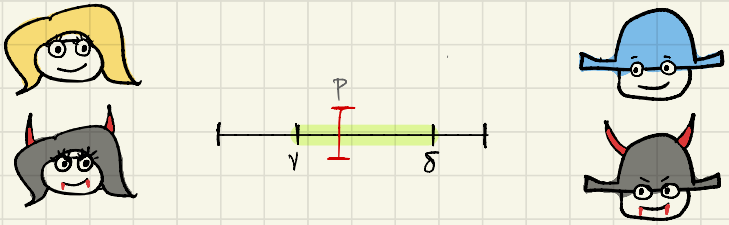

II: Alice-unaware Bob-aware

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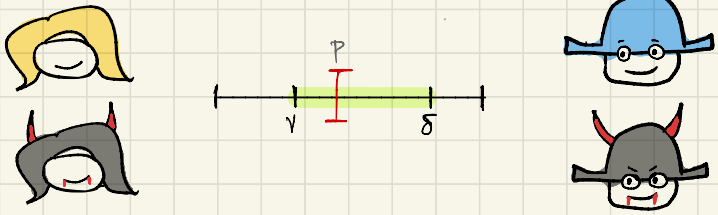
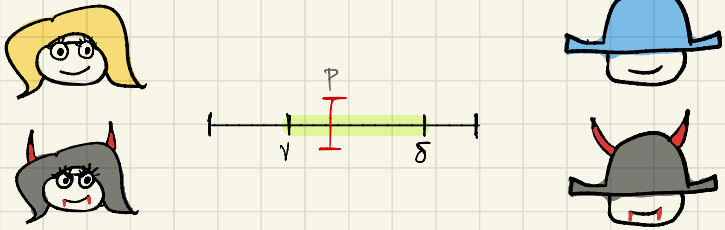


III: Alice-aware Bob-unaware

$C\text{-}BSC_{A}[\gamma,\delta]$

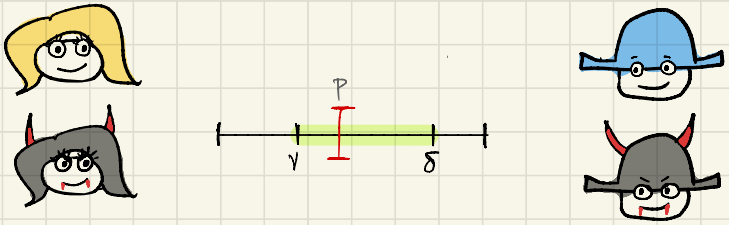


Note:
In all the variants, channel behaviour is the same for malicious and honest parties.

State Aware Variants of Compound BSCs

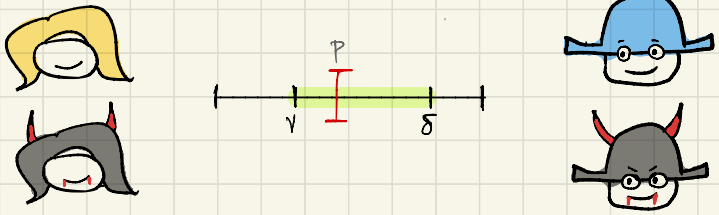
I: Alice-aware Bob-aware

$C\text{-}BSC_{A,B}[\gamma,\delta]$



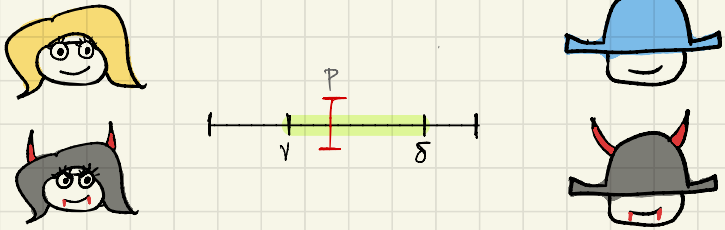
II: Alice-unaware Bob-aware

$C\text{-}BSC_{B}[\gamma,\delta]$



III: Alice-aware Bob-unaware

$C\text{-}BSC_{A}[\gamma,\delta]$



"unreliable variants"
Capabilities of Alice and Bob

① Awareness → OUR RESULT

② Control ↳ modelled in UNCs.

Unfair Noisy Channels : The UNCs

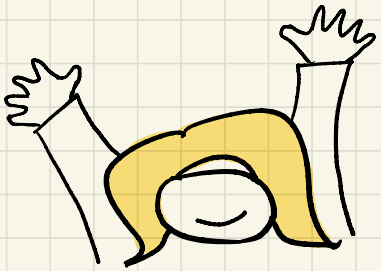
$UNC[\gamma, \delta]$

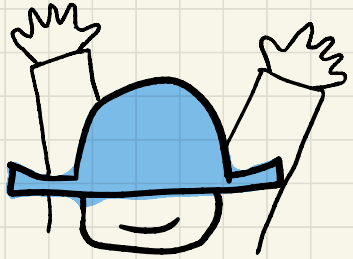
$0 \leq \gamma \leq \delta \leq \frac{1}{2}$.

[Damgard et al EUROCRYP 1988]

Unfair Noisy Channels : The UNCs

$UNC[\gamma, \delta]$ $\quad 0 \leq \gamma \leq \delta \leq \frac{1}{2}$.



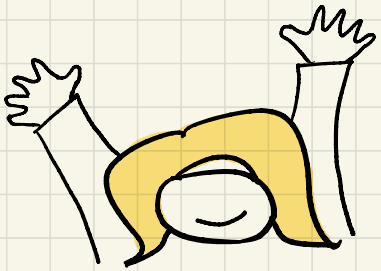


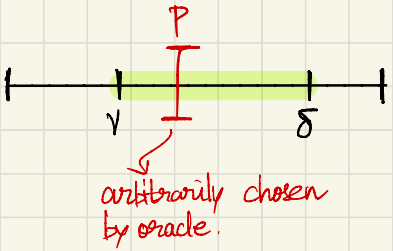
- Sender Alice and Receiver Bob
- Both have hands
- Are asked to hang them in the air.

