Practical-8

Plot the integral surfaces of a given first order PDE with initial data.

Question 1. Solve the PDE Subscript[uu, x]+ Subscript[u, y]=1/2. With the initial
condition u(s,s)=s/4,
0<=s<=1.
Solution: x=s+st/4+t^2/4, y=s+t, u=s/4+t/2.

```
sol = DSolve[{x'[t] == u[t], y'[t] == 1, u'[t] == 1/2,
    x[0] == s, y[0] == s, u[0] == s/4}, {x[t], y[t], u[t]}, t]
```

$$\left\{\left\{u[t] \to \frac{1}{4}\,(s+2\,t),\ x[t] \to \frac{1}{4}\left(4\,s+s\,t+t^2\right),\ y[t] \to s+t\right\}\right\}$$

```
Print["u[t]=", sol[[1, 1, 2]]]
```

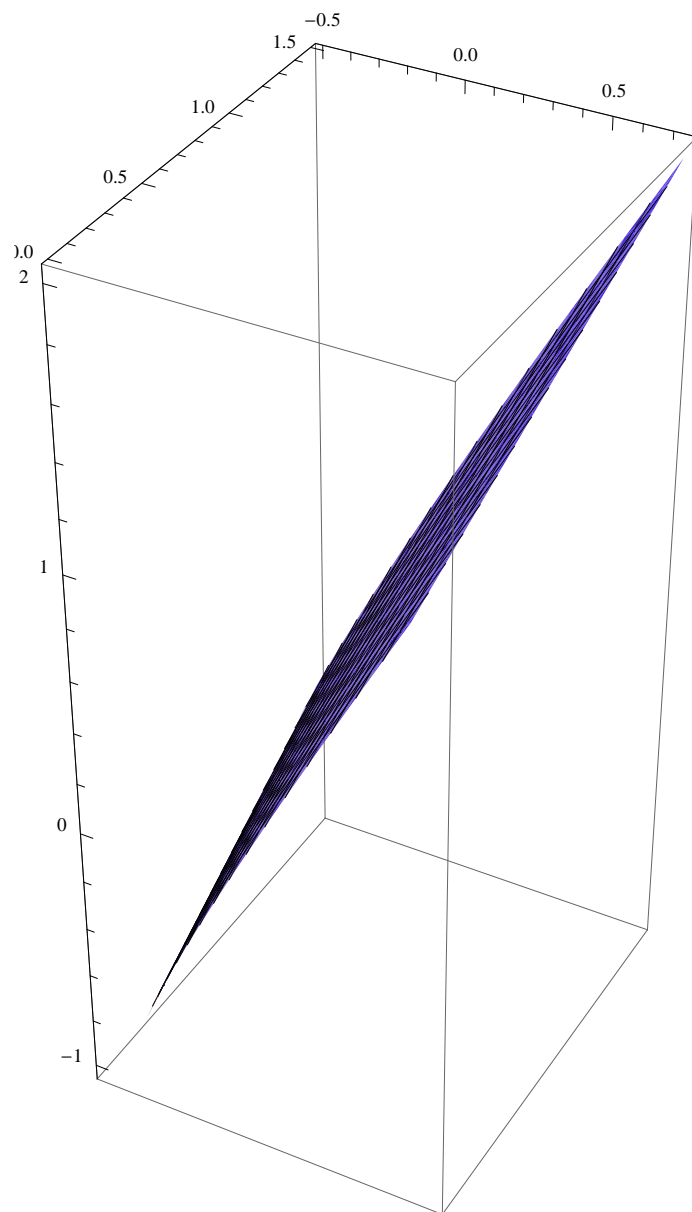$$u[t] = \frac{1}{4}\,(s+2\,t)$$

```
Print["y[t]=", sol[[1, 2, 2]]]
```
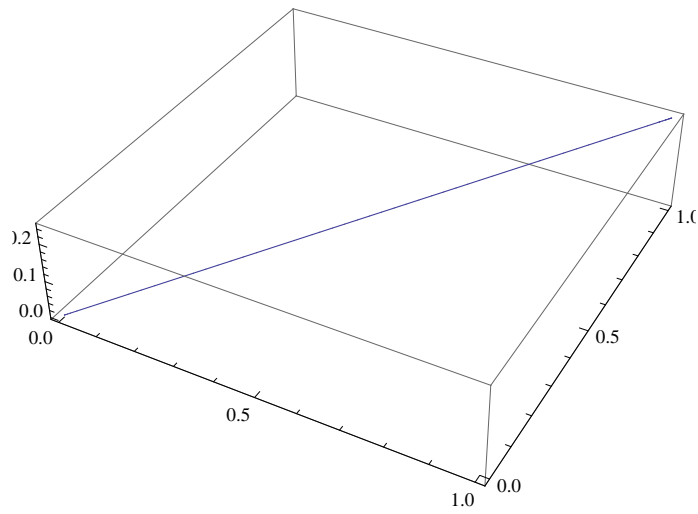
$$y[t] = \frac{1}{4}\left(4\,s+s\,t+t^2\right)$$

