

Project: Kinematics Pick & Place

Rubric Points

Here I will consider the rubric points individually and describe how I addressed each point in my implementation.

Writeup / README

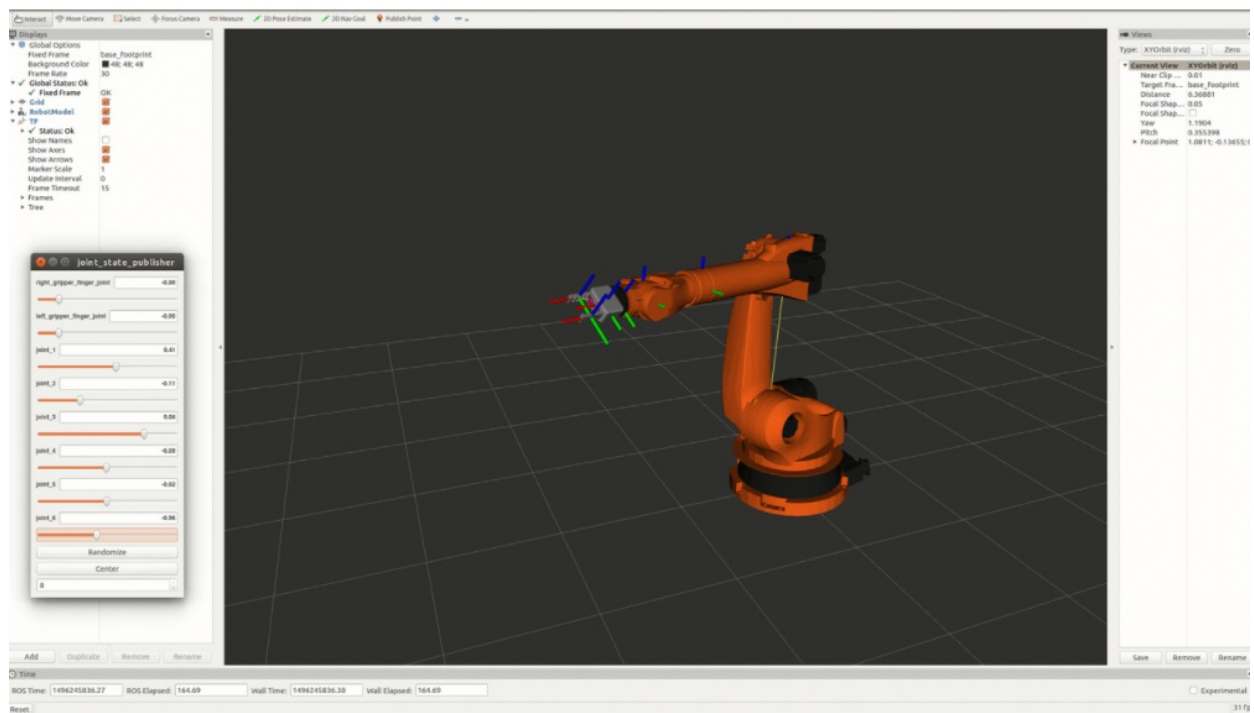
1. Provide a Writeup / README that includes all the rubric points and how you addressed each one. You can submit your writeup as markdown or pdf.