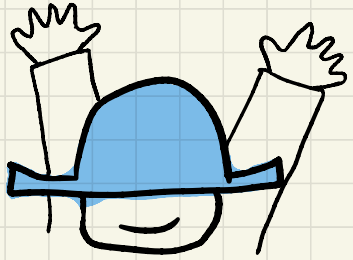
Unfair Noisy Channels : The UNCs

UNC$[\gamma, \delta]$

$0 \leq \gamma \leq \delta \leq \frac{1}{2}$







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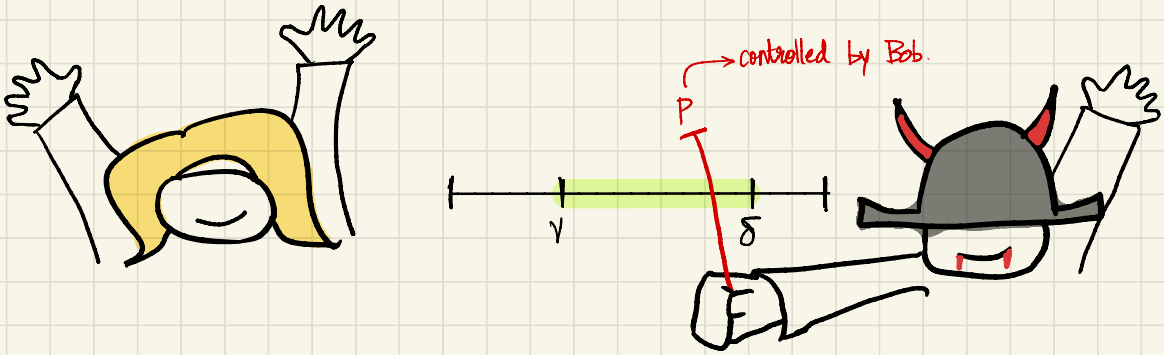

Unfair Noisy Channels : The UNCs

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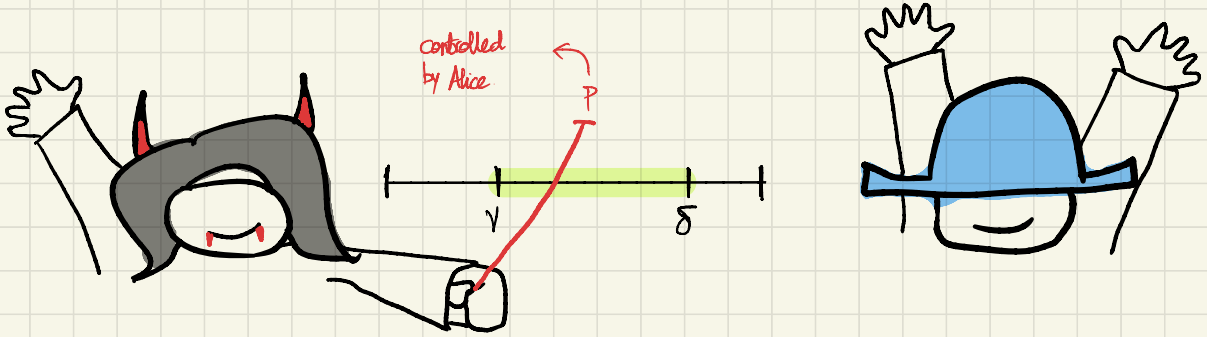

Unfair Noisy Channels : The UNCs

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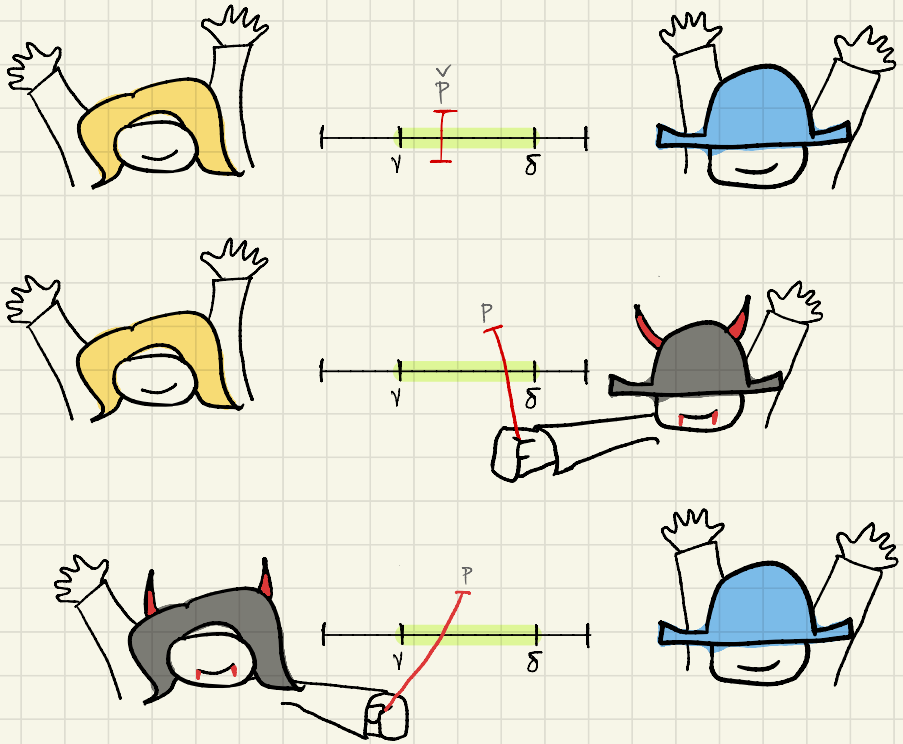


[Damgard et al. EUROCRYPT 1999]

$C_{UNC} > 0$ for $\delta < \gamma * \gamma$

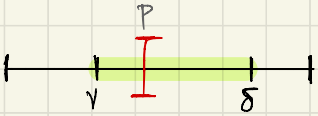
[Crepeau et al., TransIT 2020]

