

# Practical-5

## Solution of systems of ordinary differential equations

**DSolve[{eqn1,eqn2,...},{Subscript[y, 1][x],Subscript[y, 2][x] ,...},x] solve a system of differential equations for Subscript[y, i][x]**

**Question 1: find the general solution of the following linear system**

$$2 \frac{dx}{dt} - 2 \frac{dy}{dt} - 3x = t, \quad 2 \frac{dx}{dt} + 2 \frac{dy}{dt} + 3x + 8y = 2.$$

**Solution:**

$$\begin{aligned} & \text{DSolve}[\{2 x'[t] - 2 y'[t] - 3 x[t] == t, \\ & \quad 2 x'[t] + 2 y'[t] + 3 x[t] + 8 y[t] == 2\}, \{x[t], y[t]\}, t] \\ & \left\{ \left\{ x[t] \rightarrow \frac{1}{64} e^{-3t} (1 + 3 e^{4t}) \left( e^{-t} (-7 - 5t) + e^{3t} \left( \frac{19}{9} - \frac{t}{3} \right) \right) - \right. \right. \\ & \quad \frac{1}{64} e^{-3t} (-1 + e^{4t}) \left( e^{3t} \left( \frac{19}{3} - t \right) + e^{-t} (7 + 5t) \right) + \\ & \quad \left. \frac{1}{4} e^{-3t} (1 + 3 e^{4t}) C[1] - \frac{1}{2} e^{-3t} (-1 + e^{4t}) C[2] \right\}, \\ & \quad y[t] \rightarrow -\frac{3}{128} e^{-3t} (-1 + e^{4t}) \left( e^{-t} (-7 - 5t) + e^{3t} \left( \frac{19}{9} - \frac{t}{3} \right) \right) + \\ & \quad \frac{1}{128} e^{-3t} (3 + e^{4t}) \left( e^{3t} \left( \frac{19}{3} - t \right) + e^{-t} (7 + 5t) \right) - \\ & \quad \left. \left. \frac{3}{8} e^{-3t} (-1 + e^{4t}) C[1] + \frac{1}{4} e^{-3t} (3 + e^{4t}) C[2] \right\} \right\} \end{aligned}$$

**Question 2: find the general solution of the following linear system**

$$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t, \quad \frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}.$$

**Solution:**

$$\begin{aligned} & \text{DSolve}[\{x'[t] + y'[t] - 2x[t] - 4y[t] == \text{Exp}[t], \\ & \quad x'[t] + y'[t] - y[t] == \text{Exp}[4t]\}, \{x[t], y[t]\}, t] \\ & \left\{ \left\{ x[t] \rightarrow -e^t (-1 + e^{3t}) + \frac{1}{3} (3e^t (-1 + e^{3t}) + e^{-2t} C[1]) \right\}, \right. \\ & \quad \left. y[t] \rightarrow e^t (-1 + e^{3t}) - \frac{2}{9} (3e^t (-1 + e^{3t}) + e^{-2t} C[1]) \right\} \end{aligned}$$

**Question 3: find the general solution of the following linear system**  
 $dx/dt + dy/dt = -2t$ ,  $dx/dt + dy/dt - 3x - y = t^2$ .

**Solution:**

$$\begin{aligned} & \text{DSolve}[\{x'[t] + y'[t] - x[t] == -2t, \\ & \quad x'[t] + y'[t] - 3x[t] - y[t] == t^2\}, \{x[t], y[t]\}, t] \\ & \left\{ \left\{ x[t] \rightarrow -2t - t^2 + \frac{1}{4} (4(-2 + 2t + t^2) - e^{-t} C[1]) \right\}, \right. \\ & \quad \left. y[t] \rightarrow 2t + t^2 + \frac{1}{2} (-4(-2 + 2t + t^2) + e^{-t} C[1]) \right\} \end{aligned}$$

**Question 4: find the general solution of the following linear system**  
 $dx/dt + dy/dt - 3y = e^t$ ,  $dx/dt + dy/dt + x = e^{3t}$ .

**Solution:**

$$\begin{aligned} & \text{DSolve}[\{x'[t] + y'[t] - x[t] - 3y[t] == \text{Exp}[t], \\ & \quad x'[t] + y'[t] + x[t] == \text{Exp}[3t]\}, \{x[t], y[t]\}, t] \\ & \left\{ \left\{ x[t] \rightarrow -e^t (-1 + e^{2t}) + \frac{3}{16} \left( \frac{4}{3} e^t (-3 + 4e^{2t}) + e^{-3t} C[1] \right) \right\}, \right. \\ & \quad \left. y[t] \rightarrow e^t (-1 + e^{2t}) + \frac{1}{8} \left( -\frac{4}{3} e^t (-3 + 4e^{2t}) - e^{-3t} C[1] \right) \right\} \end{aligned}$$

**Question 5: find the general solution of the following linear system**  $dy/dt=0$ ,  
 $dx/dt+10x = t^2$ ,  $dz/dt+24z = e^t$ .

