ASSIGNMENT 4

MANIKANTA VALLEPU - AI20BTECH11014

Download all python codes from

```
https://github.com/manik2255/AI1103−
    PROBABILITY−AND−RANDOM−
    VARIABLES/blob/main/ASSIGNMENT_4/
    assignment_4.py
```

and latex-tikz codes from

```
https://github.com/manik2255/AI1103−
    PROBABILITY−AND−RANDOM−
    VARIABLES/blob/main/ASSIGNMENT_4/
    ASSIGNMENT_4.tex
```

1 GATE 2017 MA PROBLEM.49

Let X and Y be independent and identically distributed random variables with probability density function $f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & otherwise \end{cases}$ Then $\Pr(max(X, Y) < 2)$ equals

2 SOLUTION

Given,

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

We know,

$$\Pr(X < x) = \int_0^x f(x)\, dx \tag{2.0.2}$$

Using (2.0.1) in (2.0.2),

$$\Pr(X < x) = \int_0^x e^{-x}\, dx \tag{2.0.3}$$

We need to find $\Pr(max(X, Y) < 2)$,which is also can be written as

$$\Pr(max(X, Y) < 2) = \Pr(X < 2\ and\ Y < 2) \tag{2.0.4}$$

As X and Y be independent random variables,

$$\Pr(max(X, Y) < 2) = \Pr(X < 2)\Pr(Y < 2) \tag{2.0.5}$$

Using (2.0.3) in (2.0.5),

$$\Pr(max(X, Y) < 2) = (\int_0^2 e^{-x}\, dx)(\int_0^2 e^{-y}\, dy) \tag{2.0.6}$$

$$= [-e^{-x}]_0^2\,[-e^{-y}]_0^2 \tag{2.0.7}$$

$$= 0.748 \tag{2.0.8}$$

We need to find $\Pr(max(X, Y) < 2)$,which is also can be written as

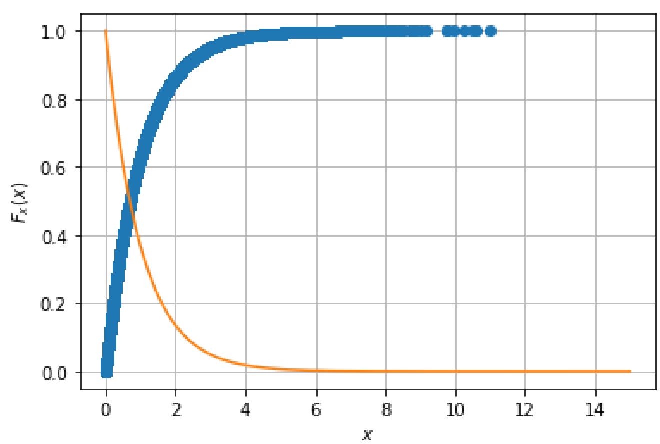

Fig. 1: CDF of random variable X

$$\Pr(max(X, Y) < 2) = \Pr(X < 2, Y < X) + \Pr(Y < 2, X \le Y) \tag{2.0.9}$$

$\Pr(X < 2, Y < X)$ can also be written as,

$$\Pr(X < 2, Y < X) = \int_0^2 e^{-x}(\int_0^x e^{-y}\, dy)\, dx \tag{2.0.10}$$

$$= \int_0^2 e^{-x}(1 - e^{-x})\, dx \tag{2.0.11}$$

$$= \left[-e^{-x} + \frac{e^{-2x}}{2}\right]_0^2 \tag{2.0.12}$$

$$= 0.3738 \tag{2.0.13}$$

similarly, $\Pr(Y < 2, X \le Y)$ can also be written as,

$$\Pr(Y < 2, X \le Y) = \int_0^2 e^{-y} \left(\int_0^y e^{-x} \, dx \right) dy \tag{2.0.14}$$

$$= \int_0^2 e^{-y} (1 - e^{-y}) \, dy \tag{2.0.15}$$

$$= \left[-e^{-y} + \frac{e^{-2y}}{2} \right]_0^2 \tag{2.0.16}$$

$$= 0.3738 \tag{2.0.17}$$

Using (2.0.13) and (2.0.17) in (2.0.9),

$$\Pr(max(X, Y) < 2) = 0.748 \tag{2.0.18}$$