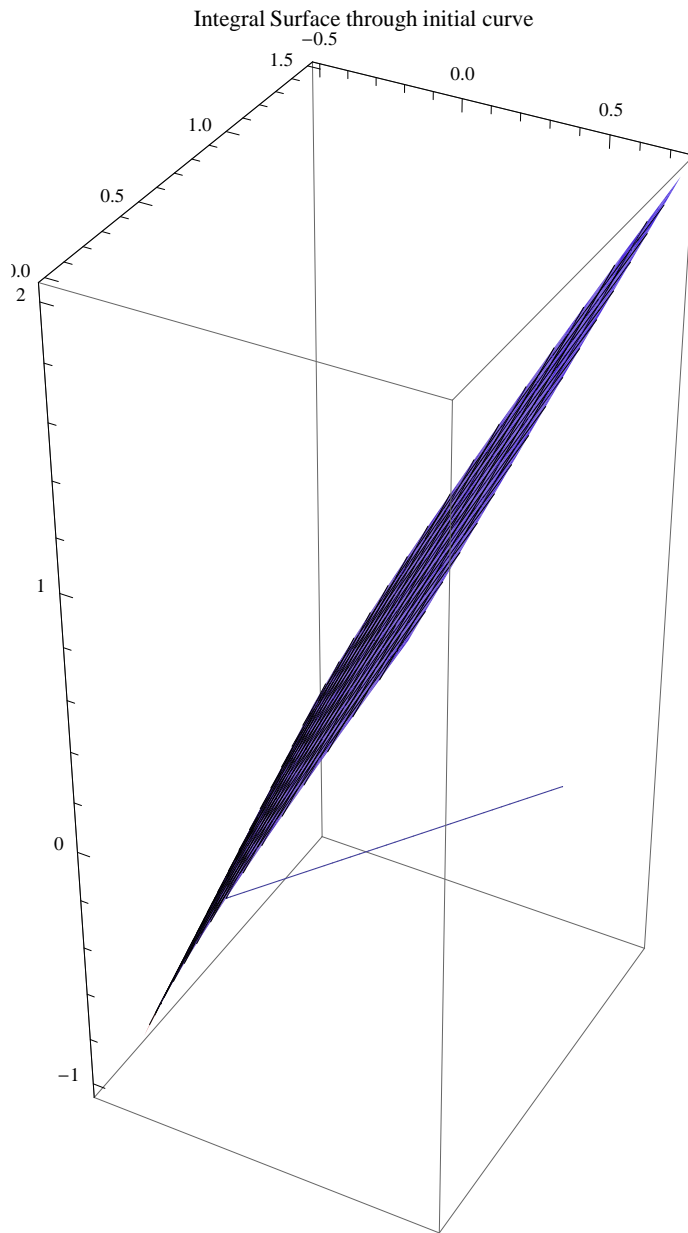```
Print["x[t]=", sol[[1, 3, 2]]]
```

$$x[t] = s+t$$

```
map = ParametricPlot3D[
  {sol[[1, 1, 2]], sol[[1, 2, 2]], sol[[1, 3, 2]]},
  {t, -1, 1}, {s, 0, 1}, PlotPoints -> 10]
```

map1 = ParametricPlot3D[{s, s, s / 4}, {s, 0, 1}]

```
Show[map, map1,
  PlotLabel -> "Integral Surface through initial curve "]
```


Integral Surface through initial curve

Question 2. Solve the PDE Subscript[u, x]- Subscript[u, y]=2. With the initial condition u(s,s)=2s,
0<=s<=1.
Solution: x=s + t, y= s - t, u=2t + 2s.

