

Gate Questions

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- 1) The energy levels of a particle of mass m in a potential of the form

[February2007]

$$V(x) = \begin{cases} \infty, & x \leq 0 \\ \frac{1}{2}m\omega^2 x^2, & x > 0 \end{cases}$$

are given, in terms of quantum number $n = 0, 1, 2, 3, \dots$, by

- a) $\left(n + \frac{1}{2}\right) \hbar\omega$
 - b) $\left(2n + \frac{1}{2}\right) \hbar\omega$
 - c) $\left(2n + \frac{3}{2}\right) \hbar\omega$
 - d) $\left(n + \frac{3}{2}\right) \hbar\omega$
- 2) The electromagnetic field due to a point charge must be described by Lienard-Weichert potentials when [February2007]
- a) the point charge is highly accelerated,
 - b) the electric and magnetic fields are not perpendicular.
 - c) the point charge is moving with velocity close to that of light.
 - d) the calculation is done for the radiation zone, i.e far away from the charge.
- 3) The strangeness quantum numbers is conserved in [February2007]
- a) strong, weak and electromagnetic interactions .
 - b) weak and electromagnetic interactions only.
 - c) strong and weak interactions only.
 - d) strong and electromagnetic interactions only.
- 4) The eigenvalues and eigenvectors of the matrix [February2007]

$$\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$$

are

- a) 6, 1 and $\begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
- b) 2, 5 and $\begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
- c) 6, 1 and $\begin{pmatrix} 1 \\ 4 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
- d) 2, 5 and $\begin{pmatrix} 1 \\ 4 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

5) A vector field is defined everywhere as $(F) = -\frac{y^2}{L}\hat{i} + z\hat{k}$. The net flux of (F) associated with a cube of side L , with one vertex at the origin and sides along the positive X , Y , and Z axes, is [February 2007]

- a) $2L^3$
- b) $4L^3$
- c) $8L^3$
- d) $10L^3$

6) If $(r) = x\hat{i} + y\hat{j}$, then [February 2007]

- a) $\nabla \cdot (r) = 0$ and $|\nabla(r)| = \frac{(r)}{r}$
- b) $\nabla \cdot (r) = 2$ and $|\nabla(r)| = \hat{r}$
- c) $\nabla \cdot (r) = 2$ and $|\nabla(r)| = \frac{(r)}{r}$
- d) $\nabla \cdot (r) = 3$ and $|\nabla(r)| = \frac{(r)}{r}$

7) Consider a vector $(p) = 2\hat{i} + 3\hat{j} + 2\hat{k}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$. The axes are rotated anti-clockwise about the Y axis by an angle of 60° . The vector (p) in the rotated coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ is [February 2007]

- a) $(1 - \sqrt{3})\hat{i}' + 3\hat{j}' + (1 + \sqrt{3})\hat{k}'$
- b) $(1 + \sqrt{3})\hat{i}' + 3\hat{j}' + (1 - \sqrt{3})\hat{k}'$
- c) $(1 - \sqrt{3})\hat{i}' + (3 + \sqrt{3})\hat{j}' + 2\hat{k}'$
- d) $(1 - \sqrt{3})\hat{i}' + (3 - \sqrt{3})\hat{j}' + 2\hat{k}'$

8) The contour integral $\oint \frac{dz}{z^4 + a^4}$ is to be evaluated on a circle of radius $2a$ centered at the origin. It will have contributions only from the points [February 2007]

- a) $\frac{1+i}{\sqrt{2}}a$ and $\frac{1-i}{\sqrt{2}}a$
- b) ia and $-ia$
- c) ia , $-ia$, $\frac{-i}{\sqrt{2}}a$ and $\frac{1-i}{\sqrt{2}}a$
- d) $\frac{1+i}{\sqrt{2}}a$, $\frac{1-i}{\sqrt{2}}a$, $\frac{-i}{\sqrt{2}}a$ and $\frac{1-i}{\sqrt{2}}a$

9) Inverse Laplace transform of $\frac{s+1}{s^2-4}$ is [February 2007]

- a) $\cos 2x + \frac{1}{2} \sin 2x$
- b) $\cos x + \frac{1}{2} \sin x$
- c) $\cosh x + \frac{1}{2} \sinh x$
- d) $\cosh 2x + \frac{1}{2} \sinh 2x$

10) The points, where the series solution of the Legendre differential equation [February 2007]

$$(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + \frac{3}{2} \left(\frac{3}{2} + 1 \right) y = 0$$

will diverge, are located at

- a) 0 and 1
- b) 0 and -1
- c) -1 and 1
- d) $\frac{3}{2}$ and $\frac{5}{2}$

- 11) Solution of the differential equation $x \frac{dy}{dx} + y = x^4$, with the boundary condition that $y = 1$ at $x = 1$, is [February2007]

- a) $y = 5x^4 - 4$
- b) $y = \frac{x^5}{5} + \frac{4x}{5}$
- c) $y = \frac{4x^4}{5} + \frac{1}{5x}$
- d) $y = \frac{x^4}{5} + \frac{4}{5x}$