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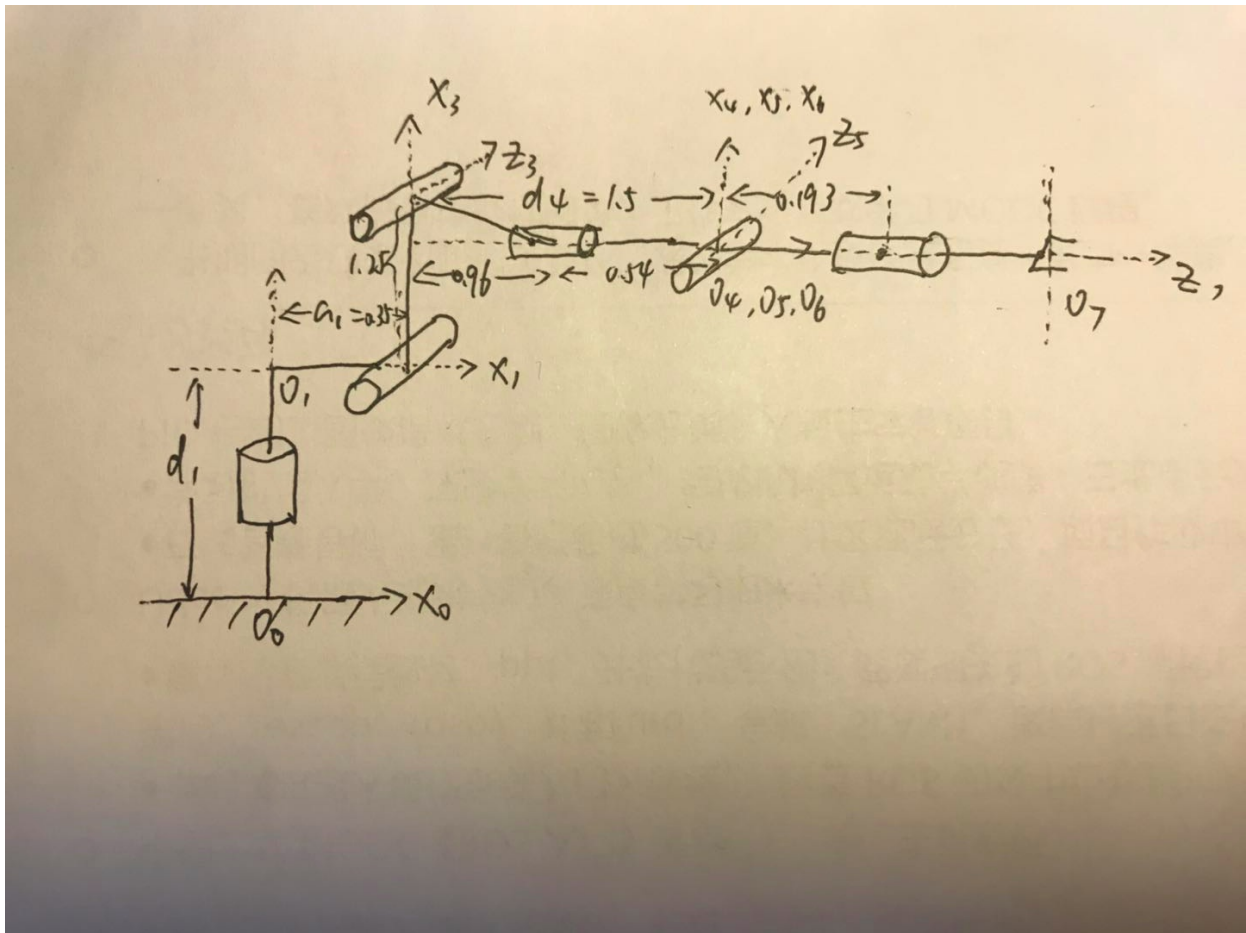
Kinematic Analysis

1. Run the forward_kinematics demo and evaluate the kr210.urdf.xacro file to perform kinematic analysis of Kuka KR210 robot and derive its DH parameters.

I run forward_kinematics demo and change the joint angle using the joint state publisher, and I can see how each angle rotates and how it affects the end-effector. The `kr210.urdf.xacro` defines the translation and rotations between links and joints, but it's not in DH format. I derived the DH params from the beginning. I think it's easier to understand and represent in the urdf format.



2. Using the DH parameter table you derived earlier, create individual transformation matrices about each joint. In addition, also generate a generalized homogeneous transform between base_link and gripper_link using only end-effector(gripper) pose.



Kinematics analysis

So the DH params can be built as follows.

Links	$\alpha(i-1)$	$a(i-1)$	$d(i)$	$\theta(i)$
0->1	0	0	0.75	0
1->2	$-\pi/2$	0.55	0	$-\pi/2 + q_2$
2->3	0	1.25	0	0
3->4	$-\pi/2$	-0.054	1.5	0
4->5	$\pi/2$	0	0	0
5->6	$-\pi/2$	0	0	0
6->EE	0	0	0.303	0

The params can be obtained according to the urdf file which only describe the position and rotation between two links.

