Gate Questions

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1) Consider a two-way fixed effects analysis of variance model without interaction effect and one observation per cell. If there are 5 factors and 4 columns, then the degrees of freedom for the error sum of squares is [February 2020]
   a) 20
   b) 19
   c) 18
   d) 17
2) Let $X_1, \dots, X_n$ be a random sample of size $n\,(\geq 2)$ from an exponential distribution with the probability density function [February 2020]
$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & x > 0 \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$
where $\theta \in (1, 2)$. Consider the problem of testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$, based on $X_1, \dots, X_n$. Which of the following statements is TRUE? [February 2020]
   a) Likelihood ratio test at level $\alpha\,(0 < \alpha < 1)$ leads to the same critical region as the corresponding most powerful test at the same level.
   b) Critical region of level $\alpha\,(0 < \alpha < 1)$ likelihood ratio test is $(x_1, \dots, x_n) : \sum_{i=1}^{n} x_i < 0.5x_{(n,1-\alpha)}$ where $x_{(n,1-\alpha)}$ is the $(1-\alpha)$-quantile of the central chi-square distribution with $2n$ degrees of freedom.
   c) Likelihood ratio test for testing H_0 against H_1 does not exist.
   d) At any fixed level $\alpha\,(0 < \alpha < 1)$, the power of the likelihood ratio test is lower than that of the most powerful test.
3) The characteristic function of a random variable X is given by
$$\varphi_X(t) = \begin{cases} \frac{\sin t \cos t}{t}, & \text{for } t \neq 0 \\ 1, & \text{for } t = 0 \end{cases} \tag{3.1}$$
Then $P\left(|X| \leq \frac{3}{4}\right) =$ ______ (correct up to two decimal places). [February 2020]
4) Let the random vector $X = (X_1, X_2, X_3, X_4)^T$ follow $N_4(\mu, \Sigma)$ distribution, where
$$\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{and} \Sigma = \begin{pmatrix} 1 & 0.7 & 0.6 & 0.1 \\ 0.7 & 1 & 0.3 & 0.4 \\ 0.6 & 0.3 & 1 & 0.8 \\ 0.1 & 0.4 & 0.8 & 1 \end{pmatrix}. \tag{4.1}$$
Then
$P(X_1 + X_2 + X_3 + X_4 > 0) =$ ___ (correct up to two decimal places). [February 2020]

5) Let $(X_n)_{n\geq 0}$ be a homogeneous Markov chain with state space $(0, 1)$ and one-step transition probability matrix $P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$. If $P(X_0 = 0) = \frac{1}{3}$, then

$$27 \times E(X_2) = \underline{\hspace{4cm}} \text{(correct up to two decimal place.)} \tag{5.1}$$

[February 2020]

6) Let E, F and G be mutually independent events with $P(E) = \frac{1}{2}$,$P(F) = \frac{1}{3}$, and $P(G) = \frac{1}{4}$. Let p be the probability that at least two of the events among E, F, and G occur.Then $12 \times p =$ ______ [February 2020]

7) Let the joint probability mass function of (X, Y, Z) be

$$P(X = x, Y = y, Z = z) = \frac{10!}{x!y!z!}(0.2)^x (0.3)^y (0.4)^z (0.1)^t \tag{7.1}$$

where $t = 10 - x - y - z$, $x, y, z = 0, 1, \ldots, 10$; $x + y + z \leq 10$. Then the variance of the random variable $Y + 2Z$ equals _ (correct up to two decimal places). [February 2020]

8) The total number of standard 4×4 Latin squares is ______. [February 2020]

9) Let X be a 4×1 random vector with $E(X) = 0$ and variance-covariance matrix

$$\Sigma = \begin{pmatrix} 1 & 0.5 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 1 \end{pmatrix}. \tag{9.1}$$

Let Y be the 4×1 random vector of principal components derived from Σ. The proportion of total variation explained by the first two principal components equals ________________ (correct up to two decimal places). [February 2020]

10) Let $X_1, \ldots, X_n$ be a random sample of size $n \geq 2$ from an exponential distribution with the probability density function

$$f(x; \theta) = \begin{cases} \theta e^{-\theta x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{10.1}$$

where $\theta \in (0, \infty)$. If $X_{(2)} = \min X_2, \ldots, X_n$, then the conditional expectation

$$E\left(\frac{X_{(2)}}{\theta} = 1 | X_1 = x, X_2 = z\right) = \underline{\hspace{4cm}}. \tag{10.2}$$

[February 2020]

11) Let $Y_i = a + bx_i + \epsilon_i, i = 1, 2, \ldots, 7$, where x_i are fixed covariates and ϵ_i are independent and identically distributed random variables with mean zero and finite variance. Suppose that a and b are the least squares estimators of a and b, respectively. Given the following data:

$$\sum_{i=1}^{7} x_i = 0, \sum_{i=1}^{7} x_i^2 = 28, \sum_{i=1}^{7} x_i y_i = 28, \sum_{i=1}^{7} y_i = 21 \text{and} \sum_{i=1}^{7} y_i^2 = 91, \tag{11.1}$$

where y_i is the observed value of $Y_i, i = 1, \ldots, 7$. Then the correlation coefficient between a and b equals ________________. [February 2020]

12) Let $0, 1, 2, 3$ be an observed sample of size 4 from $N(\theta, 5)$ distribution, where $\theta \in [2, \infty)$. Then the maximum likelihood estimate of θ based on the observed sample is ________________. [February 2020]

13) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x, y) = x^4 - 2x^2y + 16y + 17, \tag{13.1}$$

where $\mathbb{R}$ denotes the set of all real numbers. Then [February 2020]

a) f has a local minimum at $(2, 3)$
b) f has a local maximum at $(2, 3)$
c) f has a saddle point at $\left(2, \frac{4}{3}\right)$
d) f is bounded