$C_{UNC[\gamma,\delta]} \ge H(\gamma) - H\left(\frac{\delta-\gamma}{1-2\gamma}\right)$

Main Results

I:

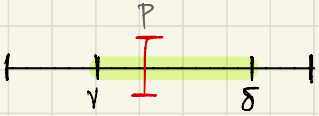
AWARE



AWARE

II:

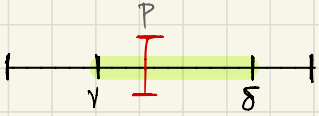
UNAWARE



AWARE

III:

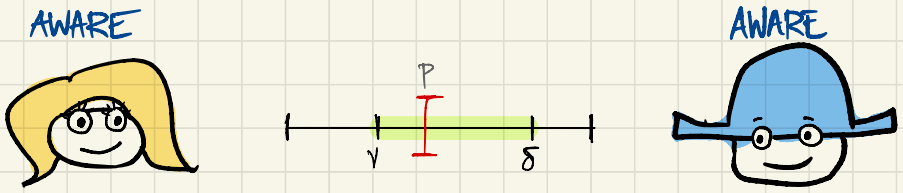
AWARE



UNAWARE

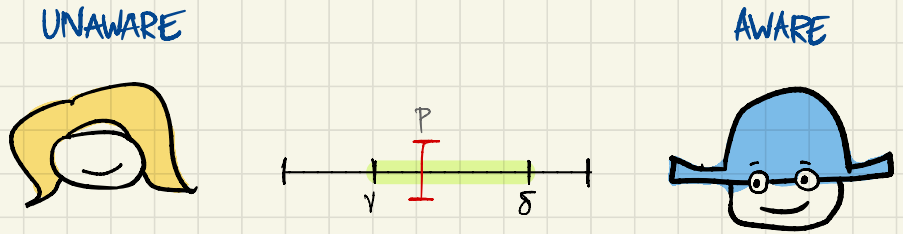
Main Results

I:



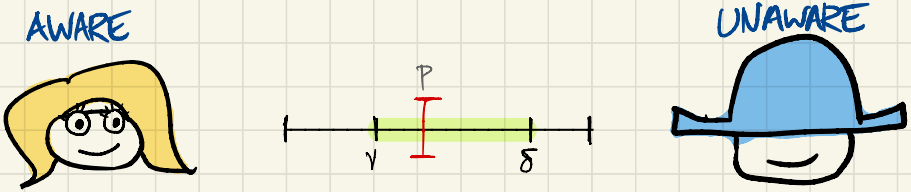
$$C_{C\text{-}BSC_{AB}} = H(\nu)$$

II:



$$C_{C\text{-}BSC_{B}} = H(\nu)$$

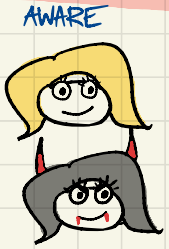
III:

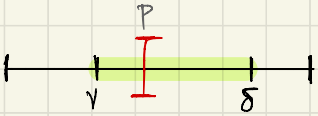


$$C_{C\text{-}BSC_{A}} \geq H(\nu) - H\left(\frac{\delta-\nu}{1-2\nu}\right)$$

Main Results

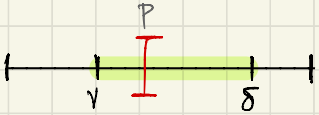
I:





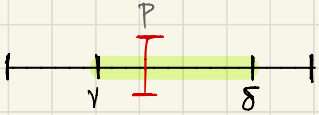
$$\mathbb{C}_{\text{C-BSC}_{1B}} = H(\gamma)$$

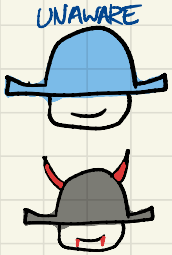
II:



$$\mathbb{C}_{\text{C-BSC}_{B}} = H(\gamma)$$

III:





$$\mathbb{C}_{\text{C-BSC}_{1}} \geq H(\gamma) - H\left(\frac{\delta-\gamma}{1-2\gamma}\right)$$

Main Results

Proofs for Commitment Capacity Expression.

for given conditions. "$\mathbb{C}_{\{conditions\}}$"

I: Achievability

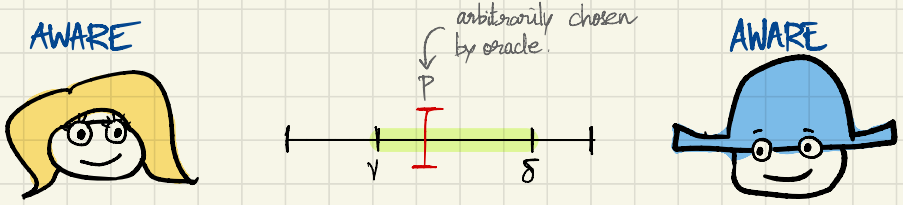
Design a protocol that is secure and has throughput $\geq \mathbb{C}_{\{conditions\}} - \epsilon$

II: Converse

Argue that any protocol with throughput $> \mathbb{C}_{\{conditions\}} + \epsilon$, is not secure.

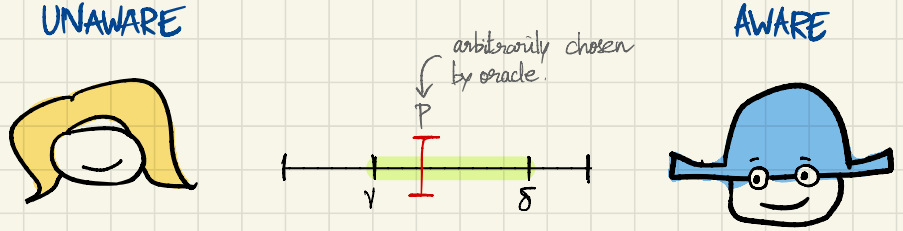
Main Results

I:



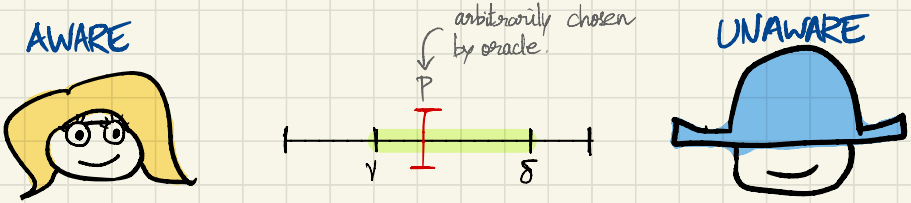
$$\mathbb{C}_{C\text{-}BSC_{1,B}} = H(\nu) = \min_{p} \mathbb{C}_{BSC(p)} = \min_{p} H(p)$$

II:



$$\mathbb{C}_{C\text{-}BSC_{B}} = H(\nu)$$

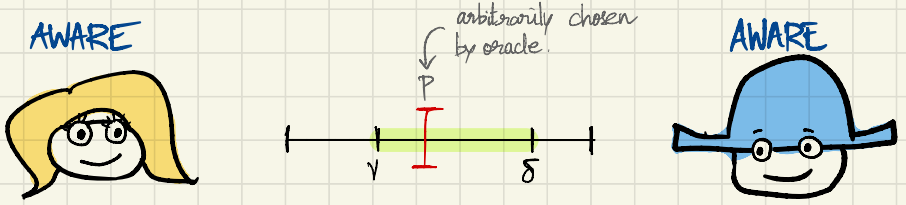
III:



$$\mathbb{C}_{C\text{-}BSC_{1}} \geq H(\nu) - H\left(\frac{\delta-\nu}{1-2\nu}\right)$$

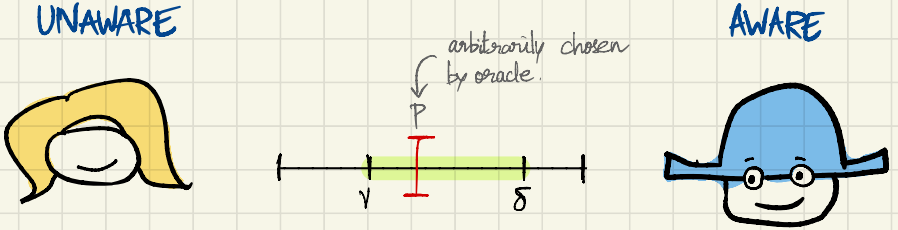
Main Results

I:



$$\mathbb{C}_{C\text{-}BSC_{1B}} = H(\nu)$$

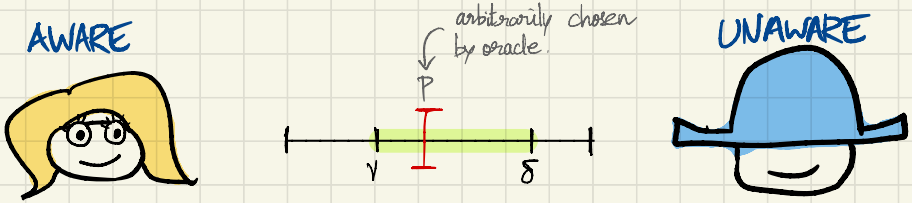
II:



$$\mathbb{C}_{C\text{-}BSC_{B}} = H(\nu)$$

→ Similar Achievability & Converse as in State-unaware case [Yadav et al NCC 2021]

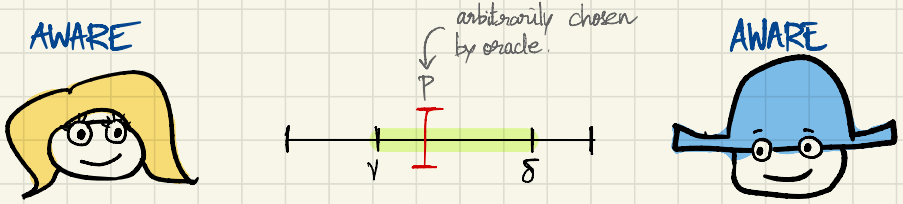
III:



$$\mathbb{C}_{C\text{-}BSC_{1}} \geqslant H(\nu) - H\left(\frac{\delta-\nu}{1-2\nu}\right)$$

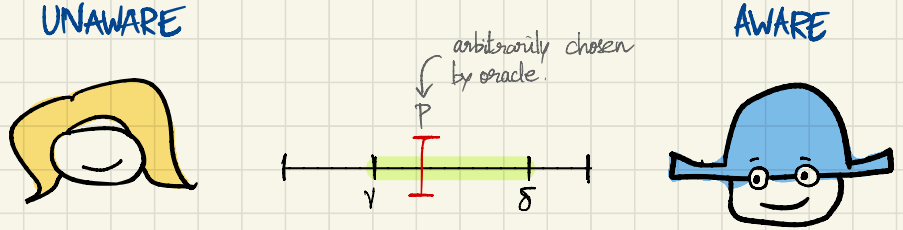
Main Results

I:



$$\mathbb{C}_{C\text{-}BSC_{1B}} = H(\nu)$$

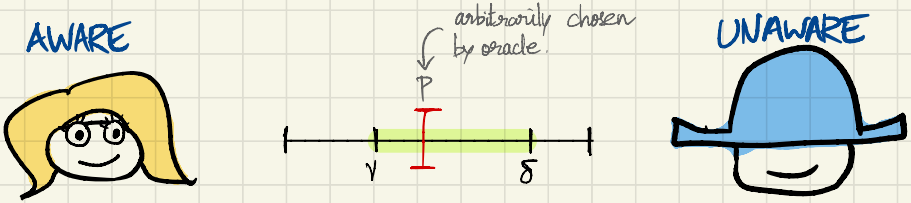
II:



$$\mathbb{C}_{C\text{-}BSC_{B}} = H(\nu)$$

AWARENESS MEETS CONTROL

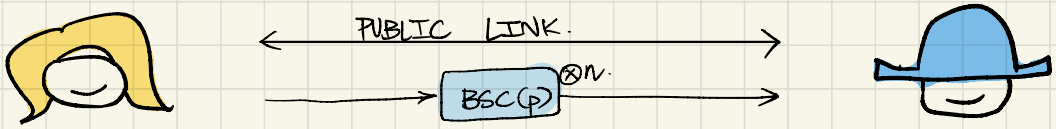
III:



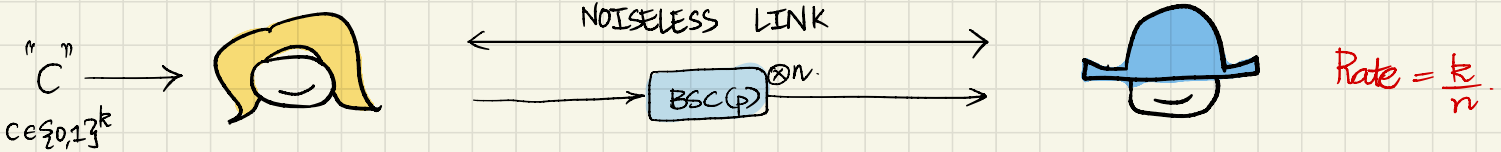
$$\mathbb{C}_{C\text{-}BSC_{1}} \geq H(\nu) - H\left(\frac{\delta-\nu}{1-2\nu}\right)$$

→ Achievability scheme [same as UNC≤]

Achievable Protocols over BSC(p)

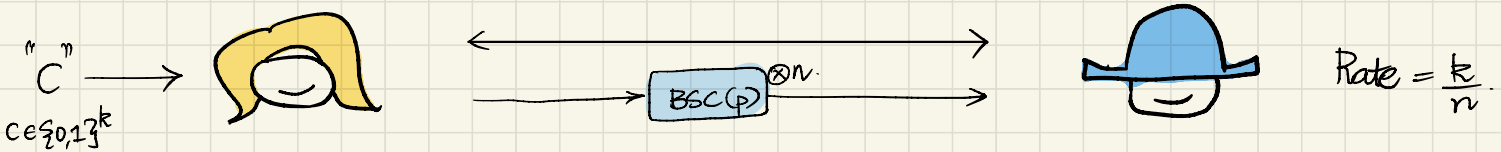


Achievable Protocols over BSC(p)



$\text{Rate} = \frac{k}{n}$.

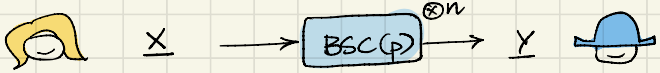
Achievable Protocols over BSC(p)



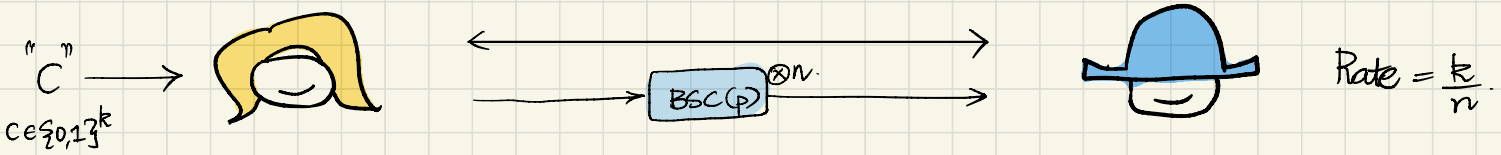
COMMIT PHASE

REVEAL PHASE

1. generates $\underline{x} \in \{0,1\}^n$ uniformly at random

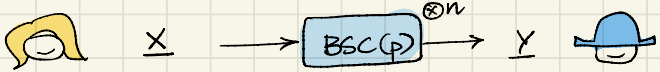


Achievable Protocols over BSC(p)



COMMIT PHASE

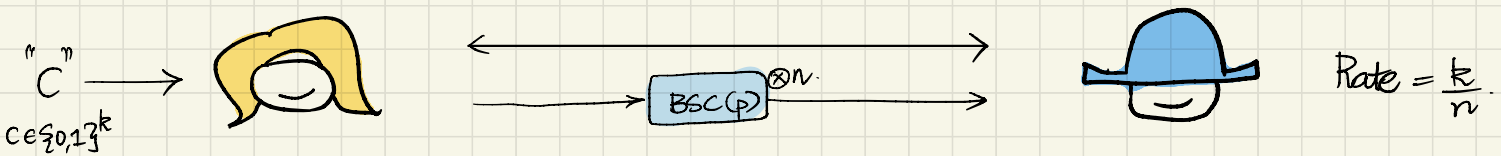
1. generates $\underline{x} \in \{0,1\}^n$ uniformly at random



2. picks Random Extractor "Ext"
$Ext : \{0,1\}^n \to \{0,1\}^k$

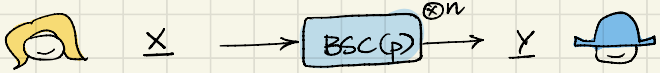
REVEAL PHASE

Achievable Protocols over BSC(p)



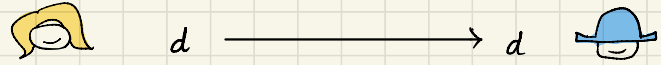
COMMIT PHASE

1. generates $\underline{x} \in \{0,1\}^n$ uniformly at random



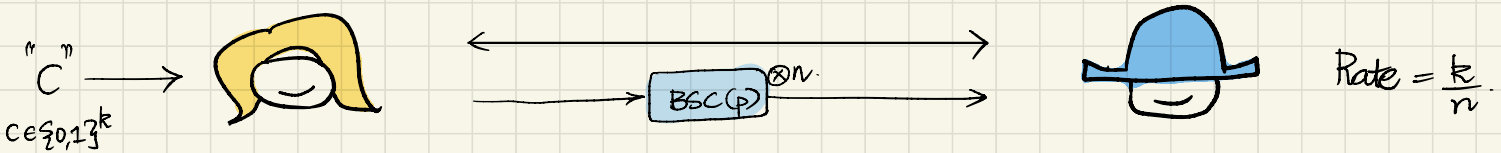
2. picks Random Extractor "Ext"
$Ext : \{0,1\}^n \to \{0,1\}^k$