Transform Matrices is as follows.

$$T_1^0 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 & a_0 \\ \sin \theta_1 \cos \theta_0 & \cos \theta_1 \cos \theta_0 & -\sin \theta_0 & -\sin \theta_0 d_1 \\ \sin \theta_1 \sin \theta_0 & \cos \theta_1 \sin \theta_0 & \cos \theta_0 & \cos \theta_0 d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & a_1 \\ \sin \theta_2 \cos \alpha_1 & \cos \theta_2 \cos \alpha_1 & -\sin \alpha_1 & -\sin \alpha_1 d_2 \\ \sin \theta_2 \sin \alpha_1 & \cos \theta_2 \sin \alpha_1 & \cos \alpha_1 & \cos \alpha_1 d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & 0 & a_2 \\ \sin \theta_3 \cos \alpha_2 & \cos \theta_3 \cos \alpha_2 & -\sin \alpha_2 & -\sin \alpha_2 d_3 \\ \sin \theta_3 \sin \alpha_2 & \cos \theta_3 \sin \alpha_2 & \cos \alpha_2 & \cos \alpha_2 d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} \cos \theta_4 & -\sin \theta_4 & 0 & a_3 \\ \sin \theta_4 \cos \alpha_3 & \cos \theta_4 \cos \alpha_3 & -\sin \alpha_3 & -\sin \alpha_3 d_4 \\ \sin \theta_4 \sin \alpha_3 & \cos \theta_4 \sin \alpha_3 & \cos \alpha_3 & \cos \alpha_3 d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^5 = \begin{bmatrix} \cos \theta_6 & -\sin \theta_6 & 0 & a_5 \\ \sin \theta_6 \cos \alpha_5 & \cos \theta_6 \cos \alpha_5 & -\sin \alpha_5 & -\sin \alpha_5 d_6 \\ \sin \theta_6 \sin \alpha_5 & \cos \theta_6 \sin \alpha_5 & \cos \alpha_5 & \cos \alpha_5 d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_G^b = \begin{bmatrix} c\theta_7 & -s\theta_7 & 0 & a_6 \\ s\theta_7 c\alpha_6 & c\theta_7 c\alpha_6 & -s\alpha_6 & -s\alpha_6 d_7 \\ s\theta_7 s\alpha_6 & c\theta_7 s\alpha_6 & c\alpha_6 & c\alpha_6 d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{gy} = \begin{bmatrix} c-\frac{\pi}{2} & 0 & s-\frac{\pi}{2} & 0 \\ 0 & 1 & 0 & 0 \\ -s-\frac{\pi}{2} & 0 & c-\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{gz} = \begin{bmatrix} c\kappa & -s\kappa & 0 & 0 \\ s\kappa & c\kappa & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{G_a}^0 = T_1^0 \cdot T_2^1 \cdot T_3^1 \cdots T_G^b \cdot T_{gy} \cdot T_{gz}$$

3. Decouple Inverse Kinematics problem into Inverse Position Kinematics and inverse Orientation Kinematics; doing so derive the equations to calculate all individual joint angles.

Extract end-effector position and orientation from request, that is p_x, p_y, p_z and roll, pitch, yaw.

$$R_r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_r & -s_r \\ 0 & s_r & c_r \end{bmatrix}$$

r — roll

p — pitch

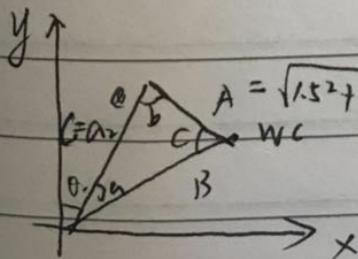
y — yaw

$$R_p = \begin{bmatrix} c_p & 0 & s_p \\ 0 & 1 & 0 \\ -s_p & 0 & c_p \end{bmatrix}$$

$$R_y = \begin{bmatrix} c_y & -s_y & 0 \\ s_y & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{rpy} = R_y \cdot R_p \cdot R_r$$

$$\text{wrist center} = WC = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} - d_7 R_{rpy} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



$$A = \sqrt{1.5^2 + 0.35^2} = 1.501, \quad c = 1.25$$

$$B = \sqrt{(WC_x^2 + WC_y^2 - 0.35)^2 + (z_{wc} - 0.75)^2}$$

$$\Rightarrow a = a \cos\left(\frac{B^2 + c^2 - A^2}{2Bc}\right), \quad b = a \cos\left(\frac{A^2 + c^2 - B^2}{2Ac}\right)$$

$$c = a \cos\left(\frac{A^2 + B^2 - C^2}{2AB}\right)$$

So the wrist center(w_x, w_y, w_z) can be derived from the end-effector position and R_{0_6} .

