ASSIGNMENT 4

MANIKANTA VALLEPU - AI20BTECH11014

Download all python codes from

```
https://github.com/manik2255/AI1103−
    PROBABILITY−AND−RANDOM−
    VARIABLES/blob/main/ASSIGNMENT_4/
    assignment_4.py
```

and latex-tikz codes from

```
https://github.com/manik2255/AI1103−
    PROBABILITY−AND−RANDOM−
    VARIABLES/blob/main/ASSIGNMENT_4/
    ASSIGNMENT_4.tex
```

1 GATE 2017 MA PROBLEM.49

Let X and Y be independent and identically distributed random variables with probability density function $f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & otherwise \end{cases}$ Then $\Pr(max(X, Y) < 2)$ equals

2 SOLUTION

Given,

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

We know,

$$\Pr(X < x) = \int_0^x f(x)\, dx \tag{2.0.2}$$

Using (2.0.1) in (2.0.2),

$$\Pr(X < x) = \int_0^x e^{-x}\, dx \tag{2.0.3}$$

Let $F_X(x)$ be the cumulative distribution function of random variable X.

$$F_X(x) = \int_0^x f(x)\, dx \tag{2.0.4}$$

$F_X(x)$ can be obtained from the uniform distribution of a random variable U on (0,1) and let $U = e^{-x}$.

$$0 < U < 1 \tag{2.0.5}$$

As for random variable X also,

$$0 < F_X(x) < 1 \tag{2.0.6}$$

This similarity between U and $F_X(x)$ is used to generate the random variable X from U.

$$F_X(x) = \Pr(X < x) \tag{2.0.7}$$
$$= \Pr(-log_e U < x) \tag{2.0.8}$$
$$= \Pr(U > e^{-x}) \tag{2.0.9}$$
$$= 1 - \Pr(U < e^{-x}) \tag{2.0.10}$$
$$= 1 - F_U(e^{-x}) \tag{2.0.11}$$

From uniform distribution,

$$F_U(x) = x, 0 < x < 1 \tag{2.0.12}$$

Using (2.0.11) in (2.0.12),

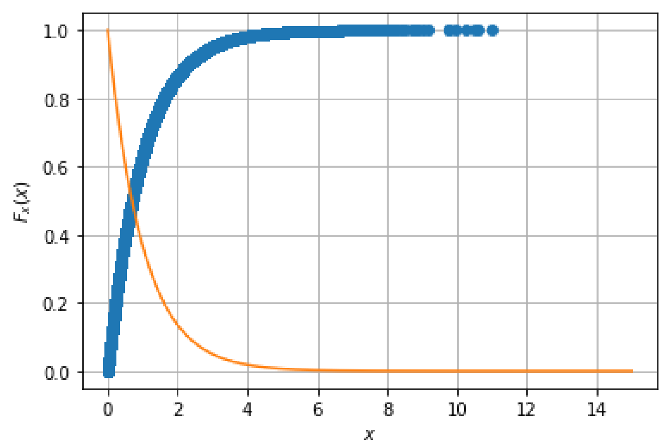

Fig. 1: CDF of random variable X

$$F_X(x) = \Pr(X < x) \tag{2.0.13}$$
$$= 1 - e^{-x} \tag{2.0.14}$$

$$\Pr(max(X, Y) < 2) = \Pr(X < 2, Y < 2) \tag{2.0.15}$$

As X and Y are independent random variables,

$$\Pr(max(X, Y) < 2) = \Pr(X < 2)\Pr(Y < 2) \tag{2.0.16}$$

using (2.0.14),

$$\Pr\left(max(X, Y) < 2\right) = (1 - e^{-2})(1 - e^{-2}) \tag{2.0.17}$$
$$= 0.748. \tag{2.0.18}$$