CSD302 : Design and Analysis of Algorithms *Spring 2020*

Programming Assignment 2

Due : 8th April 2020

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1 Problem Statement : Function Approximation

Given an integer valued function $f : D \to \mathbb{Z}$ where $D = \{1, 2, \cdots n\}$, find another function $g : D \to \mathbb{R}$ with a maximum of $k \leq n$ steps such that the sum of squares error between the 2 functions, $\Delta(f, g) = \sum_{i=1}^{n}(g(i) - f(i))^2$ is minimized.

You can lookup what a step function is on wikipedia :
`https://en.wikipedia.org/wiki/Step_function`

2 Input/Output and Constraints

2.1 Input Format

The input consists of multiple test cases. The first line of input contains a single integer t ($1 \leq t \leq 1000$) - the number of test cases. The description of the test cases follows.

The first line of each test case consists of 2 space separated integers, n and k($1 \leq n \leq 5000$, $1 \leq k \leq n$) the size of the domain of the function f, and the maximum number of steps allowed in function g.

The second line of each test case consist of n space separated integers, $f(1), f(2), \cdots, f(n)$ ($-10^6 \leq f(i) \leq 10^6$) - describing the function f.

Note : The sum of n across all test cases is less than 10^4

2.2 Output Format

For each test case output a new line with 2 numbers m - the number of steps in your solution, and $n = \Delta(f, g)$ (the sum of squares error). In the next m lines output a pair of numbers, x and $g(x)$, representing the starting position (x) of every step and the value of g at that input. Clearly the first such line would represent $< 1, g(1) >$

If there are multiple such solutions with the same value of $\Delta(f, g)$, print any one of them.

3 Sample Cases

Sample Input

```
1
9 3
1 2 3 4 5 6 7 8 9
```

Sample Output

```
3 6.0
1 2.0
4 5.0
7 8.0
```

4 Other Instructions

There is a time limit of 2 seconds for each test case.
The tests have been tested to run in time on java, and c/c++. Using python is not recommended since the tests have not been designed to ensure that python will run successfully (some tests may pass, some may not).

The submissions will be on the platform codeforces and will be auto-graded AFTER the deadline. Till then your code will be run on 2 of the provided example cases, and one **small** test case to ensure it adheres to the I/O format. Please also add your name and roll number as a comment in the source file.

Registration Link : `https://codeforces.com/contestInvitation/6523ef249c94062489ad017a7cd648d3d387fd6d`
Submission Link : `https://codeforces.com/contests/274633`