$\theta_1 = \text{atan2}(w_y, w_x)$ and θ_2 and θ_3 can be calculated by

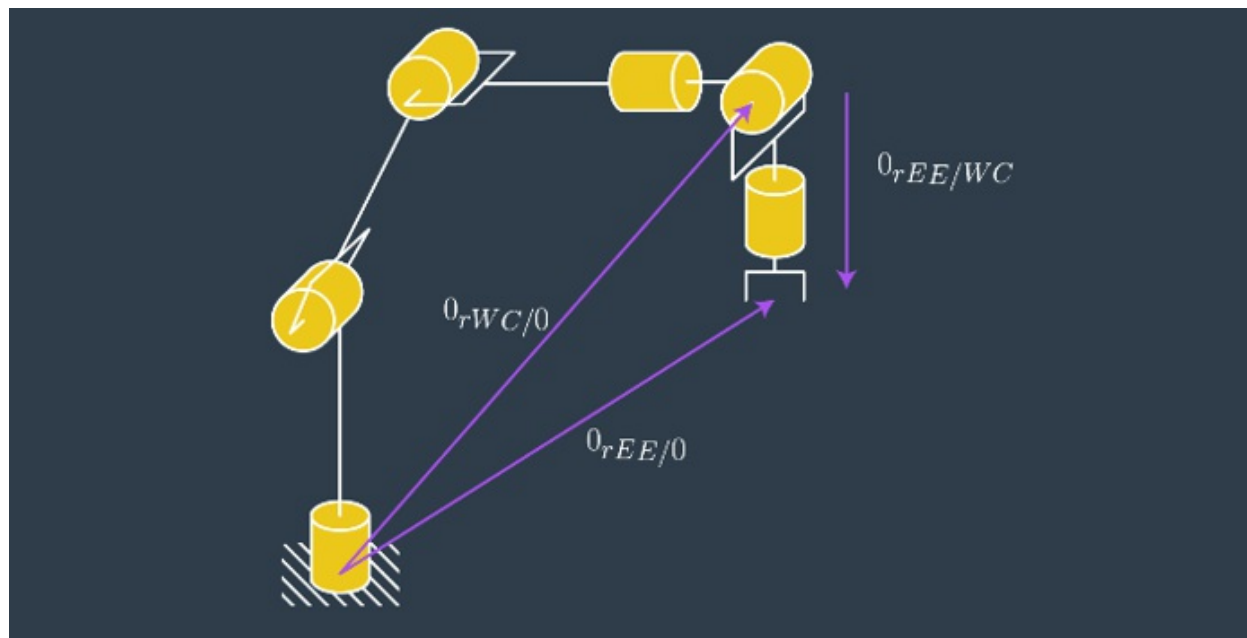
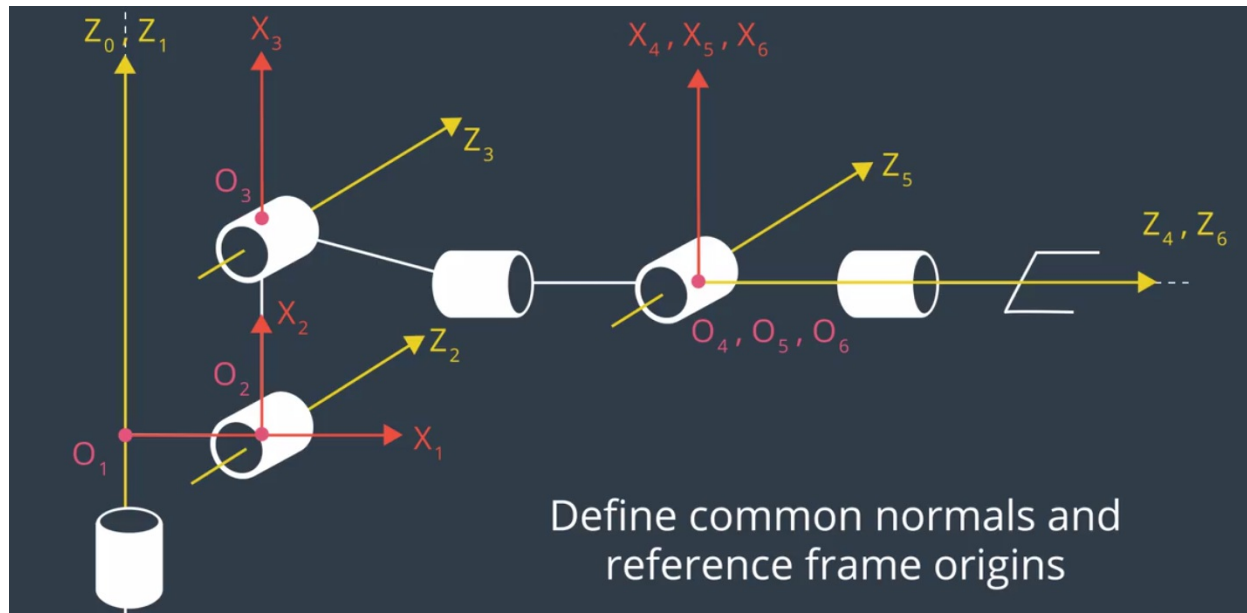
$\theta_2 = \pi/2 - \theta_2_2 - \theta_2_1,$

$\theta_3 = \pi/2 - \theta_{3_2} - \theta_{3_1}$.

