Practical-2

Plotting of second order solution of family of differential equation

command: ***DSolve[eqn,y[x]x,x]*** to soove a differential equayion fory[x]

Question 1: Solve the second order differential equation
d^2y/dx^2+7dy/dx+10y=0

Solution:

```
DSolve[y''[x] + 7 y'[x] + 10 y[x] == 0, y[x], x]
{{y[x] → e^(-5 x) C[1] + e^(-2 x) C[2]}}
```

Question 2: Solve the second order differential equation
d^2y/dx^2+y=0

Solution:

```
DSolve[y''[x] + y[x] == 0, y[x], x]
{{y[x] → C[1] Cos[x] + C[2] Sin[x]}}
```

Questiuon 3: Solve second order differential equation
d **^ 2** ***y*** **/ dx ^ 2+dy/dx-6y=0.**

Solution:

```
DSolve[y''[x] + y'[x] - 6 y[x] == 0, y[x], x]
{{y[x] → e^(-3 x) C[1] + e^(2 x) C[2]}}
```

Question 4: Solve the second order differential equation
4d^2y/dx^2+12dy/dx+9y=0

Solution:

```
DSolve[4 y''[x] + 12 y'[x] + 9 y[x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \to e^{-3x/2} C[1] + e^{-3x/2} x C[2]\right\}\right\}$$

Question 5: Solve the second order differentiual equation (*d*^2 *y* / dx^2-6dy/dx+13y=0

Solution:

```
DSolve[y''[x] - 6 y'[x] + 13 y[x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \to e^{3x} C[2] \text{Cos}[2x] + e^{3x} C[1] \text{Sin}[2x]\right\}\right\}$$

Question 6: Solve the second order differential equation d^2y/dx^2-2dy/dx+y=0

Solution:

```
DSolve[y''[x] - 2 y'[x] + y[x] == 0, y[x], x]
```

$$\{\{y[x] \to e^{x} C[1] + e^{x} x C[2]\}\}$$

Plotting of solutions of second order differentiual equations

Question 1: Solve the second order differential equation d^2y/dx^2+y=0 and plots its three solutions.

Solution:

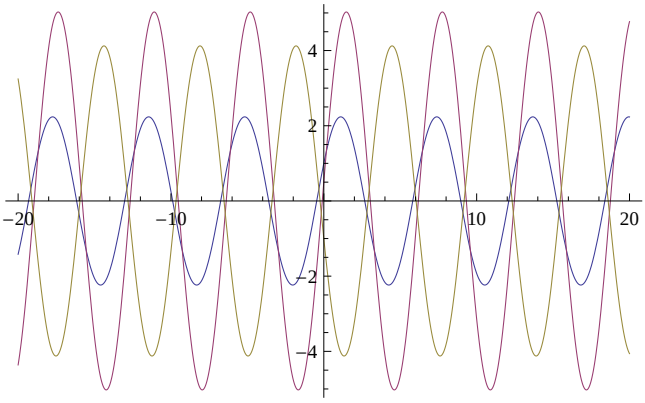
```
Sol = DSolve[y''[x] + y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 2}]
sol2 = y[x] /. Sol[[1]] /. {C[1] → 1 / 2, C[2] → 5}
sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → -4}
Plot[{sol1, sol2, sol3}, {x, -20, 20}]
```

$$\{\{y[x] \to C[1]\,\text{Cos}[x] + C[2]\,\text{Sin}[x]\}\}$$

$$\text{Cos}[x] + 2\,\text{Sin}[x]$$

$$\frac{\text{Cos}[x]}{2} + 5\,\text{Sin}[x]$$

$$-\text{Cos}[x] - 4\,\text{Sin}[x]$$



Question 2: Soolve the second order differential equation (d^2y/dx^2+dy/dx-6y=0 and plot its three solutions.

Solution:

```
Needs["PlotLegends`"]
Sol = DSolve[y''[x] + y'[x] - 6 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 2.5}]
sol2 = y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 5}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1 / 2, C[2] → 5}
Plot[{sol1, sol2, sol3}, {x, -2, 2},
 PlotStyle → {{Pink, Thickness[0.01]},
   {Green, Thick}, {Orange, Thickness[0.02]}}]
```

PlotLegend::shdw :

Symbol PlotLegend appears in multiple contexts {PlotLegends`, Global`};
definitions in context PlotLegends` may
shadow or be shadowed by other definitions. ≫

General::obspkg :

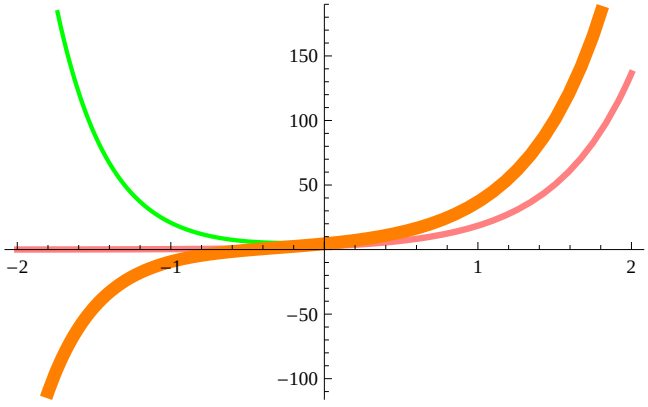
PlotLegends` is now obsolete. The legacy version being loaded
may conflict with current Mathematica functionality.
See the Compatibility Guide for updating information.

