PRACTICAL-01

Solving first order Ordinary Differential Equations

Command: Dsolve[eqn,y[x],x]
Solve a differential equation for y[x]

Dsolve[{eqn1, eqn2, ...}, {y_1[x], y_2[x],.......},x]
Solve a system of differential equations for y_1[x]

Question 1: Solve first order differential equation dy/dx+y=0
Solution:

```
DSolve[y'[x] + y[x] == 0, y[x], x]
{{y[x] → e^-x C[1]}}
```

Question-2 : Solve the differential equation dy/dx+12y=0
Solution:

```
DSolve[y'[x] + 12 y[x] == 0, y[x], x]
{{y[x] → e^-12 x C[1]}}
```

Question-3: Solve first order differential equation dz/dt=0.
Solution:

```
DSolve[z'[t] == 0, z[t], t]
```

{{z[t] → C[1]}}

Question-4: Solve first order differential equation $dz/dt + 10z = t^2$

Solution:

```
DSolve[z'[t] + 10 z[t] == t^2, z[t], t]
```

$$\left\{\left\{z[t] \to \frac{1}{500}\left(1 - 10\,t + 50\,t^2\right) + e^{-10\,t}\,C[1]\right\}\right\}$$

Question-5: Solve the first order differential equation $dy/dx + 24y = e^x$.

```
DSolve[z'[x] + 24 z[x] == e^x, z[x], x]
```

$$\left\{\left\{z[x] \to \frac{e^x}{24}\right\}\right\}$$

Question-6: Solve the first order differential equation $dy/dx + y = 0$

$dy/dt + 12y = 0$

```
DSolve[{y'[t] + y[t] == 0, x'[t] + 12 x[t] == 0},
 {y[t], x[t]}, t]
```

$\{\{x[t] \to e^{-12\,t}\,C[1],\ y[t] \to 0\}\}$

Question-7: Solve first order differential equations...

$\frac{ds}{dx} = 0$

$\frac{dv}{dt} + 10\,v = t^{\wedge}2$

$\frac{dz}{dt} + 24\,z = e^{\wedge}t$

Solution:

```
DSolve[{s'[t] == 0, v'[t] + 10 v[t] == t^2,
  w'[t] + 24 w[t] == Exp[t]}, {s[t], v[t], w[t]}, t]
```

$$\left\{\left\{s[t] \to C[1],\ v[t] \to \frac{1}{500}\left(1-10\,t+50\,t^2\right)+e^{-10\,t}\,C[2],\ w[t] \to \frac{e^t}{25}+e^{-24\,t}\,C[3]\right\}\right\}$$

Question-8: Solve first order differential equations

$$\frac{dy}{dx} + \sqrt{1 - y^2/(1-x^2) = 0.}$$

Solution :

```
DSolve[
 y'[x] + Sqrt[(1 - y[x]^2) / (1 - x^2)] ==
  0, y[x], x]
```

$$\left\{\left\{y[x] \to 1 + 2\,\mathrm{Sinh}\left[\frac{1}{2}\left(-2\,\mathrm{ArcSinh}\left[\frac{\sqrt{-1+x}}{\sqrt{2}}\right]+C[1]\right)\right]^2\right\}\right\}$$

Question-9: Solve first order differential equations.

$$\left(y - x\,\frac{dy}{dx}\right) = \left(y^2 + \frac{dy}{dx}\right)$$

Solution:

```
DSolve[(y[t] - t * y'[t]) == ((y[t])^2 + y'[t]), y[t], t]
```

$$\left\{\left\{y[t] \to \frac{1+t}{1+e^{c(1)}+t}\right\}\right\}$$

Question-10: Solve first order differential equations.

$$\frac{dy}{dx} = -y^2/(xy + x^2)$$

Solution :

```
DSolve[y'[x] == -(y[x])^2/(x y[x] + x^2), y[x], x]
```

$$\left\{\left\{y[x] \to \frac{e^{2 C[1]} - \sqrt{e^{4 C[1]} + e^{2 C[1]} x^2}}{x}\right\}, \left\{y[x] \to \frac{e^{2 C[1]} + \sqrt{e^{4 C[1]} + e^{2 C[1]} x^2}}{x}\right\}\right\}$$

Question-11: Solve first order differential equations.

$$\left(\frac{dy}{dx}\right)^2 - x^3 = 0$$

Solution :

```
DSolve[(y'[x])^2 == x^3, y[x], x]
```

$$\left\{\left\{y[x] \to -\frac{2 x^{5/2}}{5} + C[1]\right\}, \left\{y[x] \to \frac{2 x^{5/2}}{5} + C[1]\right\}\right\}$$

Question-12: Solve first order differential equations.

$x = y + (dy/dx)^2$

Solution:

```
DSolve[x == y[x] + (y'[x])^2, y[x], x]
```

$$\left\{\left\{y[x] \to -1 + x - 2\,\mathrm{ProductLog}\left[-e^{-1-\frac{x}{2}+\frac{C[1]}{2}}\right] - \mathrm{ProductLog}\left[-e^{-1-\frac{x}{2}+\frac{C[1]}{2}}\right]^2\right\}\right\}$$

Practical-2

Plotting of second order solution of family of differential equation

command: ***DSolve[eqn,y[x]x,x]*** to soove a differential equayion fory[x]

Question 1: Solve the second order differential equation d^2y/dx^2+7dy/dx+10y=0

Solution:

```
DSolve[y''[x] + 7 y'[x] + 10 y[x] == 0, y[x], x]
```

$\{\{y[x] \to e^{-5x} C[1] + e^{-2x} C[2]\}\}$

Question 2: Solve the second order differential equation d^2y/dx^2+y=0

Solution:

```
DSolve[y''[x] + y[x] == 0, y[x], x]
```

$\{\{y[x] \to C[1] \operatorname{Cos}[x] + C[2] \operatorname{Sin}[x]\}\}$

Questiuon 3: Solve second order differential equation *d* ^ 2 *y* / dx ^ 2+dy/dx-6y=0.

Solution:

```
DSolve[y''[x] + y'[x] - 6 y[x] == 0, y[x], x]
```

$\{\{y[x] \to e^{-3x} C[1] + e^{2x} C[2]\}\}$

Question 4: Solve the second order differential equation 4d^2y/dx^2+12dy/dx+9y=0

Solution:

```
DSolve[4 y''[x] + 12 y'[x] + 9 y[x] == 0, y[x], x]
```

$\{\{y[x] \to e^{-3x/2} C[1] + e^{-3x/2} x C[2]\}\}$

Question 5: Solve the second order differentiual equation (*d*^2 *y*/dx^2-6dy/dx+13y=0

Solution:

```
DSolve[y''[x] - 6 y'[x] + 13 y[x] == 0, y[x], x]
```

$\{\{y[x] \to e^{3x} C[2] \text{Cos}[2x] + e^{3x} C[1] \text{Sin}[2x]\}\}$

Question 6: Solve the second order differential equation d^2y/dx^2-2dy/dx+y=0

Solution:

```
DSolve[y''[x] - 2 y'[x] + y[x] == 0, y[x], x]
```

$\{\{y[x] \to e^{x} C[1] + e^{x} x C[2]\}\}$

Plotting of solutions of second order differentiual equations

Question 1: Solve the second order differential equation d^2y/dx^2+y=0 and plots its three solutions.

Solution:

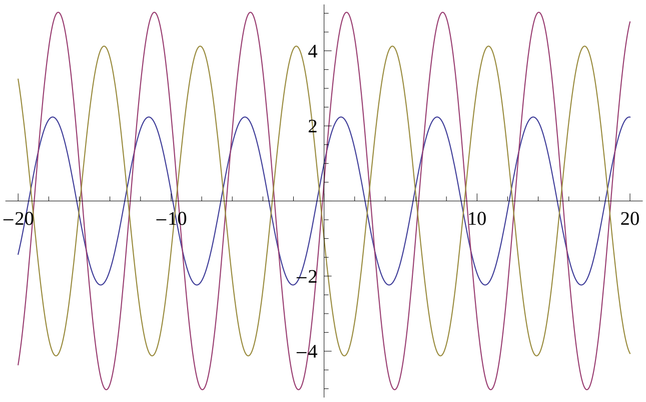
```
Sol = DSolve[y''[x] + y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 2}]
sol2 = y[x] /. Sol[[1]] /. {C[1] → 1/2, C[2] → 5}
sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → -4}
Plot[{sol1, sol2, sol3}, {x, -20, 20}]
```

{{y[x] → C[1] Cos[x] + C[2] Sin[x]}}

Cos[x] + 2 Sin[x]

$\frac{\text{Cos}[x]}{2} + 5\,\text{Sin}[x]$

-Cos[x] - 4 Sin[x]



Question 2: Soolve the second order differential equation (d^2y/dx^2+dy/dx-6y=0 and plot its three solutions.

Solution:

```
Needs["PlotLegends`"]
Sol = DSolve[y''[x] + y'[x] - 6 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 2.5}]
sol2 = y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 5}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1 / 2, C[2] → 5}
Plot[{sol1, sol2, sol3}, {x, -2, 2},
 PlotStyle → {{Pink, Thickness[0.01]},
   {Green, Thick}, {Orange, Thickness[0.02]}}]
```

PlotLegend::shdw :

Symbol PlotLegend appears in multiple contexts {PlotLegends`, Global`}; definitions in context PlotLegends` may shadow or be shadowed by other definitions. ≫

General::obspkg :

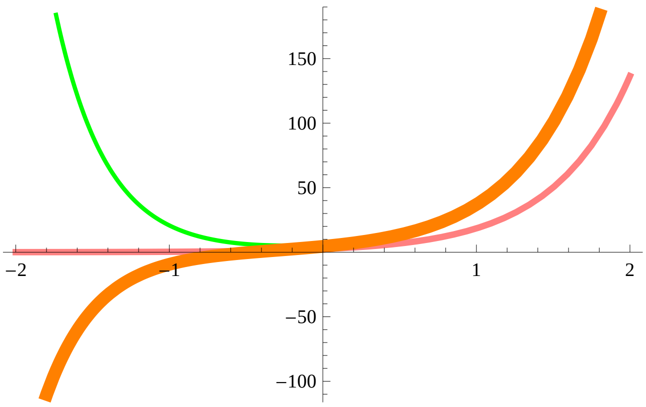
PlotLegends` is now obsolete. The legacy version being loaded may conflict with current Mathematica functionality. See the Compatibility Guide for updating information.

$$\{\{y[x] \to e^{-3x} C[1] + e^{2x} C[2]\}\}$$

$$2.5\, e^{2x}$$

$$e^{-3x} + 5\, e^{2x}$$

$$-\frac{1}{2} e^{-3x} + 5\, e^{2x}$$



Question 3: Solve the second order diffeential equation (d^2y)/dx^2+12dy/dx+9y=0 and plots its four solutions for

(1) C[1]=-1,C[2]=4
(2) C[1]=3,C[2]=6
(3) C[1]=-10, C[2]=7
(4) C[1]=-1.5, C[2]=-5

Solution:

```
sol = DSolve[4 y''[x] + 12 y'[x] + 9 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 4}]
sol2 = y[x] /. sol[[1]] /. {C[1] → 3, C[2] → 6}
sol3 = y[x] /. sol[[1]] /. {C[1] → -10, C[2] → 7}
sol4 = y[x] /. sol[[1]] /. {C[1] → -1.5, C[2] → -5}
Plot[{sol1, sol2, , sol3, sol4}, {x, -2, 2},
 PlotStyle → {{Pink, Thickness[0.01]},
   {Green, Thick}, {Orange, Thickness[0.02]},
   {Purple, Thickness[0.02]}, {Purple, Thickness[0.03]}}]
```

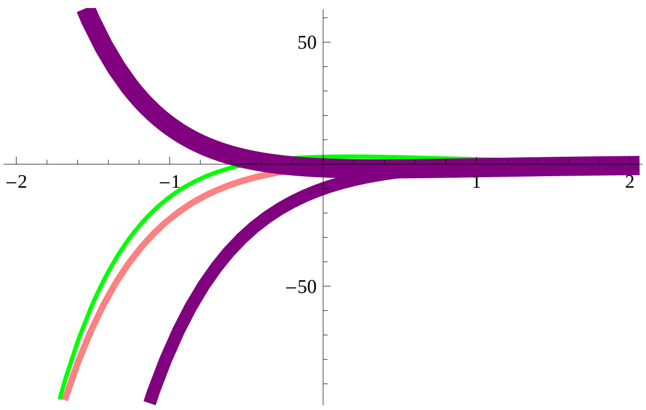
$\{\{y[x] \to e^{-3x/2} C[1] + e^{-3x/2} x C[2]\}\}$

$-e^{-3x/2} + 4 e^{-3x/2} x$

$3 e^{-3x/2} + 6 e^{-3x/2} x$

$-10 e^{-3x/2} + 7 e^{-3x/2} x$

$-1.5 e^{-3x/2} - 5 e^{-3x/2} x$



Question 4: Solve second order differential equation 4d^2y/dx^2-6dy/dx+13y=0 and plot its three solutions

Solution:

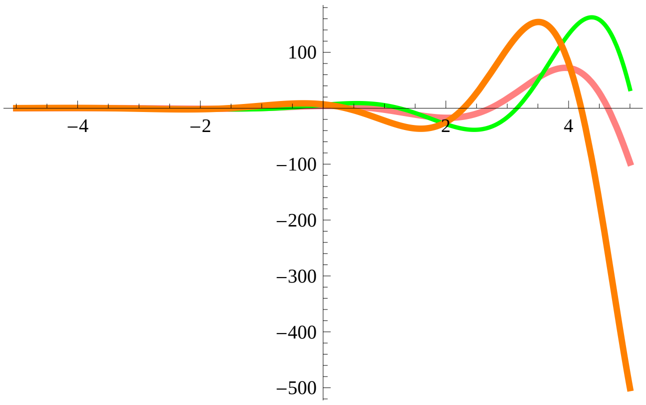
```
sol = DSolve[4 y''[x] - 6 y'[x] + 13 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 4}]
sol2 = y[x] /. sol[[1]] /. {C[1] → 3, C[2] → 6}
sol3 = y[x] /. sol[[1]] /. {C[1] → -10, C[2] -> 7}
Plot[{sol1, sol2, sol3}, {x, -5, 5},
 PlotStyle → {{Pink, Thickness[0.01]}, {Green, Thick},
   {Orange, Thickness[0.01]}}, PlotRange → All]
```

$$\left\{\left\{y[x] \to e^{3x/4} C[2] \operatorname{Cos}\left[\frac{\sqrt{43}\, x}{4}\right] + e^{3x/4} C[1] \operatorname{Sin}\left[\frac{\sqrt{43}\, x}{4}\right]\right\}\right\}$$

$$4 e^{3x/4} \operatorname{Cos}\left[\frac{\sqrt{43}\, x}{4}\right] - e^{3x/4} \operatorname{Sin}\left[\frac{\sqrt{43}\, x}{4}\right]$$

$$6 e^{3x/4} \operatorname{Cos}\left[\frac{\sqrt{43}\, x}{4}\right] + 3 e^{3x/4} \operatorname{Sin}\left[\frac{\sqrt{43}\, x}{4}\right]$$

$$7 e^{3x/4} \operatorname{Cos}\left[\frac{\sqrt{43}\, x}{4}\right] - 10 e^{3x/4} \operatorname{Sin}\left[\frac{\sqrt{43}\, x}{4}\right]$$



Question 5: Solve second order differential equation d^2y/dx^2-2dy/dx+y=0 and plot its five solutions.

