1.4.9p

EE24BTECH11013-MANIKANTA

Question:
Find the position vector of a point **R** which divides the line joining two points **P** and **Q** whose position vectors are $\mathbf{P} = (2\mathbf{a} + \mathbf{b})$ and $\mathbf{Q} = (\mathbf{a} - 3\mathbf{b})$ externally in the ratio $1 : 2$. Also, show that **P** is the midpoint of the line segment RQ.

Solution:
Let the position vectors of points **P**, **Q**, and **R** be:

$$\mathbf{P} = 2\mathbf{a} + \mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{Q} = \mathbf{a} - 3\mathbf{b} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{0.2}$$

$$\mathbf{R} = \begin{pmatrix} R_x \\ R_y \end{pmatrix} \tag{0.3}$$

Given that divides PQ externally in the ratio $1 : 2$, the position vector of **R** can be found using the section formula for external division:

$$\mathbf{R} = \frac{m\mathbf{Q} - n\mathbf{P}}{m - n} \tag{0.4}$$

where $m : n = 1 : 2$.
Substituting the values:

$$\mathbf{R} = \frac{1 \cdot \mathbf{Q} - 2 \cdot \mathbf{P}}{1 - 2} \tag{0.5}$$

$$\mathbf{R} = \frac{1 \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} - 2 \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{-1} \tag{0.6}$$

$$\mathbf{R} = \frac{\begin{pmatrix} 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix}}{-1} \tag{0.7}$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{0.8}$$

Therefore, the position vector of **R** is:

$$\mathbf{R} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{0.9}$$

Verification that **P** is the midpoint of RQ

The midpoint $\mathbf{M}$ of RQ is given by:

$$\mathbf{M} = \frac{\mathbf{R} + \mathbf{Q}}{2} \tag{0.10}$$

Substituting the known vectors:

$$\mathbf{M} = \frac{\begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix}}{2} \tag{0.11}$$

$$= \frac{\begin{pmatrix} 4 \\ 2 \end{pmatrix}}{2} \tag{0.12}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{0.13}$$

$$= \mathbf{P} \tag{0.14}$$

Hence, $\mathbf{P}$ is the midpoint of the line segment RQ.

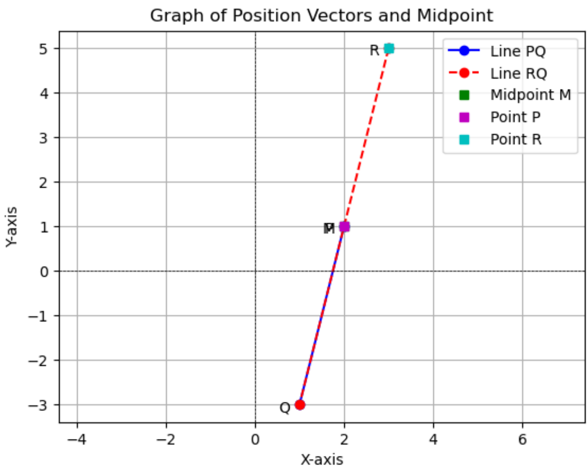

Fig. 0.1: Position Vector Division