3. $Ext(\underline{x}) = MASK$.
$c \oplus Ext(\underline{x}) =: d$



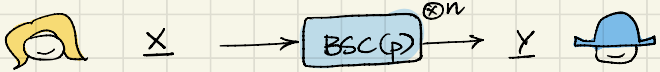
REVEAL PHASE

Achievable Protocols over BSC(p)



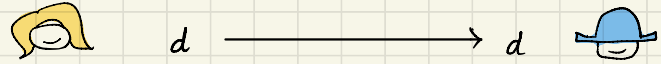
Commit Phase

1. generates $\underline{x} \in \{0,1\}^n$ uniformly at random



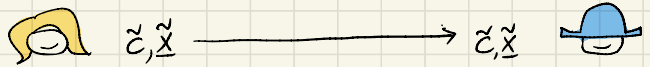
2. picks Random Extractor "Ext"
$\text{Ext} : \{0,1\}^n \to \{0,1\}^k$

3. $\text{Ext}(\underline{x}) = \text{MASK}$.
$c \oplus \text{Ext}(\underline{x}) =: d$

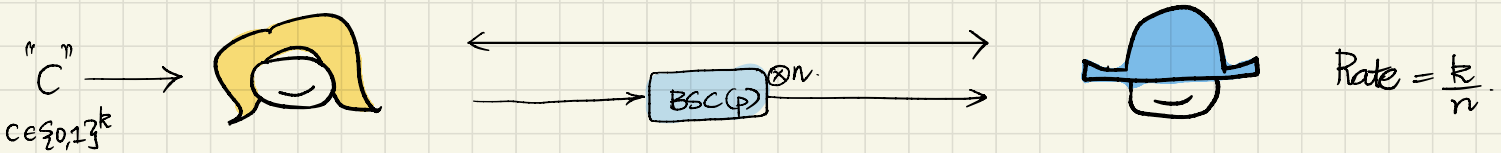


Reveal Phase

1. reveals $\tilde{c}, \underline{\tilde{x}}$

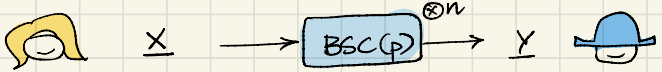


Achievable Protocols over BSC(p)



Commit Phase

1. generates $x \in \{0,1\}^n$ uniformly at random

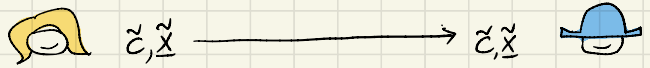


2. picks Random Extractor "Ext"
$Ext : \{0,1\}^n \to \{0,1\}^k$

3. $Ext(x) = MASK$.
$c \oplus Ext(x) =: d$

Reveal Phase

1. reveals $\tilde{c}, \tilde{x}$



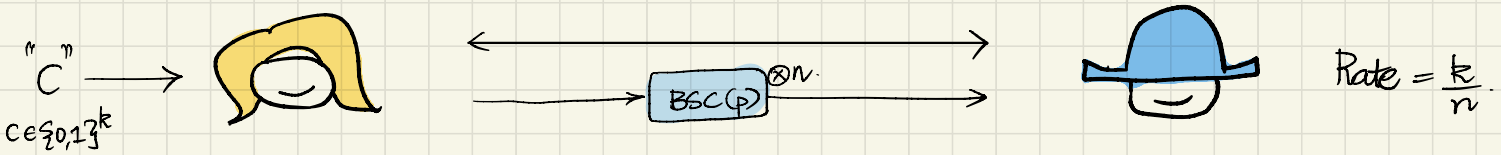
2. performs Tests. T.

(A) $\tilde{c} \oplus Ext(\tilde{x}) = d$
Are the mask and commit string consistent?

(B) $d_H(\tilde{x}, y) \in [np - n\nu, np + n\nu]$?

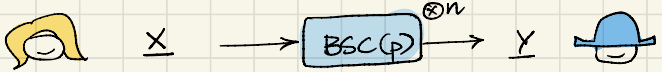
Theory of Typical sets, $\tilde{x}$ — BSC(p) — y ?

Achievable Protocols over BSC(p)



COMMIT PHASE

1. generates $x \in \{0,1\}^n$ uniformly at random



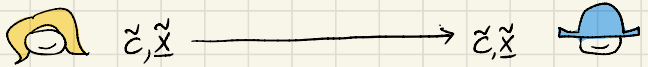
2. picks Random Extractor "Ext"
$Ext : \{0,1\}^n \to \{0,1\}^k$

3. $Ext(x) = MASK$.
$c \oplus Ext(x) =: d$

PRIVACY AMPLIFICATION (for concealment)

REVEAL PHASE

1. reveals $\tilde{c}, \tilde{x}$



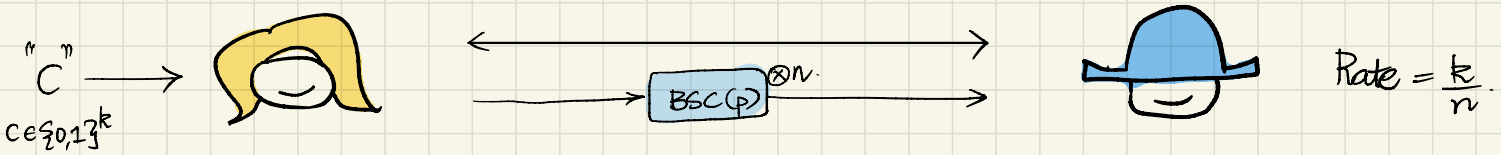
2. performs Tests. T.

→ (A) $\tilde{c} \oplus Ext(\tilde{x}) = d$
Are the mask and commit string consistent?

(B) $d_H(\tilde{x}, y) \in [np - n\nu, np + n\nu]$?