Solution:

```
sol = DSolve[y''[x] - 2 y'[x] + y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -3, C[2] → -2}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 7}
sol4 = y[x] /. sol[[1]] /. {C[1] → -6, C[2] → 1}
sol5 = y[x] /. sol[[1]] /. {C[1] → 1/5, C[2] → 2/3}
Plot[{sol1, sol2, sol3, sol4, sol5}, {x, -5, 5},
 PlotStyle -> {Thickness[0.01], Thick, Thickness[0.02],
   Thickness[0.03], Thickness[0.04]}, PlotRange -> All]
```

$\{\{y[x] \to e^{x} C[1] + e^{x} x C[2]\}\}$

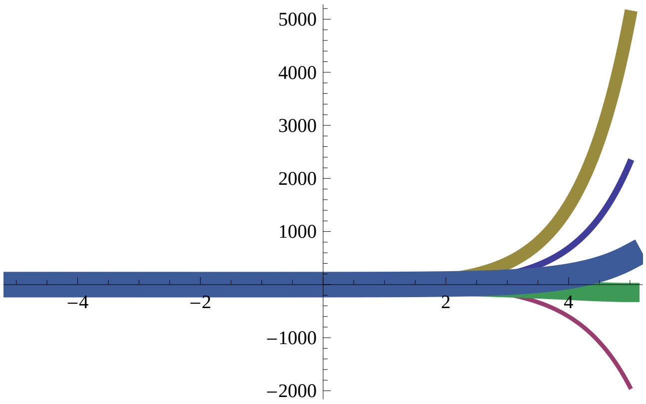
$0.5\, e^{x} + 3\, e^{x} x$

$-3\, e^{x} - 2\, e^{x} x$

$-e^{x} + 7\, e^{x} x$

$-6\, e^{x} + e^{x} x$

$\frac{e^{x}}{5} + \frac{2\, e^{x} x}{3}$



Question 6: Solve second order differential equation x^2d^2y/dx^2-xdy/dx+y=2log(x) and plot its any five solutions.

Solution:

```
Sol = DSolve[x^2 y''[x] - x y'[x] + y[x] == 2 Log[x], y[x], x]
Sol1 = Evaluate[y[x] /. [[1]] /. {C[1] -> 0.5, C[2] -> 3}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] -> -3, C[2] -> -2}
Sol3 = y[x] /. Sol[[1]] /. {C[1] -> -1, C[2] -> 7}
Sol4 = y[x] /. Sol[[1]] /. {C[1] -> -6, C[2] -> 1}
Sol5 = y[x] /. Sol[[1]] /. {C[1] -> 1/5, C[2] -> 2/3}
Plot[{Sol1, Sol2, Sol3, Sol4, Sol5}, {x, -15, 15}]
```

{{y[x] → x C[1] + x C[2] Log[x] + 2 (2 + Log[x])}}

Syntax::sntxf : "y[x] /." cannot be followed by "[[1]]".

Syntax::tsntxi : "[[1]]" is incomplete; more input is needed.

Syntax::sntxi : Incomplete expression; more input is needed .

```
Sol = DSolve[x^2 y''[x] - x y'[x] + y[x] == 2 Log[x], y[x], x]
```

{{y[x] → x C[1] + x C[2] Log[x] + 2 (2 + Log[x])}}

```
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] -> 0.5, C[2] -> 3}]
```

0.5 x + 3 x Log[x] + 2 (2 + Log[x])

```
Sol2 = y[x] /. Sol[[1]] /. {C[1] -> -3, C[2] -> -2}
```

-3 x - 2 x Log[x] + 2 (2 + Log[x])

```
Sol3 = y[x] /. Sol[[1]] /. {C[1] -> -1, C[2] -> 7}
```

-x + 7 x Log[x] + 2 (2 + Log[x])

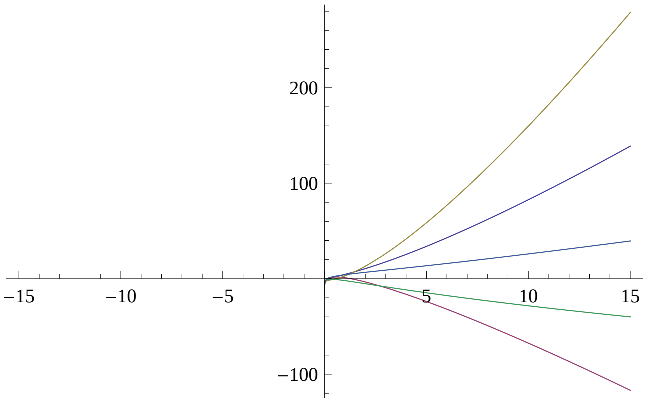
```
Sol4 = y[x] /. Sol[[1]] /. {C[1] -> -6, C[2] -> 1}
```

-6 x + x Log[x] + 2 (2 + Log[x])

```
Sol5 = y[x] /. Sol[[1]] /. {C[1] -> 1/5, C[2] -> 2/3}
```

$\frac{x}{5} + \frac{2}{3} x \, \text{Log}[x] + 2 \, (2 + \text{Log}[x])$

```
Plot[{Sol1, Sol2, Sol3, Sol4, Sol5}, {x, -15, 15}]
```



Question 7: Solve second order differential equation x^2d^2y/dx+y=3x^2 and plot its any five solutions.

Solution:

```
sol = DSolve[x^2 y''[x] + y[x] == 3 x^2, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -2, C[2] → -2}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 7}
sol4 = y[x] /. sol[[1]] /. {C[1] → -6, C[2] → 4}
sol5 = y[x] /. sol[[1]] /. {C[1] → 5, C[2] → 2/3}
Plot[{sol1, sol2, sol3, sol4, sol5}, {x, -15, 15}]
```

$$\left\{\left\{y[x] \to \sqrt{x}\, C[1] \,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + \sqrt{x}\, C[2]\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2 + \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2\right)\right\}\right\}$$

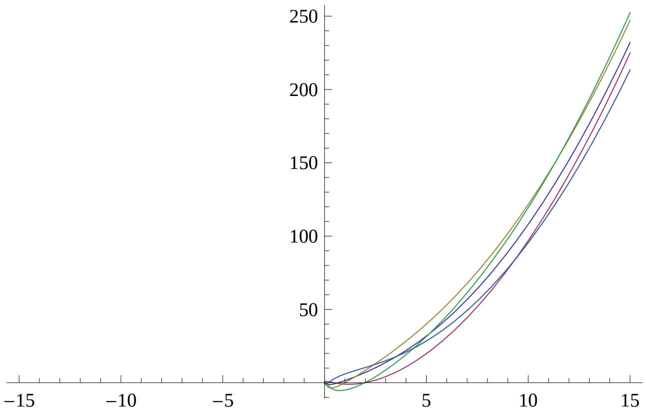
$$0.5\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + 3\sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2 + \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2\right)$$

$$-2\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] - 2\sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2 + \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2\right)$$

$$-\sqrt{x}\ \mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]+7\sqrt{x}\ \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]+$$
$$x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]^2+\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]^2\right)$$

$$-6\sqrt{x}\ \mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]+4\sqrt{x}\ \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]+$$
$$x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]^2+\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]^2\right)$$

$$5\sqrt{x}\ \mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]+\frac{2}{3}\sqrt{x}\ \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]+$$
$$x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]^2+\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\ \mathrm{Log}[x]\right]^2\right)$$



Question 8: Solve second order differential equation d^2y/dx^2=$\sqrt{1+(dy/dx)^2}$ and plot its any five solutions

Solution:

```
sol = DSolve[y''[x] == Sqrt[1 + (y'[x])^2], y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -2, C[2] → -2}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 7}
sol4 = y[x] /. sol[[1]] /. {C[1] → -6, C[2] → 4}
sol5 = y[x] /. sol[[1]] /. {C[1] → 5, C[2] → 2/3}
Plot[{sol1, sol2, sol3, sol4, sol5}, {x, -35, 35}]
```

{{y[x] → C[2] + Cosh[x] Cosh[C[1]] + Sinh[x] Sinh[C[1]]}}

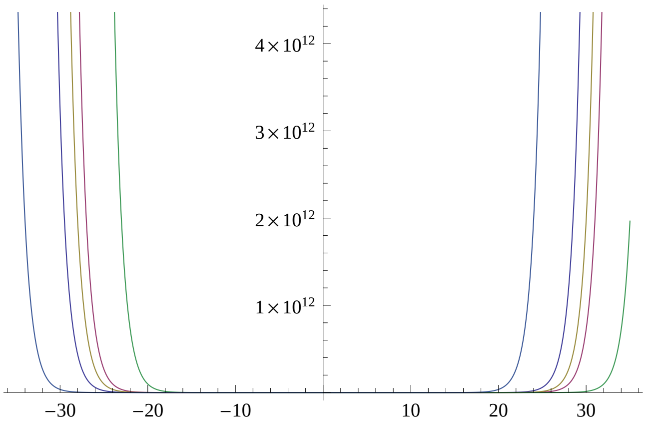
3 + 1.12763 Cosh[x] + 0.521095 Sinh[x]

-2 + Cosh[2] Cosh[x] - Sinh[2] Sinh[x]

7 + Cosh[1] Cosh[x] - Sinh[1] Sinh[x]

4 + Cosh[6] Cosh[x] - Sinh[6] Sinh[x]

$\frac{2}{3}$ + Cosh[5] Cosh[x] + Sinh[5] Sinh[x]



Question 9: Solve the second order differential equation (1+x^2)d^2y/dx^2+1+(dy/dx)^2=0 and plot its any five solutionss

Solution:

```
sol = DSolve[(1 + x^2) y''[x] + 1 + (y'[x])^2 == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -2, C[2] → -2}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 7}
sol4 = y[x] /. sol[[1]] /. {C[1] → -6, C[2] → 4}
sol5 = y[x] /. sol[[1]] /. {C[1] → 5, C[2] → 2 / 3}
Plot[{sol1, sol2, sol3, sol4, sol5}, {x, -15, 15}]
```

$$\{\{y[x] \to C[2] - x\,\mathrm{Cot}[C[1]] + \mathrm{Csc}[C[1]]^2\,\mathrm{Log}[-\mathrm{Cos}[C[1]] - x\,\mathrm{Sin}[C[1]]]\}\}$$

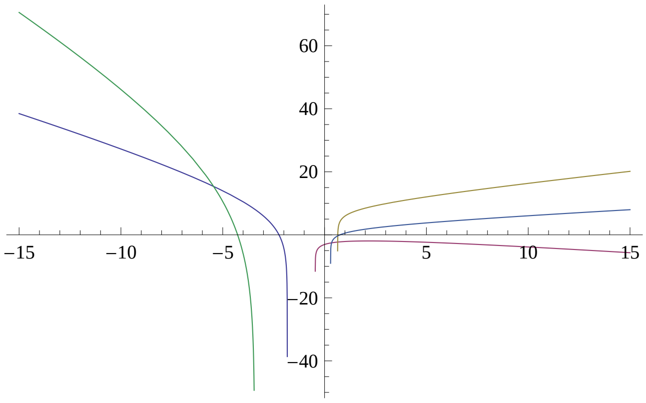
$$3 - 1.83049\,x + 4.35069\,\mathrm{Log}[-0.877583 - 0.479426\,x]$$

$$-2 + x\,\mathrm{Cot}[2] + \mathrm{Csc}[2]^2\,\mathrm{Log}[-\mathrm{Cos}[2] + x\,\mathrm{Sin}[2]]$$

$$7 + x\,\mathrm{Cot}[1] + \mathrm{Csc}[1]^2\,\mathrm{Log}[-\mathrm{Cos}[1] + x\,\mathrm{Sin}[1]]$$

$$4 + x\,\mathrm{Cot}[6] + \mathrm{Csc}[6]^2\,\mathrm{Log}[-\mathrm{Cos}[6] + x\,\mathrm{Sin}[6]]$$

$$\frac{2}{3} - x\,\mathrm{Cot}[5] + \mathrm{Csc}[5]^2\,\mathrm{Log}[-\mathrm{Cos}[5] - x\,\mathrm{Sin}[5]]$$



Practical-3

Ploting of Third order solution family of differential equation.

Question 1: Solve third order differential equation d^3y/dx^2-5d^2y-/dx^2+8dy/dx-4y=0
and plot its any three solutions.

Solution:

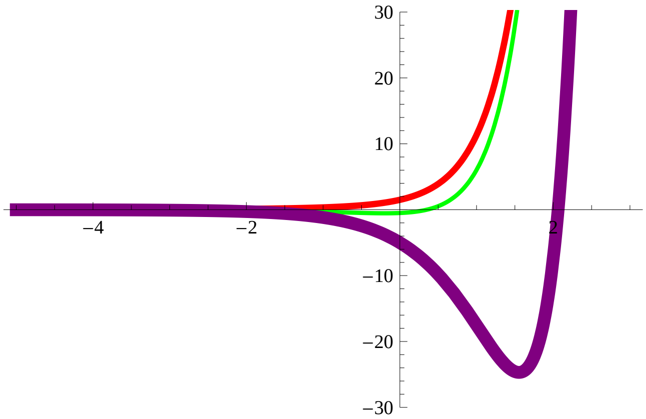
```
sol = DSolve[y'''[x] - 5 y''[x] + 8 y'[x] - 4 y[x] == 0, y[x], x]
sol1 = Evaluate[
  y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 0.5, C[3] → 2 / 3}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -1 / 2, C[2] → 0, C[3] → 1}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → -4, C[3] → 2}
Plot[{sol1, sol2, sol3}, {x, -5, 3}, PlotRange → {-30, 30},
 PlotStyle → {{Red, Thickness[0.01]},
   {Green, Thick}, {Purple, Thickness[0.02]}}]
```

$\{\{y[x] \to e^{x} C[1] + e^{2x} C[2] + e^{2x} x C[3]\}\}$

$e^{x} + 0.5\, e^{2x} + \frac{2}{3} e^{2x} x$

$-\frac{e^{x}}{2} + e^{2x} x$

$-e^{x} - 4 e^{2x} + 2 e^{2x} x$



Question 2: Solve third order differential equation d^3y/dx^3+3d^2y/dx^2-25d/dx+21y=0 and plot its any four solutions.