**Solution:**

```
DSolve[{y'[t] == 0, x'[t] + 10 x[t] == t^2,
  z'[t] + 24 z[t] == Exp[t]}, {y[t], x[t], z[t]}, t]
{ {y[t] -> C[1], x[t] -> 1/500 (1 - 10 t + 50 t^2) + e^{-10 t} C[2],
  z[t] -> e^t/25 + e^{-24 t} C[3]} }
```

**Question 6: find the general solution of the following linear system**  
 $d^2x/dt^2 + dy/dt - x + y = 1$ ,  $d^2y/dt^2 + dx/dt - x + y = 0$ .

**Solution:**

```
DSolve[{x''[t] + y'[t] - x[t] + y[t] == 1,
  y''[t] + x'[t] - x[t] + y[t] == 0}, {x[t], y[t]}, t]
{ {x[t] -> (e^t - t) (1 - e^t + t) + e^{-t} (-e^{-t} + e^t - t) (-1 + e^{2t} - e^t t) +
  e^{-t} (1 + e^t t) (-e^{-t} - e^t + t^2/2) - e^{-t} (1 - e^t + e^t t) (-e^t + t + t^2/2) +
  e^{-t} (1 + e^t t) C[1] + e^{-t} (-1 + e^{2t} - e^t t) C[2] -
  e^{-t} (1 - e^t + e^t t) C[3] + (1 - e^t + t) C[4],
  y[t] -> (e^t - t) t - e^{-t} (-e^{-t} + e^t - t) (1 - e^t + e^t t) +
  e^{-t} (1 - e^t + e^t t) (-e^{-t} - e^t + t^2/2) -
  e^{-t} (1 - 2 e^t + e^t t) (-e^t + t + t^2/2) + e^{-t} (1 - e^t + e^t t) C[1] -
  e^{-t} (1 - e^t + e^t t) C[2] - e^{-t} (1 - 2 e^t + e^t t) C[3] + t C[4]} }
```

**Question 7: find the general solution of the following linear system**  
 $d^2x/dt^2 - dy/dt - x + y = e^t$ ,  
 $dy/dt + dx/dt - 4x - y = 2e^t$ .

**Solution:**

```
DSolve[{x''[t] - y'[t] - x[t] + y[t] == Exp[t],
  x'[t] + y'[t] - 4 x[t] - y[t] == 2 Exp[t]}, {x[t], y[t]}, t]
```

$$\begin{aligned}
& \left\{ \left\{ \mathbf{x}[t] \rightarrow \right. \right. \\
& - \frac{1}{168 \sqrt{21}} \left( -3 + \sqrt{21} \right) \left( 3 + \sqrt{21} \right) e^{-\frac{1}{2} \left( -3 + \sqrt{21} \right) t} \left( -7 - 3 \sqrt{21} + \right. \\
& \quad \left. \left( -7 + 3 \sqrt{21} \right) e^{\sqrt{21} t} \right) \left( e^{\frac{1}{2} \left( -1 - \sqrt{21} \right) t} - e^{\frac{1}{2} \left( -1 + \sqrt{21} \right) t} \right) - \\
& \frac{1}{84} e^{-\frac{1}{2} \left( -3 + \sqrt{21} \right) t} \left( 3 + \sqrt{21} + \left( -3 + \sqrt{21} \right) e^{\sqrt{21} t} \right) \left( -e^{\frac{1}{2} \left( -1 - \sqrt{21} \right) t} + \right. \\
& \quad \left. \sqrt{21} e^{\frac{1}{2} \left( -1 - \sqrt{21} \right) t} + e^{\frac{1}{2} \left( -1 + \sqrt{21} \right) t} + \sqrt{21} e^{\frac{1}{2} \left( -1 + \sqrt{21} \right) t} \right) + \\
& \frac{1}{2 \sqrt{21}} \left( -e^{\frac{1}{2} \left( -1 - \sqrt{21} \right) t} + \sqrt{21} e^{\frac{1}{2} \left( -1 - \sqrt{21} \right) t} + \right. \\
& \quad \left. e^{\frac{1}{2} \left( -1 + \sqrt{21} \right) t} + \sqrt{21} e^{\frac{1}{2} \left( -1 + \sqrt{21} \right) t} \right) C[1] - \\
& \frac{\left( -3 + \sqrt{21} \right) \left( 3 + \sqrt{21} \right) \left( e^{\frac{1}{2} \left( -1 - \sqrt{21} \right) t} - e^{\frac{1}{2} \left( -1 + \sqrt{21} \right) t} \right) C[2]}{12 \sqrt{21}}, \\
& \mathbf{y}[t] \rightarrow \frac{1}{588} e^{-\frac{1}{2} \left( -3 + \sqrt{21} \right) t} \left( -7 - 3 \sqrt{21} + \left( -7 + 3 \sqrt{21} \right) e^{\sqrt{21} t} \right) \\
& \left( -42 e^t + 21 e^{\frac{1}{2} \left( -1 - \sqrt{21} \right) t} - \sqrt{21} e^{\frac{1}{2} \left( -1 - \sqrt{21} \right) t} + \right. \\
& \quad \left. 21 e^{\frac{1}{2} \left( -1 + \sqrt{21} \right) t} + \sqrt{21} e^{\frac{1}{2} \left( -1 + \sqrt{21} \right) t} \right) - \\
& \frac{1}{84 \sqrt{21}} e^{-\frac{1}{2} \left( -3 + \sqrt{21} \right) t} \left( 3 + \sqrt{21} + \left( -3 + \sqrt{21} \right) e^{\sqrt{21} t} \right) \\
& \left( -42 e^t + 21 e^{\frac{1}{2} \left( -1 - \sqrt{21} \right) t} - 11 \sqrt{21} e^{\frac{1}{2} \left( -1 - \sqrt{21} \right) t} + \right. \\
& \quad \left. 21 e^{\frac{1}{2} \left( -1 + \sqrt{21} \right) t} + 11 \sqrt{21} e^{\frac{1}{2} \left( -1 + \sqrt{21} \right) t} \right) - \frac{1}{7} e^{t - \frac{1}{2} \left( -1 + \sqrt{21} \right) t}
\end{aligned}$$

