Elements of Artificial Intelligence

1) Let $x \in Z$, $y \in Z$ and $z \in Z$

where $Z = \{, \ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

a) $\forall x, y \ [Z(x) \wedge Z(y) \Rightarrow (x \times y) \Leftrightarrow (y \times x)]$

(i e) Multiplication is commutative.

Addition is commutative.

$\forall x, y \ [Z(x) \wedge Z(y) \Rightarrow (x + y) \Leftrightarrow (y + x)]$

b) Addition is closed in Z.

$\forall x, y \ [Z(x) \wedge Z(y) \Rightarrow Z(x + y)]$

Multiplication is closed in Z.

$\forall x, y \ [Z(x) \wedge Z(y) \Rightarrow Z(x \times y)]$

c) Multiplication distributes over addition.

$\forall x, y, z \ [Z(x) \wedge Z(y) \wedge Z(z) \Rightarrow (x \times (y + z)) \Leftrightarrow (x \times y) + (x \times z)]$

d) Multiplication is associative.

$\forall (x, y, z) \ [Z(x) \wedge Z(y) \wedge Z(z) \Rightarrow ((x \times (y \times z)) \Leftrightarrow ((x \times y) \times z))]$

Addition is associative.

$\forall (x, y, z) \ [Z(x) \wedge Z(y) \wedge Z(z) \Rightarrow ((x + (y + z)) \Leftrightarrow ((x + y) + z))]$

e) Multiplication and Addition have Identity properties

$\forall x \ [Z(x) \wedge Z(0) \Rightarrow (x + 0 \Leftrightarrow x)]$

$\forall x \ [Z(x) \wedge Z(1) \Rightarrow (x \times 1 \Leftrightarrow x)]$

2) $\alpha = \forall x\, (P(x) \vee Q(x))$ and

$\beta = (\forall x\, (P(x)) \vee (\forall x\, (Q(x))$

$\alpha \Rightarrow \beta$ is equivalent to $\neg\alpha \vee \beta$ according to de-morgan's law.

$\alpha = \forall x\, [P(x) \vee Q(x)]$

Applying de morgan's laws, we get

$\neg\alpha \equiv \neg\, [\forall x (P(x) \vee Q(x))]$

$\equiv \exists x\ \neg [P(x) \vee Q(x)]$

$\equiv \exists x\, [\neg P(x) \wedge \neg Q(x)]$

$\beta = (\forall x\, (P(x)) \vee (\forall x\, Q(x))$

$\beta \equiv \forall x\, [P(x)] \vee \forall x\, [Q(x)]$

$\equiv \forall x, y\, [P(x) \vee Q(y)]$

renaming one of the variable to y

$\neg\alpha \vee \beta$ has to hold for all scenarios. Otherwise $\alpha \Rightarrow \beta$ is not true.

$(\alpha \Rightarrow \beta) \equiv \neg\alpha \vee \beta \equiv [\neg P(x) \wedge \neg Q(x)] \vee [P(x) \vee Q(y)]$

Let us have the equation in a truth table.

$P(x)$	$Q(x)$	$Q(y)$	$\neg P(x) \wedge \neg Q(x)$	$P(x) \vee Q(y)$	$\neg\alpha \vee \beta$
T	F	F	F	F	F

In this case, it is invalid.

Hence $\alpha \Rightarrow \beta$ is not true.

3) a) Hans Solo owns the Millenium Falcon.