Solution:

```
eqn = y'''[x] + 3 * y''[x] - 25 * y'[x] + 21 * y[x];
sol = DSolve[eqn == 0, y[x], x]
sol1 =
 Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 0, C[3] → 2}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -1/2, C[2] → 0, C[3] → 1}
sol3 = y[x] /. sol[[1]] /. {C[1] -> -1, C[2] → -4, C[3] → 2}
sol4 = y[x] /. sol[[1]] /. {C[1] → -0.5, C[2] → -2, C[3] → 1}
Plot[{sol1, sol2, sol3, sol4}, {x, -0.5, 0.5},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick},
   {Purple, Thickness[0.02]}, {Orange, Thickness[0.01]}}]
```

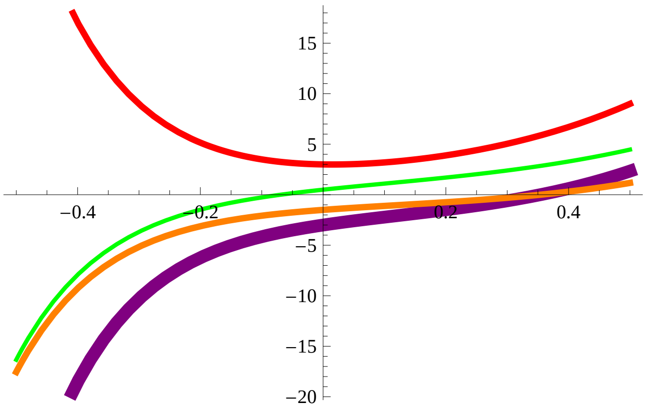
$$\{\{y[x] \to e^{-7x} C[1] + e^{x} C[2] + e^{3x} C[3]\}\}$$

$$e^{-7x} + 2 e^{3x}$$

$$-\frac{1}{2} e^{-7x} + e^{3x}$$

$$-e^{-7x} - 4 e^{x} + 2 e^{3x}$$

$$-0.5 e^{-7x} - 2 e^{x} + e^{3x}$$



Question 3: Solve third order differential equation (d^3y)/dx^3-4d^2/dx^2-25dy/dx+28y=0 and plot its any four solutions

Solution:

```
eqn = y'''[x] - 4*y''[x] - 25*y'[x] + 28*y[x]
sol = DSolve[eqn == 0, y[x], x]
sol1 =
 Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -2, C[2] → 10, C[3] → 3}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → -4, C[3] → 20}
sol4 = y[x] /. sol[[1]] /. {C[1] → -0.5, C[2] → 2, C[3] → 1}
Plot[{sol1, sol2, sol3, sol4}, {x, -0.5, 0.5},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick},
   {Purple, Thickness[0.02]}, {Orange, Thickness[0.01]}}]
```

$28\, y[x] - 25\, y'[x] - 4\, y''[x] + y^{(3)}[x]$

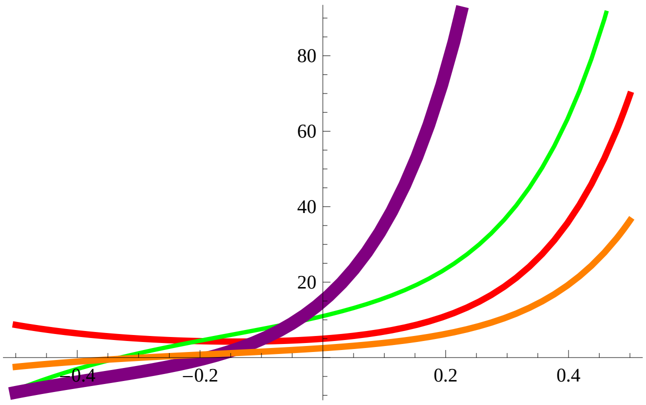
$\{\{y[x] \to e^{-4x} C[1] + e^{x} C[2] + e^{7x} C[3]\}\}$

$e^{-4x} + 2\, e^{x} + 2\, e^{7x}$

$-2\, e^{-4x} + 10\, e^{x} + 3\, e^{7x}$

$-e^{-4x} - 4\, e^{x} + 20\, e^{7x}$

$-0.5\, e^{-4x} + 2\, e^{x} + e^{7x}$



Question 4: Solve third orer differential equation d^3y/dx^3-13d^2y/dx^2+19dy/dx+33y=cos(2x) and plot its any four dolutions.

Solution:

```
eqn = y'''[x] - 13 * y''[x] + 19 * y'[x] + 33 * y[x];
sol = DSolve[eqn == Cos[2 x], y[x], x]
sol1 =
 Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -2, C[2] → 10, C[3] → 3}
sol3 = y[x] /. sol[[1]] /. {C[1] → -2, C[2] → -6, C[3] → 20}
sol4 = y[x] /. sol[[1]] /. {C[1] → -0.5, C[2] → 2, C[3] → 1}
Plot[{sol1, sol2, sol3, sol4}, {x, -0.5, 0.5},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick},
   {Purple, Thickness[0.02]}, {Orange, Thickness[0.01]}}]
```

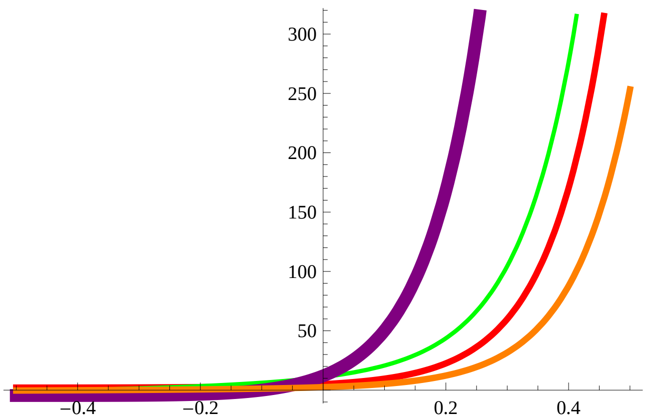
$$\left\{\left\{y[x] \to e^{-x} C[1] + e^{3x} C[2] + e^{11x} C[3] + \frac{17 \text{Cos}[2x] + 6 \text{Sin}[2x]}{1625}\right\}\right\}$$

$$e^{-x} + 2 e^{3x} + 2 e^{11x} + \frac{17 \text{Cos}[2x] + 6 \text{Sin}[2x]}{1625}$$

$$-2 e^{-x} + 10 e^{3x} + 3 e^{11x} + \frac{17 \text{Cos}[2x] + 6 \text{Sin}[2x]}{1625}$$

$$-2 e^{-x} - 6 e^{3x} + 20 e^{11x} + \frac{17 \text{Cos}[2x] + 6 \text{Sin}[2x]}{1625}$$

$$-0.5 e^{-x} + 2 e^{3x} + e^{11x} + \frac{17 \text{Cos}[2x] + 6 \text{Sin}[2x]}{1625}$$



Practical-4

Solution of Differential Equation by Variation of Parameter Method

Question 1: Solve second order differential equation
d^2y/dx^2=9y=sec(3x)
by variation of parameter method

Solution:

```
Sol = DSolve[y''[x] + 9 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 0}]
sol2 = y[x] /. sol[[1]] /. {C[1] → 0, C[2] → 1}
fs = {sol1, sol2}
wm = {fs, D[fs, x]}; wm // MatrixForm
wd = Simplify[Det[wm]]
u1 = (Integrate[-sol2 Sec[3 x], x]) / wd
u2 = (integrate[sol1 Sec[3 x], x]) / wd
yc = DSolve[y''[x] + 9 y[x] == 0, y[x], x]
yp = Evaluate[y[x] /. sol[[1]] /. {C[1] → u1, C[2] → u2}]
yg = yc + yp
```

{{y[x] → C[1] Cos[3 x] + C[2] Sin[3 x]}}

Cos[3 x]

Sin[3 x]

{Cos[3 x], Sin[3 x]}

$$\begin{pmatrix} \text{Cos}[3\,x] & \text{Sin}[3\,x] \\ -3\,\text{Sin}[3\,x] & 3\,\text{Cos}[3\,x] \end{pmatrix}$$

3

$$\frac{1}{9}\,\text{Log}[\text{Cos}[3\,x]]$$

$$\frac{1}{3}\,\text{integrate}[1,\,x]$$

{{y[x] → C[1] Cos[3 x] + C[2] Sin[3 x]}}

$$\frac{1}{9}\,\text{Cos}[3\,x]\,\text{Log}[\text{Cos}[3\,x]] + \frac{1}{3}\,\text{integrate}[1,\,x]\,\text{Sin}[3\,x]$$

$$\left\{\left\{\frac{1}{9}\,\text{Cos}[3\,x]\,\text{Log}[\text{Cos}[3\,x]] + (y[x] \to C[1]\,\text{Cos}[3\,x] + C[2]\,\text{Sin}[3\,x]) + \frac{1}{3}\,\text{integrate}[1,\,x]\,\text{Sin}[3\,x]\right\}\right\}$$

Question 2: Solve third order differential equation
d^3y/dx^3+4dy/dx=sec(2x)
by variation of parameter method

Solution:

```
sol = DSolve[y'''[x] + 4 y'[x] == 0, y[x], x]
sol1 =
 Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 0, C[3] → 0}]
sol2 = y[x] /. sol[[1]] /. {C[1] → 0, C[2] → -2, C[3] → 0}
sol3 = y[x] /. sol[[1]] /. {C[1] → 0, C[2] → 0, C[3] → 1}
fs = {sol1, sol2, sol3}
wm = {fs, D[fs, x], D[fs, {x, 2}]}; MatrixForm[wm]
wd = Simplify[Det[wm]]
a = (1 / wd (Det[{{0, sol2, sol3},
       {0, D[sol2, x], D[sol3, x]}, {Sec[2 x],
        D[sol2, {x, 2}], D[sol3, {x, 2}]}}]))  // Simplify
u1 = Integrate[a, x]
b = (1 / wd (Det[{{sol1, 0, sol3},
       {D[sol1, x], 0, D[sol3, x]}, {D[sol1, {x, 2}],
        Sec[2 x], D[sol3, {x, 2}]}}]))  // Simplify
u2 = Integrate[b, x]
c = (1 / wd (Det[{{sol1, sol2, 0},
       {D[sol1, x], D[sol2, x], 0}, {D[sol1, {x, 2}],
        D[sol2, {x, 2}], Sec[2 x]}}]))  // simplify
u3 = Integrate[c, x]
yc = Evaluate[y[x] /. sol[[1]]]
yp = Evaluate[
  y[x] /. sol[[1]] /. {C[1] → u1, C[2] → u2, C[3] → u3}]
yg =
 yc +
  yp
```

$$\left\{\left\{y[x] \to C[3] - \frac{1}{2} C[2] \, \mathrm{Cos}[2\,x] + \frac{1}{2} C[1] \, \mathrm{Sin}[2\,x]\right\}\right\}$$

Sin[2 x]

Cos[2 x]

$$1$$

$$\{\text{Sin}[2\,x],\ \text{Cos}[2\,x],\ 1\}$$

$$\begin{pmatrix} \text{Sin}[2\,x] & \text{Cos}[2\,x] & 1 \\ 2\,\text{Cos}[2\,x] & -2\,\text{Sin}[2\,x] & 0 \\ -4\,\text{Sin}[2\,x] & -4\,\text{Cos}[2\,x] & 0 \end{pmatrix}$$

$$-8$$

$$-\frac{1}{4}\,\text{Tan}[2\,x]$$

$$\frac{1}{8}\,\text{Log}[\text{Cos}[2\,x]]$$

$$-\frac{1}{4}$$

$$-\frac{x}{4}$$

$$\text{simplify}\left[-\frac{1}{8}\,\text{Sec}[2\,x]\,\left(-2\,\text{Cos}[2\,x]^2-2\,\text{Sin}[2\,x]^2\right)\right]$$

$$\int \text{simplify}\left[-\frac{1}{8}\,\text{Sec}[2\,x]\,\left(-2\,\text{Cos}[2\,x]^2-2\,\text{Sin}[2\,x]^2\right)\right] dx$$

$$C[3]-\frac{1}{2}\,C[2]\,\text{Cos}[2\,x]+\frac{1}{2}\,C[1]\,\text{Sin}[2\,x]$$

$$\frac{1}{8}\,x\,\text{Cos}[2\,x]+\int \text{simplify}\left[-\frac{1}{8}\,\text{Sec}[2\,x]\,\left(-2\,\text{Cos}[2\,x]^2-2\,\text{Sin}[2\,x]^2\right)\right] dx+\frac{1}{16}\,\text{Log}[\text{Cos}[2\,x]]\,\text{Sin}[2\,x]$$

$$\text{C}[3]+\frac{1}{8}\,\text{x}\,\text{Cos}[2\,\text{x}]-\frac{1}{2}\,\text{C}[2]\,\text{Cos}[2\,\text{x}]+$$
$$\int \text{simplify}\left[-\frac{1}{8}\,\text{Sec}[2\,\text{x}]\,\left(-2\,\text{Cos}[2\,\text{x}]^{2}-2\,\text{Sin}[2\,\text{x}]^{2}\right)\right]\,d\text{x}+$$
$$\frac{1}{2}\,\text{C}[1]\,\text{Sin}[2\,\text{x}]+\frac{1}{16}\,\text{Log}[\text{Cos}[2\,\text{x}]]\,\text{Sin}[2\,\text{x}]$$

Question 3: Solve second order differential equation
d^2y/dx^2+t=Tan(x)
by variation of parameter method

Solution:

```
Sol = DSolve[y''[x] + y[x] == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 1}
fs = {Sol1, Sol2}
wm = {fs, D[fs, x]}; wm // MatrixForm
wd = Simplify[Det[wm]]
u1 = (Integrate[-Sol2 Tan[x], x]) / wd
u2 = (Integrate[Sol1 Tan[x], x]) / wd
yc = DSolve[y''[x] + y[x] == 0, y[x], x]
yp = Simplify[
  Evaluate[y[x] /. Sol[[1]] /. {C[1] → u1, C[2] → u2}]]
yg =
 yc +
  yp
```

{{y[x] → C[1] Cos[x] + C[2] Sin[x]}}

Cos[x]

Sin[x]

{Cos[x], Sin[x]}

$$\begin{pmatrix} \text{Cos}[x] & \text{Sin}[x] \\ -\text{Sin}[x] & \text{Cos}[x] \end{pmatrix}$$

1

$$\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sin}[x]$$

-Cos[x]

{{y[x] → C[1] Cos[x] + C[2] Sin[x]}}

$$\text{Cos}[x]\left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]\right)$$

$$\left\{\left\{\text{Cos}[x]\left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]\right) + (y[x] \to C[1]\,\text{Cos}[x] + C[2]\,\text{Sin}[x])\right\}\right\}$$

Question 4: Solve third order differential equation
$$d^{\wedge}3\, y/\text{dx}^{\wedge}3 - 6\, d^{\wedge}2\, y/\text{dx}^{\wedge}2 + 11\, \text{dy}/\text{dx} - 6\, y = e^{x}$$
by variation of parameter method

Solution:

```
Sol = DSolve[y'''[x] - 6 y''[x] + 11 y'[x] - 6 y[x] == 0, y[x], x]
Sol1 =
 Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0, C[3] → 0}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 1, C[3] → 0}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 0, C[3] → 1}
fs = {Sol1, Sol2, Sol3}
wm = {fs, D[fs, x], D[fs, {x, 2}]}; MatrixForm[wm]
wd = Simplify[Det[wm]]
a = (1 / wd (Det[
      {{0, Sol2, Sol3}, {0, D[Sol2, x], D[Sol3, x]}, {Exp[x],
        D[Sol2, {x, 2}], D[Sol3, {x, 2}]}}])) // Simplify
u1 = Integrate[a, x]
b = (1 / wd (Det[{{Sol1, 0, Sol3},
       {D[Sol1, x], 0, D[Sol3, x]}, {D[Sol1, {x, 2}],
        Exp[x], D[Sol3, {x, 2}]}}])) // Simplify
u2 = Integrate[b, x]
c = (1 / wd (Det[{{Sol1, Sol2, 0},
       {D[Sol1, x], D[Sol2, x], 0}, {D[Sol1, {x, 2}],
        D[Sol2, {x, 2}], Exp[x]}}])) // Simplify
u3 = Integrate[c, x]
yc = Evaluate[y[x] /. Sol[[1]]]
yp = Evaluate[
  y[x] /. Sol[[1]] /. {C[1] → u1, C[2] → u2, C[3] → u3}]
yg =
 yc +
  yp
```

$\{\{y[x] \to e^{x} C[1] + e^{2x} C[2] + e^{3x} C[3]\}\}$

e^{x}

e^{2x}

e^{3x}

$\{e^{x}, e^{2x}, e^{3x}\}$

$$\begin{pmatrix} e^{x} & e^{2x} & e^{3x} \\ e^{x} & 2 e^{2x} & 3 e^{3x} \\ e^{x} & 4 e^{2x} & 9 e^{3x} \end{pmatrix}$$

$2 e^{6x}$

$$\frac{1}{2}$$

$$\frac{x}{2}$$

$$-e^{-x}$$

$$e^{-x}$$

$$\frac{e^{-2x}}{2}$$

$$-\frac{1}{4}e^{-2x}$$

$$e^{x}C[1]+e^{2x}C[2]+e^{3x}C[3]$$

$$\frac{3e^{x}}{4}+\frac{e^{x}x}{2}$$

$$\frac{3e^{x}}{4}+\frac{e^{x}x}{2}+e^{x}C[1]+e^{2x}C[2]+e^{3x}C[3]$$

Question 5: Solve second order differential equation
d^2y/dx^2-2y=4x-8
given condition is y[1]=4,y’[1]=-1 by variation of parameter method

