Gate Questions

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1) The number of subgroups of a cyclic group of order 12 is ______ [February 2022]
2) The radius of convergence of the series
$$\sum_{n\geq 0} 3^{n+1} z^{2n}, z \in \mathbb{C} \tag{2.1}$$
[February 2022]
3) The number of zeroes of the polynomial
$$2z^7 - 7z^5 + 2z^3 - z + 1 \tag{3.1}$$
in the unit disc $\{z \in \mathbb{C} : |z| < 1\}$ is ___________ [February 2022]
4) If $P(x)$ is a polynomial of degree 5 and
$$\alpha = \sum_{i=0}^{6} P(x_i) \left(\prod_{\substack{j=0 \\ j\neq i}}^{6} \left(x_i - x_j \right)^{-1} \right), \tag{4.1}$$
where $x_0, x_1, \ldots, x_6$ are distinct points in the interval $[2, 3]$, then the value of $\alpha^2 - \alpha + 1$ is ___________. [February 2022]
5) If the function $f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$, $x \neq 0, y \neq 0$ attains its local minimum value at the point (a, b), then the value of $a^3 + b^3$ is ___________. (round off to two decimal places). [February 2022]
6) The maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$ is___________. [February 2022]
7) If the ordinary differential equation
$$x^2 \frac{d^2\phi}{dx^2} + x\frac{d\phi}{dx} + x^2\phi = 0, x > 0 \tag{7.1}$$
has a solution of the form $\phi(x) = x^r \sum_{n=0}^{\infty} a_n x^n$, where a_n's are constants and $a_0 \neq 0$, then the value of $r^2 + 1$ is ___________. [February 2022]
8) The Bessel functions $J_\alpha(x)$, $x > 0$, $\alpha \in \mathbb{R}$ satisfy $J_{\alpha-1}(x) + J_{\alpha+1}(x) = \frac{2\alpha}{x} J_\alpha(x)$. Then, the value of $\left(\pi J_{\frac{3}{2}}(\pi)\right)^2$ is ___________. [February 2022]
9) The partial differential equation
$$7\frac{\partial^2 u}{\partial x^2} + 16\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0 \tag{9.1}$$
is transformed to
$$A\frac{\partial^2 u}{\partial \xi^2} + B\frac{\partial^2 u}{\partial \xi \partial \eta} + C\frac{\partial^2 u}{\partial \eta^2} = 0, \tag{9.2}$$

using $\xi = y - 2x$ and $\eta = 7y - 2x$. Then, the value of $\frac{1}{12^3}\left(B^2 - 4AC\right)$ is ___________. [February 2022]

10) Let $\mathbb{R}[X]$ denote the ring of polynomials in X with real coefficients. Then, the quotient ring $\mathbb{R}[X] / \left(X^4 + 4\right)$ is [February 2022]

a) a field
b) an integral domain, but not a field
c) not an integral domain, but also has 0 as the the only nilpotent element
d) a ring which contains non zero nilpotent elements

11) Consider the following conditions on two proper non-zero ideals J_1 and J_2 of a non-zero commutative ring R.

P: For any $r_1, r_2 \in R$, there exists a unique $r \in R$ such that $r - r_1 \in J_1$ and $r - r_2 \in J_2$.

Q: $J_1 + J_2 = R$ Then, which of the following statements is TRUE? [February 2022]

a) **P** implies **Q** does not imply **P**
b) **Q** implies **P** but **P** does not imply **Q**
c) **P** implies **Q** and **Q** implies **P**
d) **P** does not imply **Q** and **Q** does not imply **P**

12) **P**: Suppose that $\sum_{n=0}^{\infty} a_n x^n$ converges at $x = -3$ and diverges at $x = 6$. Then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges.

Q: The interval of convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n \log_e n}$ is $[-4, 4]$.

Which of the following statements is TRUE? [February 2022]

a) **P** is true and **Q** is true
b) **P** is false and **Q** is false
c) **P** is true and **Q** is false
d) **P** is false and **Q** is true

13) Let $f : [-\pi, \pi] \to \mathbb{R}$ be a continuous function such that $f(x) > \frac{f(0)}{2}$, $|x| < \delta$ for some δ satisfying $0 < \delta < \pi$. Define $P_{n,\delta}(x) = (1 + \cos x - \cos\delta)^n$, for $n = 1, 2, 3, \ldots$ Then, which of the following statements is TRUE? [February 2022]

a) $\lim_{n\to\infty} \int_0^{2\delta} f(x)\, P_{n,\delta}(x)\, dx = 0$
b) $\lim_{n\to\infty} \int_{-2\delta}^{0} f(x)\, P_{n,\delta}(x)\, dx = 0$
c) $\lim_{n\to\infty} \int_{-\delta}^{\delta} f(x)\, P_{n,\delta}(x)\, dx = 0$
d) $\lim_{n\to\infty} \int_{[-\pi,\pi]\setminus[-\delta,\delta]} f(x)\, P_{n,\delta}(x)\, dx = 0$