- 12) Match the following [February2007]

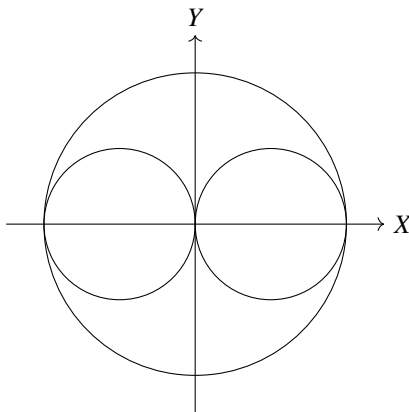
- | | |
|--------------------------|------------------------------------|
| P. rest mass | 1. timelike vector |
| Q. charge | 2. Lorentz invariant |
| R. four-momentum | 3. tensor of rank 2 |
| S. electromagnetic field | 4. conserved and Lorentz invariant |

- a) $P - 2, Q - 4, R - 3, S - 1$
- b) $P - 4, Q - 2, R - 1, S - 3$
- c) $P - 2, Q - 4, R - 1, S - 3$
- d) $P - 4, Q - 2, R - 3, S - 1$

- 13) The moment of inertia of a uniform sphere of radius r about an axis passing through its center is given by [February2007]

$$\frac{2}{5} \left(\frac{4\pi}{3} r^3 \rho \right)$$

A rigid sphere of uniform mass density ρ and radius R has two smaller spheres of radius $R/2$ hollowed out of it, as shown in the figure. The moment of inertia of the resulting body about the Y axis is:



- a) $\frac{\pi \rho R^5}{4}$
- b) $\frac{5\pi \rho R^5}{12}$
- c) $\frac{7\pi \rho R^5}{12}$
- d) $\frac{3\pi \rho R^5}{4}$

- 14) The Lagrangian of a particle of mass m is [February2007]

$$L = \frac{m}{2} \left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 + \left(\frac{dz}{dt} \right)^2 \right] - \frac{V}{2} (x^2 + y^2) + W \sin \omega t$$

where V , W , and ω are constants. The conserved quantities are:

- energy and z-component of linear momentum only.
 - energy and z-component of angular momentum only.
 - z-components of both linear and angular momenta only.
 - energy and z-components of both linear and angular momenta.
- 15) Three particles of mass m , each situated at $x_1(t)$, $x_2(t)$, and $x_3(t)$ respectively, are connected by two springs of spring constant k and un-stretched length ℓ . The system is free to oscillate only in one dimension along the straight line joining all three particles. The Lagrangian of the system is: [February2007]

$$\text{a) } L = \frac{m}{2} \left[\left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dx_2}{dt} \right)^2 + \left(\frac{dx_3}{dt} \right)^2 \right] - \frac{k}{2} (x_1 - x_2 - \ell)^2 - \frac{k}{2} (x_3 - x_2 - \ell)^2$$

$$\text{b) } L = \frac{m}{2} \left[\left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dx_2}{dt} \right)^2 + \left(\frac{dx_3}{dt} \right)^2 \right] - \frac{k}{2} (x_1 - x_3 - \ell)^2 - \frac{k}{2} (x_3 - x_2 - \ell)^2$$

$$\text{c) } L = \frac{m}{2} \left[\left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dx_2}{dt} \right)^2 + \left(\frac{dx_3}{dt} \right)^2 \right] - \frac{k}{2} (x_1 - x_2 + \ell)^2 - \frac{k}{2} (x_3 - x_2 + \ell)^2$$

$$\text{d) } L = \frac{m}{2} \left[\left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dx_2}{dt} \right)^2 + \left(\frac{dx_3}{dt} \right)^2 \right] - \frac{k}{2} (x_1 - x_2 - \ell)^2 - \frac{k}{2} (x_3 - x_2 + \ell)^2$$

- 16) The Hamiltonian of a particle is $H = \frac{p^2}{2m} + pq$, where q is the generalized coordinate and p is the corresponding canonical momentum. The Lagrangian is [February2007]

$$\text{a) } \frac{m}{2} \left(\frac{dq}{dt} + q \right)^2$$

$$\text{b) } \frac{m}{2} \left(\frac{dq}{dt} - q \right)^2$$

$$\text{c) } \frac{m}{2} \left(\frac{dq}{dt} \right)^2 + q \frac{dq}{dt} - q^2$$

$$\text{d) } \frac{m}{2} \left(\frac{dq}{dt} \right)^2 - q \frac{dq}{dt} + q^2$$

- 17) A toroidal coil has N closely-wound turns. Assume the current through the coil to be I and the toroid is filled with a magnetic material of relative permittivity μ_r . The magnitude of magnetic induction (B) inside the toroid, at a radial distance r from the axis, is given by [February2007]

$$\text{a) } \mu_r \mu_0 N I r$$

$$\text{b) } \frac{\mu_r \mu_0 N I}{r}$$

$$\text{c) } \frac{\mu_r \mu_0 N I}{2\pi r}$$

$$\text{d) } \frac{2\pi \mu_r \mu_0 N I r}{r}$$