ASSIGNMENT 3

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Download all python codes from

```
https://github.com/manik2255/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/blob/main/ASSIGNMENT_3/assign_3.py
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and latex-tikz codes from

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https://github.com/manik2255/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/blob/main/ASSIGNMENT_3/ASSIGNMENT_3.tex
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1 GATE 2017 MA PROBLEM.47

Let X and Y be independent and identically distributed random variables with probability mass function $p(n) = 2^{-n}$,$n = 1, 2, \dots$
Then $pr(X \geq 2Y)$ equals

2 Solution

given,

$$pr(X = x) = 2^{-x}, x = 1, 2, \dots \tag{2.0.1}$$
$$pr(Y = y) = 2^{-y}, y = 1, 2, \dots \tag{2.0.2}$$

We need to find $pr(X \geq 2Y)$,which is also can be written as

$$pr(X \geq 2Y) = \sum_{y=1}^{\infty} pr(X \geq 2y|Y = y) \tag{2.0.3}$$

as,X and Y are independent random variables

$$pr(X \geq 2Y) = \sum_{y=1}^{\infty} pr(X \geq 2y)pr(Y = y) \tag{2.0.4}$$
$$= \sum_{y=1}^{\infty}(1 - pr(X < 2y))pr(Y = y) \tag{2.0.5}$$

using (2.0.1) and (2.0.2) in (2.0.5),

$$= \sum_{y=1}^{\infty}(1 - \sum_{i=1}^{2y-1} 2^{-i})(2^{-y}) \tag{2.0.6}$$
$$= \sum_{y=1}^{\infty}(1 - (1 - 2^{-(2y-1)}))(2^{-y}) \tag{2.0.7}$$
