In [22]:

```
pip install ucimlrepo
```

```
Requirement already satisfied: ucimlrepo in /Users/manikanr/anaconda3/lib/python3.8/site-
packages (0.0.3)
WARNING: You are using pip version 21.0.1; however, version 24.0 is available.
You should consider upgrading via the '/Users/manikanr/anaconda3/bin/python -m pip instal
l --upgrade pip' command.
Note: you may need to restart the kernel to use updated packages.
```

# Team 7 : Manikandan Ramalingam & Muhammed Haris

In [33]:

```
from ucimlrepo import fetch_ucirepo

# fetch dataset
adult = fetch_ucirepo(id=2)

# data (as pandas dataframes)
X = adult.data.features
y = adult.data.targets

# metadata
print(adult.metadata)

# variable information
print(adult.variables)
```


```
{'uci_id': 2, 'name': 'Adult', 'repository_url': 'https://archive.ics.uci.edu/dataset/2/a
dult', 'data_url': 'https://archive.ics.uci.edu/static/public/2/data.csv', 'abstract': 'P
redict whether income exceeds $50K/yr based on census data. Also known as "Census Income"
dataset. ', 'area': 'Social Science', 'tasks': ['Classification'], 'characteristics': ['M
ultivariate'], 'num_instances': 48842, 'num_features': 14, 'feature_types': ['Categorical
', 'Integer'], 'demographics': ['Age', 'Income', 'Education Level', 'Other', 'Race', 'Sex
'], 'target_col': ['income'], 'index_col': None, 'has_missing_values': 'yes', 'missing_va
lues_symbol': 'NaN', 'year_of_dataset_creation': 1996, 'last_updated': 'Mon Aug 07 2023',
'dataset_doi': '10.24432/C5XW20', 'creators': ['Barry Becker', 'Ronny Kohavi'], 'intro_pa
per': None, 'additional_info': {'summary': 'Extraction was done by Barry Becker from the
1994 Census database.  A set of reasonably clean records was extracted using the followin
g conditions: ((AAGE>16) && (AGI>100) && (AFNLWGT>1)&& (HRSWK>0))\r\n\r\nPrediction task
is to determine whether a person makes over 50K a year.\r\n', 'purpose': None, 'funded_by
': None, 'instances_represent': None, 'recommended_data_splits': None, 'sensitive_data':
None, 'preprocessing_description': None, 'variable_info': 'Listing of attributes:\r\n\r\n
>50K, <=50K.\r\n\r\nage: continuous.\r\nworkclass: Private, Self-emp-not-inc, Self-emp-in
c, Federal-gov, Local-gov, State-gov, Without-pay, Never-worked.\r\nfnlwgt: continuous.\r
\neducation: Bachelors, Some-college, 11th, HS-grad, Prof-school, Assoc-acdm, Assoc-voc,
9th, 7th-8th, 12th, Masters, 1st-4th, 10th, Doctorate, 5th-6th, Preschool.\r\neducation-n
um: continuous.\r\nmarital-status: Married-civ-spouse, Divorced, Never-married, Separated
, Widowed, Married-spouse-absent, Married-AF-spouse.\r\noccupation: Tech-support, Craft-r
epair, Other-service, Sales, Exec-managerial, Prof-specialty, Handlers-cleaners, Machine-
op-inspct, Adm-clerical, Farming-fishing, Transport-moving, Priv-house-serv, Protective-s
erv, Armed-Forces.\r\nrelationship: Wife, Own-child, Husband, Not-in-family, Other-relati
ve, Unmarried.\r\nrace: White, Asian-Pac-Islander, Amer-Indian-Eskimo, Other, Black.\r\ns
ex: Female, Male.\r\ncapital-gain: continuous.\r\ncapital-loss: continuous.\r\nhours-per-
week: continuous.\r\nnative-country: United-States, Cambodia, England, Puerto-Rico, Canad
a, Germany, Outlying-US(Guam-USVI-etc), India, Japan, Greece, South, China, Cuba, Iran, H
onduras, Philippines, Italy, Poland, Jamaica, Vietnam, Mexico, Portugal, Ireland, France,
Dominican-Republic, Laos, Ecuador, Taiwan, Haiti, Columbia, Hungary, Guatemala, Nicaragua
, Scotland, Thailand, Yugoslavia, El-Salvador, Trinadad&Tobago, Peru, Hong, Holand-Nether
lands.', 'citation': None}}
```


```
               name     role         type      demographic  \
0               age  Feature      Integer              Age
1         workclass  Feature  Categorical           Income
2            fnlwgt  Feature      Integer             None
3         education  Feature  Categorical  Education Level
4     education-num  Feature      Integer  Education Level
```

```
4    education-num  Feature     Integer  Education Level
5   marital-status  Feature  Categorical           Other
6       occupation  Feature  Categorical           Other
7     relationship  Feature  Categorical           Other
8             race  Feature  Categorical            Race
9              sex  Feature       Binary             Sex
10    capital-gain  Feature      Integer            None
11    capital-loss  Feature      Integer            None
12  hours-per-week  Feature      Integer            None
13  native-country  Feature  Categorical           Other
14          income   Target       Binary          Income

                                          description units missing_values
0                                                 N/A  None             no
1   Private, Self-emp-not-inc, Self-emp-inc, Feder...  None            yes
2                                                None  None             no
3    Bachelors, Some-college, 11th, HS-grad, Prof-...  None             no
4                                                None  None             no
5   Married-civ-spouse, Divorced, Never-married, S...  None             no
6   Tech-support, Craft-repair, Other-service, Sal...  None            yes
7   Wife, Own-child, Husband, Not-in-family, Other...  None             no
8   White, Asian-Pac-Islander, Amer-Indian-Eskimo,...  None             no
9                                        Female, Male.  None             no
10                                               None  None             no
11                                               None  None             no
12                                               None  None             no
13  United-States, Cambodia, England, Puerto-Rico,...  None            yes
14                                         >50K, <=50K.  None             no
```

In [2]:

```
import pandas as pd

#Fetch data from csv file with pandas
adultData = pd.read_csv('https://archive.ics.uci.edu/static/public/2/data.csv', sep=',')
adultData.head()
#print(adultData.count())
```

Out[2]:

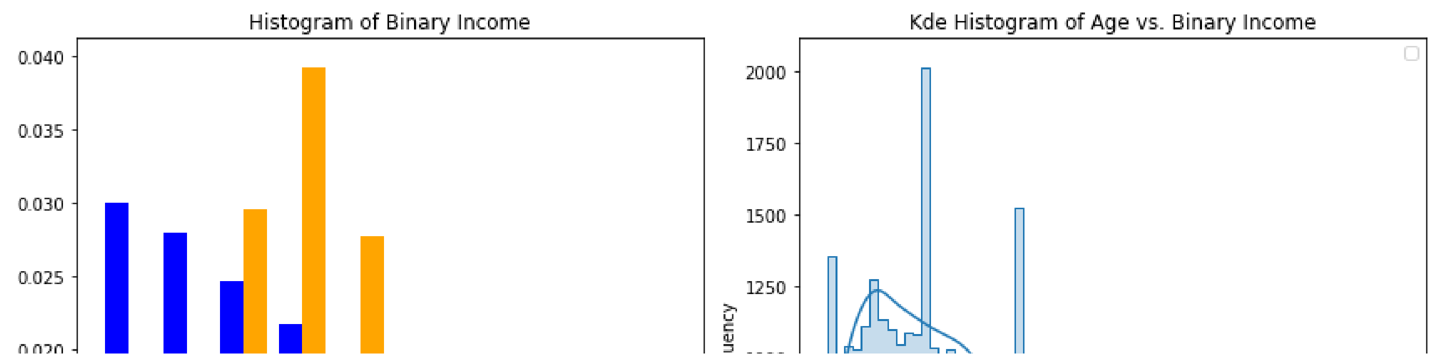
| | age | workclass | fnlwgt | education | education-num | marital-status | occupation | relationship | race | sex | capital-gain | capital-loss | hours per wee |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 39 | State-gov | 77516 | Bachelors | 13 | Never-married | Adm-clerical | Not-in-family | White | Male | 2174 | 0 | 4 |
| **1** | 50 | Self-emp-not-inc | 83311 | Bachelors | 13 | Married-civ-spouse | Exec-managerial | Husband | White | Male | 0 | 0 | 1 |
| **2** | 38 | Private | 215646 | HS-grad | 9 | Divorced | Handlers-cleaners | Not-in-family | White | Male | 0 | 0 | 4 |
| **3** | 53 | Private | 234721 | 11th | 7 | Married-civ-spouse | Handlers-cleaners | Husband | Black | Male | 0 | 0 | 4 |
| **4** | 28 | Private | 338409 | Bachelors | 13 | Married-civ-spouse | Prof-specialty | Wife | Black | Female | 0 | 0 | 4 |