$$Own(H, M)$$

b) Princess Leia is unhappy

$$Unhappy(L)$$

c) Princess Leia loves Han Solo

$$Loves(L, H)$$

d) $\forall x, [Owns(x, M) \vee Unhappy(x) \Rightarrow Visits(x, O)]$

e) $\forall x, [Visits(x, O) \Rightarrow Wise(x)]$

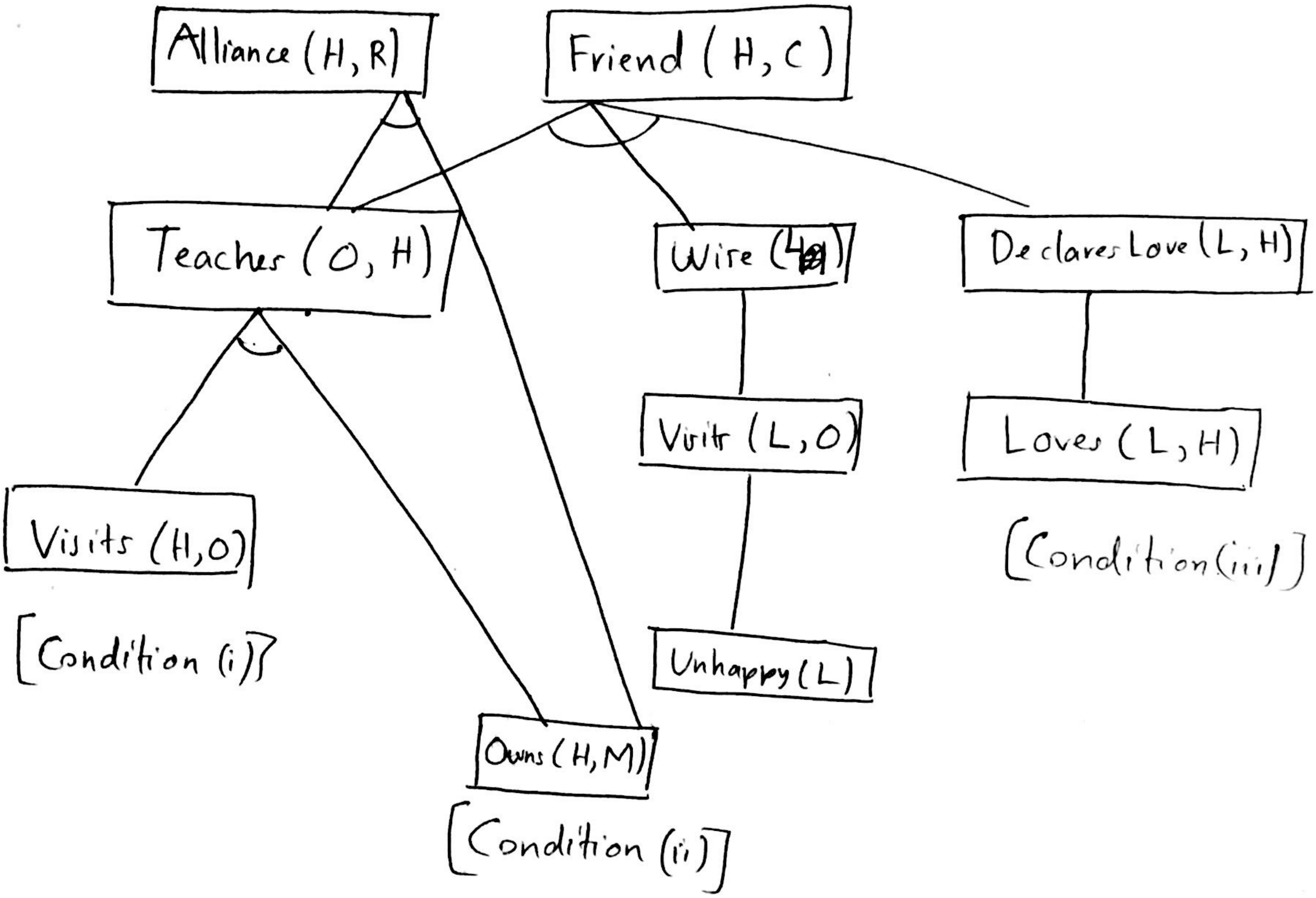
f) $\forall x, [Owns(x, M) \wedge Visits(x, O) \Rightarrow Teaches(O, x)]$

g) $\forall x, [(Unhappy(x) \vee Owns(x, M) \wedge Teaches(O, x)) \Rightarrow Alliance(x, R)]$