$$= \sum_{y=1}^{\infty} 2^{-(3y-1)} \tag{2.0.8}$$
$$= \frac{2}{7} \tag{2.0.9}$$

Finding the probability using cumulative distribution, Let $F_X(x)$ be the cumulative distribution function of random variable X.

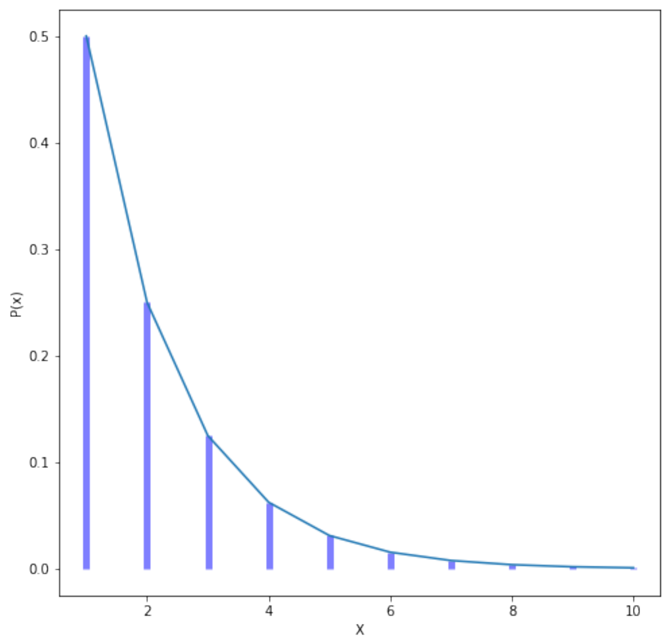

Fig. 1: Pmf of random variable X

$$F_X(x) = \Pr(X < x) \tag{2.0.10}$$

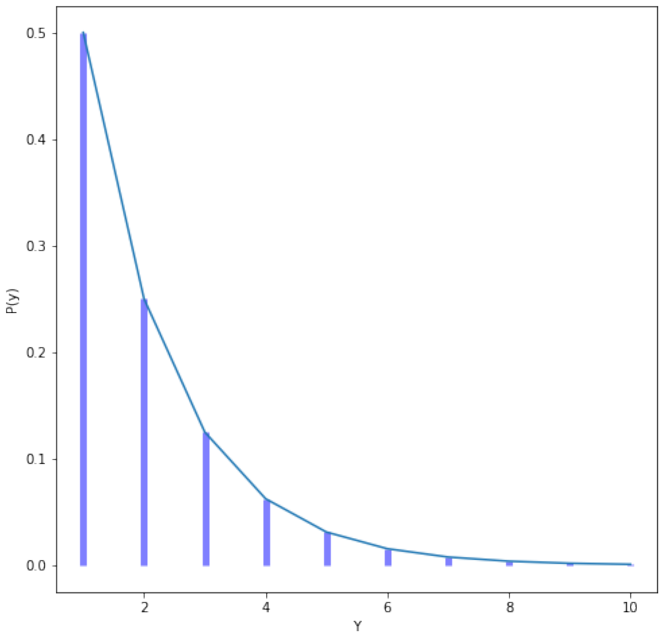

Fig. 2: Pmf of random variable Y

As it is a discrete probability distribution, $F_X(x)$ can be written as,

$$F_X(x) = \sum_{i=1}^{x-1} \Pr\left(X = x\right) \tag{2.0.11}$$

using (2.0.1) in (2)

$$F_X(x) = \sum_{i=1}^{x-1} 2^{-i} \tag{2.0.12}$$

$$= 1 - 2^{-(x-1)} \tag{2.0.13}$$

Cumulative distribution function (CDF) of random variable X is given by,

$$F_X(x) = 1 - 2^{-(x-1)}, x = 1, 2, \ldots \tag{2.0.14}$$

We need to find $pr(X \geq 2Y)$,which is also can be written as

$$pr(X \geq 2Y) = \sum_{y=1}^{\infty} pr(X \geq 2y|Y = y) \tag{2.0.15}$$

As,X and Y are independent random variables

$$pr(X \geq 2Y) = \sum_{y=1}^{\infty} pr(X \geq 2y)pr(Y = y) \tag{2.0.16}$$

$$= \sum_{y=1}^{\infty}(1 - pr(X < 2y))pr(Y = y) \tag{2.0.17}$$

using (2.0.2) and (2) in (2.0.17),

$$pr(X \geq 2Y) = \sum_{y=1}^{\infty}(1 - (1 - 2^{-(2y-1)}))(2^{-y}) \tag{2.0.18}$$

$$= \sum_{y=1}^{\infty} 2^{-(3y-1)} \tag{2.0.19}$$

$$= \frac{2}{7} \tag{2.0.20}$$

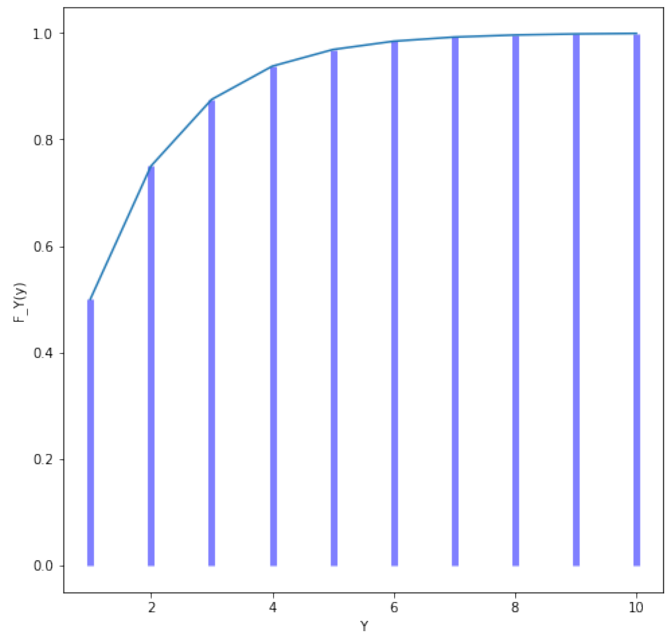

Fig. 3: cdf of random variable Y