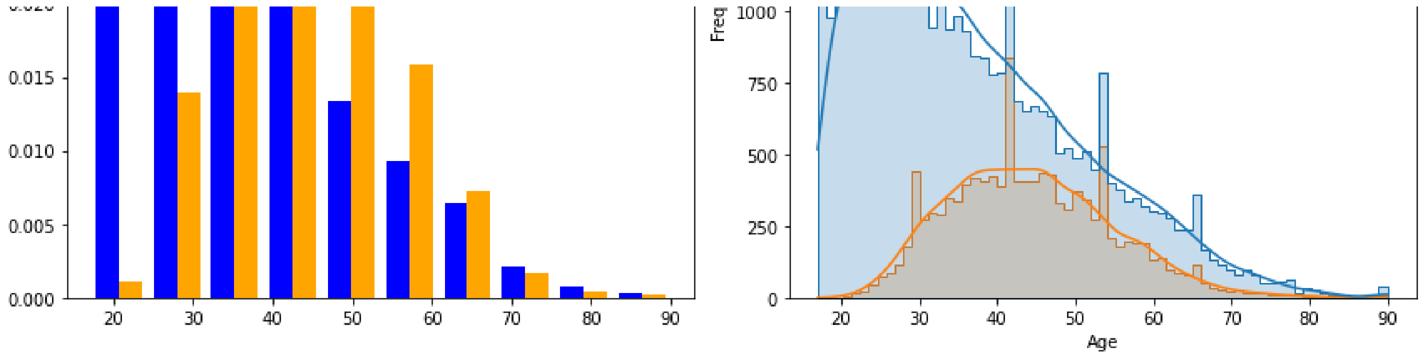
In [120]:

```
from sklearn.preprocessing import LabelEncoder
import matplotlib.pyplot as plt
import seaborn as sns

# Filter/cleanse the columns which has null values
filteredAdultData = adultData[adultData.notnull().all(axis=1)]

# Filter/cleanse the columns that are considered irrelvant
```

```python
# Assumptions
# fnlwgt - Weightage is not considered to avoid bias
# education - Removed as eduction-num is equivalent
# marital status and relationship wasn't considered too

filteredAdultData = filteredAdultData.drop(columns=['fnlwgt', 'education', 'marital-stat
us', 'relationship'])
filteredAdultData.head()

# Define a function to map income values to binary values
def incomeToBinary(income):
    if income == '<=50K' or income == '<=50K.':
        return 0
    elif income == '>50K' or income == '>50K.':
        return 1

# Apply the function to transform the 'income' column to binary
filteredAdultData['income'] = filteredAdultData['income'].apply(lambda x: incomeToBinary
(x))

filteredAdultData.head()
# Drop the original 'income' column if needed
# df.drop(columns=['income'], inplace=True)

# Create KDE plot with histogram
#sns.histplot(data=filteredAdultData, x='age', hue='income', kde=True, element='step', fi
ll=True)
# Separate data for income <= 50 and income > 50

# Create a figure and axes with two subplots
#fig, axs = plt.subplots(1, 2, figsize=(12, 6))

def plotHistogram(continuousVariable, yLabel, axis, axs):
    incomeZero = filteredAdultData[filteredAdultData['income'] == 0][continuousVariable]
    incomeOne = filteredAdultData[filteredAdultData['income'] == 1][continuousVariable]

    axs[axis].hist([incomeZero, incomeOne], bins=10, color=['blue', 'orange'], label=['I
ncome <= 50', 'Income > 50'], density='true')
    axs[axis].set_title('Histogram of Binary Income')
    plt.xlabel(yLabel)
    plt.ylabel('Frequency')
    return plt

# Create a figure and axes with two subplots
fig, axs = plt.subplots(1, 2, figsize=(12, 6))

plt = plotHistogram('age', 'Age', 0, axs)
sns.histplot(data=filteredAdultData, x='age', hue='income', kde=True, element='step', fil
l=True)
axs[1].set_title('Kde Histogram of Age vs. Binary Income')
# Add labels and title
plt.xlabel('Age')
plt.ylabel('Frequency')
plt.legend()

# Show the plot
plt.tight_layout()
plt.show()
```

```
No handles with labels found to put in legend.
```



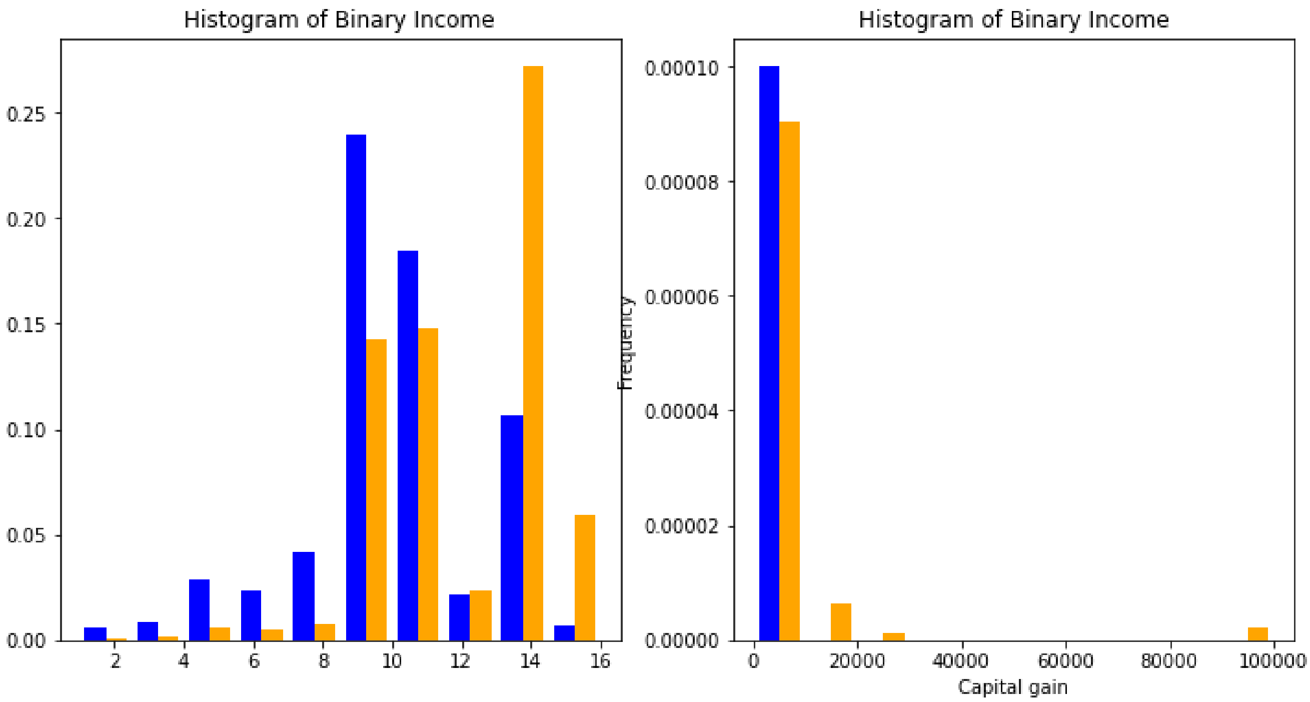


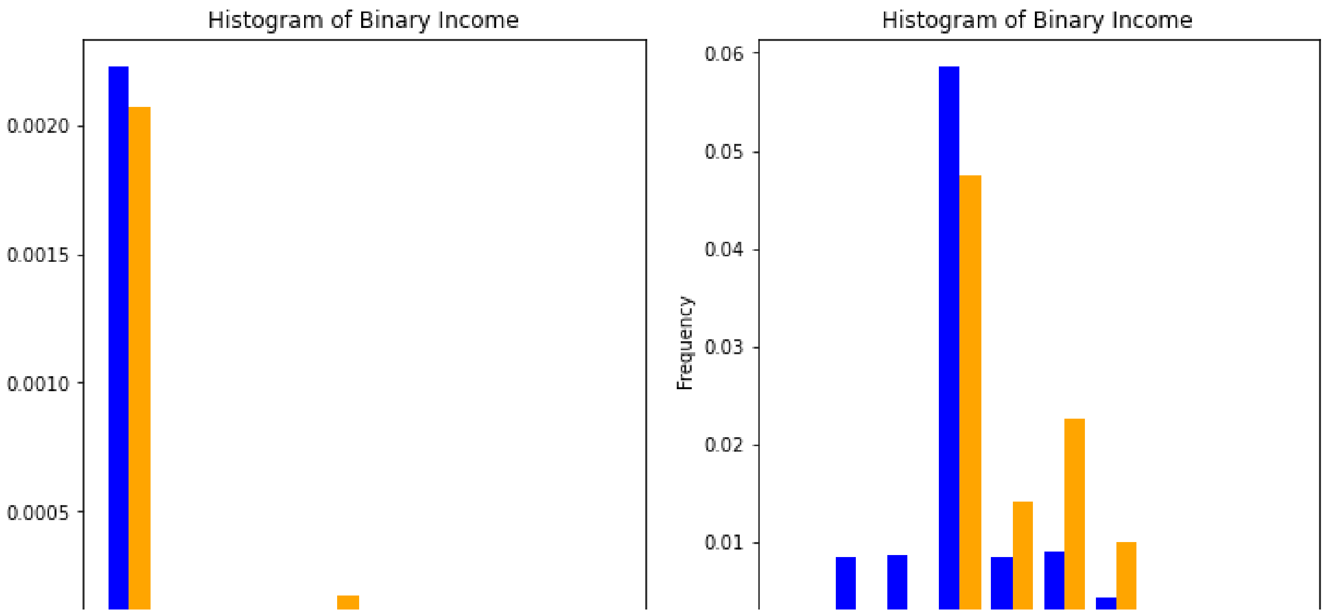

**From 35 years - 50 years, the proportion of incomes > 50 shows the normal distribution whereas the incomes <=50 shows long right tail with gamma distribution.**

