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ASSIGNMENT-3

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1)

Total number of ways the parakeets can land = 8!

$$\left.\begin{array}{l}\text{Probability that no parakeet of same} \\ \text{color land together}\end{array}\right\} = \frac{4!\,4!\,(2)}{8!} \quad \left[\begin{array}{l}4 \text{ are green} \\ 4 \text{ are blue}\end{array}\right]$$

$$= \frac{(\cancel{4}\times\cancel{3}\times\cancel{2})\times(\cancel{4}\times\cancel{3}\times\cancel{2})\times(\cancel{2})}{\cancel{8}\times 7\times\cancel{6}\times 5\times\cancel{4}\times\cancel{3}\times\cancel{2}\times 1}$$

$$\left.\begin{array}{l}\text{Probability that no parakeet of same} \\ \text{color land together}\end{array}\right\} = 1/35$$

2) $\left.\begin{array}{l}\text{Probability that a given CPU core} \\ \text{has a manufacturing defect}\end{array}\right\} = 0.3$

a) $\left.\begin{array}{l}\text{Probability that a given CPU has} \\ \text{8 functioning cores} \\ \text{(Extreme CPU)}\end{array}\right\} = (1-0.3)^8$

$= (0.7)^8$

$= 0.05764$

b) $\left.\begin{array}{l}\text{Probability that a given CPU} \\ \text{made is Extreme} \\ \text{(8 CPU cores}\end{array}\right\} = (0.7)^8$

$= 0.05764$ (Same as above one)

$\left.\begin{array}{l}\text{Out of 1000 CPUs, no. of Extreme} \\ \text{CPUs}\end{array}\right\} = 1000 \times 0.05764$

$\simeq 58$

Probability that a given CPU is Advanced (≥ 4 CPU cores) $\Big\}$

$$= 8C_4 (0.7)^4 (0.3)^4 + 8C_5 (0.7)^5 (0.3)^3 + 8C_6 (0.7)^6 (0.3)^2 + 8C_7 (0.7)^7 (0.3)^1$$

$$= \frac{\overset{2}{\cancel{8}}\times 7\times\cancel{6}\times 5}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1}(0.2401)(0.0081) + \frac{8\times 7\times\cancel{6}}{\cancel{3}\times\cancel{2}\times 1}(0.168)(0.027) + \frac{\overset{4}{\cancel{8}}\times 7}{\cancel{2}\times 1}(0.1176)(0.09) + 8(0.0823)(0.3)$$

$$= (0.1361) + (0.254) + \cancel{88}(0.2963) + (0.19752)$$

$$= 0.88395$$

Out of 1000 CPUs, no. of Advanced CPUs $\Big\}$ $= 0.883 \times 1000$

$\simeq 884$

Probability that a given CPU is great $\Big\}$

$$= 8C_1 (0.7)^1 (0.3)^7 + 8C_2 (0.7)^2 (0.3)^6 + 8C_3 (0.7)^3 (0.3)^5$$

(Even though a CPU with more than 4 functioning cores can be considered great, it would be a loss for the company to sell it as great)

$$= 8\times(0.7)(0.00021) + 28(0.49)(0.000729) + 56(0.343)(0.00243)$$

$$= 0.001224 + 0.010 + 0.04667$$

$$= 0.05789$$

Probability that a given CPU is great $= 0.05789$

No. of CPU with more than 1 functioning core and less than 4 functioning core in 1000 CPUs $\simeq 58$

c) Revenue for the company $= (58 \times 50) + (58 \times 1000) + (884 \times 100)$

$$= 149300$$

3) Probability that the judge votes guilty when the person is guilty $= 0.7$

$$P(J_1 / G) = 0.7$$

$$P(J_2 / G) = 0.7$$

$$P(J_3 / G) = 0.7$$

Probability that the accused is guilty $= 0.7$

$$P(G) = 0.7$$

Probability that the judge votes guilty when accused is innocent $\Big\} = 0.2$

$$P(J_1 / \bar{G}) = 0.2.$$

$$P(J_2 / \bar{G}) = 0.2$$

$$(P(J_3 / \bar{G})) = 0.2.$$

a) When Judge 1 votes guilty, probability that the person is guilty $\Bigg\} = \dfrac{P(J_1 / G) . P(G)}{P(J_1)}$

$$P(G | J_1) = \frac{P(J_1 / G). P(G)}{P(J_1 / G). P(G) + P(J_1 | \bar{G}). P(\bar{G})}$$

$$= \frac{(0.7)(0.7)}{(0.7)(0.7) + (0.2)(0.3)}$$

$$= \frac{(0.49)}{(0.49 + 0.06)} = 0.89$$

b) Probability that the person is guilty when all 3 judges vote guilty $\Bigg\} = P(G | J_1, J_2, J_3)$

By applying Bayes rule and total chain of probability, we get,

$$P(G | J_1, J_2, J_3) = \frac{P(J_1, J_2, J_3 | G) . P(G)}{P(J_1 | G) P(J_2 | G) P(J_3 | G) P(G) + P(\bar{G}). P(J_1 | \bar{G}) P(J_2 | \bar{G}) P(J_3 | \bar{G})}$$

$$P(G \mid J_1, J_2, J_3) = \frac{P(J_1|G).P(J_2|G).P(J_3|G).P(G)}{P(J_1|G).P(J_2|G).P(J_3|G)P(G) + P(\bar{G}).P(J_1|\bar{G})\,P(J_2|\bar{G}).P(J_3|\bar{G})}$$

$$= \frac{(0.7)\,(0.7)(0.7)(0.7)}{(0.7)(0.7)(0.7)(0.7) + (0.2)(0.2)(0.2)(0.3)}$$

$$= 0.9901$$

c) Probability that the Judge 3 voter guilty when Judge 1 and Judge 2 vote innocent (Conditional independent events)

$$= \cancel{P(J_3|\bar{J})}$$

$$= P(J_3 \mid \bar{J_1}, \bar{J_2})$$

$$= \frac{P(\bar{J_1}, \bar{J_2} \mid J_3) \times P(J_3)}{P(\bar{J_1}, \bar{J_2})}$$

$$= \frac{P(\bar{J_1}|G).P(\bar{J_2}|G).P(G)\;P(J_3|G)}{P(\bar{J_1}, \bar{J_2})}$$

$$P(\bar{J_1}|G).P(\bar{J_2}|G).P(J_3|G).P(G) = (0.3)(0.3)\times(0.7)\times(0.9) + (0.8)(0.8)\times(0.2)(0.3)$$

$$= 0.0825.$$

$$P(\bar{J_1}, \bar{J_2}) = P(\bar{J_1}|G) \times P(\bar{J_2}|G) \times P(G) + P(\bar{J_1}|\bar{G}) \times P(\bar{J_2}|\bar{G}).P(\bar{G})$$

$$= (0.3)(0.3)(0.7) + (0.8)\times(0.8)(0.3)$$

$$= 0.255$$

Probability that Judge 3 votes guilty when Judge 1 and Judge 2 vote innocent $= \dfrac{0.0825}{0.253}$

$= 0.326.$