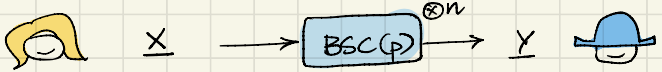
Theory of Typical sets, $\tilde{x}$ — BSC(p) — y ?

Achievable Protocols over BSC(p)



COMMIT PHASE

1. generates $\underline{x} \in \{0,1\}^n$ uniformly at random

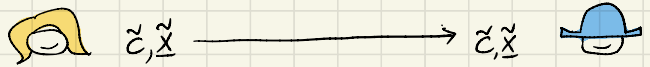


2. picks Random Extractor "Ext"
$\text{Ext} : \{0,1\}^n \to \{0,1\}^k$

3. $\text{Ext}(\underline{x}) = \text{MASK}$.
$c \oplus \text{Ext}(\underline{x}) =: d$

REVEAL PHASE

1. reveals $\tilde{c}, \underline{\tilde{x}}$



2. performs Tests. T.

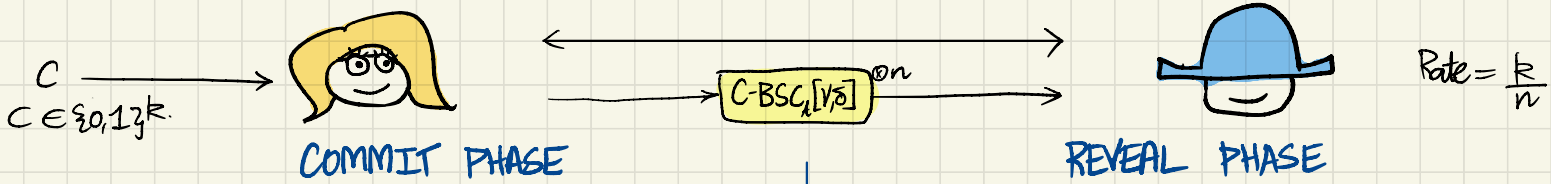
(A) $\tilde{c} \oplus \text{Ext}(\underline{\tilde{x}}) = d$
Are the mask and commit string consistent?

→ (B) $d_H(\underline{\tilde{x}}, \underline{y}) \in [np - n\nu, np + n\nu]$?

Theory of Typical sets, $\underline{\tilde{x}}$ — BSC(p) — $\underline{y}$?

↳ soundness, bindingness.

Achievable Protocol over C-BSC$_A$[γ,δ]



1. generates $\underline{X}$ → C-BSC$_A$[γ,δ] (⊗n) → $\underline{Y}$

4. picks Ext → Ext

5. $C \oplus Ext(X)$
   d → d

1. reveals $\tilde{C}, \underline{\tilde{X}}$

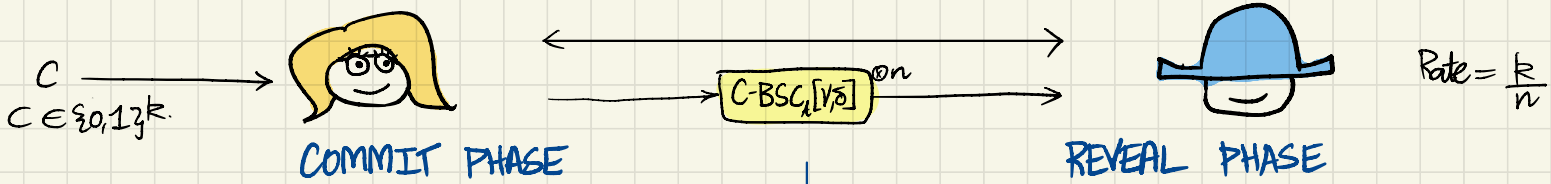
   $\tilde{C}, \underline{\tilde{X}}$ → $\tilde{C}, \underline{\tilde{X}}$

2. performs Tests. T.

(A) $\tilde{C} \oplus Ext(\underline{\tilde{X}}) = d$
Are the mask and commit string consistent?

(B) $d_H(\underline{\tilde{X}}, \underline{Y}) \in [n\gamma - n\epsilon, n\delta + n\epsilon]$

Achievable Protocol over C-BSC$_A$[γ,δ].



1. generates $\underline{X}$ → C-BSC$_A$[γ,δ] (⊗n) → $\underline{Y}$

4. picks Ext → Ext

5. $C \oplus Ext(X)$
   d → d

1. reveals $\tilde{C}, \underline{\tilde{X}}$

   $\tilde{C}, \underline{\tilde{X}}$ → $\tilde{C}, \underline{\tilde{X}}$

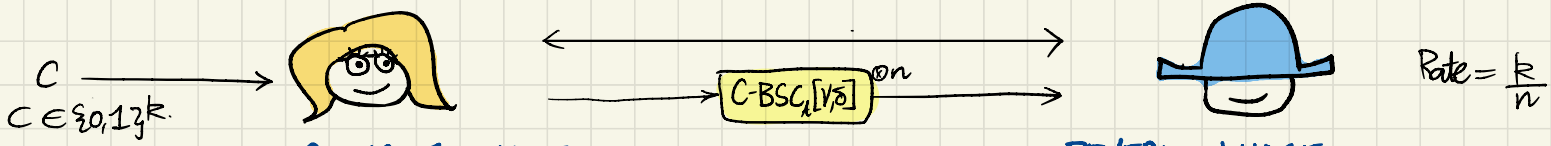
2. performs Tests. T.

(A) $\tilde{C} \oplus Ext(\underline{\tilde{X}}) = d$
Are the mask and commit string consistent?