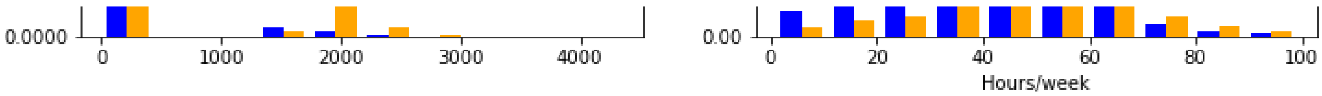
## Plot the relationships of other continuous variables of education (yrs), capital-gain and hours-per-week to income

In [4]:

```
fig, axs = plt.subplots(1, 2, figsize=(12, 6))
plt = plotHistogram('education-num', 'Education (yrs)', 0, axs)
plt = plotHistogram('capital-gain', 'Capital gain', 1, axs)
fig, axs = plt.subplots(1, 2, figsize=(12, 6))
plt = plotHistogram('capital-loss', 'Capital loss', 0, axs)
plt = plotHistogram('hours-per-week', 'Hours/week', 1, axs)
```






**From above histograms, we can say positive relationships exist between number of years of education and income greater than 50K.**

In [5]:

```
# Filter and cleanse the data by converting the categorical variables to numeric codes
# Initialize LabelEncoder
label_encoder = LabelEncoder()

# Encode each categorical variable with integer values
for column in ['workclass', 'occupation', 'race', 'sex', 'native-country']:
    filteredAdultData[column] = label_encoder.fit_transform(filteredAdultData[column])

# Describe the data set for continuous and discrete data
filteredAdultData.describe()
filteredAdultData.head()
```

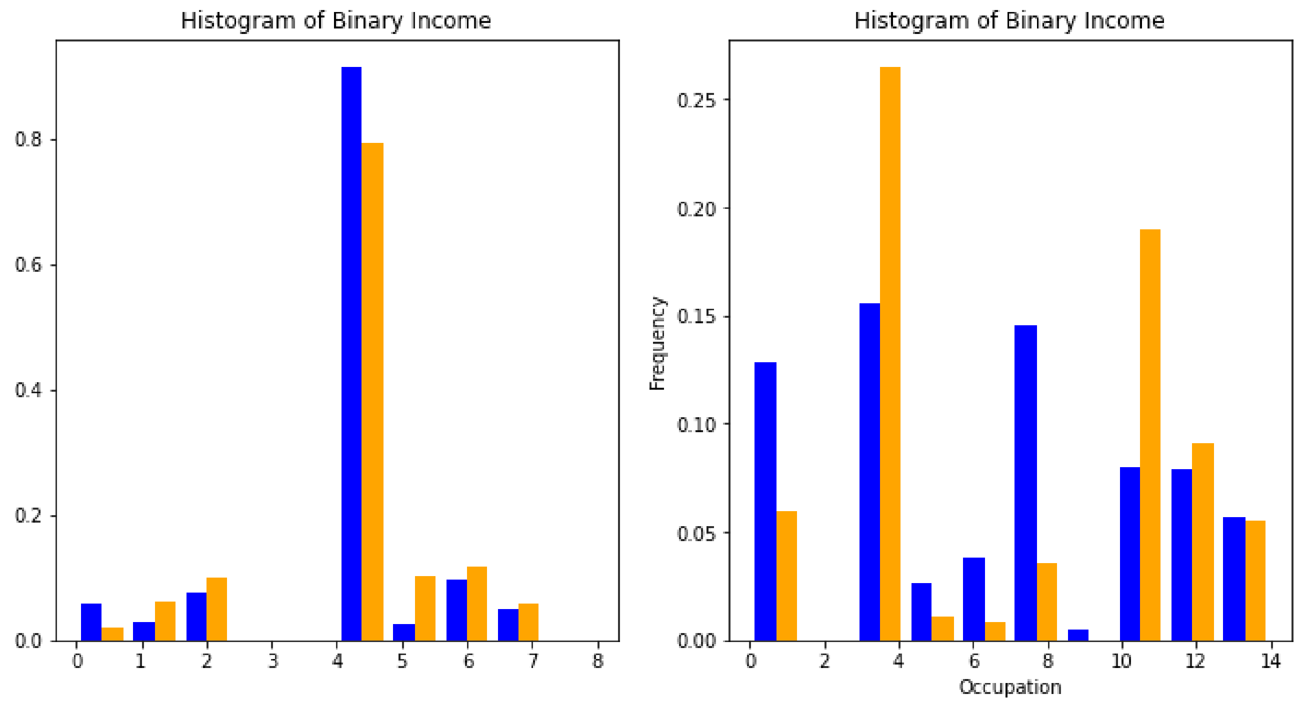
Out[5]:

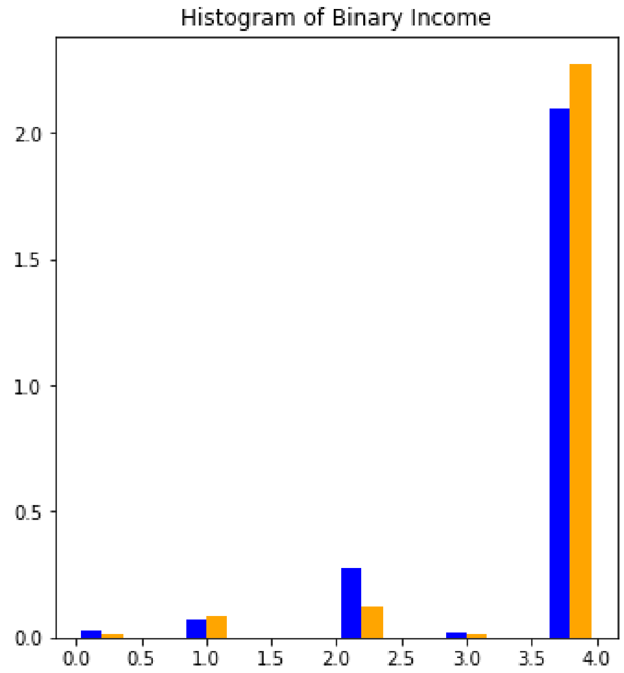
| | age | workclass | education-num | occupation | race | sex | capital-gain | capital-loss | hours-per-week | native-country | income |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 39 | 7 | 13 | 1 | 4 | 1 | 2174 | 0 | 40 | 39 | 0 |
| **1** | 50 | 6 | 13 | 4 | 4 | 1 | 0 | 0 | 13 | 39 | 0 |
| **2** | 38 | 4 | 9 | 6 | 4 | 1 | 0 | 0 | 40 | 39 | 0 |
| **3** | 53 | 4 | 7 | 6 | 2 | 1 | 0 | 0 | 40 | 39 | 0 |
| **4** | 28 | 4 | 13 | 10 | 2 | 0 | 0 | 0 | 40 | 5 | 0 |

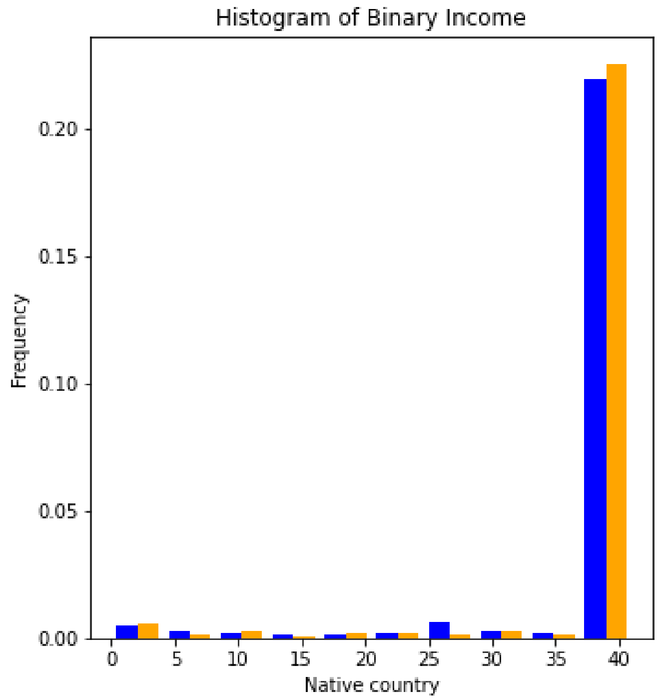
## Plot the corresponding histograms on frequency distribution for other explanatory variables

In [6]:

```
fig, axs = plt.subplots(1, 2, figsize=(12, 6))
plt = plotHistogram('workclass', 'Workclass', 0, axs)
plt = plotHistogram('occupation', 'Occupation', 1, axs)
fig, axs = plt.subplots(1, 2, figsize=(12, 6))
plt = plotHistogram('race', 'Race', 0, axs)
plt = plotHistogram('native-country', 'Native country', 1, axs)
```






**From the other explanatory variables, we can see occupation and workclass to some extent has a little relevance to income. Its better to find the correlation of all variables to rule out the possibilities.**

