ASSIGNMENT 4

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Download all python codes from

https://github.com/manik2255/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/blob/main/ASSIGNMENT%204/assign_4.py

and latex-tikz codes from

https://github.com/manik2255/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/blob/main/ASSIGNMENT%204/ASSIGNMENT_4.tex

1 GATE 2017 MA PROBLEM.49

Let X and Y be independent and identically distributed random variables with probability density function $f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & otherwise \end{cases}$ Then $\Pr(max(X, Y) < 2)$ equals

2 SOLUTION

Given,

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

We know,

$$\Pr(X < x) = \int_0^x f(x)\,dx \tag{2.0.2}$$

Using (2.0.1) in (2.0.2),

$$\Pr(X < x) = \int_0^x e^{-x}\,dx \tag{2.0.3}$$

We need to find $\Pr(max(X, Y) < 2)$,which is also can be written as

$$\Pr(max(X, Y) < 2) = \Pr(x < 2\ and\ y < 2) \tag{2.0.4}$$

As X and Y be independent random variables,

$$\Pr(max(X, Y) < 2) = \Pr(x < 2)\Pr(y < 2) \tag{2.0.5}$$

Using (2.0.3) in (2.0.5),

$$\Pr(max(X, Y) < 2) = (\int_0^2 e^{-x}\,dx)(\int_0^2 e^{-y}\,dy) \tag{2.0.6}$$

$$= [-e^{-x}]_0^2\,[-e^{-y}]_0^2 \tag{2.0.7}$$

$$= [(-e^{-2}) - (-e^{0})][(-e^{-2}) - (-e^{0})] \tag{2.0.8}$$

$$= (1 - e^{-2})^2 \tag{2.0.9}$$

$$= 0.748 \tag{2.0.10}$$

Finding the probability using uniform distribution, Let $F_X(x)$ be the cumulative distribution function of random variable X.

$$F_X(x) = \int_0^x f(x)\,dx \tag{2.0.11}$$

$F_X(x)$ can be obtained from the uniform distribution of a random variable U on (0,1) and let U=e^{-x}.

$$0 < U < 1 \tag{2.0.12}$$

As for random variable X also,

$$0 < F_X(x) < 1 \tag{2.0.13}$$

This similarity between U and $F_X(x)$ is used to generate the random variable X from U.

$$F_X(x) = \Pr(X < x) \tag{2.0.14}$$

$$= \Pr(-\log_e U < x) \tag{2.0.15}$$

$$= \Pr(U < e^{-x}) \tag{2.0.16}$$

$$= F_U(e^{-x}) \tag{2.0.17}$$

From uniform distribution,

$$F_U(x) = x, 0 < x < 1 \tag{2.0.18}$$

In the figure 1,orange colour graph represents the pdf of the random variable X and blue colour graph represents the cdf of the random variable X. Using (2.0.18) in (2.0.17), Cumulative distribution

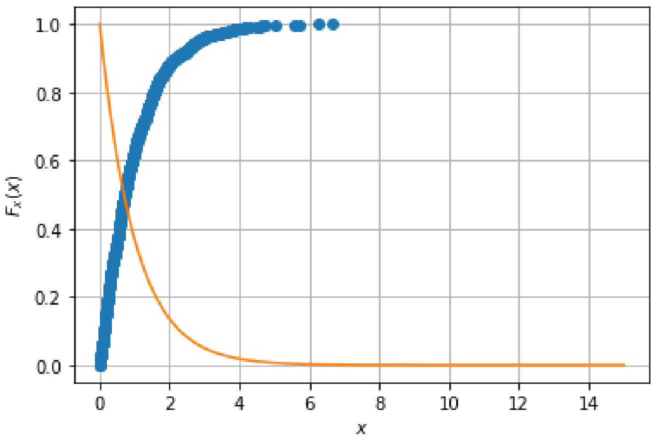

Fig. 1: CDF of random variable X

function (CDF) of random variable X is,

$$F_X(x) = \Pr(X < x) \tag{2.0.19}$$
$$= 1 - e^{-x}, 0 < x < \infty \tag{2.0.20}$$

Now we have to find $\Pr(max(X, Y) < 2)$,which is also can be written as

$$\Pr(max(X, Y) < 2) = \Pr(x < 2 \; and \; y < 2) \tag{2.0.21}$$

As X and Y be independent random variables,

$$\Pr(max(X, Y) < 2) = \Pr(x < 2)\Pr(y < 2) \tag{2.0.22}$$

Using (2.0.20),

$$\Pr(max(X, Y) < 2) = (1 - e^{-2})(1 - e^{-2}) \tag{2.0.23}$$
$$= (1 - e^{-2})^2 \tag{2.0.24}$$
$$= 0.748 \tag{2.0.25}$$