```
sol = DSolve[{x'[t] == 1, y'[t] == -1, u'[t] == 2, x[0] == s,
    y[0] == s, u[0] == 2 * s}, {x[t], y[t], u[t]}, t]
```
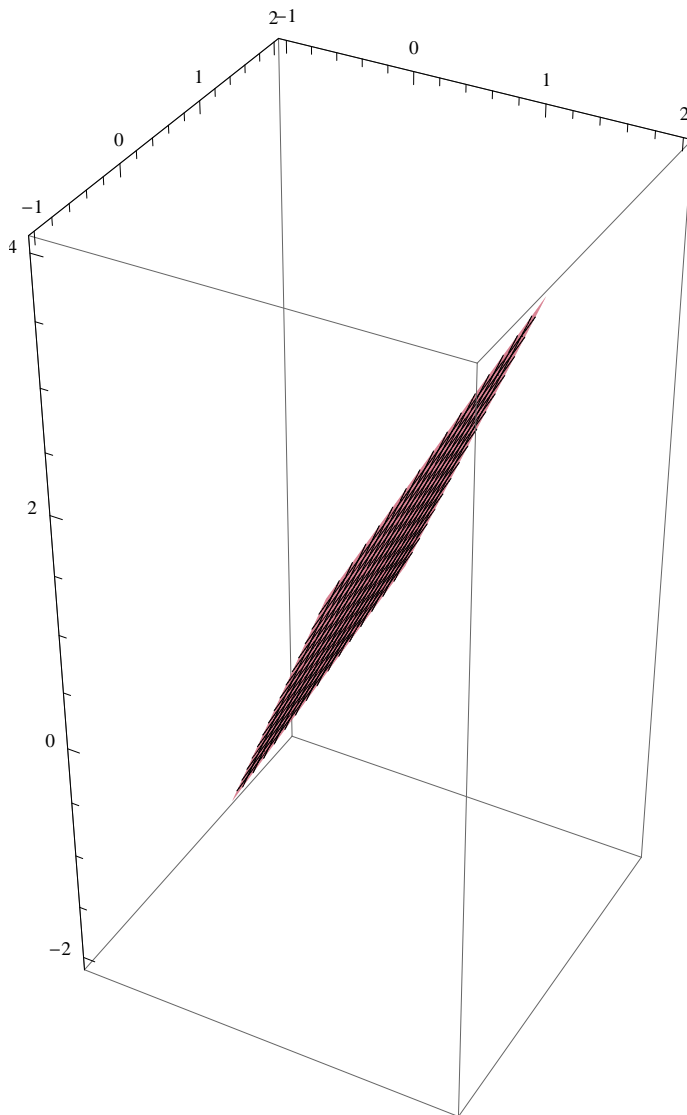
$\{\{x[t] \to s + t, y[t] \to s - t, u[t] \to 2 \, (s + t)\}\}$