$\theta_4 = \text{atan2}(\text{new_R3_6}[2,2], -\text{new_R3_6}[0,2])$

$\theta_5 = \text{atan2}(\sin_{\theta_5}, \text{new_R3_6}[1,2])$

$\theta_6 = \text{atan2}(-\text{new_R3_6}[1,1], \text{new_R3_6}[1,0])$



Project Implementation

1. Fill in the `IK_server.py` file with properly commented python code for calculating Inverse Kinematics based on previously performed Kinematic Analysis. Your code must guide the robot to

successfully complete 8/10 pick and place cycles. Briefly discuss the code you implemented and your results.

The trajectory was drawn correctly , and the arm followed it.

Here is the main implementation of my code.

It can successfully pick 8 out of 10 objects, while sometimes reaches the pick place but fails to hold the object tightly.

```
# Extract end-effector position and orientation from request
# px,py,pz = end-effector position
# roll, pitch, yaw = end-effector orientation
px = req.poses[x].position.x
py = req.poses[x].position.y
pz = req.poses[x].position.z

roll, pitch, yaw = tf.transformations.euler_from_quaternion(
    [req.poses[x].orientation.x, req.poses[x].orientation.y,
    req.poses[x].orientation.z, req.poses[x].orientation.w])

# Calculate joint angles using Geometric IK method
R_roll = Matrix([[ 1,      0,      0],
                  [ 0,   cos(roll), -sin(roll)],
                  [ 0,   sin(roll),  cos(roll)]])

R_pitch = Matrix([[ cos(pitch), 0,  sin(pitch)],
                  [      0,    1,      0],
                  [-sin(pitch), 0,  cos(pitch)]])

R_yaw = Matrix([[ cos(yaw), -sin(yaw),      0],
                 [ sin(yaw),  cos(yaw),      0],
                 [ 0,          0,          1]])

R0_6 = R_yaw*R_pitch*R_roll*R_corr

# Calculate wc point
wx = px - 0.303 * R0_6[0,2]
wy = py - 0.303 * R0_6[1,2]
wz = pz - 0.303 * R0_6[2,2]
a1=0.35
a2=1.25
a3=-0.054
d1=0.75
d4=1.50

# theta1
theta1 = atan2(wy, wx)

#distance
dis_2_3=a2
dis_2_4=sqrt((sqrt(wx**2+wy**2)-a1)**2+(wz-d1)**2)
dis_3_4=sqrt(d4**2+a3**2)
```

```

# theta2
cos_theta2_1=(dis_2_4**2+dis_2_3**2-dis_3_4**2)/(2*dis_2_4*dis_2_3)
sin_theta2_1=sqrt(1-cos_theta2_1**2)
theta2_1 = atan2(sin_theta2_1,cos_theta2_1)
sin_theta2_2=(wz-d1)/(dis_2_4)
cos_theta2_2=sqrt(1-sin_theta2_2**2)
theta2_2 = atan2(sin_theta2_2,cos_theta2_2)
theta2 = pi/2-theta2_2-theta2_1

#theta3
theta3_1=atan2(a3,d4)
cos_theta3_2 = (dis_2_3**2+dis_3_4**2-dis_2_4**2)/(2*dis_2_3*dis_3_4)
sin_theta3_2 = sqrt(1-cos_theta3_2**2)
theta3_2 = atan2(sin_theta3_2,cos_theta3_2)
theta3 = pi/2 - theta3_2-theta3_1

# Calculate R0_3
R0_3 = T0_3.evalf(subs={q1:theta1, q2:theta2, q3:theta3})[0:3, 0:3]

# Calculate new_R3_6
new_R3_6 = R0_3.transpose() * R0_6

# theta4,theta5,theta6
theta4=atan2(new_R3_6[2,2],-new_R3_6[0,2])
sin_theta5=sqrt(new_R3_6[2,2]**2+new_R3_6[0,2]**2)
theta5=atan2(sin_theta5, new_R3_6[1,2])
theta6=atan2(-new_R3_6[1,1],new_R3_6[1,0])

```

The result of IK_server.py code:

