ASSIGNMENT 7

MANIKANTA VALLEPU - AI20BTECH11014

Download all python codes from

https://github.com/manik2255/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/blob/main/ASSIGNMENT_7/assign_7.py

and latex-tikz codes from

https://github.com/manik2255/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/blob/main/ASSIGNMENT_7/ASSIGNMENT_7.tex

1 GATE 2021 ST PROBLEM.14

Let A and b be two events such that $\Pr(B) = \frac{3}{4}$ and $\Pr(A + B') = \frac{1}{2}$.If A and B are independent, then $\Pr(A)$ equals

2 SOLUTION

Given,

$$\Pr(B) = \frac{3}{4} \tag{2.0.1}$$

$$\Pr(A + B') = \frac{1}{2} \tag{2.0.2}$$

we know that,

$$\Pr(B') = 1 - \Pr(B) \tag{2.0.3}$$

using (2.0.1) in (2.0.3),

$$\Pr(B') = \frac{1}{4} \tag{2.0.4}$$

we know that,

$$\Pr(A + B') = \Pr(A) + \Pr(B') - \Pr(A, B') \tag{2.0.5}$$

A and B are independent $\iff$ A and B' are independent

$$\Pr(A + B') = \Pr(A) + \Pr(B') - \Pr(A)\Pr(B') \tag{2.0.6}$$

using (2.0.2) and (2.0.4) in (2.0.6),

$$\frac{1}{2} = \Pr(A) + \frac{1}{4} - \frac{\Pr(A)}{4} \tag{2.0.7}$$

$$\frac{1}{4} = \frac{3\Pr(A)}{4} \tag{2.0.8}$$

$$\therefore \Pr(A) = \frac{1}{3} \tag{2.0.9}$$

From DeMorgan's Law, $(A + B')'$ can be written as,

$$(A + B')' = A'.B \tag{2.0.10}$$

$$\Pr((A + B')') = \Pr(A'.B) \tag{2.0.11}$$

As,

$$(A + B').(A + B')' = 0 \tag{2.0.12}$$

$$\Pr(A + B') + \Pr((A + B')') = 1 \tag{2.0.13}$$

using (2.0.11) in (2.0.13),

$$\Pr(A + B') + \Pr(A'.B) = 1 \tag{2.0.14}$$

A and B are independent $\iff$ A' and B are independent

$$\Pr(A'.B) = \Pr(A')\Pr(B) \tag{2.0.15}$$

using (2.0.14) in (2.0.15),

$$\Pr(A + B') + \Pr(A')\Pr(B) = 1 \tag{2.0.16}$$

using (2.0.1) and (2.0.2) in (2.0.16),

$$\frac{1}{2} + \Pr(A')\frac{3}{4} = 1 \tag{2.0.17}$$

$$\Pr(A') = \frac{2}{3} \tag{2.0.18}$$

As,

$$A.A' = 0 \tag{2.0.19}$$

$$\Pr(A) + \Pr(A') = 1 \tag{2.0.20}$$

using (2.0.18) in (2.0.20),

$$\Pr(A) + \frac{2}{3} = 1 \tag{2.0.21}$$

$$\Pr(A) = \frac{1}{3} \tag{2.0.22}$$