ASSIGNMENT 10

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Download all latex-tikz codes from

https://github.com/manik2255/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ASSIGNMENT_10/ASSIGNMENT_10.tex

1 CSIR UGC NET EXAM (Dec 2015), PROBLEM.53

Let $X_i's$ be independent random variables such that $X_i's$ are symmetric about 0 and $var(X_i) = 2i - 1$,for $i \geq 1$.then,

$$\lim_{n\to\infty} \Pr\left(X_1 + X_2 + \cdots + X_n > n\log n\right)$$

1) does not exist.
2) equals $\frac{1}{2}$.
3) equals 1.
4) equals 0.

2 SOLUTION

Let $X = X_1 + X_2 + \cdots + X_n$, as $X_i's$ are symmetric about 0. The mean of X is given by,

$$E[X] = 0 \tag{2.0.1}$$

the variance of X is given by,

$$var[X] = \sum_{i=1}^{n}(2i - 1) \tag{2.0.2}$$
$$= \frac{2n(n+1)}{2} - n \tag{2.0.3}$$
$$= n^2 \tag{2.0.4}$$

the standard deviation,

$$\sigma_X = n \tag{2.0.5}$$

Applying Chebyshev's Inequality for the random variable X, for any $k > 0$

$$\Pr\left(|X - E[X]| > k\sigma_X\right) \leq \frac{1}{k^2} \tag{2.0.6}$$

let $k = \log n$, using (2.0.1) and (2.0.5) in (2.0.6),

$$\Pr\left(|X| > n\log n\right) \leq \frac{1}{(\log n)^2} \tag{2.0.7}$$

$$\Pr\left(X > n\log n\right) + \Pr\left(X < -n\log n\right) \leq \frac{1}{(\log n)^2} \tag{2.0.8}$$

As, X is symmetric about 0,

$$\Pr\left(X > n\log n\right) = \Pr\left(X < -n\log n\right) \tag{2.0.9}$$

using (2.0.9) in (2.0.8),

$$2\Pr\left(X > n\log n\right) \leq \frac{1}{(\log n)^2} \tag{2.0.10}$$

$$\Pr\left(X > n\log n\right) \leq \frac{1}{2(\log n)^2} \tag{2.0.11}$$

as any probability is greater than 0,

$$0 < \Pr\left(X > n\log n\right) \leq \frac{1}{2(\log n)^2} \tag{2.0.12}$$

applying sandwich principle to (2.0.12),

$$\lim_{n\to\infty} 0 < \lim_{n\to\infty} \Pr\left(X > n\log n\right) \leq \lim_{n\to\infty} \frac{1}{2(\log n)^2} \tag{2.0.13}$$

$$\lim_{n\to\infty} \Pr\left(X_1 + X_2 + \cdots + X_n > n\log n\right) = 0 \tag{2.0.14}$$

Hence the option.4 is correct.