$$\begin{aligned}
& \left( (7 + 2\sqrt{21}) e^t + (7 - 2\sqrt{21}) e^{t+\sqrt{21}t} + 7 e^{\frac{1}{2}(-1+\sqrt{21})t} t \right) + \\
& \frac{1}{42} \left( -42 e^t + 21 e^{\frac{1}{2}(-1-\sqrt{21})t} - 11\sqrt{21} e^{\frac{1}{2}(-1-\sqrt{21})t} + \right. \\
& \quad \left. 21 e^{\frac{1}{2}(-1+\sqrt{21})t} + 11\sqrt{21} e^{\frac{1}{2}(-1+\sqrt{21})t} \right) C[1] + \\
& \frac{1}{42} \left( -42 e^t + 21 e^{\frac{1}{2}(-1-\sqrt{21})t} - \sqrt{21} e^{\frac{1}{2}(-1-\sqrt{21})t} + \right. \\
& \quad \left. 21 e^{\frac{1}{2}(-1+\sqrt{21})t} + \sqrt{21} e^{\frac{1}{2}(-1+\sqrt{21})t} \right) C[2] + e^t C[3] \} \}
\end{aligned}$$

**Question 8: find the general solution of the following linear system**  
 $\mathbf{dx/dt} + 2\mathbf{x} - 3\mathbf{y} = t$ ,  
 $\mathbf{dy/dt} - 3\mathbf{x} + 2\mathbf{y} = e^{2t}$ .

**Solution:**

**DSolve[{x'[t] + 2 x[t] - 3 y[t] == t,**

**y'[t] - 3 x[t] + 2 y[t] == Exp[2 t]}, {x[t], y[t]}, t]**

$$\begin{aligned}
& \left\{ \left\{ x[t] \rightarrow \frac{1}{4} e^{-5t} (-1 + e^{6t}) \left( e^t + \frac{e^{7t}}{7} + e^{-t} (-1 - t) + e^{5t} \left( \frac{1}{25} - \frac{t}{5} \right) \right) + \right. \right. \\
& \quad \frac{1}{4} e^{-5t} (1 + e^{6t}) \left( e^t - \frac{e^{7t}}{7} + e^{-t} (-1 - t) + e^{5t} \left( -\frac{1}{25} + \frac{t}{5} \right) \right) + \\
& \quad \left. \frac{1}{2} e^{-5t} (1 + e^{6t}) C[1] + \frac{1}{2} e^{-5t} (-1 + e^{6t}) C[2] \right\}, \\
& y[t] \rightarrow \frac{1}{4} e^{-5t} (1 + e^{6t}) \left( e^t + \frac{e^{7t}}{7} + e^{-t} (-1 - t) + e^{5t} \left( \frac{1}{25} - \frac{t}{5} \right) \right) + \\
& \quad \frac{1}{4} e^{-5t} (-1 + e^{6t}) \left( e^t - \frac{e^{7t}}{7} + e^{-t} (-1 - t) + e^{5t} \left( -\frac{1}{25} + \frac{t}{5} \right) \right) + \\
& \quad \left. \frac{1}{2} e^{-5t} (-1 + e^{6t}) C[1] + \frac{1}{2} e^{-5t} (1 + e^{6t}) C[2] \right\} \}
\end{aligned}$$