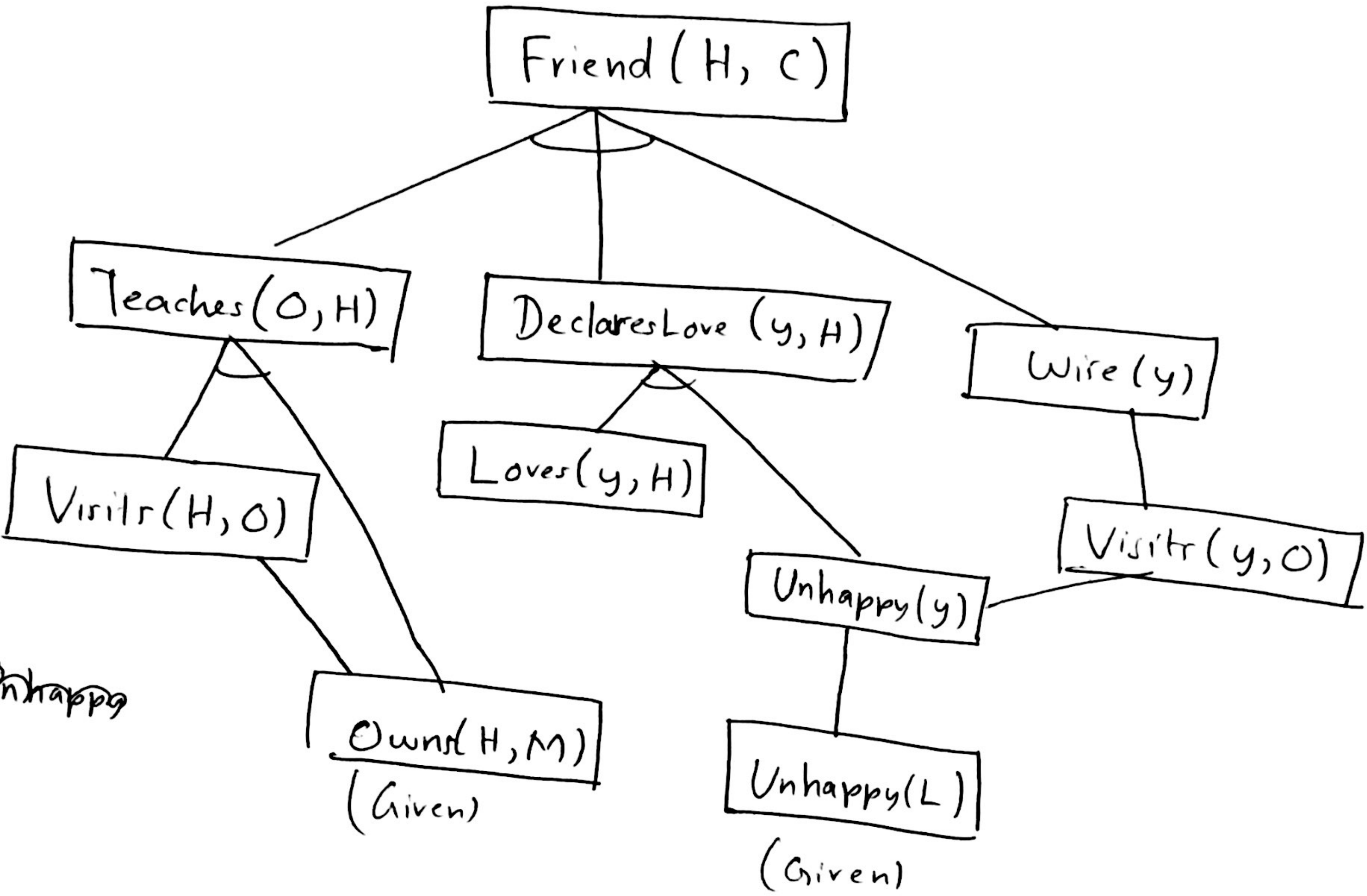
h) $\forall x, y [Unhappy(x) \wedge Loves(x, y) \Rightarrow DeclaresLove(x, y)]$

i) $\forall x, y, [(Teaches(O, x) \wedge DeclaresLove(y, x) \wedge Wise(y)) \Rightarrow Friend(x, C)]$

3) (i) Forward Chaining:-



3) (ii) Backward Chaining :-



Unhappy

4) a) $Able(z) \wedge Willing(z) \Rightarrow Prevents(z)$

b) $\neg Able(z) \wedge Prevents(z) \Rightarrow Impotent(z)$

c) $\neg Willing(z) \wedge Prevent(z) \Rightarrow Malevolent(z)$

d) $\neg Prevents(z)$

e). $Exists(z) \Rightarrow \neg [Impotent(z) \wedge Malevolent(z)]$

$Exists(z) \Rightarrow [\neg Impotent(z) \vee \neg Malevolent(z)]$

~~Able(z)~~

$[A \Rightarrow B \equiv \neg A \vee B]$

a) $(Able(z) \wedge Willing(z)) \Rightarrow Prevents(z)$

$\neg(Able(z) \wedge Willing(z)) \vee Prevents(z)$

$\boxed{\neg Able(z) \vee \neg Willing(z) \vee Prevents(z)}$

b) $(\neg Able(z) \wedge Prevents(z)) \Rightarrow Impotent(z)$.

$\neg(\neg Able(z) \wedge Prevents(z)) \vee Impotent(z)$.

