PRACTICAL-01

Solving first order Ordinary Differential Equations

Command: Dsolve [eqn,y[x],x] Solve a differential equation for y[x]

Dsolve[{eqn1, eqn2,}, { $y_1[x]$, $y_2[x]$,......},x] Solve a system of differential equations for $y_1[x]$

Question 1: Solve first order differential equation dy/dx+y=0

Solution:

DSolve[y'[x] + y[x] == 0, y[x], x]
$$\{ \{y[x] \rightarrow e^{-x} C[1] \} \}$$

Question-2 : Solve the differential equation dy/dx+12y=0 Solution:

DSolve[y'[x] + 12 y[x] == 0, y[x], x]
$$\{\{y[x] \rightarrow e^{-12x}C[1]\}\}$$

Question-3: Solve first order differential equation dz/dt=0. Solution:

DSolve[z'[t] == 0, z[t], t]
$$\{ \{z[t] \rightarrow C[1] \} \}$$

Question-4: Solve first order differential equation $dz/dt + 10z = t^2$ Solution:

DSolve
$$[z'[t] + 10 z[t] == t^2, z[t], t]$$

$$\left\{ \left\{ z[t] \rightarrow \frac{1}{500} \left(1 - 10 t + 50 t^2 \right) + e^{-10 t} C[1] \right\} \right\}$$

Question-5: Solve the first order differential equation $dy/dx + 24 y = e^x$.

DSolve[z'[x] + 24 z[x] == e^x, z[x], x]
$$\left\{ \left\{ z[x] \rightarrow \frac{e^{x}}{24} \right\} \right\}$$

Question-6: Solve the first order differential equation dy/dx + y = 0 $dy/dt + 12 \ y = 0$

Question-7: Solve first order differential equations...

$$\frac{\mathrm{d}s}{\mathrm{d}x} = 0$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} + 10\,v = t^{\wedge}2$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} + 24\,z = e^{\wedge}t$$

Solution:

DSolve
$$\left[\left\{s'[t] = 0, v'[t] + 10v[t] = t^2, w'[t] + 24w[t] = Exp[t]\right\}, \left\{s[t], v[t], w[t]\right\}, t\right]$$

$$\left\{\left\{s[t] \rightarrow C[1], v[t] \rightarrow \frac{1}{500} \left(1 - 10t + 50t^2\right) + e^{-10t}C[2], w[t] \rightarrow \frac{e^t}{25} + e^{-24t}C[3]\right\}\right\}$$

Question-8: Solve first order differential equations

$$\frac{dy}{dx} + \sqrt{1 - y^2/(1 - x^2)} = 0.$$

Solution:

DSolve[
y'[x] + Sqrt[(1 - y[x]^2) / (1 - x^2)] == 0, y[x], x]
$$\left\{ \left\{ y[x] \to 1 + 2 \sinh \left[\frac{1}{2} \left(-2 \operatorname{Arcsinh} \left[\frac{\sqrt{-1+x}}{\sqrt{2}} \right] + C[1] \right) \right]^2 \right\} \right\}$$

Question-9: Solve first order differential equations.

$$\left(y - x \frac{\mathrm{d}y}{\mathrm{d}x}\right) = \left(y^2 + \frac{\mathrm{d}y}{\mathrm{d}x}\right)$$

Solution:

DSolve
$$[(y[t] - t * y'[t]) == ((y[t])^2 + y'[t]), y[t], t]$$

$$\{\{y[t] \rightarrow \frac{1+t}{1+e^{c(1)}+t}\}\}$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = -y^2/(xy + x^2)$$

Solution:

DSolve
$$[y'[x] = -(y[x])^2 / (xy[x] + x^2), y[x], x]$$

$$\{\{y[x] \rightarrow \frac{e^{2C[1]} - \sqrt{e^{4C[1]} + e^{2C[1]} x^2}}{x}\},$$

$$\{y[x] \rightarrow \frac{e^{2C[1]} + \sqrt{e^{4C[1]} + e^{2C[1]} x^2}}{x}\}\}$$

Question-11: Solve first order differential equations.

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - x^3 = 0$$

Solution:

DSolve
$$[(y'[x])^2 = x^3, y[x], x]$$

 $\{\{y[x] \rightarrow -\frac{2x^{5/2}}{5} + C[1]\}, \{y[x] \rightarrow \frac{2x^{5/2}}{5} + C[1]\}\}$

Question-12: Solve first order differential equations.

$$x = y + (dy/dx)^2$$

Solution:

DSolve
$$[\mathbf{x} = \mathbf{y}[\mathbf{x}] + (\mathbf{y}'[\mathbf{x}])^2, \mathbf{y}[\mathbf{x}], \mathbf{x}]$$

$$\left\{ \left\{ \mathbf{y}[\mathbf{x}] \rightarrow -1 + \mathbf{x} - 2 \operatorname{ProductLog}\left[-e^{-1-\frac{\mathbf{x}}{2} + \frac{C[1]}{2}} \right] - \operatorname{ProductLog}\left[-e^{-1-\frac{\mathbf{x}}{2} + \frac{C[1]}{2}} \right]^2 \right\} \right\}$$