ASSIGNMENT 2

MANIKANTA VALLEPU - AI20BTECH11014

Download all python codes from

https://github.com/manik2255/AI1103–PROBABILITY–AND–RANDOM–VARIABLES/blob/main/ASSIGNMENT_2/ASSIGNMENT_2_GRAPH.py

and latex-tikz codes from

https://github.com/manik2255/AI1103–PROBABILITY–AND–RANDOM–VARIABLES/blob/main/ASSIGNMENT_2/ASSIGNMENT_2.tex

1 Problem.GATE.14

A continuous random variable X has a probability density function $f(x) = e^{-x}, 0 < x < \infty$. Then $P(X > 1)$ is

2 Solution

Given,

$$f(x) = e^{-x}, 0 < x < \infty \tag{2.0.1}$$

We have to find $\Pr(X > 1)$,

$$\Pr(X > 1) = \int_1^\infty f(x)\,dx \tag{2.0.2}$$

Using (2.0.1) in (2.0.2)

$$\begin{aligned}\Pr(X > 1) &= \int_1^\infty e^{-x}\,dx && (2.0.3)\\ &= [-e^{-x}]_1^\infty && (2.0.4)\\ &= (-e^{-\infty}) - (-e^{-1}) && (2.0.5)\\ &= e^{-1} && (2.0.6)\\ &= \frac{1}{e} && (2.0.7)\\ \implies \Pr(X > 1) &= 0.368 && (2.0.8)\end{aligned}$$

Finding the probability using uniform distribution, Let $F_X(x)$ be the cumulative distribution function of random variable X.

$$F_X(x) = \int_0^x f(x)\,dx \tag{2.0.9}$$

$F_X(x)$ can be obtained from the uniform distribution of a random variable U on (0,1) and let U=e^{-x}.

$$0 < U < 1 \tag{2.0.10}$$

As for random variable X also,

$$0 < F_X(x) < 1 \tag{2.0.11}$$

This similarity between U and $F_X(x)$ is used to generate the random variable X from U.

$$\begin{aligned}F_X(x) &= \Pr(X < x) && (2.0.12)\\ &= \Pr(-\log_e U < x) && (2.0.13)\\ &= \Pr(U < e^{-x}) && (2.0.14)\\ &= F_U(e^{-x}) && (2.0.15)\end{aligned}$$

From uniform distribution,

$$F_U(x) = x, 0 < x < 1 \tag{2.0.16}$$

In the figure 0,orange colour graph represents the pdf of the random variable X and blue colour graph represents the cdf of the random variable X.

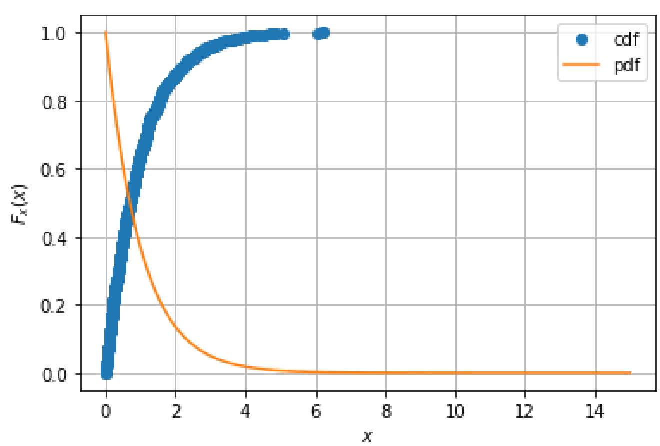

Fig. 0: CDF of random variable X

Using (2.0.16) in (2.0.15), Cumulative distribution function (CDF) of random variable X is,

$$\begin{aligned}F_X(x) &= \Pr(X < x) && (2.0.17)\\ &= 1 - e^{-x}, 0 < x < \infty && (2.0.18)\end{aligned}$$

Now we have to find $\Pr(X > 1)$,

$$\Pr(X > 1) = 1 - \Pr(X < 1) \tag{2.0.19}$$

Using (2.0.18),

$$\Pr(X > 1) = 1 - (1 - e^{-1}) \tag{2.0.20}$$
$$\Pr(X > 1) = e^{-1} \tag{2.0.21}$$
$$\implies \Pr(X > 1) = 0.368 \tag{2.0.22}$$