## Plot the correlation matrix of all explanatory variables against income

In [7]:

```
filteredAdultData.corr()
```

Out[7]:

| | age | workclass | education-num | occupation | race | sex | capital-gain | capital-loss | hours-per-week | native-country | incom |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **age** | 1.000000 | 0.019250 | 0.033327 | -0.014986 | 0.027481 | 0.086811 | 0.078006 | 0.057772 | 0.079306 | -0.002065 | 0.23198 |
| **workclass** | 0.019250 | 1.000000 | 0.041272 | 0.196509 | 0.055403 | 0.085466 | 0.033021 | 0.011103 | 0.114724 | -0.004796 | 0.03589 |
| **education-num** | 0.033327 | 0.041272 | 1.000000 | 0.104945 | 0.031123 | 0.007802 | 0.125569 | 0.081799 | 0.143727 | 0.062447 | 0.33284 |
| **occupation** | -0.014986 | 0.196509 | 0.104945 | 1.000000 | 0.004717 | 0.069735 | 0.022018 | 0.017622 | 0.060587 | -0.007589 | 0.06664 |
| **race** | 0.027481 | 0.055403 | 0.031123 | 0.004717 | 1.000000 | 0.087625 | 0.011930 | 0.017947 | 0.040628 | 0.133863 | 0.07106 |
| **sex** | 0.086811 | 0.085466 | 0.007802 | 0.069735 | 0.087625 | 1.000000 | 0.047300 | 0.045509 | 0.230097 | -0.009341 | 0.21532 |
| **capital-gain** | 0.078006 | 0.033021 | 0.125569 | 0.022018 | 0.011930 | 0.047300 | 1.000000 | -0.031691 | 0.082279 | 0.000272 | 0.22250 |
| **capital-loss** | 0.057772 | 0.011103 | 0.081799 | 0.017622 | 0.017947 | 0.045509 | -0.031691 | 1.000000 | 0.055360 | 0.000140 | 0.14907 |
| **hours-per-week** | 0.079306 | 0.114724 | 0.143727 | 0.060587 | 0.040628 | 0.230097 | 0.082279 | 0.055360 | 1.000000 | 0.000418 | 0.22752 |
| **native-country** | -0.002065 | -0.004796 | 0.062447 | -0.007589 | 0.133863 | -0.009341 | 0.000272 | 0.000140 | 0.000418 | 1.000000 | 0.01539 |
| **income** | 0.231986 | 0.035898 | 0.332845 | 0.066642 | 0.071068 | 0.215325 | 0.222509 | 0.149078 | 0.227527 | 0.015395 | 1.00000 |

**From the correlation matrix above, we can see number of years of education as more relevant to income. Also, age, number of hours of work, capital-gain, sex as close second.**

## Let's try to calculate Probability for few scenarios

#### 1. Probability of getting income >50K for the whole sample

In [20]:

```
sampleProbForIncomeGreaterThanFifty = filteredAdultData[filteredAdultData['income'] == 1
].count()/filteredAdultData.count()
print("\nProbability of income >50K in sample is {}".format(round(sampleProbForIncomeGrea
terThanFifty['income'], 2)))

print("\nProbability of income <=50K in sample is {}\n".format(round(1 - sampleProbForIn
comeGreaterThanFifty['income'], 2)))
print("From above, sample probability of getting income > 50K is only 24%\n")
```

```
Probability of income >50K in sample is 0.24

Probability of income <=50K in sample is 0.76

From above sample probability of getting income > 50K is only 24%
```

#### 2. Probability of getting income >50k or <=50k if we take samples after and before mean age

In [47]:

```
print("\nThe mean age of sample is {}".format(round(filteredAdultData['age'].mean(), 0)))

# Filter the data with age >= 39 and income > 50K
def probabilityBasedOnAge(age, income, incomeRange, ageRange, filteredAdultDataAgeGrThanM
ean, count,
                          dataType):
    sampleProbForIncGrThanFiftyAfterMeanAge = filteredAdultDataAgeGrThanMean[dataType].co
unt()/count
    print("\nProbability of income {} if {} is {} in sample is {}".format(incomeRange, d
ataType, ageRange, round(sampleProbForIncGrThanFiftyAfterMeanAge['income'], 2)))
    print("From above, sample probability of getting income {} if the {} is {} is {}%\n"
.format(incomeRange, dataType, ageRange,

round(sampleProbForIncGrThanFiftyAfterMeanAge['income'], 2)*100))

# Filter the data with age >= 39 and income > 50K
filteredAdultDataAgeGrThanMean = filteredAdultData[(filteredAdultData['age'] >= 39)
                                                    & (filteredAdultData['income'] == 1)
]
countOfAllDataGreater = filteredAdultData[filteredAdultData['age'] >= 39].count()
probabilityBasedOnAge(39, 1, '>50K', '>=39', filteredAdultDataAgeGrThanMean, countOfAllD
ataGreater, 'age')

# Filter the data with age >= 39 and income < 50K. This should be (1-p)
filteredAdultDataAgeGrThanMean = filteredAdultData[(filteredAdultData['age'] >= 39)
                                                    & (filteredAdultData['income'] == 0)
]
probabilityBasedOnAge(39, 0, '<50K', '>=39', filteredAdultDataAgeGrThanMean, countOfAllD
ataGreater, 'age')

# Filter the data with age < 39 and income > 50K
filteredAdultDataAgeGrThanMean = filteredAdultData[(filteredAdultData['age'] < 39)
                                                    & (filteredAdultData['income'] == 1)
]
countOfAllDataLesser = filteredAdultData[filteredAdultData['age'] < 39].count()
probabilityBasedOnAge(39, 1, '>50K', '<39', filteredAdultDataAgeGrThanMean, countOfAllDa
taLesser, 'age')

# Filter the data with age >= 39 and income < 50K. This should be (1-p)
filteredAdultDataAgeGrThanMean = filteredAdultData[(filteredAdultData['age'] < 39)
                                                    & (filteredAdultData['income'] == 0)
]
probabilityBasedOnAge(39, 0, '<50K', '<39', filteredAdultDataAgeGrThanMean, countOfAllDa
taLesser, 'age')
```

```
The mean age of sample is 39.0

Probability of income >50K if age is >=39 in sample is 0.35
From above, sample probability of getting income >50K if the age is >=39 is 35.0%


Probability of income <50K if age is >=39 in sample is 0.65
From above, sample probability of getting income <50K if the age is >=39 is 65.0%


Probability of income >50K if age is <39 in sample is 0.15
From above, sample probability of getting income >50K if the age is <39 is 15.0%


Probability of income <50K if age is <39 in sample is 0.85
From above, sample probability of getting income <50K if the age is <39 is 85.0%
```

**From above, we can conclude the probability of getting higher salaries after mean age of 39 is higher ie. 35% compared to 15% if age is less than 39.**

## Probability based on Number of years of Education

#### Probability of getting income >50k or <=50k if we take samples after and before mean years of education

In [45]:

```
print("\nThe mean number of years of education of sample is {}".format(round(filteredAdul
tData['education-num'].mean(), 0)))
filteredAdultData['education-num'].describe()
```

```
The mean number of years of education of sample is 10.0
```

Out[45]:

```
count    47621.000000
mean        10.090821
std          2.568320
min          1.000000
25%          9.000000
50%         10.000000
75%         12.000000
max         16.000000
Name: education-num, dtype: float64
```

In [50]:

```
# Filter the data with number of years of education >= 10 and income > 50K
filteredAdultDataAgeGrThanMean = filteredAdultData[(filteredAdultData['education-num'] >
= 10)
                                                   & (filteredAdultData['income'] == 1)
]
countOfAllDataGreater = filteredAdultData[filteredAdultData['education-num'] >= 10].coun
t()
probabilityBasedOnAge(39, 1, '>50K', '>=10', filteredAdultDataAgeGrThanMean, countOfAllD
ataGreater, 'education-num')

# Filter the data with number of years of education >= 10 and income < 50K. This should b
e (1-p)
filteredAdultDataAgeGrThanMean = filteredAdultData[(filteredAdultData['education-num'] >
= 10)
                                                   & (filteredAdultData['income'] == 0)
]
probabilityBasedOnAge(39, 0, '<50K', '>=10', filteredAdultDataAgeGrThanMean, countOfAllD
ataGreater, 'education-num')

# Filter the data with number of years of education < 10 and income > 50K
filteredAdultDataAgeGrThanMean = filteredAdultData[(filteredAdultData['education-num'] <
10)
                                                   & (filteredAdultData['income'] == 1)
```

```
]
countOfAllDataLesser = filteredAdultData[filteredAdultData['education-num'] < 10].count(
)
probabilityBasedOnAge(39, 1, '>50K', '<10', filteredAdultDataAgeGrThanMean, countOfAllDa
taLesser, 'education-num')

# Filter the data with number of years of education < 10 and income < 50K. This should be
(1-p)
filteredAdultDataAgeGrThanMean = filteredAdultData[(filteredAdultData['education-num'] <
10)
                                                    & (filteredAdultData['income'] == 0)
]
probabilityBasedOnAge(39, 0, '<50K', '<10', filteredAdultDataAgeGrThanMean, countOfAllDa
taLesser, 'education-num')
```

```
Probability of income >50K if education-num is >=10 in sample is 0.33
From above, sample probability of getting income >50K if the education-num is >=10 is 33.
0%


Probability of income <50K if education-num is >=10 in sample is 0.67
From above, sample probability of getting income <50K if the education-num is >=10 is 67.
0%


Probability of income >50K if education-num is <10 in sample is 0.13
From above, sample probability of getting income >50K if the education-num is <10 is 13.0
%


Probability of income <50K if education-num is <10 in sample is 0.87
From above, sample probability of getting income <50K if the education-num is <10 is 87.0
%
```