(B) $d_H(\underline{\tilde{X}}, \underline{Y}) \in [n\gamma - n\epsilon, n\delta + n\epsilon]$

To account for $p \in [\gamma, \delta]$

Achievable Protocol over C-BSC$_A$[V,δ]



COMMIT PHASE

1. Alice generates $\underline{X}$ → C-BSC$_A$[V,δ] ⊗n → $\underline{Y}$
2. Bob picks $G_1 : \{0,1\}^n \to \{0,1\}^{\lceil nH(K) \rceil}$, sends G_1 to Alice
   Alice: $G_1(\underline{X})$ → $G_1(\underline{X})$
3. Alice picks $G_2 : \{0,1\}^n \to \{0,1\}^{nR_2}$, sends G_2 to Bob
   Alice: $G_2(\underline{X})$ → $G_2(\underline{X})$
4. Alice picks Ext → Ext
5. Alice: $C \oplus \text{Ext}(\underline{X}) = d$ → d

REVEAL PHASE

1. Alice reveals $\tilde{C}, \tilde{\underline{X}}$
   $\tilde{C}, \tilde{\underline{X}}$ → $\tilde{C}, \tilde{\underline{X}}$
2. Bob performs Tests. T.

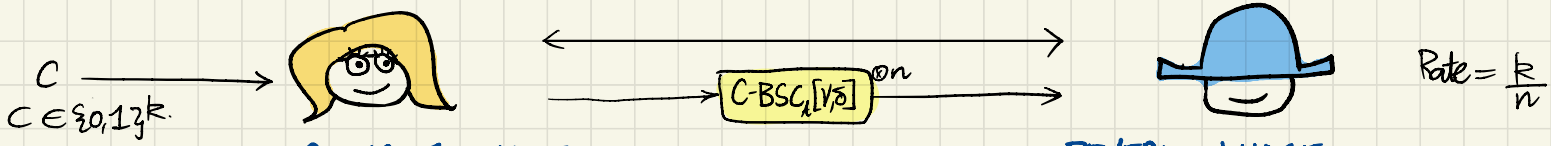
(A) $\tilde{C} \oplus \text{Ext}(\tilde{\underline{X}}) = d$
Are the mask and commit string consistent?

(B) $d_H(\tilde{\underline{X}}, \underline{Y}) \in [nV - n\epsilon, n\delta + n\epsilon]$

(C) $G_1(\tilde{\underline{X}}) = G_1(\underline{X})$, $G_2(\tilde{\underline{X}}) = G_2(\underline{X})$.

Extra-steps for bindingness ⇒ loss in throughput

Achievable Protocol over C-BSC$_A$[V,δ]



COMMIT PHASE

1. generates $\underline{X}$ → C-BSC$_A$[V,δ] ⊗n → $\underline{Y}$
2. picks $G_1 : \{0,1\}^n \to \{0,1\}^{n(H(K)-\frac{\alpha}{2})}$; G_1
   $G_1(\underline{X})$ → $G_1(\underline{X})$
3. picks $G_2 : \{0,1\}^n \to \{0,1\}^{n\alpha_2}$; G
   $G_2(\underline{X})$ → $G_2(\underline{X})$
4. picks Ext → Ext
5. $C \oplus Ext(\underline{X})$
   d → d

REVEAL PHASE

1. reveals $\tilde{C}, \tilde{\underline{X}}$
   $\tilde{C}, \tilde{\underline{X}}$ → $\tilde{C}, \tilde{\underline{X}}$
2. performs Tests. T.

(A) $\tilde{C} \oplus Ext(\tilde{\underline{X}}) = d$
Are the mask and commit string consistent?

(B) $d_H(\tilde{\underline{X}}, \underline{Y}) \in [nV - n\epsilon, n\delta + n\epsilon]$

(C) $G_1(\tilde{\underline{X}}) = G_1(\underline{X})$, $G_2(\tilde{\underline{X}}) = G_2(\underline{X})$.

OBSERVATIONS

Unreliable Noisy Channels – "Capabilities"

① AWARENESS
(publicly exercised)

② CONTROL
(privately exercised)

OBSERVATIONS

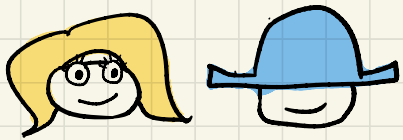
Unreliable Noisy Channels – "Capabilities"

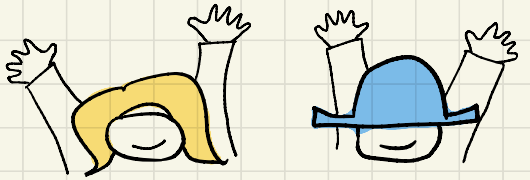
① AWARENESS (publicly exercised)

② CONTROL (privately exercised)

$C\text{-}BSC_{\lambda}[\gamma,\delta]$ $\equiv$ $UNC[\gamma,\delta]$

$\mathbb{C}_{C\text{-}BSC_{\lambda}}$ $\equiv$ $\mathbb{C}_{UNC}$





OBSERVATIONS

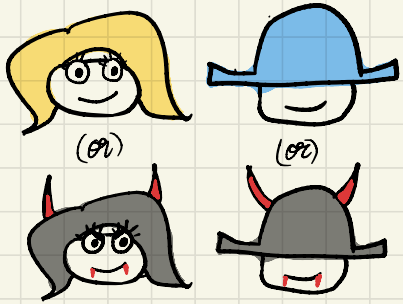
Unreliable Noisy Channels – "Capabilities"

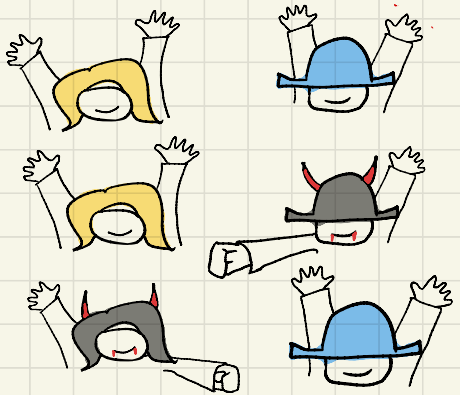
① AWARENESS
(publicly exercised)

② CONTROL
(privately exercised)

$C\text{-}BSC_\lambda[\gamma,\delta]$ $\quad=\quad$ $UNC[\gamma,\delta]$

$C_{C\text{-}BSC_\lambda}$ $\quad=\quad$ C_{UNC}





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