ASSIGNMENT 3

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Download all python codes from

https://github.com/manik2255/AI1103−
PROBABILITY−AND−RANDOM−
VARIABLES/blob/main/ASSIGNMENT_2/
ASSIGNMENT_2_GRAPH.py

and latex-tikz codes from

https://github.com/manik2255/AI1103−
PROBABILITY−AND−RANDOM−
VARIABLES/blob/main/ASSIGNMENT_2/
ASSIGNMENT_2.tex

1 GATE 2017 MA PROBLEM.47

Let X and Y be independent and identically distributed random variables with probability mass function $p(n) = 2^{-n}$, $n = 1, 2, \ldots$
Then $pr(X \geq 2Y)$ equals

2 SOLUTION

given,

$$pr(X = x) = 2^{-x}, x = 1, 2, \ldots \tag{2.0.1}$$
$$pr(Y = y) = 2^{-y}, y = 1, 2, \ldots \tag{2.0.2}$$

We need to find $pr(X \geq 2Y)$, which is also can be written as

$$pr(X \geq 2Y) = \sum_{y=1}^{\infty} pr(X \geq 2y | Y = y) \tag{2.0.3}$$

as, X and Y are independent random variables

$$pr(X \geq 2Y) = \sum_{y=1}^{\infty} pr(X \geq 2y) pr(Y = y) \tag{2.0.4}$$

$$= \sum_{y=1}^{\infty} (1 - pr(X < 2y)) pr(Y = y)$$

$$\tag{2.0.5}$$
$$\tag{2.0.6}$$

using (2.0.1) and (2.0.2) in (2.0.5),

$$= \sum_{y=1}^{\infty} (1 - \sum_{i=1}^{2y-1} 2^{-i})(2^{-2y}) \tag{2.0.7}$$

$$= \sum_{y=1}^{\infty} (1 - (1 - 2^{-(2y-1)}))(2^{-2y}) \tag{2.0.8}$$

$$= \sum_{y=1}^{\infty} 2^{-(3y-1)} \tag{2.0.9}$$

$$= \frac{2}{7} \tag{2.0.10}$$