**From the above probability distribution, conclusion is that if number of years of education is greater than 10 years (mean), the probability of getting income more than 50K is 33%. It is only 13% if the education is below 10 years. At the same time, if the number of years of education is more than 10 years, there is no drastic increase in percentiles as 67% is still below 50K.**

### Joint probability Distribution

#### Joint probability distribution of income >50K when Age > 39 and Education > 10

In [55]:

```
# Assuming these are independent events, the probability of both events occurring P(A n B
) = P(A) * P(B)
print("\nThe joint probability distribution of income >50K with Age >=39 and education >=
10 years is {}".format(round(0.35*0.33, 2)))
```

```
The joint probability distribution of income >50K with Age >=39 and education >= 10 years
is 0.12
```

#### Joint probability distribution of income >50K when Age > 39 or Education > 10

In [56]:

```
# Assuming these are independent events, the probability of either events occurring P(A U
B) = P(A) + P(B) - P(A n B)
print("\nThe joint probability distribution of income >50K with Age >=39 or education >=
10 years is {}".format(round(0.35+0.33-0.11, 2)))
```

```
The joint probability distribution of income >50K with Age >=39 or education >= 10 years
is 0.57
```

#### Joint probability distribution of income <50K when Age < 39 and Education < 10

In [57]:

```
# Assuming these are independent events, the probability of both events occurring P(A n B
) = P(A) * P(B)
print("\nThe joint probability distribution of income <50K with Age <39 and education < 1
0 years is {}".format(round(0.87*0.85, 2)))
```

```
The joint probability distribution of income <50K with Age <39 and education < 10 years i
s 0.74
```

#### Joint probability distribution of income <50K when Age < 39 or Education < 10

In [58]:

```
# Assuming these are independent events, the probability of either events occurring P(A U
B) = P(A) + P(B) - P(A n B)
print("\nThe joint probability distribution of income <50K with Age <39 or education < 10
years is {}".format(round(0.87+0.85-0.74, 2)))
```

```
The joint probability distribution of income <50K with Age <39 or education < 10 years is
0.98
```

## If time permits, add Bayesian inference here with posterior probability based on prior probabilities

### Normal Distribution of Population Sample

#### 1. Population Distribution of income >50K if education-num is >=10 in sample is 0.33

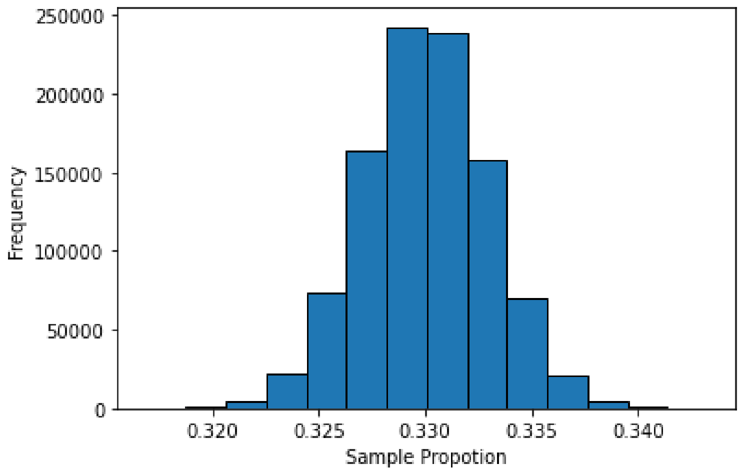
**The data has an income of either <=50K or >50K. So, the binomial probabilty distribution would be right here.**

In [65]:

```
import numpy as np
import statistics
import matplotlib.pyplot as plt
import math

# Count of all observations with inceome > 50K and eductation-num >=10
noOfObs = countOfAllDataGreater
proportionInThatSample = 0.33

def plotNormal(countInSample, sampleProportion, incomeRange):
   # Do binomial distribution for a single random sample
   y1 = np.random.binomial(countInSample, sampleProportion, 1)
   print("People with income {} when sample gets used was {}".format(incomeRange, y1))
   print("Mean of the sample {}".format(y1*sampleProportion))

   # Do binomial distribution for multiple samples randomly selected with each sample siz
e a million times
   y2 = np.random.binomial(countInSample, sampleProportion, 1000000)/countInSample
   print("People with income {} when multiple samples selected randomly were used was {}"
         .format(incomeRange, statistics.mean(y2)))
   print("Standard Deviation {}".format(round(statistics.stdev(y2), 4)))
   print("Standard Error {}".format(round(statistics.stdev(y2)/math.sqrt(countInSample),
8)))
   plt.hist(y2, bins=14, edgecolor='black')
   plt.xlabel('Sample Propotion')
   plt.ylabel('Frequency')
   plt.show()
plotNormal(countOfAllDataGreater['income'], 0.33, '>50K')
```

```
People with income >50K when sample gets used was [8566]
Mean of the sample [2826.78]
People with income >50K when multiple samples selected randomly were used was 0.329995008
19104755
Standard Deviation 0.0029
Standard Error 1.8080e-05
```

Standard Error 1.808e-05

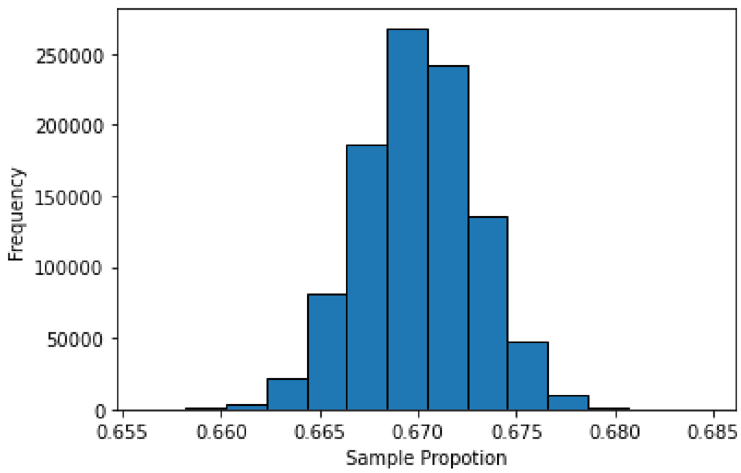



**2. Population Distribution of income <50K if education-num is >=10 in sample is (1-0.33) = 0.67**

In [66]:

```
plotNormal(countOfAllDataGreater['income'], 0.67, '<50K')
```

```
People with income <50K when sample gets used was [17331]
Mean of the sample [11611.77]
People with income <50K when multiple samples selected randomly were used was 0.669998748
3848638
Standard Deviation 0.0029
Standard Error 1.811e-05
```

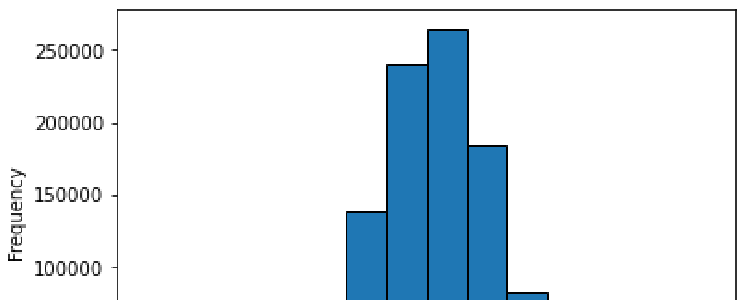


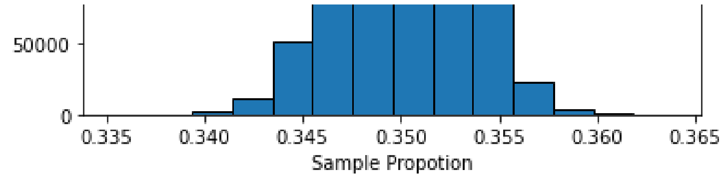

**3. Probability of income >50K if age is >=39 in sample is 0.35**

In [67]:

```
plotNormal(countOfAllDataGreater['income'], 0.35, '>50K')
```

```
People with income >50K when sample gets used was [9059]
Mean of the sample [3170.65]
People with income >50K when multiple samples selected randomly were used was 0.350001330
4876173
Standard Deviation 0.003
Standard Error 1.834e-05
```



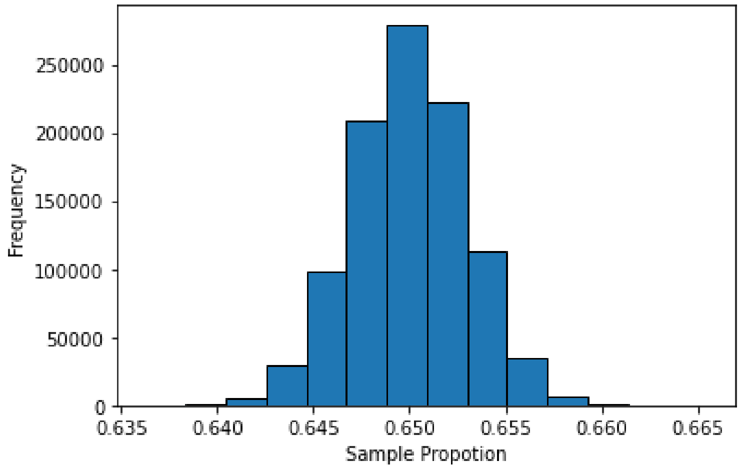



#### 4. Probability of income <50K if age is >=39 in sample is (1-0.35) = 0.65

In [68]:

```
plotNormal(countOfAllDataGreater['income'], 0.65, '<50K')
```

```
People with income <50K when sample gets used was [16821]
Mean of the sample [10933.65]
People with income <50K when multiple samples selected randomly were used was 0.650003680
5491463
Standard Deviation 0.003
Standard Error 1.831e-05
```




