ASSIGNMENT 9

MANIKANTA VALLEPU - AI20BTECH11014

Download all python codes from

```
https://github.com/manik2255/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ASSIGNMENT_9/assign_9.py
```

and latex-tikz codes from

```
https://github.com/manik2255/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ASSIGNMENT_9/ASSIGNMENT_9.tex
```

1 GATE 2021 (ME-SET1)PROBLEM.33

Customers arrive at a shop according to Poisson distribution with a mean of 10 customers/hour. The manager notes that no customer arrives for the first 3 minutes after the shop opens. The probability that a customer arrives within the next 3 minutes is

2 Solution

Given, 10 customers arrive in a time interval of an hour $\iff$ $\frac{t}{6}$ customers arrive in a time interval of t minutes.

$$\lambda = \frac{t}{6} \tag{2.0.1}$$

Let X denotes the number of customers in first t minutes,Y denotes the number of customers in second t minutes. according to poisson distribution,

$$\Pr(X = x) = e^{-\lambda}\frac{\lambda^x}{x!} \tag{2.0.2}$$

using (2.0.1) in (2.0.2),

$$\Pr(X = x) = e^{-\frac{t}{6}}\frac{(\frac{t}{6})^x}{x!} \tag{2.0.3}$$

the probability that a customer arrives within the next t minutes given that no customer arrives for the first t minutes after the shop opens,which can also be written as,

$$\Pr(Y \neq 0|X = 0) = \frac{\Pr(Y \neq 0, X = 0)}{\Pr(X = 0)} \tag{2.0.4}$$

TABLE 0: Probability distribution for values of X and Y

	P(X)	P(Y)
0	$e^{-\frac{t}{6}}$	$e^{-\frac{t}{6}}$
1	$\frac{te^{-\frac{t}{6}}}{6}$	$\frac{te^{-\frac{t}{6}}}{6}$

As the arrival of customers in second t minutes does not depend on the arrival of customers in first t minutes, X and Y are independent,

$$\Pr(Y \neq 0|X = 0) = \frac{\Pr(Y \neq 0)\Pr(X = 0)}{\Pr(X = 0)} \tag{2.0.5}$$
$$= \Pr(Y \neq 0) \tag{2.0.6}$$
$$= 1 - \Pr(Y = 0) \tag{2.0.7}$$

using (2.0.3),

$$\Pr(Y \neq 0|X = 0) = 1 - e^{-\frac{t}{6}} \tag{2.0.8}$$

we need to find the probability for t=3,the required probability is given by,

$$= 1 - e^{-\frac{1}{2}} \tag{2.0.9}$$
$$= 0.3935 \tag{2.0.10}$$

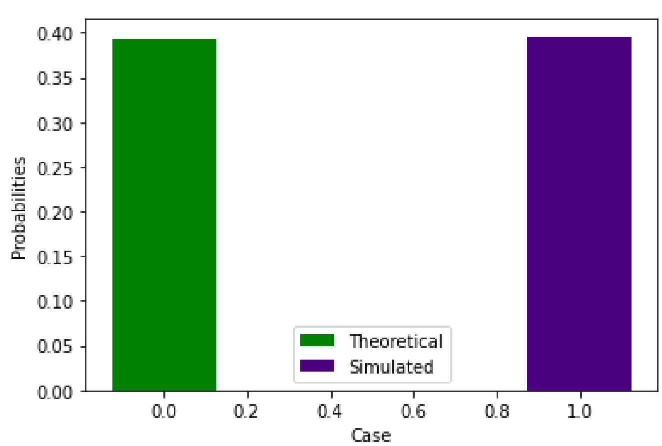

Fig. 1: Theoretical vs simulation