```
Print["x[t]=", sol[[1, 1, 2]]]
```
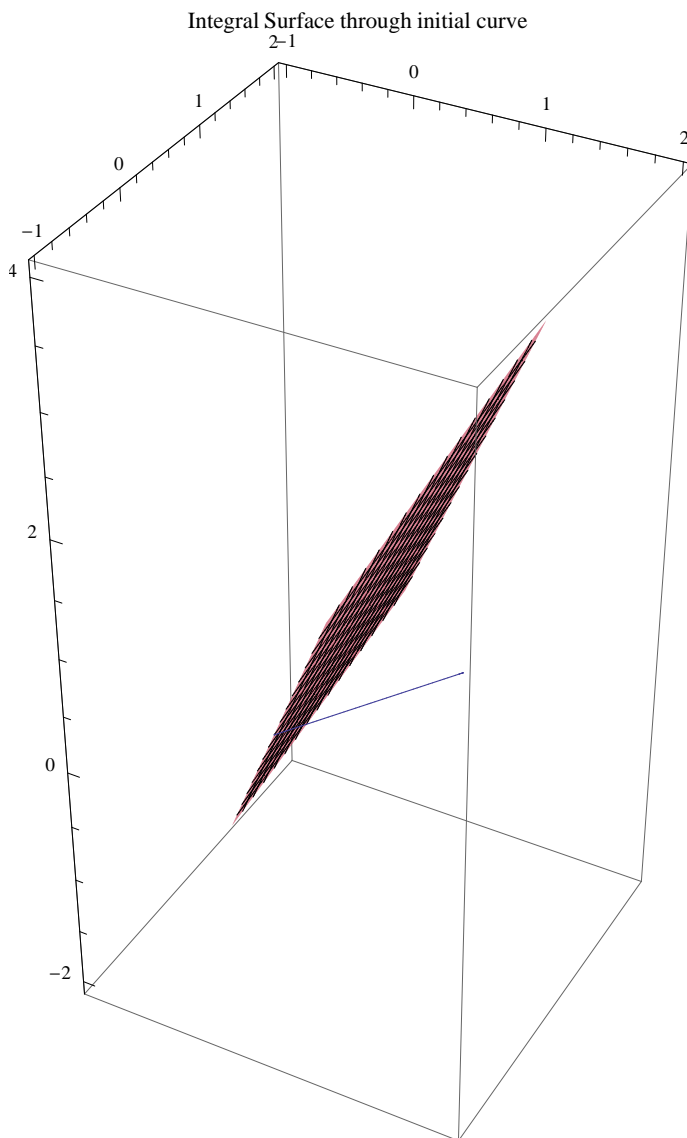
$x[t] = s + t$

```
Print["y[t]=", sol[1, 2, 2]]
```

$y[t] = s - t$

```
Print["u[t]=", sol[1, 3, 2]]
```

$u[t] = 2 (s + t)$

```
map = ParametricPlot3D[
   {sol[1, 1, 2], sol[1, 2, 2], sol[1, 3, 2]},
   {t, -1, 1}, {s, 0, 1}, PlotPoints -> 100]
```

```
Show[map, map1,
 PlotLabel -> "Integral Surface through initial curve "]
```

Integral Surface through initial curve

Question 3. Solve the PDE Subscript[u, x]+ Subscript[u, y]=1. With the initial condition u(s,s)=sin(s),
0<=s<=1.
Solution: x=s + t, y= s + t, u=t + sin(s).

