ASSIGNMENT 6

MANIKANTA VALLEPU - AI20BTECH11014

Download all latex-tikz codes from

https://github.com/manik2255/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ASSIGNMENT_6/ASSIGNMENT_6.tex

1 GATE 2020 ST PROBLEM.43

Let (X,Y) be a random vector such that, for any $y > 0$, the conditional probability density function of X given $Y = y$ is

$$f_{X|Y=y}(x) = ye^{-yx} , x > 0.$$

If the marginal probability density function of Y is

$$g(y) = ye^{-y} , y > 0$$

then $E(Y|x = 1) =$

2 SOLUTION

Given, the conditional probability density function of X given $Y = y$,

$$f_{X|Y=y}(x) = ye^{-yx} , x > 0 \tag{2.0.1}$$

and, the marginal probability density function of Y,

$$g(y) = ye^{-y} , y > 0 \tag{2.0.2}$$

let the joint probability density function of (X,Y) be $f_{X,Y}(x, y)$. We know that,

$$f_{X|Y=y}(x) = \frac{f_{X,Y}(x, y)}{g(y)} \tag{2.0.3}$$

using (2.0.1) and (2.0.2) in (2.0.3),

$$f_{X,Y}(x, y) = y^2 e^{-y(x+1)} \tag{2.0.4}$$

let the marginal probability density function of X be $f_X(x)$, as we know ,

$$f_X(x) = \int_0^\infty f_{X,Y}(x, y)\, dy \tag{2.0.5}$$

using (2.0.4) in (2.0.5),

$$f_X(x) = \int_0^\infty y^2 e^{-y(x+1)}\, dy \tag{2.0.6}$$

$$= \frac{2}{(x + 1)^3} \tag{2.0.7}$$

The conditional probability density function of Y given $X = x$ is given by,

$$f_{Y|X=x}(y) = \frac{f_{X,Y}(x, y)}{f_X(x)} \tag{2.0.8}$$

using (2.0.4) and (2.0.7) in (2.0.8),

$$f_{Y|X=x}(y) = \frac{y^2 e^{-y(x+1)}(x + 1)^3}{2} \tag{2.0.9}$$

The conditional probability density function of Y given $X = 1$ is given by,

$$f_{Y|X=1}(y) = 4y^2 e^{-2y} \tag{2.0.10}$$

We need to find $E(Y|X = 1)$ which is given by,

$$E(Y|X = 1) = \int_0^\infty y f_{Y|X=1}(y)\, dy \tag{2.0.11}$$

using (2.0.10) in (2.0.11),

$$E(Y|X = 1) = \int_0^\infty 4y^3 e^{-2y}\, dy \tag{2.0.12}$$

$$= 1.5 \tag{2.0.13}$$