

ASSIGNMENT 10

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Download all latex-tikz codes from

https://github.com/manik2255/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ASSIGNMENT_10/ASSIGNMENT_10.tex

1 CSIR UGC NET EXAM (Dec 2015),
PROBLEM.53

Let X'_i 's be independent random variables such that X'_i 's are symmetric about 0 and $\text{var}(X_i) = 2i - 1$, for $i \geq 1$. then,

$$\lim_{n \rightarrow \infty} \Pr(X_1 + X_2 + \cdots + X_n > n \log n)$$

1) does not exist. 3) equals 1.

2) equals $\frac{1}{2}$. 4) equals 0.

2 SOLUTION

Let $X = X_1 + X_2 + \cdots + X_n$, as X'_i 's are symmetric about 0. The mean of X is given by,

$$E[X] = 0 \quad (2.0.1)$$

the variance of X is given by,

$$\text{var}[X] = \sum_{i=1}^n (2i - 1) \quad (2.0.2)$$

$$= \frac{2n(n+1)}{2} - n \quad (2.0.3)$$

$$= n^2 \quad (2.0.4)$$

the standard deviation,

$$\sigma_X = n \quad (2.0.5)$$

Applying Chebyshev's Inequality for the random variable X, for any $k > 0$

$$\Pr(|X - E[X]| > k\sigma_X) \leq \frac{1}{k^2} \quad (2.0.6)$$

let $k = \log n$, using (2.0.1) and (2.0.5) in (2.0.6),

$$\Pr(X > n \log n) \leq \frac{1}{(\log n)^2} \quad (2.0.7)$$

as any probability is greater than 0,

$$0 < \Pr(X > n \log n) \leq \frac{1}{(\log n)^2} \quad (2.0.8)$$

applying sandwich principle to (2.0.8),

$$\lim_{n \rightarrow \infty} 0 < \lim_{n \rightarrow \infty} \Pr(X > n \log n) \leq \lim_{n \rightarrow \infty} \frac{1}{(\log n)^2} \quad (2.0.9)$$

$$\lim_{n \rightarrow \infty} \Pr(X_1 + X_2 + \cdots + X_n > n \log n) = 0 \quad (2.0.10)$$

Hence the option.4 is correct.