ASSIGNMENT 9

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Download all python codes from

https://github.com/manik2255/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/blob/main/ASSIGNMENT_9/assign_9.py

and latex-tikz codes from

https://github.com/manik2255/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/blob/main/ASSIGNMENT_9/ASSIGNMENT_9.tex

1 GATE 2021 (ME-SET1)PROBLEM.33

Customers arrive at a shop according to Poisson distribution with a mean of 10 customers/hour. The manager notes that no customer arrives for the first 3 minutes after the shop opens. The probability that a customer arrives within the next 3 minutes is

2 SOLUTION

Given, 10 customers arrive in a time interval of an hour $\iff$ $\frac{1}{2}$ customers arrive in a time interval of 3 minutes.

$$\lambda = \frac{1}{2} \tag{2.0.1}$$

Let X denotes the number of customers in first 3 minutes,Y denotes the number of customers in second 3 minutes. according to poisson distribution,

$$\Pr(X = x) = e^{-\lambda}\frac{\lambda^x}{x\,!} \tag{2.0.2}$$

using (2.0.1) in (2.0.2),

$$\Pr(X = x) = e^{-\frac{1}{2}}\frac{(\frac{1}{2})^x}{x\,!} \tag{2.0.3}$$

we need to find the probability that a customer arrives within the next 3 minutes given that no customer arrives for the first 3 minutes after the shop opens,which can also be written as,

$$\Pr(Y \neq 0|X = 0) = \frac{\Pr(Y \neq 0, X = 0)}{\Pr(X = 0)} \tag{2.0.4}$$

As the arrival of customers in second 3 minutes does not depend on the arrival of customers in first 3 minutes, X and Y are independent,

$$\Pr(Y \neq 0|X = 0) = \frac{\Pr(Y \neq 0)\Pr(X = 0)}{\Pr(X = 0)} \tag{2.0.5}$$
$$= \Pr(Y \neq 0) \tag{2.0.6}$$
$$= 1 - \Pr(Y = 0) \tag{2.0.7}$$

using (2.0.3),

$$\Pr(Y \neq 0|X = 0) = 1 - e^{-\frac{1}{2}}\frac{(\frac{1}{2})^0}{0\,!} \tag{2.0.8}$$
$$= 1 - e^{-\frac{1}{2}} \tag{2.0.9}$$
$$= 0.3935 \tag{2.0.10}$$

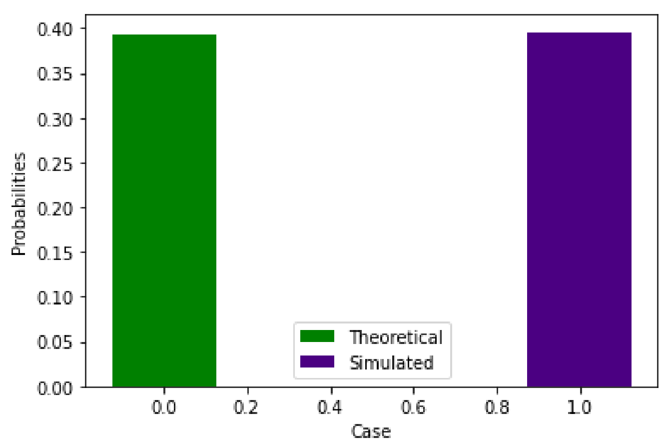

Fig. 1: Theoretical vs simulation