**According to Central Limit Theorem, the sample proportion would reach the population proportion. From above, with US population of 330 million, from above sample distribution, the population size where probability of income >50K for ages > 39 would be closer to 103 million. Also, the population size where probability of income >50K where education >= 10 years is approximately 100 million (considering education alone).**

### Maximum Likelihood Estimates of Population Proportion

**As per the central limit theorem, the maximum likelihood estimates of population proportion would fall in the same range of sample proportion. So, we can have below inferences on the population estimates with respect to ML 1. Probability proportion of income >50K if education-num is >=10 in sample is 0.33 From above, ML estimate of getting income >50K if the education-num is >=10 is 33.0% 2. Probability proportion of income <50K if education-num is >=10 in sample is 0.67 From above, ML estimate of getting income <50K if the education-num is >=10 is 67.0% 3. Probability proportion of income >50K if education-num is <10 in sample is 0.13 From above, ML estimate of getting income >50K if the education-num is <10 is 13.0% 4. Probability of income <50K if education-num is <10 in sample is 0.87 From above, ML estimate of of getting income <50K if the education-num is <10 is 87.0%**

### Finding the confidence intervals and standard errors for population proportion

#### Wald and Score confidence intervals for income >50K if education-num is >=10 for 95% confidence level

In [75]:

```
import pandas as pd
from statsmodels.stats.proportion import proportion_confint

def findIntervals(sampleFilteredCount, sampleTotalCount):
    # Find 95% Wald and Score Confidence interval
    print("\nThe confidence interval is {}".format(proportion_confint(sampleFilteredCount
, # Number of students believe
                    sampleTotalCount, # Total Number of students
```

```
                       alpha=0.05, method="normal")))
    print("\nThe Score confidence interval is {}".format(proportion_confint(sampleFiltere
dCount, # Number of students believe
                       sampleTotalCount, # Total Number of students
                       alpha=0.05, method="wilson")))

filteredData = filteredAdultData[(filteredAdultData['education-num'] >= 10)
                                                              & (filteredAdultData['income'] == 1)
]
findIntervals(filteredData['income'].count(), countOfAllDataGreater['income'])
```

```
The confidence interval is (0.329212000349972, 0.34068493858249993)

The Score confidence interval is (0.32923675095889376, 0.34070894547303027)
```

#### Wald and Score confidence intervals for income <50K if education-num is >=10 for 95% confidence level

In [79]:

```
filteredData2 = filteredAdultData[(filteredAdultData['education-num'] >= 10)
                                                              & (filteredAdultData['income'] == 0)
]
findIntervals(filteredData2['income'].count(), countOfAllDataGreater['income'])
```

```
The confidence interval is (0.6593150614175001, 0.6707879996500281)

The Score confidence interval is (0.6592910545269699, 0.6707632490411064)
```

#### Wald and Score confidence intervals for income >50K if education-num is <10 for 95% confidence level

In [80]:

```
filteredData3 = filteredAdultData[(filteredAdultData['education-num'] < 10)
                                                              & (filteredAdultData['income'] == 1)
]
findIntervals(filteredData3['income'].count(), countOfAllDataLesser['income'])
```

```
The confidence interval is (0.12646454463750048, 0.13545894150766305)

The Score confidence interval is (0.12653003462600917, 0.1355245884381265)
```

#### Wald and Score confidence intervals for income <50K if education-num is <10 for 95% confidence level

In [95]:

```
filteredData4 = filteredAdultData[(filteredAdultData['education-num'] < 10)
                                                              & (filteredAdultData['income'] == 0)
]
findIntervals(filteredData3['income'].count(), countOfAllDataLesser['income'])
```

```
The confidence interval is (0.864541058492337, 0.8735354553624994)

The Score confidence interval is (0.8644754115618736, 0.8734699653739908)
```

**From the ML estimates, 95% Wald and Score confidence intervals, the interval percentage is approximately same as probabilities perdicted above. Also, the confidence interval is more narrow which shows the values are more precise and accurate.**

## Hypothetical Sample size required to get the confidence level of 95% and standard error within 0.05

**Since normal population distributions are symmetric, the propotion of 0.5 would give the maximum value. Also, the probability of 0.95 and standard error of 0.05 denotes it is 95% confidence interval where z-score would be 1.96. So, the standard error based on propotion is z-score * sqrt((propotion(1-propotion))/n). Here, to find n, on substitution, it is z-score**2(propotion)(1-propotion)/(stadard error*2). The below python code was used to compute answer.**

In [86]:

```
zScore = 1.96
propotion = 0.33
stdError = 0.05
print("\nThe approximate sample size to be used is {}".format(round(zScore**2*propotion*
(1-propotion)/(stdError**2),0)))
```

```
The approximate sample size to be used is 340.0
```

## Standard Error, Margin of Error, upper and lower quantile

In [88]:

```
# The standard error, margin of error can be obtained by se = sqrt(p(1-p)/n) and me = z-s
core * se
def findErrorValues(proportion, totalCount):
    se = math.sqrt(proportion*(1-proportion)/totalCount)
    zScore = 1.96 # 95% confidence interval
    print("\nThe standard error is {}".format(se))
    me = se*1.96
    print("The margin of error is {}\n".format(me))
    print("The lower quantile is {}".format(proportion - me))
    print("The upper quantile is {}".format(proportion + me))
```

#### For income >50K if education-num is >=10 for 95% confidence level

In [89]:

```
findErrorValues(0.33, filteredData['income'].count())
```

```
The standard error is 0.005038314736557789
The margin of error is 0.009875096883653267

The lower quantile is 0.3201249031163467
The upper quantile is 0.3398750968836533
```

#### For income <50K if education-num is >=10 for 95% confidence level

In [91]:

```
findErrorValues(0.67, filteredData2['income'].count())
```

```
The standard error is 0.003575581190359032
The margin of error is 0.007008139133103703

The lower quantile is 0.6629918608668963
The upper quantile is 0.6770081391331038
```

#### For income >50K if education-num is <10 for 95% confidence level

In [93]:

```
findErrorValues(0.13, filteredData3['income'].count())
```

```
The standard error is 0.0024536586465425045
The margin of error is 0.004809170947223309

The lower quantile is 0.1251908290527767
The upper quantile is 0.1348091709472233
```

#### For income <50K if education-num is <10 for 95% confidence level

In [96]:

```
findErrorValues(0.87, filteredData4['income'].count())
```

```
The standard error is 0.0024536586465425045
```

```
The margin of error is 0.004809170947223309

The lower quantile is 0.8651908290527767
The upper quantile is 0.8748091709472233
```

## Significance Tests, P-value, Z-test and Chi-squared Tests

#### Z-test for education >= 10 years the proportion of income > 50K, H0: Proportion to be 0.20 Ha: Proportion not equal to 0.20

In [113]:

```
from statsmodels.stats.proportion import proportions_ztest, proportion_confint

def proportionCheck(successCount, totCount, state, piValue):
    stat, pVal = proportions_ztest(successCount, totCount, piValue)
    print("\nFor income {}, the z-test is {}, one-sided P-value assuming distribution "
          "as symmetric and normal is {}".format(state, round(stat, 2), pVal/2)) # Divid
e by 2 assuming symmetric and normal
    confidenceInterval = proportion_confint(successCount, totCount)
    print("\nFor income {}, the confidence interval is between {} - {}".format(state, ro
und(confidenceInterval[0], 2),
                                                                          round(confidenceInte
rval[1], 2)))
proportionCheck(filteredData['income'].count(), countOfAllDataGreater['income'], ">50K",
0.20)
#proportionCheck(stateBSuccessCount, stateBTotCount, "B")
print("\nSince the P-value is so low, the H0 can be rejected that population proportion g
etting income is not equal"
      " to 0.20. Infact, from confidence interval, it is in narrow range of 0.32 - 0.34.\
n")

proportionCheck(filteredData['income'].count(), countOfAllDataGreater['income'], ">50K",
0.60) # High value check

print("\nSince the P-value is so low, the H0 can be rejected that population proportion g
etting income is not equal"
      " to 0.60. Infact, from confidence interval, it is in narrow range of 0.32 - 0.34.\
n")
```

```
For income >50K, the z-test is 46.11, one-sided P-value assuming distribution as symmetri
c and normal is 0.0

For income >50K, the confidence interval is between 0.33 - 0.34

Since the P-value is so low, the H0 can be rejected that population proportion getting in
come is not equal to 0.20. Infact from confidence interval, it is in narrow range of 0.32
- 0.34.


For income >50K, the z-test is -90.56, one-sided P-value assuming distribution as symmetr
ic and normal is 0.0

For income >50K, the confidence interval is between 0.33 - 0.34
```

#### Chi-squared Test for the association between number of years of education and income. H0: No relation between education and income Ha: Significant relationship between education and income

In [105]:

```
import pandas as pd
from scipy.stats import chi2_contingency

# Create a contingency table (cross-tabulation)
contingencyTable = pd.crosstab(filteredAdultData['education-num'], filteredAdultData['in
come'])
print(contingencyTable)
```

```
income                0      1
```

```
education-num
1                  77     1
2                 231     8
3                 468    26
4                 852    60
5                 695    40
6                1251    85
7                1656    90
8                 586    47
9               12970  2474
10               8471  2041
11               1513   521
12               1159   407
13               4608  3273
14               1176  1434
15                210   609
16                157   425
```

