Practical - 6

Solution of Cauchy problem for first order PDE

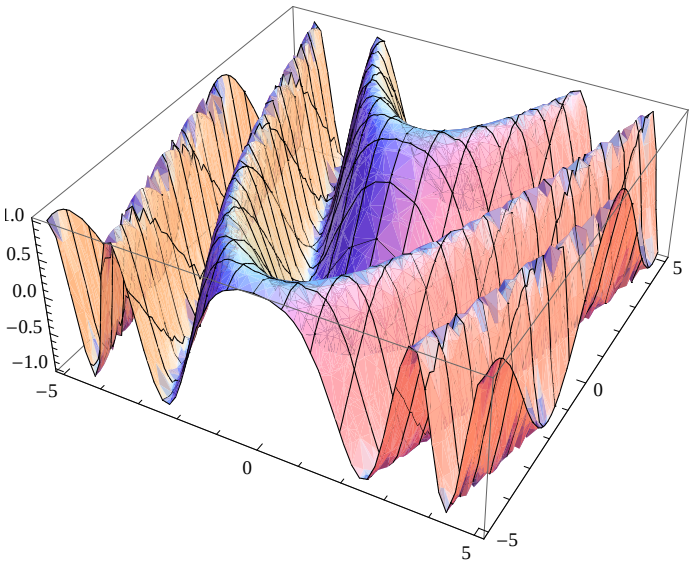
Question-1

```
eqn1 = D[u[x, y], x] + x * D[u[x, y], y] == 0
```

$$x\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$$

```
sol1 =
 u[x, y] /. DSolve[{eqn1, u[0, y] == Sin[y]}, u[x, y], {x, y}]
Plot3D[sol1, {x, -5, 5}, {y, -5, 5}]
```

$$\left\{\text{Sin}\left[\frac{1}{2}\left(-x^2 + 2\,y\right)\right]\right\}$$



Question-2

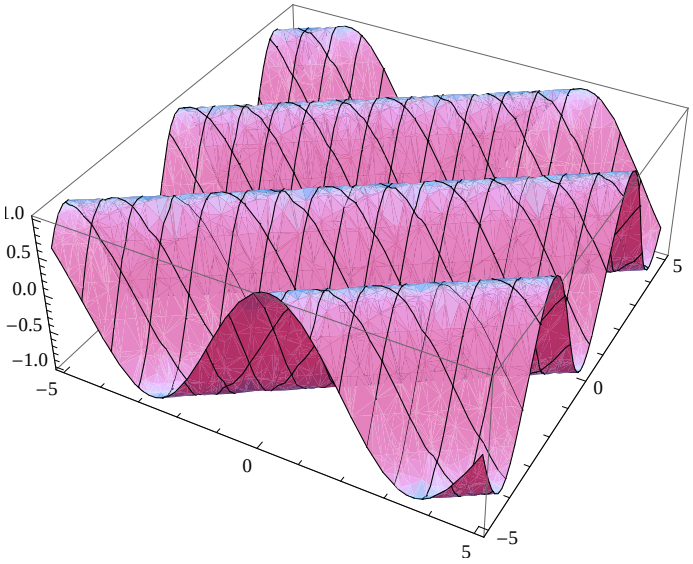
```
eqn2 = 3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0
```

$$2\, u^{(0,1)}[x, y] + 3\, u^{(1,0)}[x, y] == 0$$

```
sol2 =
 u[x, y] /. DSolve[{eqn2, u[x, 0] == Sin[x]}, u[x, y], {x, y}]
```

$$\left\{\text{Sin}\left[\frac{1}{2}\left(2\,x - 3\,y\right)\right]\right\}$$

```
Plot3D[sol2, {x, -5, 5}, {y, -5, 5}]
```



Question-3

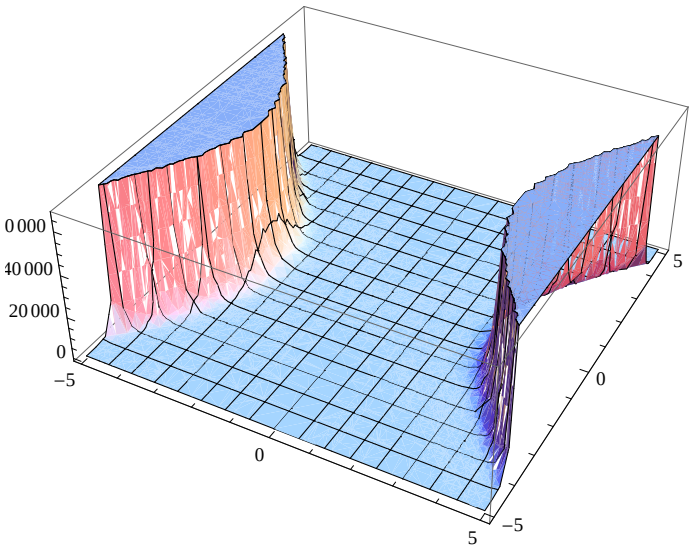
```
eqn3 = y * D[u[x, y], x] + x * D[u[x, y], y] == 0
```