```
sol = DSolve[{x'[t] == 1, y'[t] == 1, u'[t] == 1, x[0] == s,
    y[0] == s, u[0] == Sin[s]}, {x[t], y[t], u[t]}, t]
```

$\{\{x[t] \rightarrow s + t, y[t] \rightarrow s + t, u[t] \rightarrow t + \text{Sin}[s]\}\}$
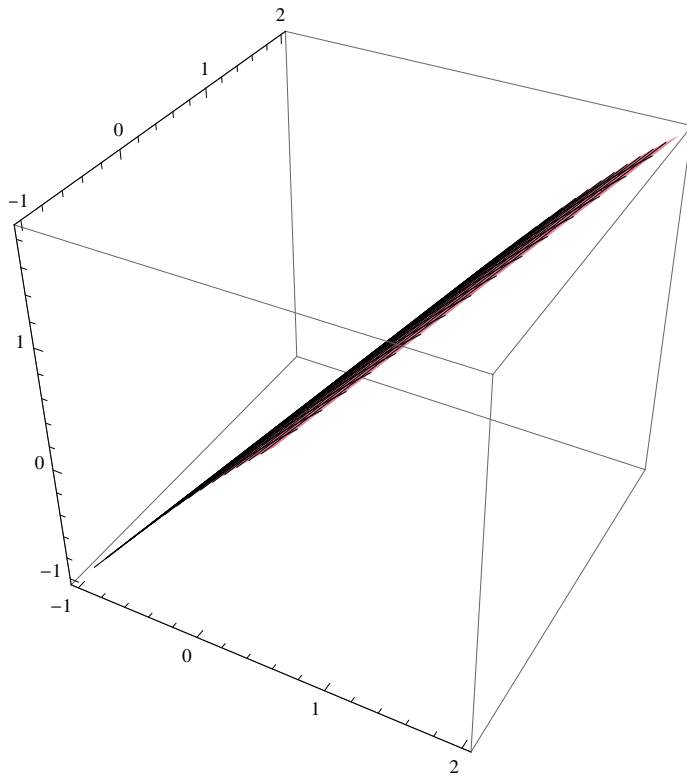
```
Print["u[t]=", sol[[1, 3, 2]]]
```

$u[t]=t + \text{Sin}[s]$
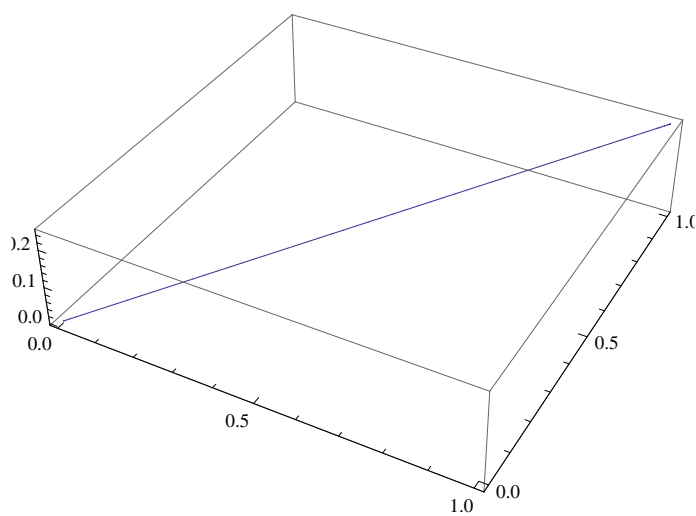
```
Print["x[t]=", sol[1, 1, 2]]
```

$x[t] = s + t$
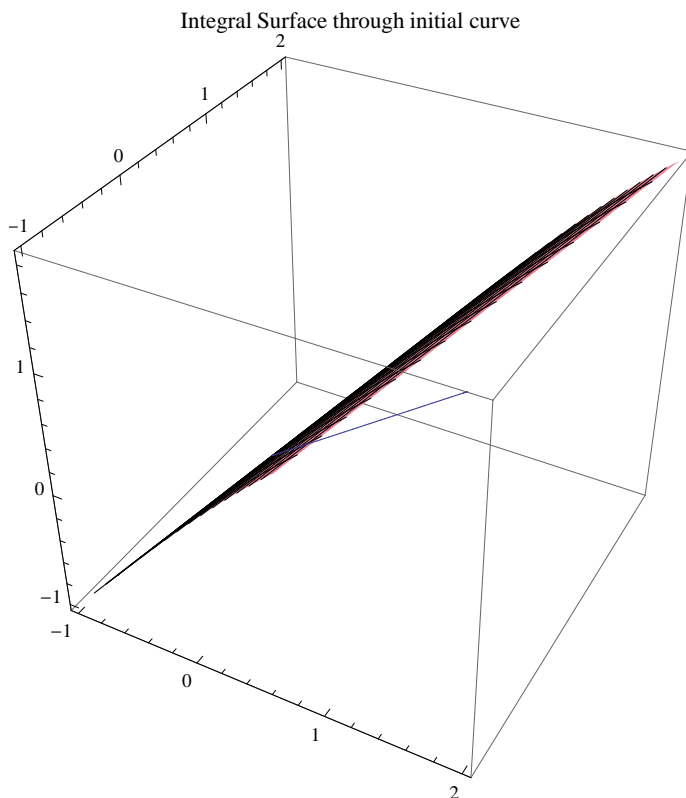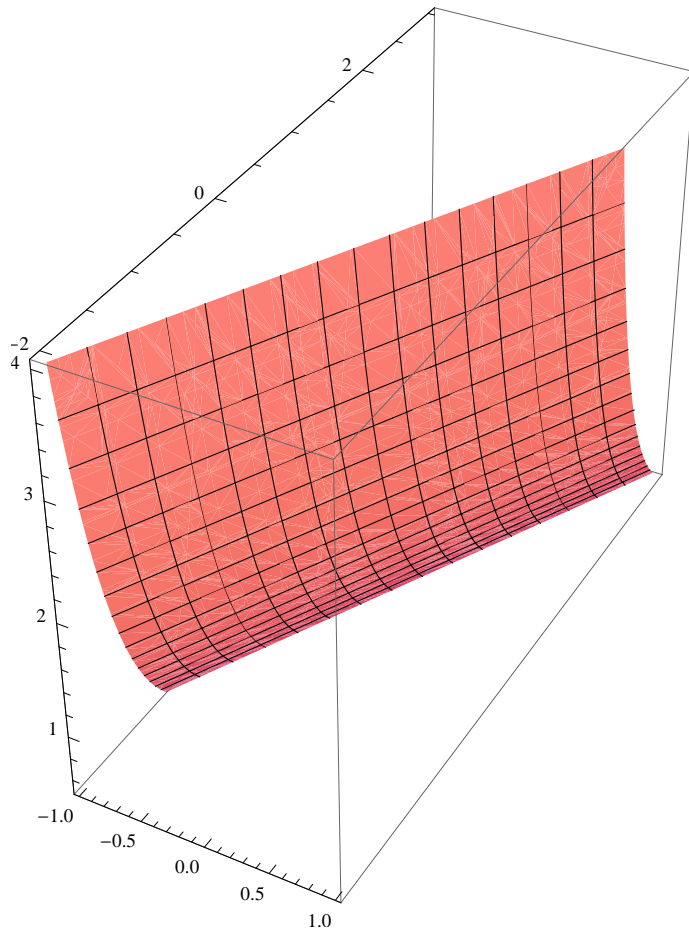
```
Print["y[t]=", sol[1, 2, 2]]
```
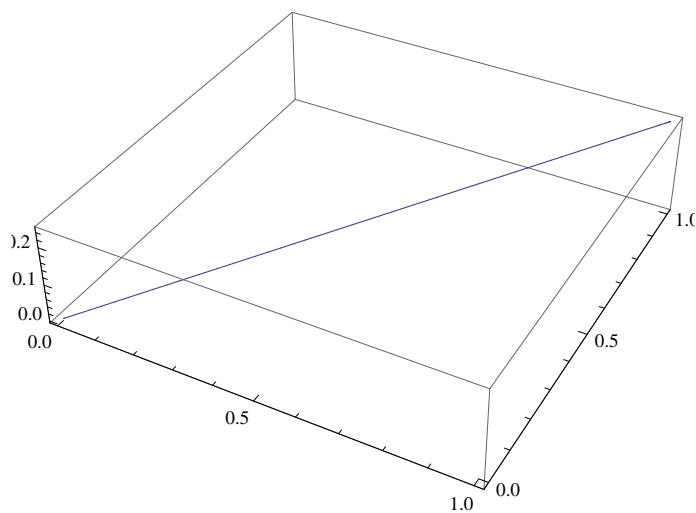
$y[t] = s + t$

