ASSIGNMENT 11

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Download all python codes from

https://github.com/manik2255/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/blob/main/ASSIGNMENT_11/assign_11.py

and latex-tikz codes from

https://github.com/manik2255/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/blob/main/ASSIGNMENT_11/ASSIGNMENT_11.tex

1 CSIR UGC NET EXAM (June 2015),PROBLEM.53

Assume that $X \sim Binomial(n, p)$ for some $n \geq 1$ and $0 < p < 1$ and $Y \sim poisson(\lambda)$for some $\lambda > 0$.Suppose E[X]=E[Y].Then

1) $var(X) = Var(Y)$
2) $var(X) < Var(Y)$
3) $var(Y) < Var(X)$
4) $Var(X)$ may be larger or smaller than $Var(Y)$ depending on the values of n,p and λ

2 SOLUTION

For the random variable

$$X \sim Binomial(n, p)$$

As we know,

$$E[X] = np \tag{2.0.1}$$
$$Var(X) = np(1-p) \tag{2.0.2}$$

for the random variable

$$Y \sim poisson(\lambda)$$

As we know,

$$E[Y] = \lambda \tag{2.0.3}$$
$$Var(Y) = \lambda \tag{2.0.4}$$

given that,

$$E[X] = E[Y] \tag{2.0.5}$$
$$np = \lambda \tag{2.0.6}$$

from (2.0.2),

$$Var(X) = np(1-p) \tag{2.0.7}$$

using (2.0.6),

$$Var(X) = \lambda(1-p) \tag{2.0.8}$$

using (2.0.4),

$$Var(X) = Var(Y)(1-p) \tag{2.0.9}$$
$$\frac{Var(X)}{Var(Y)} = 1-p \tag{2.0.10}$$

as,

$$1-p < 1 \tag{2.0.11}$$
$$\frac{Var(X)}{Var(Y)} < 1 \tag{2.0.12}$$
$$Var(X) < Var(Y) \tag{2.0.13}$$

$\therefore Var(Y) > Var(X)$, independent of n,p and λ.

1) $var(X) = Var(Y)$
   using (2.0.2) and (2.0.4),
   $$np(1-p) = \lambda \tag{2.0.14}$$
   using (2.0.6),
   $$np(1-p) = np \tag{2.0.15}$$
   $$1-p = 1 \tag{2.0.16}$$
   $$p = 0 \tag{2.0.17}$$
   which is wrong as per the question($0 < p < 1$). hence the option is incorrect.
2) $var(X) < Var(Y)$
   using (2.0.2) and (2.0.4),
   $$np(1-p) < \lambda \tag{2.0.18}$$

using (2.0.6),

$$np(1-p) < np \tag{2.0.19}$$
$$1-p < 1 \tag{2.0.20}$$
$$p > 0 \tag{2.0.21}$$

which is true as per the question($0 < p < 1$). hence the option is correct.

3) $var(Y) < Var(X)$
using (2.0.2) and (2.0.4),

$$np(1-p) > \lambda \tag{2.0.22}$$

using (2.0.6),

$$np(1-p) > np \tag{2.0.23}$$
$$1-p > 1 \tag{2.0.24}$$
$$p < 0 \tag{2.0.25}$$

which is wrong as per the question($0 < p < 1$). hence the option is incorrect.

4) $Var(X)$ may be larger or smaller than $Var(Y)$ depending on the values of n,p and λ.
Wrong, since we have shown that irrespective of the values of lambda,n, and p, $var(y) > var(x)$