$$x\, u^{(0,1)}[x, y] + y\, u^{(1,0)}[x, y] == 0$$

```
sol3 = u[x, y] /.
  DSolve[{eqn3, u[0, y] == Exp[-y^2]}, u[x, y], {x, y}]
```

$$\left\{e^{x^2-y^2}\right\}$$

```
Plot3D[sol3, {x, -5, 5}, {y, -5, 5}]
```



Question 4 : ∂xu[x,y] + 2*∂yu[x,y] = 1 + u[x, y] u[x, y] = Sin[x] on y = 3*x + 1;

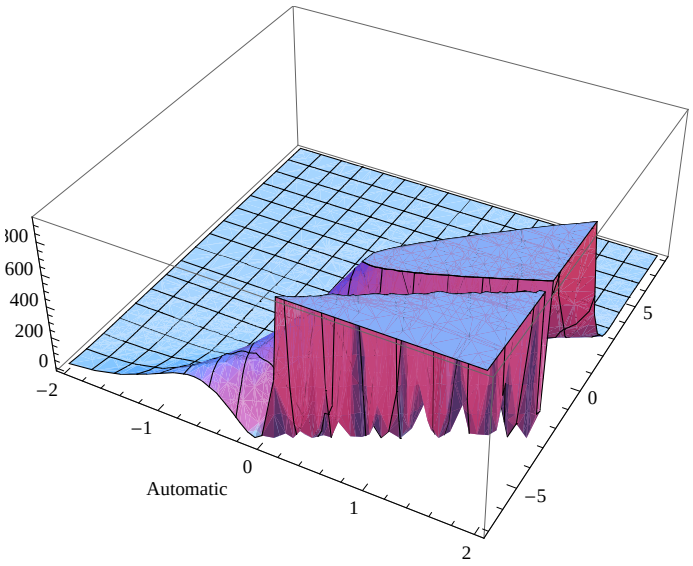
```
A = D[u[x, y], x] + 2 * D[u[x, y], y] == 1 + u[x, y]
```

$$2\,u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1 + u[x, y]$$

```
sol = DSolve[{A, u[x, 3 * x + 1] == Sin[x]}, u[x, y], {x, y}]
```

$$\left\{\left\{u[x, y] \to -e^{-y}\left(-e^{1+3x} + e^{y} + e^{1+3x}\,\text{Sin}[1 + 2x - y]\right)\right\}\right\}$$

```
Plot3D[u[x, y] /. sol, {x, -2, 2},
 {y, -7, 8}, AxesLabel → {Automatic}]
```



Question 5. Solve the PDE Subscript[uu, x]+ Subscript-t[u, y]=1/2. With the initial condition u(s,s)=s/4,0<=s<=1.

Solution: x=s+st/4=t^2/4, y=s+t, u=s/4+t/2

```
sol = DSolve[{x'[t] == u[t], y'[t] == 1, u'[t] == 1 / 2,
    x[0] == s, y[0] == s, u[0] == s / 4}, {x[t], y[t], u[t]}, t]
```

$$\left\{\left\{u[t] \to \frac{1}{4}(s + 2t),\ x[t] \to \frac{1}{4}\left(4s + st + t^2\right),\ y[t] \to s + t\right\}\right\}$$

```
Print["u[t]=", sol[[1, 1, 2]]]
```

$$u[t] = \frac{1}{4}(s + 2t)$$

```
Print["y[t]=", sol[[1, 2, 2]]]
```

$$y[t] = \frac{1}{4}\left(4s + st + t^2\right)$$

```
Print["x[t]=", sol[[1, 3, 2]]]
```

$$x[t] = s + t$$

```
map = ParametricPlot3D[
  {sol[[1, 1, 2]], sol[[1, 2, 2]], sol[[1, 3, 2]]},
  {t, -1, 1}, {s, 0, 1}, PlotPoints → 10]
```