In [110]:

```python
# Perform the chi-squared test for independence
def chiSquaredTest(contingencyTable):
    chi2Stat, pVal, dof, expected = chi2_contingency(contingencyTable)

    # Output the results
    print("Chi-squared statistic:", chi2Stat)
    print("p-value:", pVal)
    print("Degrees of freedom:", dof)
    print("Expected frequencies:\n", expected)
contingencyTable = pd.crosstab(filteredAdultData['education-num'], filteredAdultData['income'])
chiSquaredTest(contingencyTable)
```

**From the Chi-squared results above, as expected, since the p-value is negligible, there is a strong relationship between number of years of education and income.**

#### Chi-squared Test for the association between age and income. H0: No relation between age and income Ha: Significant relationship between age and income

In [112]:

```python
contingencyTableAge = pd.crosstab(filteredAdultData['age'], filteredAdultData['income'])
chiSquaredTest(contingencyTableAge)
```

```
Chi-squared statistic: 5003.703068883917
p-value: 0.0
Degrees of freedom: 73
Expected frequencies:
 [[4.24283404e+02 1.35716596e+02]
 [6.04603851e+02 1.93396149e+02]
 [7.44011256e+02 2.37988744e+02]
 [7.87954894e+02 2.52045106e+02]
 [7.83409000e+02 2.50591000e+02]
 [8.52355053e+02 2.72644947e+02]
 [9.80397724e+02 3.13602276e+02]
 [8.90995149e+02 2.85004851e+02]
 [8.90237500e+02 2.84762500e+02]
 [8.57658596e+02 2.74341404e+02]
 [9.11451670e+02 2.91548330e+02]
 [9.52364713e+02 3.04635287e+02]
 [9.06148128e+02 2.89851872e+02]
 [9.56910607e+02 3.06089393e+02]
 [9.93277756e+02 3.17722244e+02]
 [9.35696436e+02 2.99303564e+02]
 [9.97823649e+02 3.19176351e+02]
 [9.72821234e+02 3.11178766e+02]
 [1.00009660e+03 3.19903404e+02]
 [1.00691544e+03 3.22084564e+02]
 [9.53880011e+02 3.05119989e+02]
 [9.44788224e+02 3.02211776e+02]
 [9.08421075e+02 2.90578925e+02]
```

```
 [8.88722202e+02 2.84277798e+02]
 [9.19028160e+02 2.93971840e+02]
 [8.72811575e+02 2.79188425e+02]
 [8.27352639e+02 2.64647361e+02]
 [7.99319628e+02 2.55680372e+02]
 [8.15987904e+02 2.61012096e+02]
 [8.24322043e+02 2.63677957e+02]
 [8.05380819e+02 2.57619181e+02]
 [6.32636862e+02 2.02363138e+02]
 [6.30363915e+02 2.01636085e+02]
 [6.48547490e+02 2.07452510e+02]
 [6.50820436e+02 2.08179564e+02]
 [5.47780181e+02 1.75219819e+02]
 [5.34142500e+02 1.70857500e+02]
 [4.58377607e+02 1.46622393e+02]
 [4.59892904e+02 1.47107096e+02]
 [4.22010458e+02 1.34989542e+02]
 [4.05342181e+02 1.29657819e+02]
 [4.08372777e+02 1.30627223e+02]
 [3.87158606e+02 1.23841394e+02]
 [3.28819638e+02 1.05180362e+02]
 [3.29577287e+02 1.05422713e+02]
 [2.82603053e+02 9.03969467e+01]
 [2.45478255e+02 7.85217446e+01]
 [2.42447660e+02 7.75523404e+01]
 [1.96231075e+02 6.27689255e+01]
 [1.66682766e+02 5.33172340e+01]
 [1.66682766e+02 5.33172340e+01]
 [1.25769723e+02 4.02302766e+01]
 [1.09101447e+02 3.48985532e+01]
 [9.16755213e+01 2.93244787e+01]
 [8.18260851e+01 2.61739149e+01]
 [8.33413830e+01 2.66586170e+01]
 [7.34919468e+01 2.35080532e+01]
 [5.45507234e+01 1.74492766e+01]
 [4.84895319e+01 1.55104681e+01]
 [4.77318830e+01 1.52681170e+01]
 [3.63671489e+01 1.16328511e+01]
 [2.27294681e+01 7.27053191e+00]
 [2.04565213e+01 6.54347872e+00]
 [2.65177128e+01 8.48228723e+00]
 [2.57600638e+01 8.23993616e+00]
 [1.13647340e+01 3.63526595e+00]
 [7.57648936e+00 2.42351064e+00]
 [8.33413830e+00 2.66586170e+00]
 [3.78824468e+00 1.21175532e+00]
 [7.57648936e-01 2.42351064e-01]
 [1.51529787e+00 4.84702127e-01]
 [3.78824468e+00 1.21175532e+00]
 [7.57648936e-01 2.42351064e-01]
 [4.09130426e+01 1.30869574e+01]]
```

**From the Chi-squared results above, as expected, since the p-value is negligible, there is a strong relationship between age and income. So Age and number of years of education play a significant role in income proportion.**

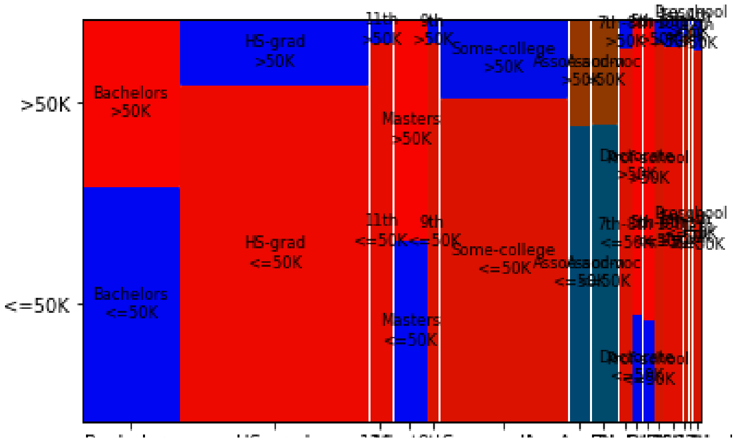
#### Find estmated expected frequencies, standardized residuals and the corresponding Mosaic plot

In [119]:

```python
from statsmodels.graphics.mosaicplot import mosaic
import statsmodels.api as sm

filteredAdultDataNew = adultData[adultData.notnull().all(axis=1)]

# Filter/cleanse the columns that are considered irrelvant
# Assumptions
# fnlwgt - Weightage is not considered to avoid bias
# education - Removed as eduction-num is equivalent
# marital status and relationship wasn't considered too

filteredAdultDataNew = filteredAdultDataNew.drop(columns=['fnlwgt', 'marital-status', 're
```

```
lationship'])
filteredAdultDataNew.head()

# Define a function to map income values to binary values
def incomeToBinary(income):
    if income == '<=50K' or income == '<=50K.':
        return 0
    elif income == '>50K' or income == '>50K.':
        return 1

# Apply the function to transform the 'income' column to binary
filteredAdultDataNew['income'] = filteredAdultDataNew['income'].apply(lambda x: incomeToB
inary(x))

filteredAdultDataNew.head()

# Table on Propotions expected frequencies
crossPropTable = pd.crosstab(filteredAdultDataNew['education'], filteredAdultDataNew['inc
ome'], normalize='index')
crossPropTable
crossTable = pd.crosstab(filteredAdultDataNew['education'], filteredAdultDataNew['income'
], margins=False)
resultH0Table = sm.stats.Table(crossTable)
print("\n")
print(resultH0Table.fittedvalues)

# Calculate Residuals
resultH0Table.standardized_resids

# Mosaic Plot
# First convert the numeric values to strings
filteredAdultDataNew.loc[filteredAdultDataNew['income'] == 0, 'income'] = '<=50K'
filteredAdultDataNew.loc[filteredAdultDataNew['income'] == 1, 'income'] = '>50K'
fig, _ = mosaic(filteredAdultDataNew, ['education','income'], statistic=True)
print("\nFrom the mosaic plot below,")
```

```
income                   0            1
education
10th           1012.218979   323.781021
11th           1322.855043   423.144957
12th            479.591777   153.408223
1st-4th         181.078096    57.921904
5th-6th         374.278575   119.721425
7th-8th         690.975830   221.024170
9th             556.871968   178.128032
Assoc-acdm     1186.478234   379.521766
Assoc-voc      1541.057937   492.942063
Bachelors      5971.031268  1909.968732
Doctorate       440.951681   141.048319
HS-grad       11701.130174  3742.869826
Masters        1977.463724   632.536276
Preschool        59.096617    18.903383
Prof-school     620.514479   198.485521
Some-college   7964.405619  2547.594381

From the mosaic plot below,
```



Bachelors HS-grad

## Scatterplot and Regression Model fit

### Regression Model Fit between number of years of education and income

**Since the number of years of education is a continuous variable and the target income is an explanatory variable with values <=50K and >50K, the logit regression model would be the right fit for this.**