Solution:

```
Sol = DSolve[x^2 y''[x] - 2 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0}]
sol2 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 1}
fs = {sol1, sol2}
wm = {fs, D[fs, x]}; wm // MatrixForm
wd = Simplify[Det[wm]]
u1 = (Integrate[-sol2 (4 x - 8), x]) / wd
u2 = (Integrate[sol1 (4 x - 8), x]) / wd
yc = DSolve[x^2 y''[x] - 2 y[x] == 0, y[x], x]
yp = Simplify[
  Evaluate[y[x] /. Sol[[1]] /. {C[1] → u1, C[2] → u2}]]
yg =
 yc +
  yp
```

$$\left\{\left\{y[x] \to x^2\, C[1] + \frac{C[2]}{x}\right\}\right\}$$

$$x^2$$

$$\frac{1}{x}$$

$$\left\{x^2,\ \frac{1}{x}\right\}$$

$$\begin{pmatrix} x^2 & \frac{1}{x} \\ 2\,x & -\frac{1}{x^2} \end{pmatrix}$$

$$-3$$

$$\frac{1}{3}\,(4\,x - 8\,\mathrm{Log}[x])$$

$$-\frac{4}{3}\left(-\frac{2\,x^3}{3} + \frac{x^4}{4}\right)$$

$$\left\{\left\{y[x] \to x^2\, C[1] + \frac{C[2]}{x}\right\}\right\}$$

$$\frac{1}{9}\,x^2\,(8 + 9\,x - 24\,\mathrm{Log}[x])$$

$$\left\{\left\{\frac{1}{9}\,x^2\,(8 + 9\,x - 24\,\mathrm{Log}[x]) + \left(y[x] \to x^2\, C[1] + \frac{C[2]}{x}\right)\right\}\right\}$$

Question 6: Solve second order differential equation

$d^2 y/dx^2 + y = \mathrm{Tan}(x)\,\mathrm{Sec}(x)$

by variation of parameter method

Solution:

```
Sol = DSolve[y''[x] + y[x] == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 1}
fs = {Sol1, Sol2}
wm = {fs, D[fs, x]}; wm // MatrixForm
wd = Simplify[Det[wm]]
u1 = (Integrate[-Sol2 Tan[x] Sec[x], x]) / wd
u2 = (Integrate[Sol1 Tan[x] Sec[x], x]) / wd
yc = DSolve[y''[x] + y[x] == 0, y[x], x]
yp = Simplify[
  Evaluate[y[x] /. Sol[[1]] /. {C[1] → u1, C[2] → u2}]]
yg =
 yc +
  yp
```

```
{{y[x] → C[1] Cos[x] + C[2] Sin[x]}}
Cos[x]
Sin[x]
{Cos[x], Sin[x]}
```

$$\begin{pmatrix} \mathrm{Cos}[x] & \mathrm{Sin}[x] \\ -\mathrm{Sin}[x] & \mathrm{Cos}[x] \end{pmatrix}$$

```
1
x - Tan[x]
-Log[Cos[x]]
{{y[x] → C[1] Cos[x] + C[2] Sin[x]}}
x Cos[x] - (1 + Log[Cos[x]]) Sin[x]
{{x Cos[x] + (y[x] → C[1] Cos[x] + C[2] Sin[x]) -
   (1 + Log[Cos[x]]) Sin[x]}}
```

Question 7: Solve second order differential equation

$d^{\wedge}2\, y/\mathrm{dx}^{\wedge}2 - 2\,\mathrm{dy}/\mathrm{dx} + 5\, y = e^x \mathrm{Tan}(2x)$

by variation of parameter method

Solution:

```
Sol = DSolve[y''[x] - 2 y'[x] + 5 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0}]
sol2 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 1}
fs = {sol1, sol2}
wm = {fs, D[fs, x]}; wm // MatrixForm
wd = Simplify[Det[wm]]
u1 = (Integrate[-sol2 Exp[x] Tan[2 x], x]) / wd
u2 = (Integrate[sol1 Exp[x] Tan[2 x], x]) / wd
yc = DSolve[y''[x] - 2 y'[x] + 5 y[x] == 0, y[x], x]
yp = Simplify[
  Evaluate[y[x] /. Sol[[1]] /. {C[1] → u1, C[2] → u2}]]
yg =
 yc +
  yp
```

$\{\{y[x] \to e^x C[2]\, \mathrm{Cos}[2x] + e^x C[1]\, \mathrm{Sin}[2x]\}\}$

$e^x\, \mathrm{Sin}[2x]$

$e^x\, \mathrm{Cos}[2x]$

$\{e^x\, \mathrm{Sin}[2x],\ e^x\, \mathrm{Cos}[2x]\}$

$$\begin{pmatrix} e^x \mathrm{Sin}[2x] & e^x \mathrm{Cos}[2x] \\ 2 e^x \mathrm{Cos}[2x] + e^x \mathrm{Sin}[2x] & e^x \mathrm{Cos}[2x] - 2 e^x \mathrm{Sin}[2x] \end{pmatrix}$$

$-2 e^{2x}$

$$\frac{1}{8}(-\mathrm{Cos}[2x] + \mathrm{Sin}[2x])$$

$$\frac{1}{8}\left(\mathrm{Cos}[2x] - (2 - 2i)\, e^{2ix}\ \mathrm{Hypergeometric2F1}\left[\frac{1}{2} - \frac{i}{2}, 1, \frac{3}{2} - \frac{i}{2}, -e^{4ix}\right] + \mathrm{Sin}[2x]\right)$$

$\{\{y[x] \to e^x C[2]\, \mathrm{Cos}[2x] + e^x C[1]\, \mathrm{Sin}[2x]\}\}$

$$\frac{1}{8}\, e^{x} \left(1 - (1 - i)\left(1 + e^{4 i x}\right)\right.$$
$$\left.\text{Hypergeometric2F1}\left[\frac{1}{2} - \frac{i}{2},\, 1,\, \frac{3}{2} - \frac{i}{2},\, -e^{4 i x}\right]\right)$$

$$\left\{\left\{\frac{1}{8}\, e^{x} \left(1 - (1 - i)\left(1 + e^{4 i x}\right)\right.\right.\right.$$
$$\left.\text{Hypergeometric2F1}\left[\frac{1}{2} - \frac{i}{2},\, 1,\, \frac{3}{2} - \frac{i}{2},\, -e^{4 i x}\right]\right) +$$
$$\left.\left.(y[x] \to e^{x}\, C[2]\, \text{Cos}[2\, x] + e^{x}\, C[1]\, \text{Sin}[2\, x])\right\}\right\}$$

Questin 8: Solve second order differential equation
$d^2 y/dx^2 + 3\,dy/dx + 3 y = e^x/x$
by variation of parameter method

Solution:

```
Sol = DSolve[y''[x] + 3 y'[x] + 2 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0}]
sol2 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 1}
fs = {sol1, sol2}
wm = {fs, D[fs, x]}; wm // MatrixForm
wd = Simplify[Det[wm]]
u1 = (Integrate[sol2 Exp[-x] / x, x]) / wd
u2 = (Integrate[sol1 Exp[-x] / x, x]) / wd
yc = DSolve[y''[x] + 3 y'[x] + 2 y[x] == 0, y[x], x]
yp = Simplify[
  Evaluate[y[x] /. Sol[[1]] /. {C[1] → u1, C[2] → u2}]]
yg =
 yc +
  yp
```

$\{\{y[x] \to e^{-2x} C[1] + e^{-x} C[2]\}\}$

e^{-2x}

e^{-x}

$\{e^{-2x}, e^{-x}\}$

$\begin{pmatrix} e^{-2x} & e^{-x} \\ -2 e^{-2x} & -e^{-x} \end{pmatrix}$

e^{-3x}

e^{3x} ExpIntegralEi$[-2x]$

e^{3x} ExpIntegralEi$[-3x]$

$\{\{y[x] \to e^{-2x} C[1] + e^{-x} C[2]\}\}$

$e^{x} (e^{x}$ ExpIntegralEi$[-3x]$ + ExpIntegralEi$[-2x])$

$\{\{e^{x} (e^{x}$ ExpIntegralEi$[-3x]$ + ExpIntegralEi$[-2x]) + (y[x] \to e^{-2x} C[1] + e^{-x} C[2])\}\}$

Practical-5

Solution of systems of ordinary differential equations

DSolve[{eqn1,eqn2,…},{Subscript[y, 1][x],Subscript[y, 2][x] ,…},x] solve a system of differential equations for Subscript[y, i][x]

Question 1: find the general solution of the following linear system
2 dx/dt-2dy/dt-3x = t, 2dx/dt+2dy/dt+3x+8y = 2.

Solution:

```
DSolve[{2 x'[t] - 2 y'[t] - 3 x[t] == t,
  2 x'[t] + 2 y'[t] + 3 x[t] + 8 y[t] == 2}, {x[t], y[t]}, t]
```

$$\Big\{\Big\{x[t] \to \frac{1}{64} e^{-3t}\left(1+3e^{4t}\right)\left(e^{-t}(-7-5t)+e^{3t}\left(\frac{19}{9}-\frac{t}{3}\right)\right) - \frac{1}{64} e^{-3t}\left(-1+e^{4t}\right)\left(e^{3t}\left(\frac{19}{3}-t\right)+e^{-t}(7+5t)\right) + \frac{1}{4} e^{-3t}\left(1+3e^{4t}\right) C[1] - \frac{1}{2} e^{-3t}\left(-1+e^{4t}\right) C[2],$$

$$y[t] \to -\frac{3}{128} e^{-3t}\left(-1+e^{4t}\right)\left(e^{-t}(-7-5t)+e^{3t}\left(\frac{19}{9}-\frac{t}{3}\right)\right) + \frac{1}{128} e^{-3t}\left(3+e^{4t}\right)\left(e^{3t}\left(\frac{19}{3}-t\right)+e^{-t}(7+5t)\right) - \frac{3}{8} e^{-3t}\left(-1+e^{4t}\right) C[1] + \frac{1}{4} e^{-3t}\left(3+e^{4t}\right) C[2]\Big\}\Big\}$$

Question 2: find the general solution of the following linear system
dx/dt+dy/dt-2x-4y = e^t, dx/dt+dy/dt-y = e^4t.

Solution:

```
DSolve[{x'[t] + y'[t] - 2 x[t] - 4 y[t] == Exp[t],
  x'[t] + y'[t] - y[t] == Exp[4 t]}, {x[t], y[t]}, t]
```

$$\left\{\left\{x[t] \to -e^{t}\left(-1+e^{3t}\right)+\frac{1}{3}\left(3 e^{t}\left(-1+e^{3t}\right)+e^{-2t} C[1]\right),\right.\right.$$
$$\left.\left. y[t] \to e^{t}\left(-1+e^{3t}\right)-\frac{2}{9}\left(3 e^{t}\left(-1+e^{3t}\right)+e^{-2t} C[1]\right)\right\}\right\}$$

Question 3: find the general solution of the following linear system dx/dt +dy/dtx = -2t, dx/dt+dy/dt-3x-y = t^2.

Solution:

```
DSolve[{x'[t] + y'[t] - x[t] == -2 t,
  x'[t] + y'[t] - 3 x[t] - y[t] == t^2}, {x[t], y[t]}, t]
```

$$\left\{\left\{x[t] \to -2t-t^{2}+\frac{1}{4}\left(4\left(-2+2t+t^{2}\right)-e^{-t} C[1]\right),\right.\right.$$
$$\left.\left. y[t] \to 2t+t^{2}+\frac{1}{2}\left(-4\left(-2+2t+t^{2}\right)+e^{-t} C[1]\right)\right\}\right\}$$

Question 4: find the general solution of the following linear system dx/dt +dy/dtx -3y= e^t, dx/dt+dy/dt+x = e^3t.

Solution:

```
DSolve[{x'[t] + y'[t] - x[t] - 3 y[t] == Exp[t],
  x'[t] + y'[t] + x[t] == Exp[3 t]}, {x[t], y[t]}, t]
```

$$\left\{\left\{x[t] \to -e^{t}\left(-1+e^{2t}\right)+\frac{3}{16}\left(\frac{4}{3} e^{t}\left(-3+4 e^{2t}\right)+e^{-3t} C[1]\right),\right.\right.$$
$$\left.\left. y[t] \to e^{t}\left(-1+e^{2t}\right)+\frac{1}{8}\left(-\frac{4}{3} e^{t}\left(-3+4 e^{2t}\right)-e^{-3t} C[1]\right)\right\}\right\}$$

Question 5: find the general solution of the following linear system dy/dt=0, dx/dt+10x = t^2, dz/dt+24z = e^t..

Solution:

```
DSolve[{y'[t] == 0, x'[t] + 10 x[t] == t^2,
  z'[t] + 24 z[t] == Exp[t]}, {y[t], x[t], z[t]}, t]
```

$$\left\{\left\{y[t] \to C[1],\ x[t] \to \frac{1}{500}\left(1-10\,t+50\,t^2\right)+e^{-10\,t}\,C[2],\ z[t] \to \frac{e^t}{25}+e^{-24\,t}\,C[3]\right\}\right\}$$

Question 6: find the general solution of the following linear system d^2x/dt^2+dy/dt-x+y = 1, d^2y/dt^2+ dx/dt-x+y = 0..

Solution:

```
DSolve[{x''[t] + y'[t] - x[t] + y[t] == 1,
  y''[t] + x'[t] - x[t] + y[t] == 0}, {x[t], y[t]}, t]
```

$$\Big\{\Big\{x[t] \to \left(e^t - t\right)\left(1 - e^t + t\right) + e^{-t}\left(-e^{-t} + e^t - t\right)\left(-1 + e^{2\,t} - e^t\,t\right) + e^{-t}\left(1 + e^t\,t\right)\left(-e^{-t} - e^t + \frac{t^2}{2}\right) - e^{-t}\left(1 - e^t + e^t\,t\right)\left(-e^t + t + \frac{t^2}{2}\right) + e^{-t}\left(1 + e^t\,t\right)C[1] + e^{-t}\left(-1 + e^{2\,t} - e^t\,t\right)C[2] - e^{-t}\left(1 - e^t + e^t\,t\right)C[3] + \left(1 - e^t + t\right)C[4],$$

$$y[t] \to \left(e^t - t\right)t - e^{-t}\left(-e^{-t} + e^t - t\right)\left(1 - e^t + e^t\,t\right) + e^{-t}\left(1 - e^t + e^t\,t\right)\left(-e^{-t} - e^t + \frac{t^2}{2}\right) - e^{-t}\left(1 - 2\,e^t + e^t\,t\right)\left(-e^t + t + \frac{t^2}{2}\right) + e^{-t}\left(1 - e^t + e^t\,t\right)C[1] - e^{-t}\left(1 - e^t + e^t\,t\right)C[2] - e^{-t}\left(1 - 2\,e^t + e^t\,t\right)C[3] + t\,C[4]\Big\}\Big\}$$

Question 7: find the general solution of the following linear system d^2x/dt^2-dy/dt-x+y = e^t,
dy/dt+ dx/dt-4x-y = 2e^t.

