ASSIGNMENT 2

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1 PROBLEM.GATE.14

A continuous random variable X has a probability density function $f(x) = e^{-x}, 0 < x < \infty$. Then $P(X > 1)$ is

2 SOLUTION

Given,

$$f(x) = e^{-x}, 0 < x < \infty \qquad (2.0.1)$$

We have to find $\Pr(X > 1)$,

$$\Pr(X > 1) = \int_1^\infty f(x)\,dx \qquad (2.0.2)$$

Using (2.0.1) in (2.0.2)

$$\Pr(X > 1) = \int_1^\infty e^{-x}\,dx \qquad (2.0.3)$$
$$= \left[-e^{-x}\right]_1^\infty \qquad (2.0.4)$$
$$= (-e^{-\infty}) - (-e^{-1}) \qquad (2.0.5)$$
$$= e^{-1} \qquad (2.0.6)$$
$$= \frac{1}{e} \qquad (2.0.7)$$
$$\implies \Pr(X > 1) = 0.368 \qquad (2.0.8)$$

Finding the probability using uniform distribution, Let $F_X(x)$ be the cumulative distribution function of random variable X.

$$F_X(x) = \int_0^x f(x)\,dx \qquad (2.0.9)$$

$F_X(x)$ can be obtained from the uniform distribution of a random variable U on (0,1) and let U=e^{-x}.

$$0 < U < 1 \qquad (2.0.10)$$

As for random variable X also,

$$0 < F_X(x) < 1 \qquad (2.0.11)$$

This similarity between U and $F_X(x)$ is used to generate the random variable X from U.
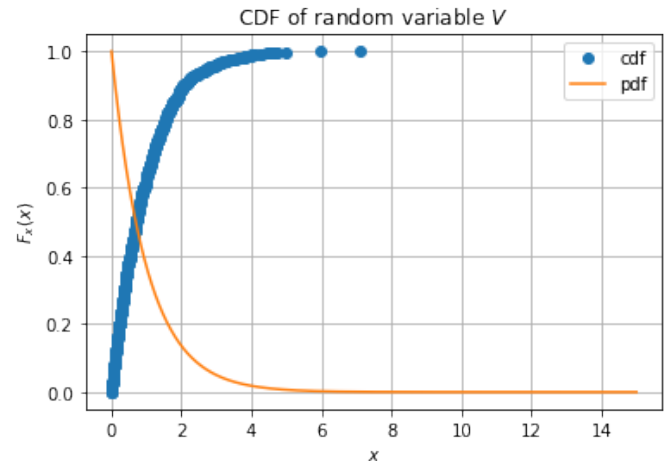
$$F_X(x) = \Pr(X < x) \qquad (2.0.12)$$
$$= \Pr(-\log_e U < x) \qquad (2.0.13)$$
$$= \Pr(U < e^{-x}) \qquad (2.0.14)$$
$$= F_U(e^{-x}) \qquad (2.0.15)$$

From uniform distribution,

$$F_U(x) = x, 0 < x < 1 \qquad (2.0.16)$$

Using (2.0.16) in (2.0.15),
Cumulative distribution function (CDF) of random variable X is,

$$F_X(x) = \Pr(X < x) \qquad (2.0.17)$$
$$= 1 - e^{-x}, 0 < x < \infty \qquad (2.0.18)$$

Now we have to find $\Pr(X > 1)$,

$$\Pr(X > 1) = 1 - \Pr(X < 1) \qquad (2.0.19)$$

Using (2.0.18),

$$\Pr(X > 1) = 1 - (1 - e^{-1}) \qquad (2.0.20)$$
$$\Pr(X > 1) = e^{-1} \qquad (2.0.21)$$
$$\implies \Pr(X > 1) = 0.368 \qquad (2.0.22)$$