```
map = ParametricPlot3D[{sol[1, 1, 2],
    sol[1, 2, 2], sol[1, 3, 2]}, {t, -1, 1}, {s, 0, 1}]
```



```
map1 = ParametricPlot3D[{s, s, s / 4}, {s, 0, 1}]
```

```
Show[map, map1,
 PlotLabel -> "Integral Surface through initial curve "]
```

Integral Surface through initial curve

Question 4. Solve the PDE Subscript[u, x]+ 2Subscript[u, y]=0. With the initial
condition u(0,s)=4E^(-2s),
0<=s<=1.
Solution: u=4E^(-2s), y= s + 2t, x=t.

```
sol = DSolve[{x'[t] == 1, y'[t] == 2, u'[t] == 0, x[0] == 0,
   y[0] == s, u[0] == 4 * E ^ (-2 * s)}, {x[t], y[t], u[t]}, t]
```

$$\left\{\left\{x[t] \to t, \ y[t] \to s + 2\,t, \ u[t] \to 4\,e^{-2\,s}\right\}\right\}$$

```
Print["u[t]=", sol[[1, 3, 2]]]
```

$u[t] = 4\,e^{-2\,s}$

```
Print["x[t]=", sol[[1, 1, 2]]]
```

$x[t] = t$

```
Print["y[t]=", sol[[1, 2, 2]]]
```

$y[t] = s + 2\,t$

map = ParametricPlot3D[{sol〚1, 1, 2〛,
sol〚1, 2, 2〛, sol〚1, 3, 2〛}, {t, -1, 1}, {s, 0, 1}]

map1 = ParametricPlot3D[{s, s, s / 4}, {s, 0, 1}]

```
Show[map, map1,
  PlotLabel -> "Integral Surface through initial curve "]
```


Integral Surface through initial curve