Solution:

```
DSolve[{x''[t] - y'[t] - x[t] + y[t] == Exp[t],
  x'[t] + y'[t] - 4 x[t] - y[t] == 2 Exp[t]}, {x[t], y[t]}, t]
```

$$\Big\{\Big\{x[t] \to -\frac{1}{168\sqrt{21}}\left(-3+\sqrt{21}\right)\left(3+\sqrt{21}\right)e^{-\frac{1}{2}\left(-3+\sqrt{21}\right)t}\left(-7-3\sqrt{21}+\left(-7+3\sqrt{21}\right)e^{\sqrt{21}\,t}\right)\left(e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}-e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}\right)-$$

$$\frac{1}{84}e^{-\frac{1}{2}\left(-3+\sqrt{21}\right)t}\left(3+\sqrt{21}+\left(-3+\sqrt{21}\right)e^{\sqrt{21}\,t}\right)\left(-e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}+\sqrt{21}\,e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}+e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}+\sqrt{21}\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}\right)+$$

$$\frac{1}{2\sqrt{21}}\left(-e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}+\sqrt{21}\,e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}+e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}+\sqrt{21}\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}\right)C[1]-$$

$$\frac{\left(-3+\sqrt{21}\right)\left(3+\sqrt{21}\right)\left(e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}-e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}\right)C[2]}{12\sqrt{21}},$$

$$y[t] \to \frac{1}{588}e^{-\frac{1}{2}\left(-3+\sqrt{21}\right)t}\left(-7-3\sqrt{21}+\left(-7+3\sqrt{21}\right)e^{\sqrt{21}\,t}\right)\left(-42\,e^{t}+21\,e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}-\sqrt{21}\,e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}+21\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}+\sqrt{21}\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}\right)-$$

$$\frac{1}{84\sqrt{21}}e^{-\frac{1}{2}\left(-3+\sqrt{21}\right)t}\left(3+\sqrt{21}+\left(-3+\sqrt{21}\right)e^{\sqrt{21}\,t}\right)\left(-42\,e^{t}+21\,e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}-11\sqrt{21}\,e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}+21\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}+11\sqrt{21}\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}\right)-\frac{1}{7}e^{t-\frac{1}{2}\left(-1+\sqrt{21}\right)t}$$

$$\left(\left(7+2\sqrt{21}\right)e^{t}+\left(7-2\sqrt{21}\right)e^{t+\sqrt{21}\,t}+7\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}\,t\right)+$$

$$\frac{1}{42}\left(-42\,e^{t}+21\,e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}-11\sqrt{21}\,e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}+\right.$$
$$\left.21\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}+11\sqrt{21}\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}\right)C[1]+$$

$$\frac{1}{42}\left(-42\,e^{t}+21\,e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}-\sqrt{21}\,e^{\frac{1}{2}\left(-1-\sqrt{21}\right)t}+\right.$$
$$\left.21\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}+\sqrt{21}\,e^{\frac{1}{2}\left(-1+\sqrt{21}\right)t}\right)C[2]+e^{t}C[3]\}\}$$

Question 8: find the general solution of the following linear system dx/dt+2x-3y
= t, dy/dt-3x+2y = e^2t.

Solution:

```
DSolve[{x'[t] + 2 x[t] - 3 y[t] == t,
  y'[t] - 3 x[t] + 2 y[t] == Exp[2 t]}, {x[t], y[t]}, t]
```

$$\Big\{\Big\{x[t]\to\frac{1}{4}e^{-5t}\left(-1+e^{6t}\right)\left(e^{t}+\frac{e^{7t}}{7}+e^{-t}(-1-t)+e^{5t}\left(\frac{1}{25}-\frac{t}{5}\right)\right)+$$
$$\frac{1}{4}e^{-5t}\left(1+e^{6t}\right)\left(e^{t}-\frac{e^{7t}}{7}+e^{-t}(-1-t)+e^{5t}\left(-\frac{1}{25}+\frac{t}{5}\right)\right)+$$
$$\frac{1}{2}e^{-5t}\left(1+e^{6t}\right)C[1]+\frac{1}{2}e^{-5t}\left(-1+e^{6t}\right)C[2],$$

$$y[t]\to\frac{1}{4}e^{-5t}\left(1+e^{6t}\right)\left(e^{t}+\frac{e^{7t}}{7}+e^{-t}(-1-t)+e^{5t}\left(\frac{1}{25}-\frac{t}{5}\right)\right)+$$
$$\frac{1}{4}e^{-5t}\left(-1+e^{6t}\right)\left(e^{t}-\frac{e^{7t}}{7}+e^{-t}(-1-t)+e^{5t}\left(-\frac{1}{25}+\frac{t}{5}\right)\right)+$$
$$\frac{1}{2}e^{-5t}\left(-1+e^{6t}\right)C[1]+\frac{1}{2}e^{-5t}\left(1+e^{6t}\right)C[2]\Big\}\Big\}$$

Practical - 6

Solution of Cauchy problem for first order PDE

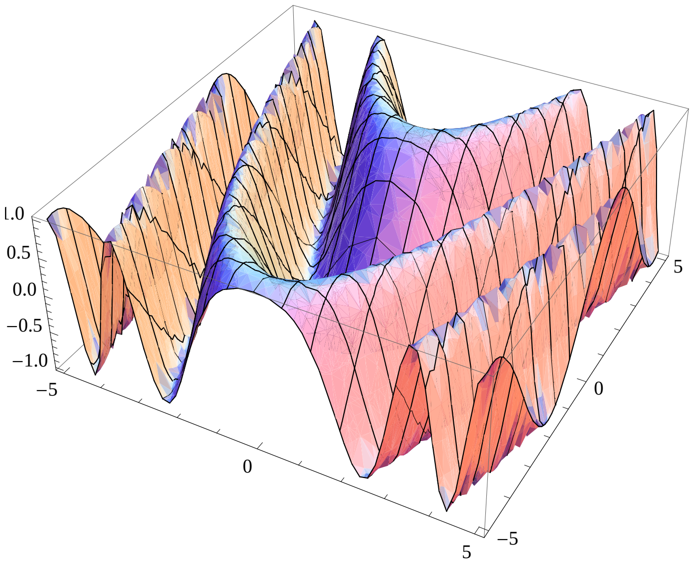
Question-1

```
eqn1 = D[u[x, y], x] + x * D[u[x, y], y] == 0
```

$x\,u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$

```
sol1 =
 u[x, y] /. DSolve[{eqn1, u[0, y] == Sin[y]}, u[x, y], {x, y}]
Plot3D[sol1, {x, -5, 5}, {y, -5, 5}]
```

$\left\{\text{Sin}\left[\frac{1}{2}\left(-x^2 + 2\,y\right)\right]\right\}$



Question-2

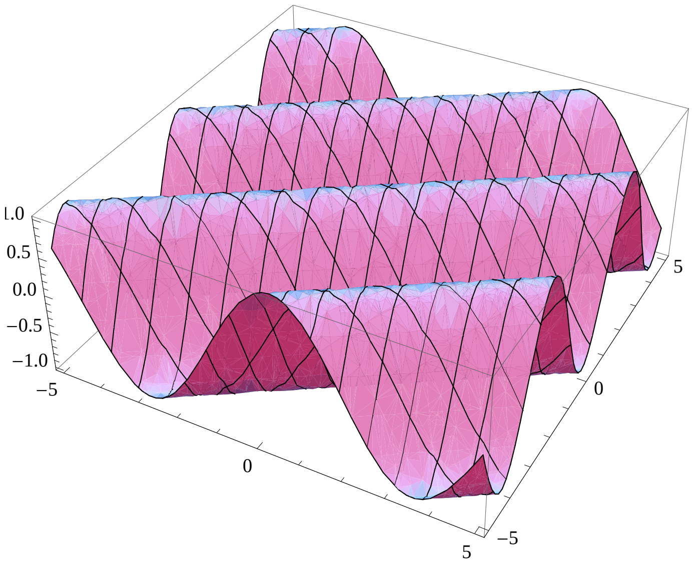
```
eqn2 = 3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0
```

$2\,u^{(0,1)}[x, y] + 3\,u^{(1,0)}[x, y] == 0$

```
sol2 =
 u[x, y] /. DSolve[{eqn2, u[x, 0] == Sin[x]}, u[x, y], {x, y}]
```

$\left\{\text{Sin}\left[\frac{1}{2}\,(2\,x - 3\,y)\right]\right\}$

```
Plot3D[sol2, {x, -5, 5}, {y, -5, 5}]
```



Question-3

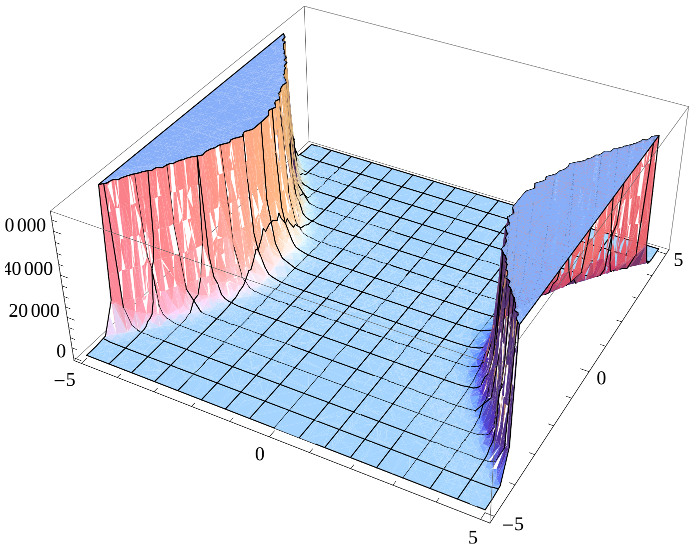
```
eqn3 = y * D[u[x, y], x] + x * D[u[x, y], y] == 0
```

$x\, u^{(0,1)}[x, y] + y\, u^{(1,0)}[x, y] == 0$

```
sol3 = u[x, y] /.
  DSolve[{eqn3, u[0, y] == Exp[-y^2]}, u[x, y], {x, y}]
```

$\{e^{x^2-y^2}\}$

```
Plot3D[sol3, {x, -5, 5}, {y, -5, 5}]
```



Question 4 : ∂xu[x,y] + 2*∂yu[x,y] = 1 + u[x, y] u[x, y] = Sin[x] on y = 3*x + 1;

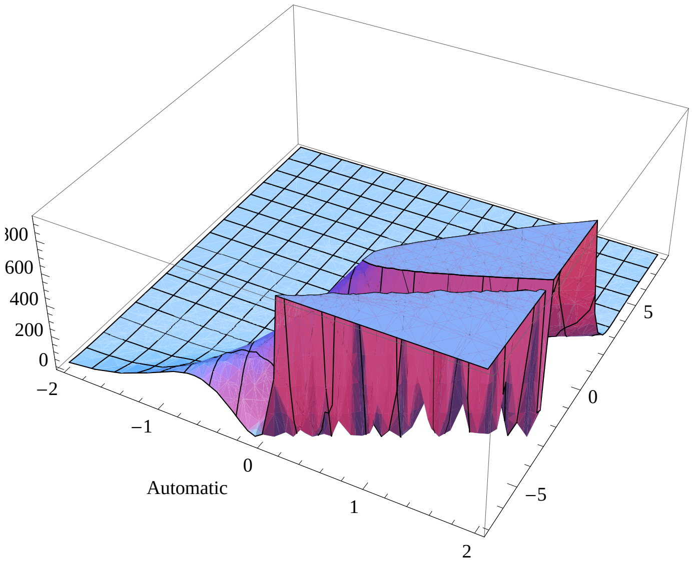
```
A = D[u[x, y], x] + 2*D[u[x, y], y] == 1 + u[x, y]
```

$2\,u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1 + u[x, y]$

```
sol = DSolve[{A, u[x, 3*x + 1] == Sin[x]}, u[x, y], {x, y}]
```

$\{\{u[x, y] \to -e^{-y}\left(-e^{1+3x} + e^{y} + e^{1+3x}\,\mathrm{Sin}[1 + 2x - y]\right)\}\}$

```
Plot3D[u[x, y] /. sol, {x, -2, 2},
 {y, -7, 8}, AxesLabel → {Automatic}]
```



Question 5. Solve the PDE Subscript[uu, x]+ Subscript[u, y]=1/2. With the initial condition u(s,s)=s/4,0<=s<=1.
Solution: x=s+st/4=t^2/4, y=s+t, u=s/4+t/2

```
sol = DSolve[{x'[t] == u[t], y'[t] == 1, u'[t] == 1/2,
   x[0] == s, y[0] == s, u[0] == s/4}, {x[t], y[t], u[t]}, t]
```

$\{\{u[t] \to \frac{1}{4}(s + 2t),\ x[t] \to \frac{1}{4}\left(4s + st + t^2\right),\ y[t] \to s + t\}\}$

```
Print["u[t]=", sol[[1, 1, 2]]]
```

$u[t] = \frac{1}{4}(s + 2t)$

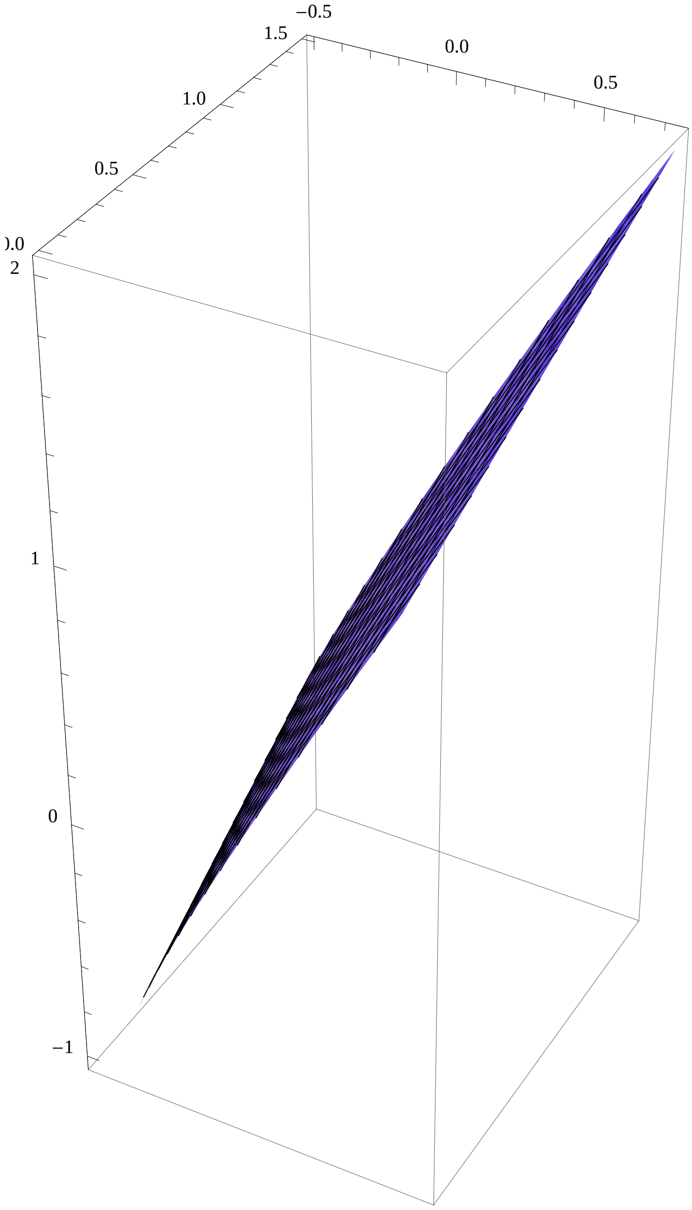
```
Print["y[t]=", sol[[1, 2, 2]]]
```

$y[t] = \frac{1}{4}\left(4s + st + t^2\right)$

```
Print["x[t]=", sol[[1, 3, 2]]]
```

$x[t] = s + t$

```
map = ParametricPlot3D[
  {sol[[1, 1, 2]], sol[[1, 2, 2]], sol[[1, 3, 2]]},
  {t, -1, 1}, {s, 0, 1}, PlotPoints → 10]
```



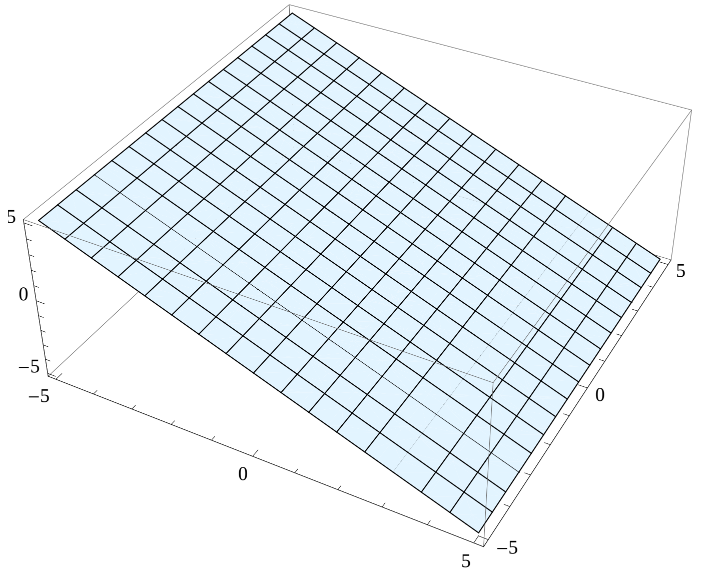
Practical-7

Plotting the characteristics for the first order PDE.