In [125]:

```
# Add a constant term for the intercept
filteredAdultData['intercept'] = 1

# Define the independent variable (X) and dependent variable (y)
X = filteredAdultData[['education-num', 'intercept']]
y = filteredAdultData['income']

# Fit the logistic regression model
model = sm.Logit(y, X).fit()

# Calculate the predicted probabilities
predictedProbs = model.predict(X)

# Print the summary of the model
print(model.summary())
```

```
Optimization terminated successfully.
         Current function value: 0.492559
         Iterations 6
                           Logit Regression Results
==============================================================================
Dep. Variable:                 income   No. Observations:                47621
Model:                          Logit   Df Residuals:                    47619
Method:                           MLE   Df Model:                            1
Date:                Sat, 24 Feb 2024   Pseudo R-squ.:                  0.1105
Time:                        22:13:13   Log-Likelihood:                -23456.
converged:                       True   LL-Null:                       -26371.
Covariance Type:            nonrobust   LLR p-value:                     0.000
=================================================================================
                    coef    std err          z      P>|z|      [0.025      0.975]
---------------------------------------------------------------------------------
education-num     0.3608      0.005     69.760      0.000       0.351       0.371
intercept        -4.9744      0.058    -85.345      0.000      -5.089      -4.860
=================================================================================
```

**Since the co-eff of number of years of education is 0.36, it denotes a positive linear relationship with income.**

In [146]:

```
# Define the logistic regression equation
def logistic_regression(education, intercept, coefEducation):
    # Calculate the log odds (logit)
    logOdds = intercept + coefEducation * education
    # Calculate the probability of high income (1) using the logistic function
    probability = np.exp(logOdds) / (1 + np.exp(logOdds))
    return probability

# Example values (replace with your data)
intercept = -4.97
coefEducation = 0.36

# Calculate probabilities for each education value
probabilities = [logistic_regression(education, intercept, coefEducation)
                 for education in filteredAdultData['education-num']]
count = 0
# Output the probabilities
for education, prob in zip(filteredAdultData['education-num'], probabilities):
    if prob > 0.60:
        count += 1
```

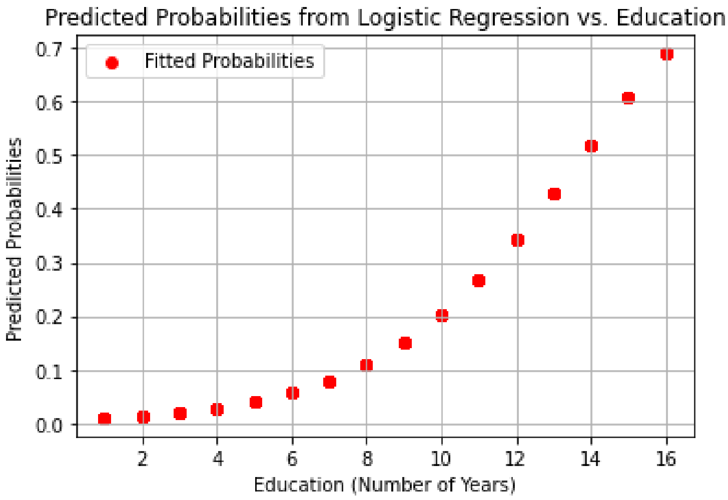
```
print("The number of entries with probability greater than 60% is {}".format(count))
```

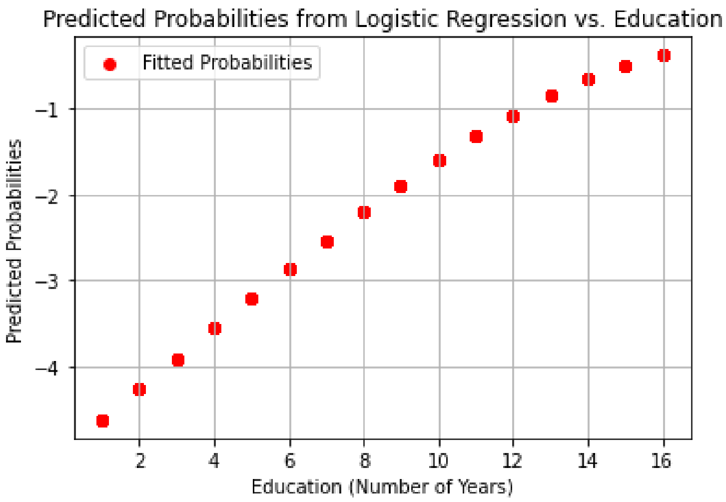
```
The number of entries with probability greater than 60% is 1401
```

**From the data above, the highest level of education results in income > 50K probability more than 60%.**

In [140]:

```
import matplotlib.pyplot as plt

# Calculate the predicted probabilities
predictedProbs = model.predict(X)

# Plot the logistic regression curve
plt.scatter(filteredAdultData['education-num'], predictedProbs, color='red', label='Fitt
ed Probabilities')
plt.xlabel('Education (Number of Years)')
plt.ylabel('Predicted Probabilities')
plt.title('Predicted Probabilities from Logistic Regression vs. Education')
plt.grid(True)
plt.legend()
plt.show()

# Plot the logistic regression curve
plt.scatter(filteredAdultData['education-num'], np.log(predictedProbs), color='red', lab
el='Fitted Probabilities')
plt.xlabel('Education (Number of Years)')
plt.ylabel('Predicted Probabilities')
plt.title('Predicted Probabilities from Logistic Regression vs. Education')
plt.grid(True)
plt.legend()
plt.show()
```



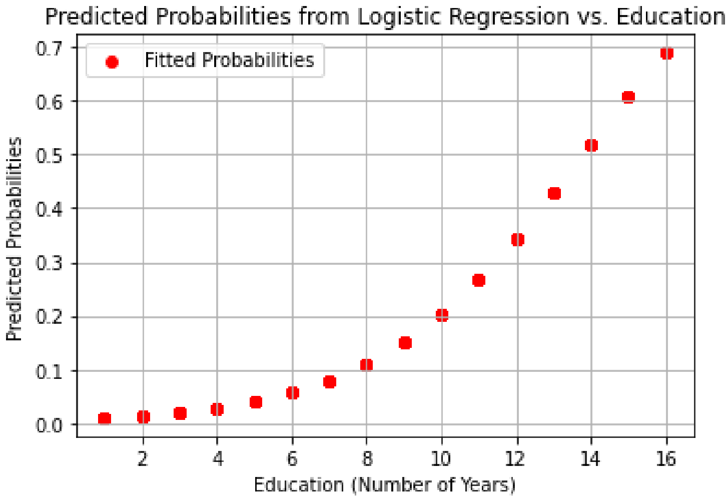



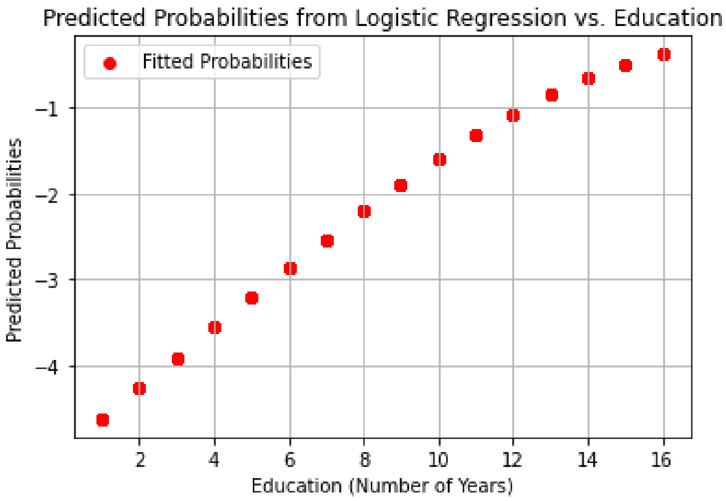

**Since the predicated probabilities was exponential, the predicated probabilities was fitted with log(Y) and it shows the linear relationship. The linear relationship shows there is a strong relationship between higher number of years of education and getting higher incomes.**

**Generalised Models Regression Fitting**

In [144]:

```
# Fit the logistic regression model using glm()
model = sm.GLM(y, X, family=sm.families.Binomial()).fit()

# Calculate the predicted probabilities
predictedProbs = model.predict(X)

# Plot the logistic regression curve
plt.scatter(filteredAdultData['education-num'], predictedProbs, color='red', label='Fitt
ed Probabilities')
plt.xlabel('Education (Number of Years)')
plt.ylabel('Predicted Probabilities')
plt.title('Predicted Probabilities from Logistic Regression vs. Education')
plt.grid(True)
plt.legend()
plt.show()

# Plot the logistic regression curve
plt.scatter(filteredAdultData['education-num'], np.log(predictedProbs), color='red', lab
el='Fitted Probabilities')
plt.xlabel('Education (Number of Years)')
plt.ylabel('Predicted Probabilities')
plt.title('Predicted Probabilities from Logistic Regression vs. Education')
plt.grid(True)
plt.legend()
plt.show()
```




**So, the Generalized Linear Model gives almost the same model fit like the normalized linear models. It shows strong linear relationship. So, conclusion is higher number of years of education results in getting higher incomes.**

**References: Agresti, Alan; Kateri, Maria. Foundations of Statistics for Data Scientists: With R and Python (Chapman & Hall/CRC Texts in Statistical Science) (p. 316). CRC Press. Kindle Edition. OpenAI. (2024, January). ChatGPT [GPT-3.5]. ChatGPT. https://openai.com/chatgpt**