$\boxed{Able(z) \vee (\neg Prevents(z)) \vee Impotent(z)}$

c) $\neg Willing(z) \wedge Prevents(z) \Rightarrow Malevolent(z)$

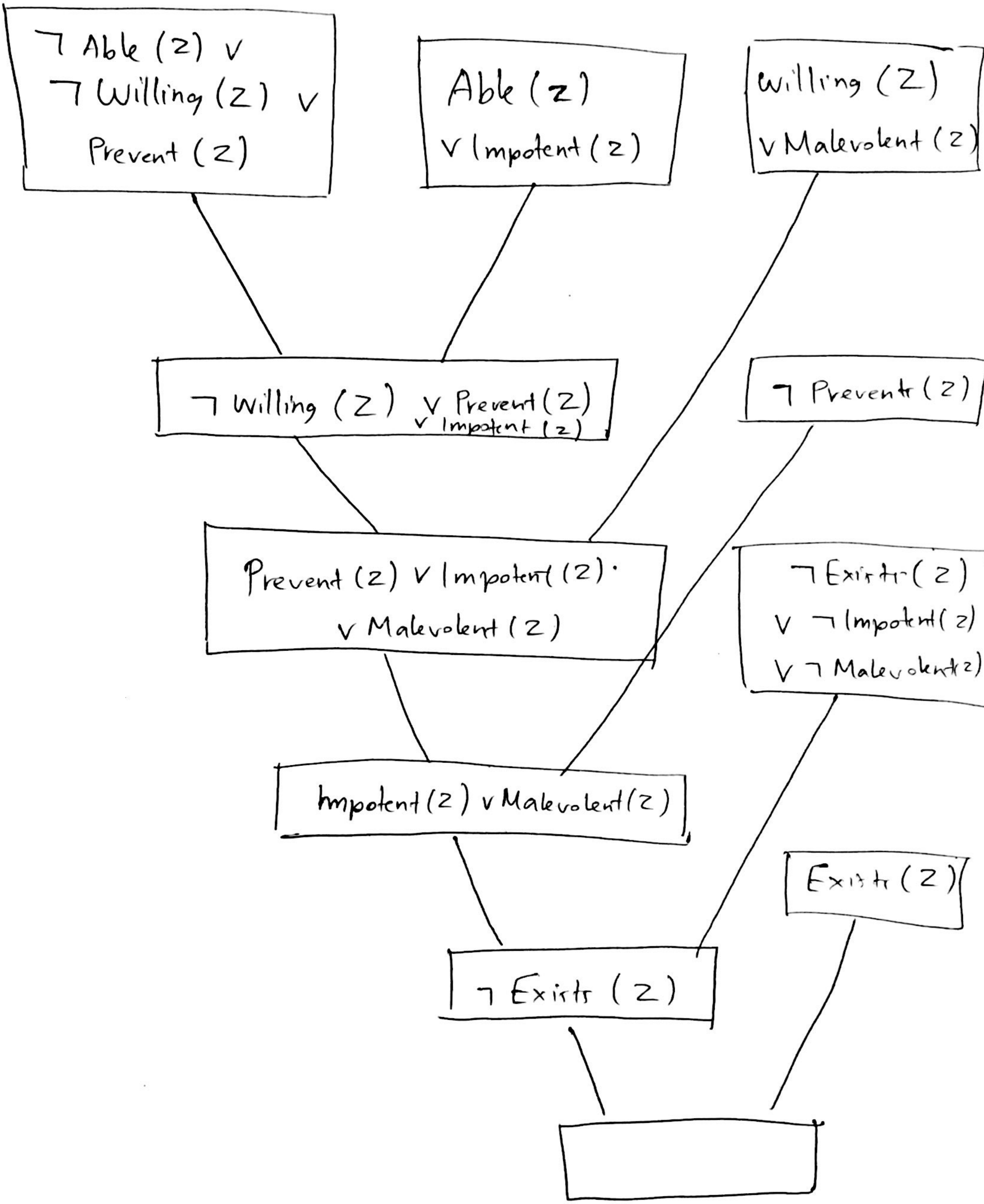
$\neg(\neg Willing(z) \wedge Prevents(z)] \vee Malevolent(z)$

$\boxed{Willing(z) \vee (\neg Prevents(z)) \vee Malevolent(z)}$

d) $\neg Prevents(z)$.

e) $Exists(z) \Rightarrow [\neg Impotent(z) \vee \neg Malevolent(z)]$

$\boxed{\neg(Exists(z)) \vee (\neg Impotent(z)) \vee (\neg Malevolent(z))}$



Hence by resolution, Zeus does not exist.

5 c) $(\forall x\ (P(x) \vee Q(x))) \Rightarrow ((\forall x\ P(x)) \vee (\exists x\ Q(x)))$

Taking negation of the statement, we have.

$$\neg\left(\forall x(P(x) \cancel{\vee} Q(x))) \Rightarrow ((\forall x\ P(x)) \vee (\exists x\ Q(x)))\right)$$

$$\neg\left(\cancel{\forall x}\ \neg(\forall x(P(x) \vee Q(x))) \vee ((\forall x\ P(x)) \vee (\exists x\ Q(x)))\right)$$

$$(\forall x(P(x) \vee Q(x))) \vee (\neg((\forall x\ P(x)) \vee (\exists x\ Q(x))))$$

$$(\forall x\ (P(x) \vee Q(x))) \vee ((\exists x\ \neg P(x)) \wedge (\forall x\ \neg Q(x)))$$

$$\forall x\ (P(x) \vee Q(x)) \vee (\exists x \neg(P(x) \vee Q(x))$$

which is invalid

$\therefore$ Hence by contradiction, we have

$$(\forall x\ (P(x) \vee Q(x))) \Rightarrow ((\forall x\ P(x)) \vee (\exists x\ Q(x)))$$