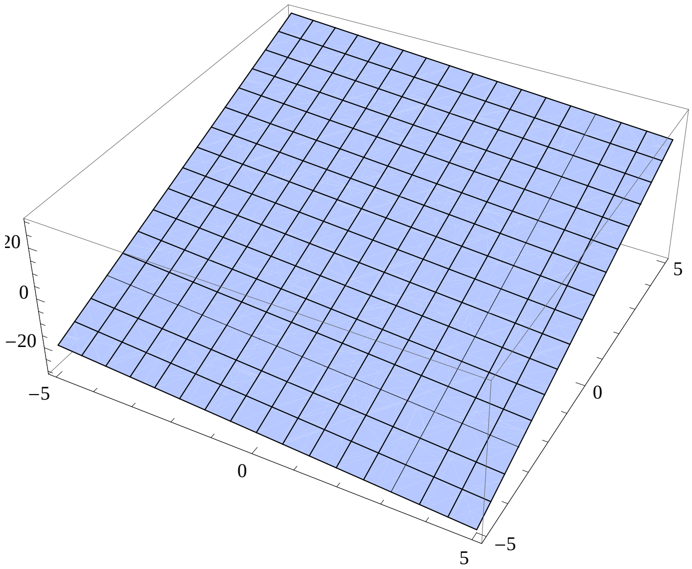
Find Characteristic Equation of the Curve (u-y)*ux+y*uy=x+y
dx/(u-y) = dy/y = du/(x+y)

On taking I+ III and II ,
we get (u+x)/y=C1
On taking I + II=III,
we get (x+y)^2-u^2=C2
On Integrate to plot this some particular values

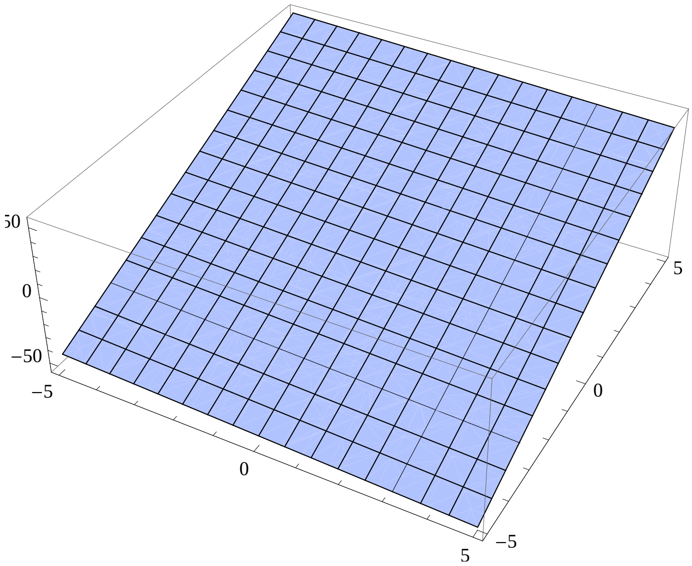
```
F0 = Plot3D[-x, {x, -5, 5}, {y, -5, 5}, PlotPoints -> 10]
```



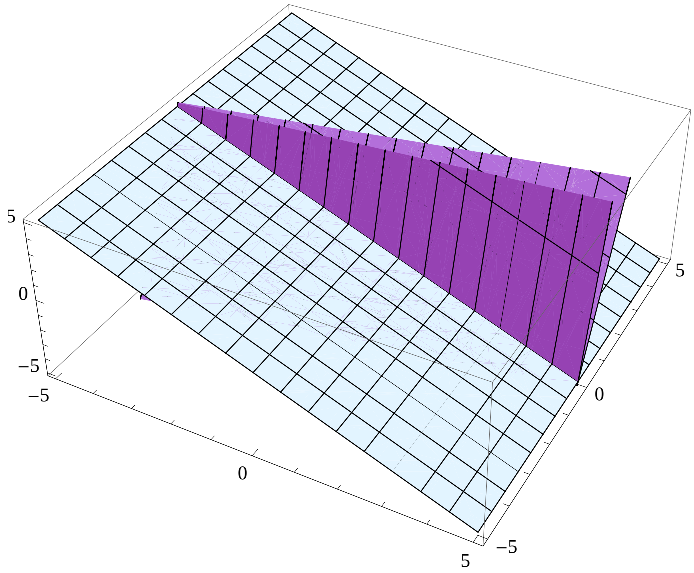
```
F1 = Plot3D[5 * y - x, {x, -5, 5},
  {y, -5, 5}, PlotPoints -> 10]
```



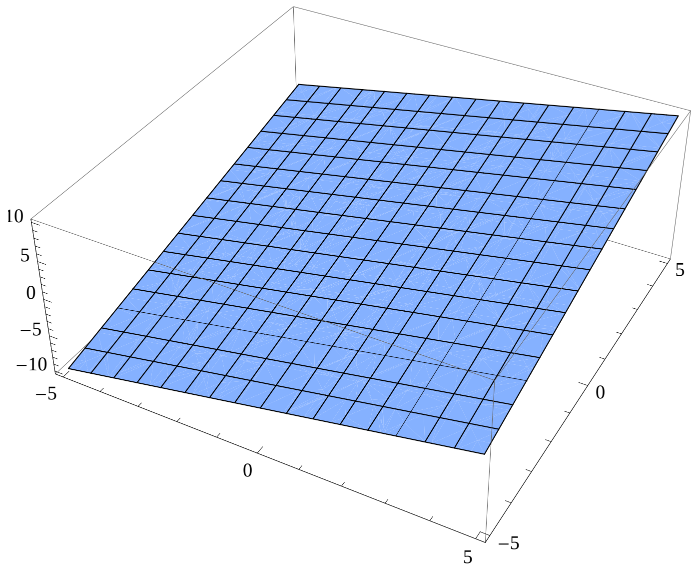

```
F2 = Plot3D[10 * y - x,
  {x, -5, 5}, {y, -5, 5}, PlotPoints -> 10]
```



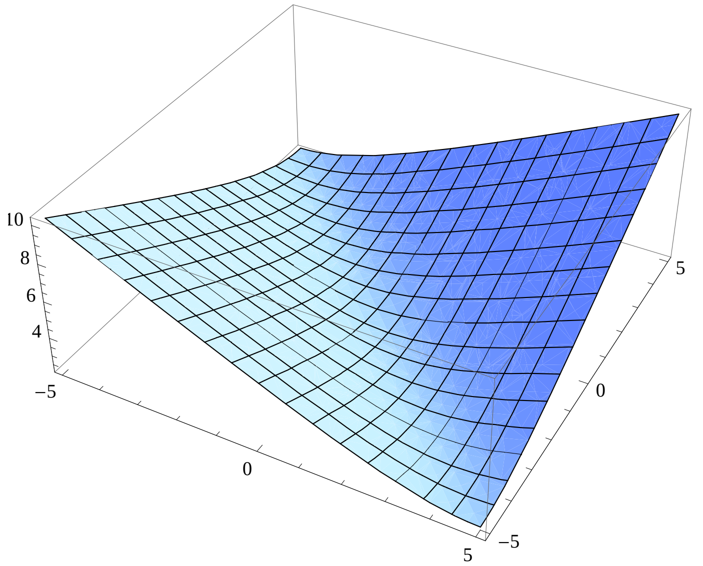
```
G1 = Show[F0, F1, F2]
```



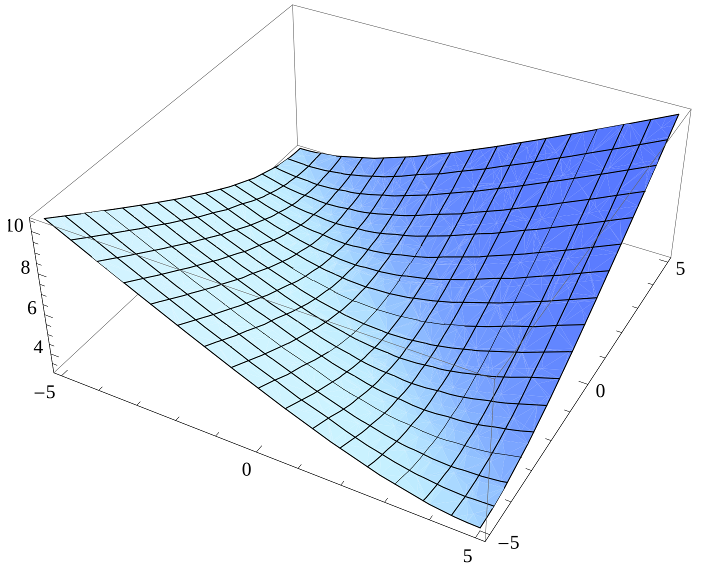

```
H0 = Plot3D[x + y, {x, -5, 5}, {y, -5, 5}, PlotPoints -> 10]
```



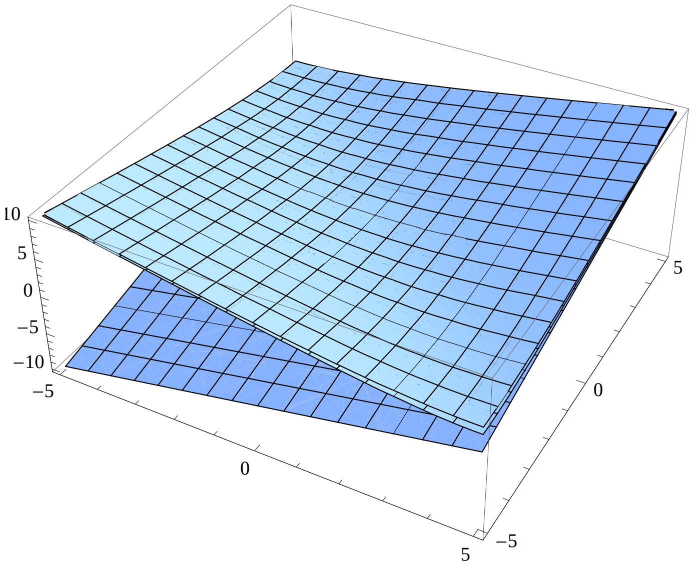
```
H1 = Plot3D[Sqrt[(x + y)^2 + 5],
  {x, -5, 5}, {y, -5, 5}, PlotPoints -> 10]
```




```
H2 = Plot3D[Sqrt[(x + y)^2 + 10],
  {x, -5, 5}, {y, -5, 5}, PlotPoints -> 10]
```



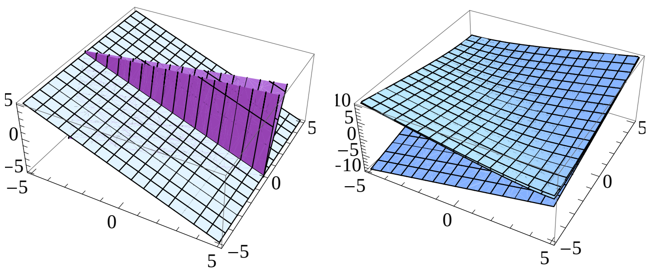
```
G2 = Show[H0, H1, H2]
```




```
Show[GraphicsArray[{G1, G2}]]
```

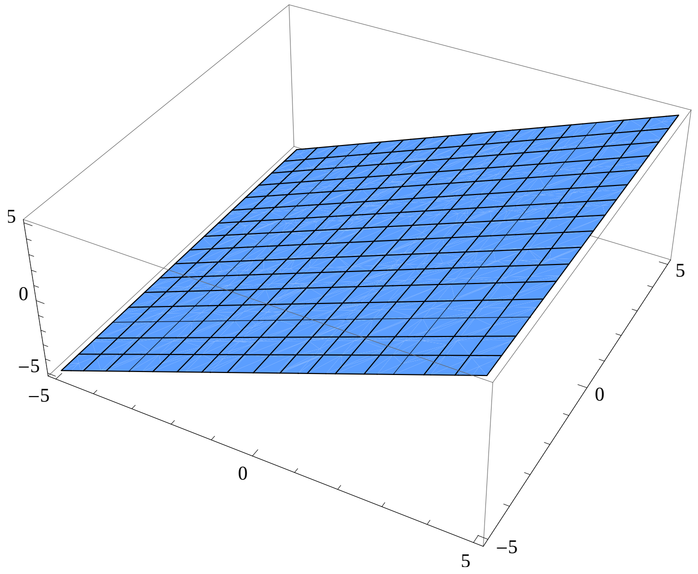
GraphicsArray::obs :

GraphicsArray is obsolete. Switching to GraphicsGrid. ≫

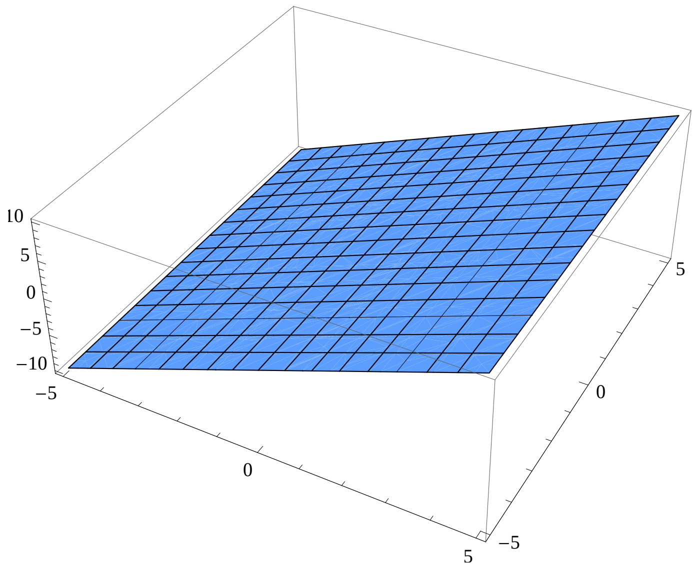


Find Characteristic Equation of the Curve (x*ux + y*uy = u)
dx/(x) = dy/(y) = du/(u)
On taking I and III ,
we get x/u = C1
On taking II = III,
we get y/u = C2
On Integrate to plot this some particular values

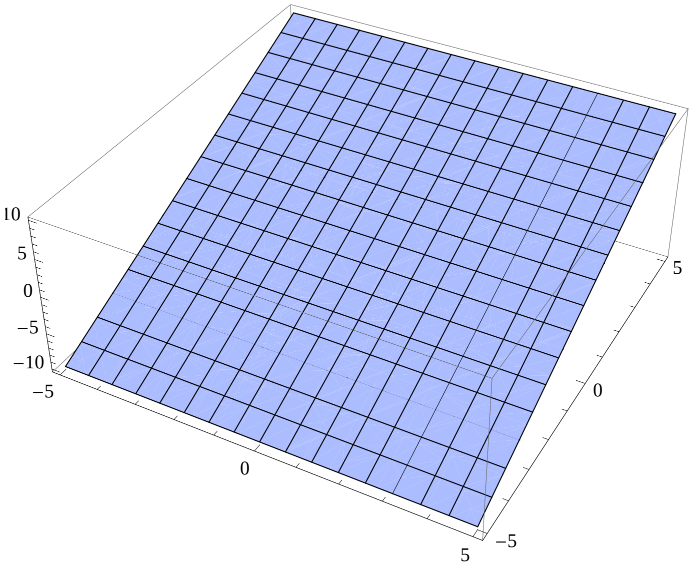
```
F0 = Plot3D[x, {x, -5, 5}, {y, -5, 5}, PlotPoints -> 10]
```



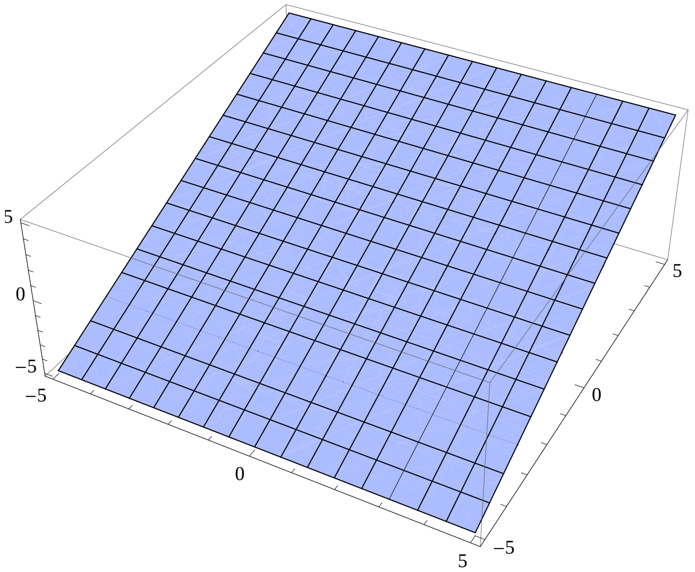

```
F1 = Plot3D[2 x, {x, -5, 5},
  {y, -5, 5}, PlotPoints -> 10]
```



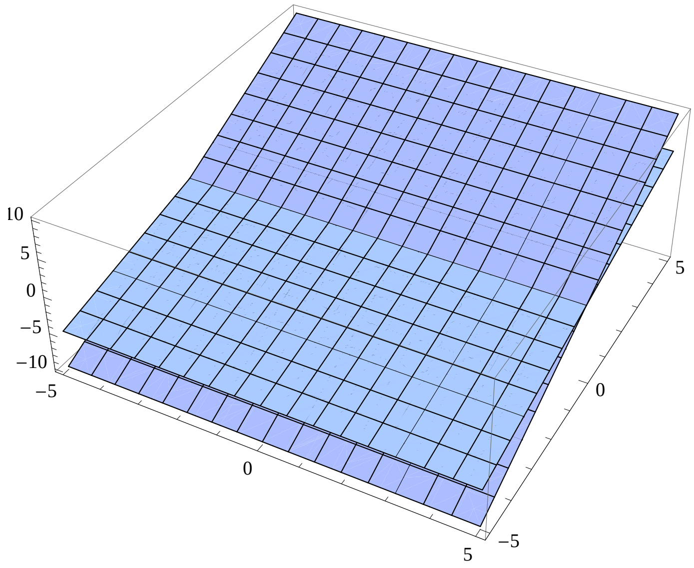
```
H0 = Plot3D[2 y, {x, -5, 5},
    {y, -5, 5}, PlotPoints → 10] H0 =
  Plot3D[2 y, {x, -5, 5}, {y, -5, 5}, PlotPoints -> 10]
```



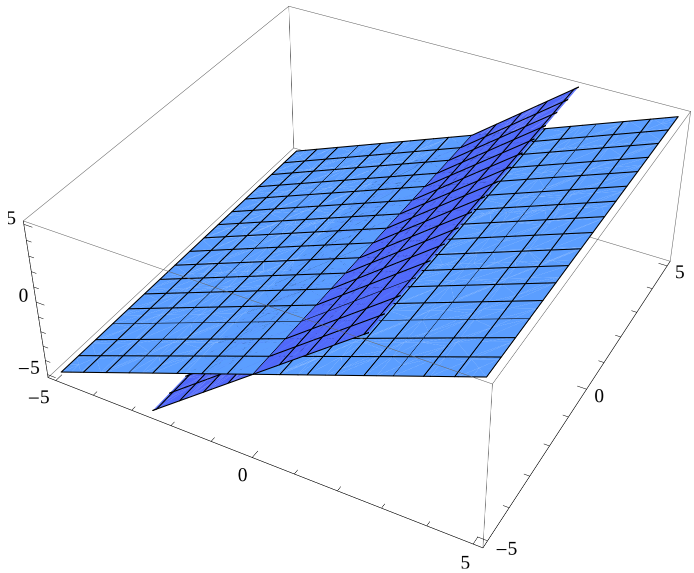

```
H1 = Plot3D[y, {x, -5, 5}, {y, -5, 5}, PlotPoints -> 10]
```



```
G2 = Show[H0, H1]
```



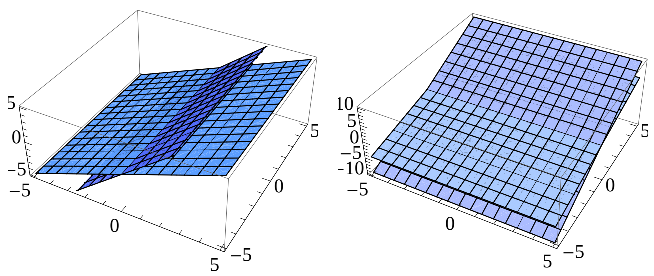

```
G1 = Show[F0, F1]
```




```
Show[GraphicsArray[{G1, G2}]]
```

GraphicsArray::obs :

GraphicsArray is obsolete. Switching to GraphicsGrid. ≫



Practical-8

Plot the integral surfaces of a given first order PDE with initial data.

Question 1. Solve the PDE Subscript[uu, x]+ Subscript[u, y]=1/2. With the initial
condition u(s,s)=s/4,
0<=s<=1.
Solution: x=s+st/4+t^2/4, y=s+t, u=s/4+t/2.

```
sol = DSolve[{x'[t] == u[t], y'[t] == 1, u'[t] == 1/2,
   x[0] == s, y[0] == s, u[0] == s/4}, {x[t], y[t], u[t]}, t]
```

$$\left\{\left\{u[t] \to \frac{1}{4}(s+2t),\ x[t] \to \frac{1}{4}\left(4s+st+t^2\right),\ y[t] \to s+t\right\}\right\}$$

```
Print["u[t]=", sol[[1, 1, 2]]]
```

$u[t]=\frac{1}{4}(s+2t)$

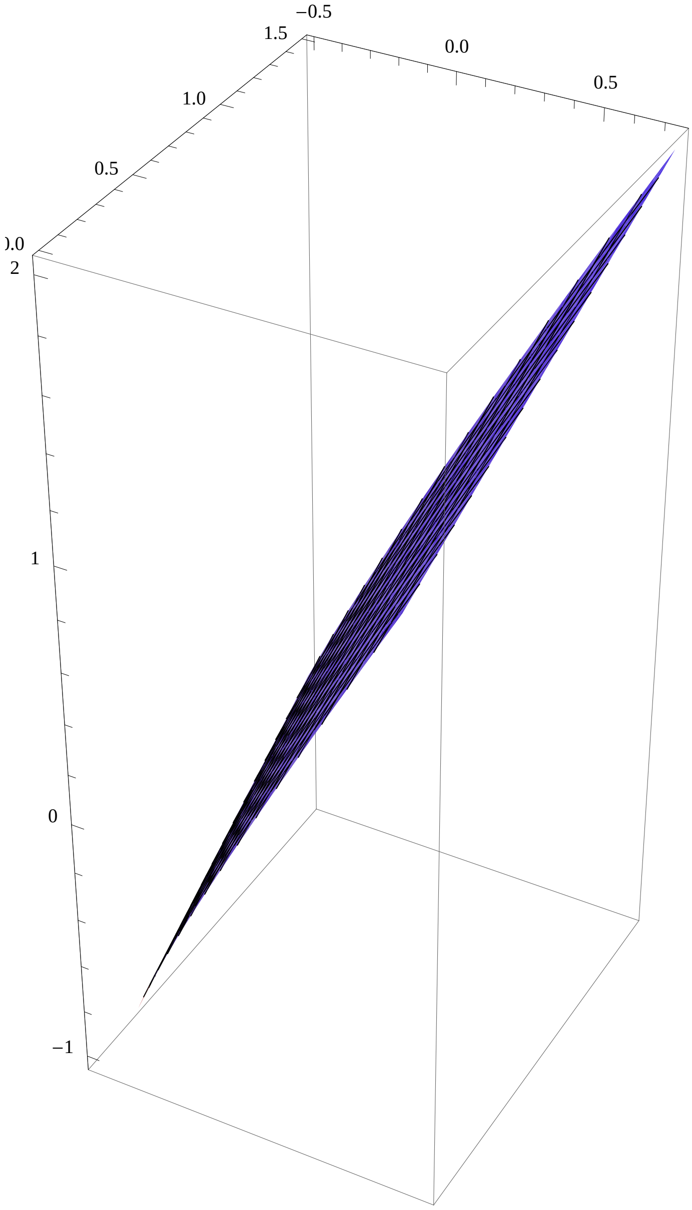
```
Print["y[t]=", sol[[1, 2, 2]]]
```

$y[t]=\frac{1}{4}\left(4s+st+t^2\right)$

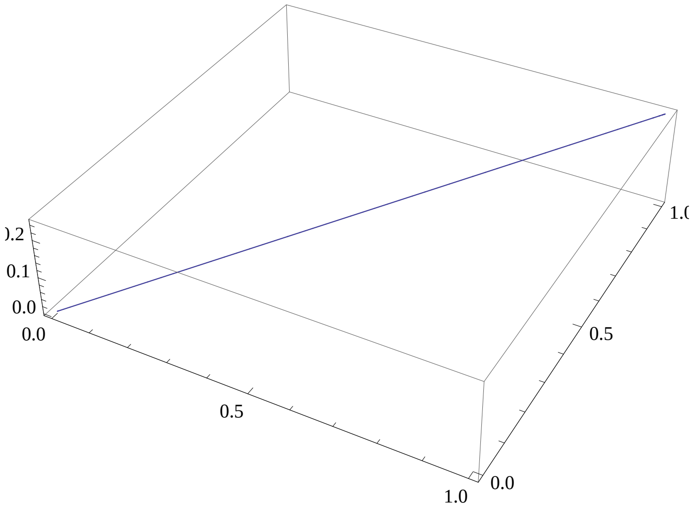
```
Print["x[t]=", sol[[1, 3, 2]]]
```

$x[t]=s+t$

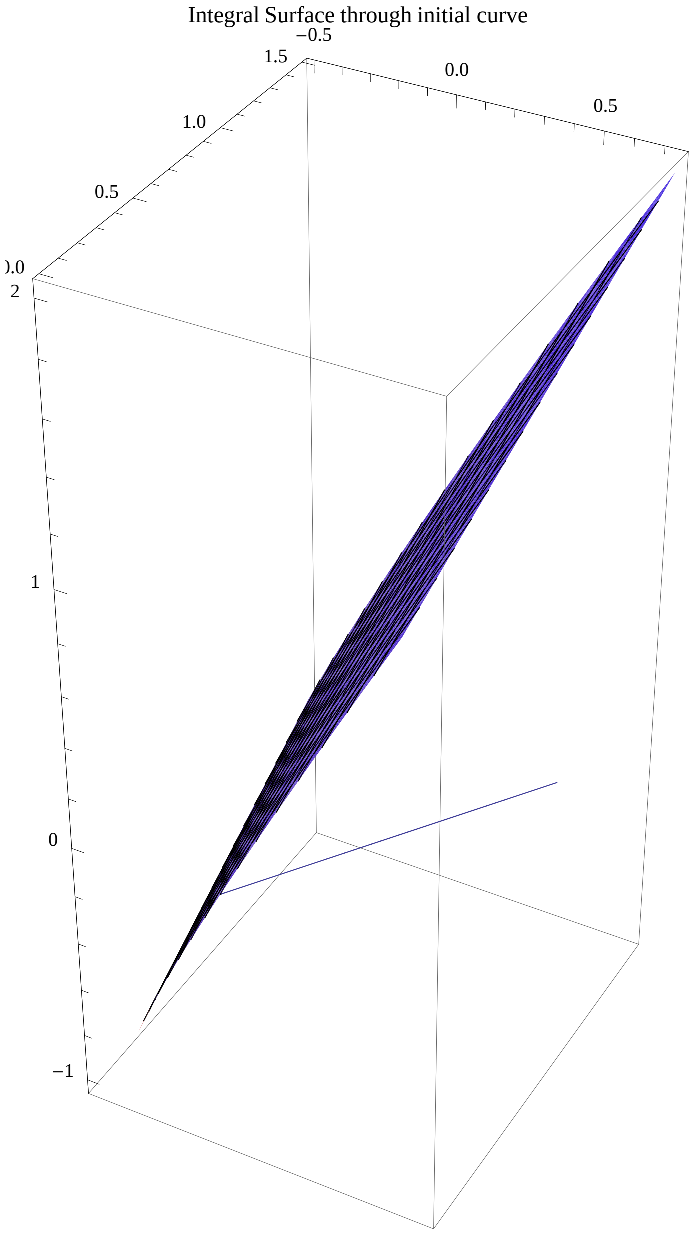
```
map = ParametricPlot3D[
  {sol[[1, 1, 2]], sol[[1, 2, 2]], sol[[1, 3, 2]]},
  {t, -1, 1}, {s, 0, 1}, PlotPoints -> 10]
```



```
map1 = ParametricPlot3D[{s, s, s / 4}, {s, 0, 1}]
```



```
Show[map, map1,
 PlotLabel -> "Integral Surface through initial curve "]
```



Question 2. Solve the PDE Subscript[u, x]- Subscript[u, y]=2. With the initial condition u(s,s)=2s,
0<=s<=1.
Solution: x=s + t, y= s - t, u=2t + 2s.

```
sol = DSolve[{x'[t] == 1, y'[t] == -1, u'[t] == 2, x[0] == s,
   y[0] == s, u[0] == 2*s}, {x[t], y[t], u[t]}, t]
```

```
{{x[t] → s + t, y[t] → s - t, u[t] → 2 (s + t)}}
```

```
Print["x[t]=", sol[[1, 1, 2]]]
```

```
x[t]=s + t
```

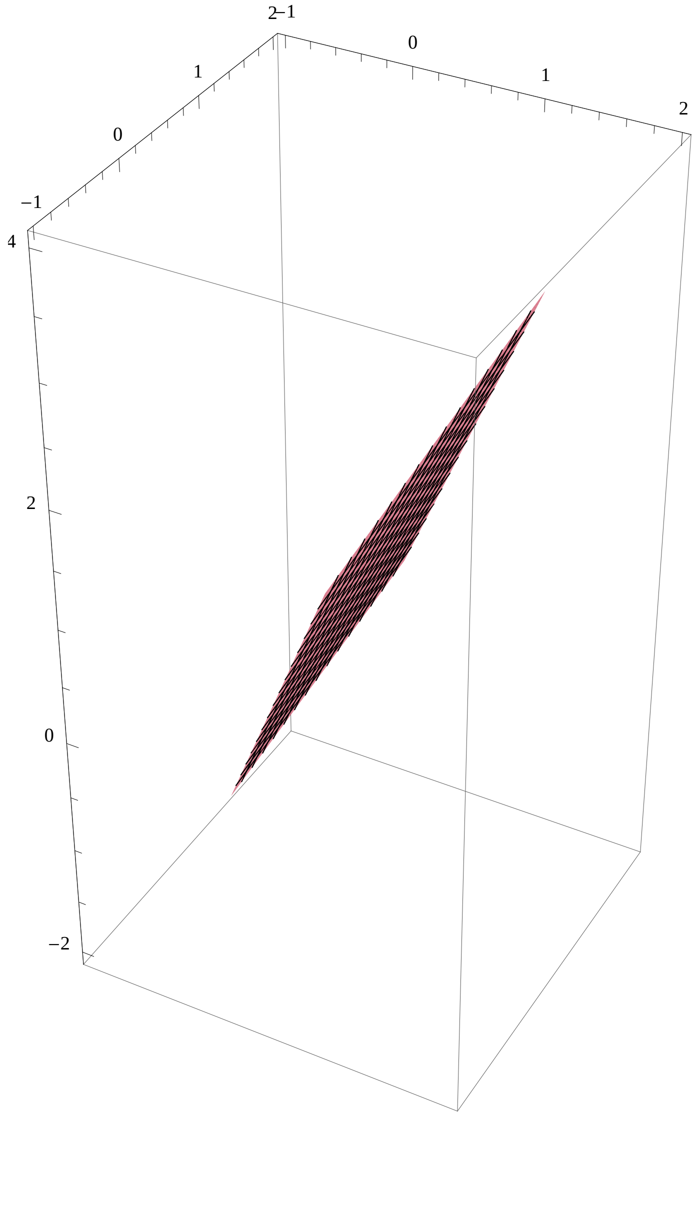
```
Print["y[t]=", sol⟦1, 2, 2⟧]
```

y[t]=s − t

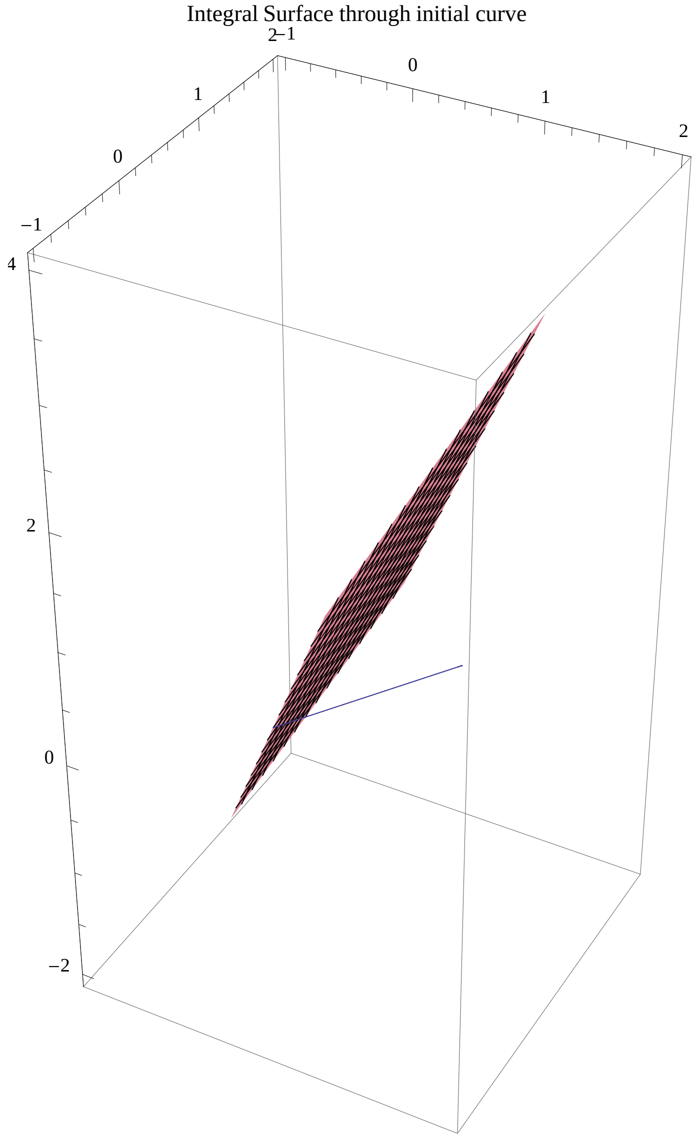
```
Print["u[t]=", sol⟦1, 3, 2⟧]
```

u[t]=2 (s + t)

```
map = ParametricPlot3D[
  {sol⟦1, 1, 2⟧, sol⟦1, 2, 2⟧, sol⟦1, 3, 2⟧},
  {t, -1, 1}, {s, 0, 1}, PlotPoints -> 100]
```



```
Show[map, map1,
 PlotLabel -> "Integral Surface through initial curve "]
```



Question 3. Solve the PDE Subscript[u, x]+ Subscript[u, y]=1. With the initial condition u(s,s)=sin(s),
0<=s<=1.
Solution: x=s + t, y= s + t, u=t + sin(s).

```
sol = DSolve[{x'[t] == 1, y'[t] == 1, u'[t] == 1, x[0] == s,
   y[0] == s, u[0] == Sin[s]}, {x[t], y[t], u[t]}, t]
```

{{x[t] → s + t, y[t] → s + t, u[t] → t + Sin[s]}}

```
Print["u[t]=", sol[[1, 3, 2]]]
```

u[t]=t + Sin[s]

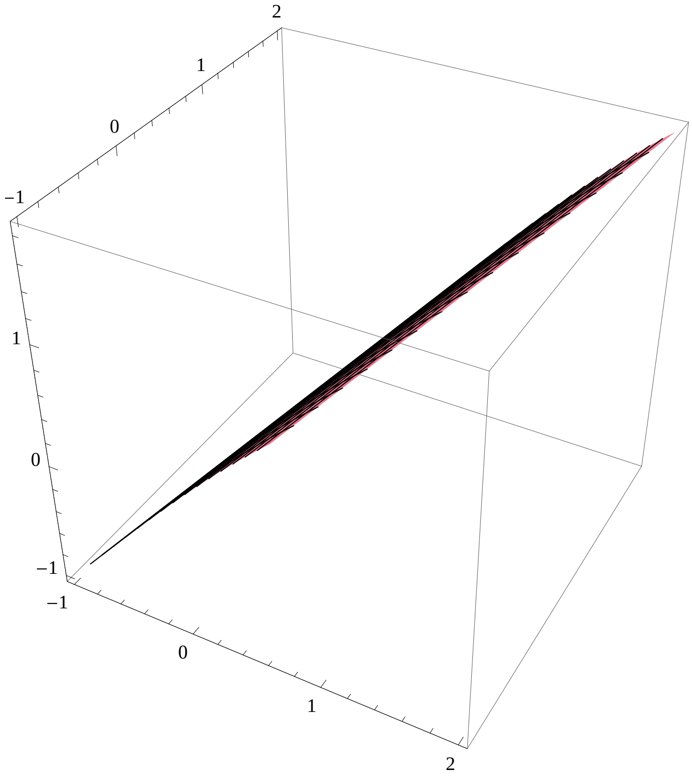
```
Print["x[t]=", sol⟦1, 1, 2⟧]
```

x[t]=s + t

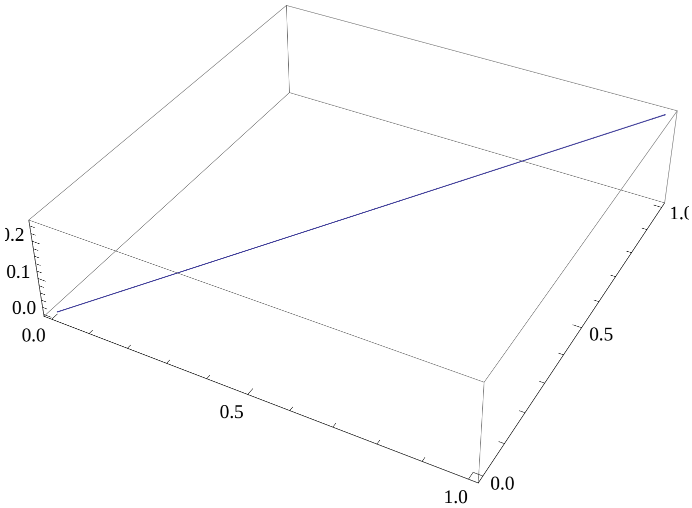
```
Print["y[t]=", sol⟦1, 2, 2⟧]
```

y[t]=s + t

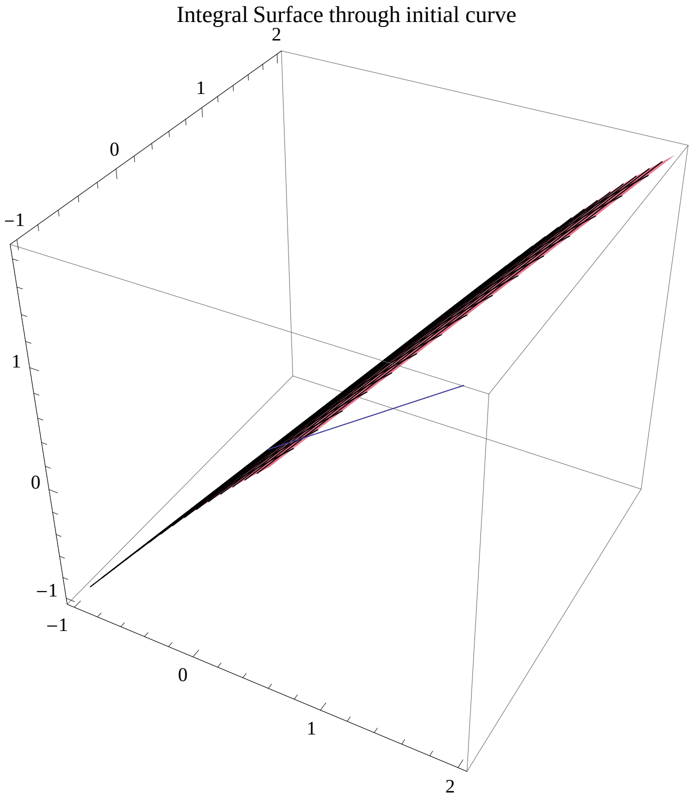
```
map = ParametricPlot3D[{sol⟦1, 1, 2⟧,
   sol⟦1, 2, 2⟧, sol⟦1, 3, 2⟧}, {t, -1, 1}, {s, 0, 1}]
```




```
map1 = ParametricPlot3D[{s, s, s / 4}, {s, 0, 1}]
```



```
Show[map, map1,
 PlotLabel -> "Integral Surface through initial curve "]
```



Question 4. Solve the PDE Subscript[u, x]+ 2Subscript[u, y]=0. With the ini-tial
condition u(0,s)=4E^(-2s),
0<=s<=1.
Solution: u=4E^(-2s), y= s + 2t, x=t.

```
sol = DSolve[{x'[t] == 1, y'[t] == 2, u'[t] == 0, x[0] == 0,
   y[0] == s, u[0] == 4*E^(-2*s)}, {x[t], y[t], u[t]}, t]
```

$\{\{x[t] \to t,\ y[t] \to s + 2\,t,\ u[t] \to 4\,e^{-2\,s}\}\}$

```
Print["u[t]=", sol[[1, 3, 2]]]
```

$u[t]=4\,e^{-2\,s}$

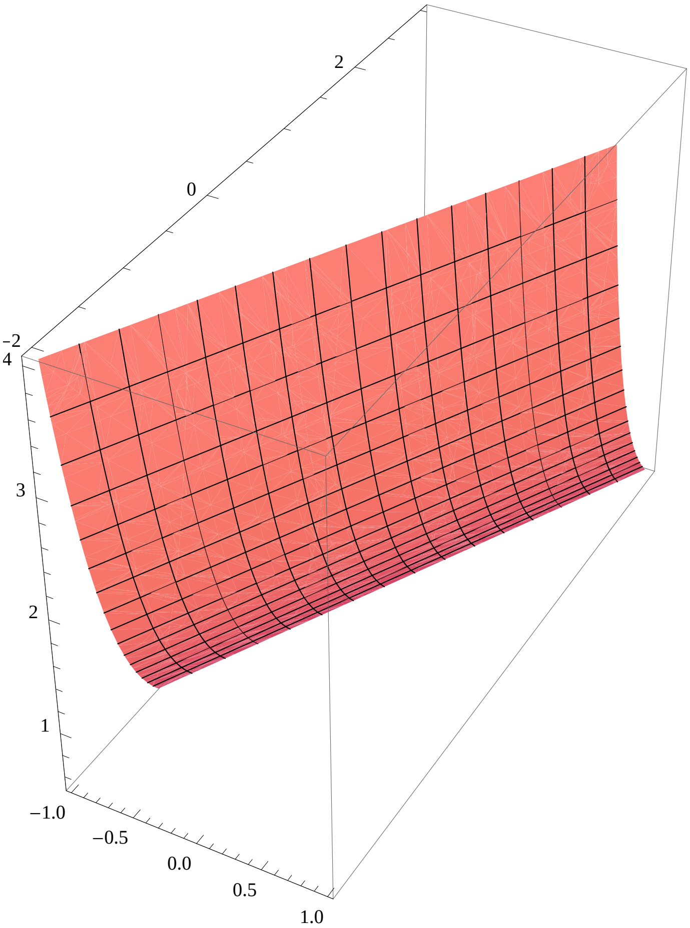
```
Print["x[t]=", sol[[1, 1, 2]]]
```

$x[t]=t$

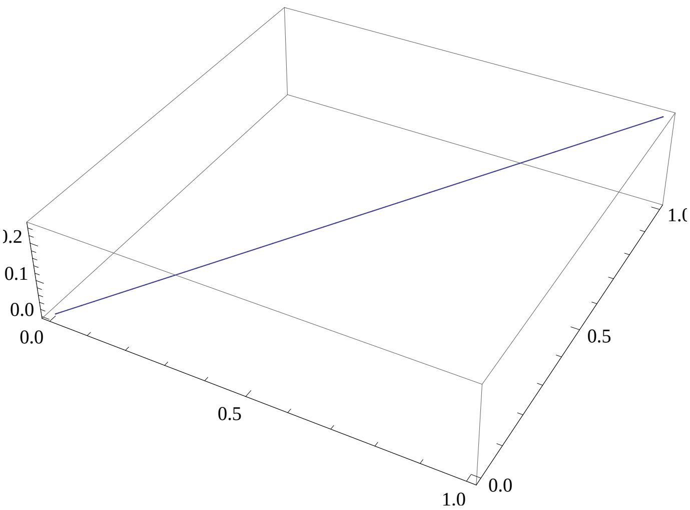
```
Print["y[t]=", sol[[1, 2, 2]]]
```

$y[t]=s+2\,t$

```
map = ParametricPlot3D[{sol⟦1, 1, 2⟧,
   sol⟦1, 2, 2⟧, sol⟦1, 3, 2⟧}, {t, -1, 1}, {s, 0, 1}]
```




```
map1 = ParametricPlot3D[{s, s, s / 4}, {s, 0, 1}]
```



```
Show[map, map1,
 PlotLabel -> "Integral Surface through initial curve "]
```

Integral Surface through initial curve