$\{\{y[x] \to e^{-3x} C[1] + e^{2x} C[2]\}\}$

$2.5\, e^{2x}$

$e^{-3x} + 5\, e^{2x}$

$-\frac{1}{2} e^{-3x} + 5\, e^{2x}$



Question 3: Solve the second order diffeential equation (d^2y)/dx^2+12dy/dx+9y=0 and plots its four solutions for

(1) C[1]=-1,C[2]=4
(2) C[1]=3,C[2]=6
(3) C[1]=-10, C[2]=7
(4) C[1]=-1.5, C[2]=-5

Solution:

```
sol = DSolve[4 y''[x] + 12 y'[x] + 9 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 4}]
sol2 = y[x] /. sol[[1]] /. {C[1] → 3, C[2] → 6}
sol3 = y[x] /. sol[[1]] /. {C[1] → -10, C[2] → 7}
sol4 = y[x] /. sol[[1]] /. {C[1] → -1.5, C[2] → -5}
Plot[{sol1, sol2, , sol3, sol4}, {x, -2, 2},
 PlotStyle → {{Pink, Thickness[0.01]},
    {Green, Thick}, {Orange, Thickness[0.02]},
    {Purple, Thickness[0.02]}, {Purple, Thickness[0.03]}}]
```

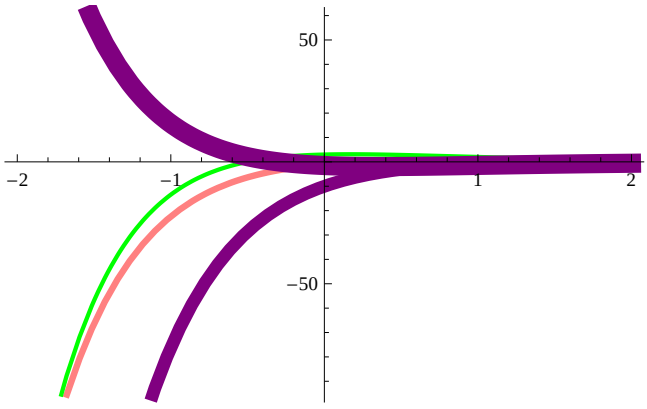
$$\{\{y[x] \to e^{-3x/2} C[1] + e^{-3x/2} x C[2]\}\}$$

$$-e^{-3x/2} + 4 e^{-3x/2} x$$

$$3 e^{-3x/2} + 6 e^{-3x/2} x$$

$$-10 e^{-3x/2} + 7 e^{-3x/2} x$$

$$-1.5 e^{-3x/2} - 5 e^{-3x/2} x$$



Question 4: Solve second order differential equation 4d^2y/dx^2-6dy/dx+13y=0 and plot its three solutions

Solution:

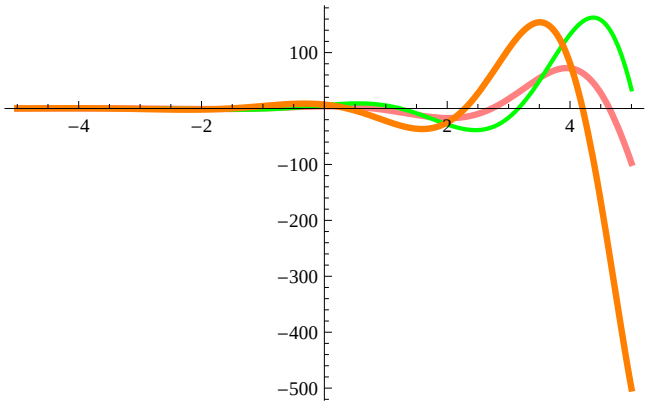
```
sol = DSolve[4 y''[x] - 6 y'[x] + 13 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 4}]
sol2 = y[x] /. sol[[1]] /. {C[1] → 3, C[2] → 6}
sol3 = y[x] /. sol[[1]] /. {C[1] → -10, C[2] -> 7}
Plot[{sol1, sol2, sol3}, {x, -5, 5},
 PlotStyle → {{Pink, Thickness[0.01]}, {Green, Thick},
   {Orange, Thickness[0.01]}}, PlotRange → All]
```

$$\left\{\left\{y[x] \to e^{3x/4} C[2] \cos\left[\frac{\sqrt{43}\,x}{4}\right] + e^{3x/4} C[1] \sin\left[\frac{\sqrt{43}\,x}{4}\right]\right\}\right\}$$

$$4\, e^{3x/4} \cos\left[\frac{\sqrt{43}\,x}{4}\right] - e^{3x/4} \sin\left[\frac{\sqrt{43}\,x}{4}\right]$$

$$6\, e^{3x/4} \cos\left[\frac{\sqrt{43}\,x}{4}\right] + 3\, e^{3x/4} \sin\left[\frac{\sqrt{43}\,x}{4}\right]$$

$$7\, e^{3x/4} \cos\left[\frac{\sqrt{43}\,x}{4}\right] - 10\, e^{3x/4} \sin\left[\frac{\sqrt{43}\,x}{4}\right]$$



Question 5: Solve second order differential equation d^2y/dx^2-2dy/dx+y=0 and plot its five solutions.

Solution:

```
sol = DSolve[y''[x] - 2 y'[x] + y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -3, C[2] → -2}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 7}
sol4 = y[x] /. sol[[1]] /. {C[1] → -6, C[2] → 1}
sol5 = y[x] /. sol[[1]] /. {C[1] → 1 / 5, C[2] → 2 / 3}
Plot[{sol1, sol2, sol3, sol4, sol5}, {x, -5, 5},
 PlotStyle -> {Thickness[0.01], Thick, Thickness[0.02],
   Thickness[0.03], Thickness[0.04]}, PlotRange -> All]
```

$$\{\{y[x] \to e^x C[1] + e^x x C[2]\}\}$$

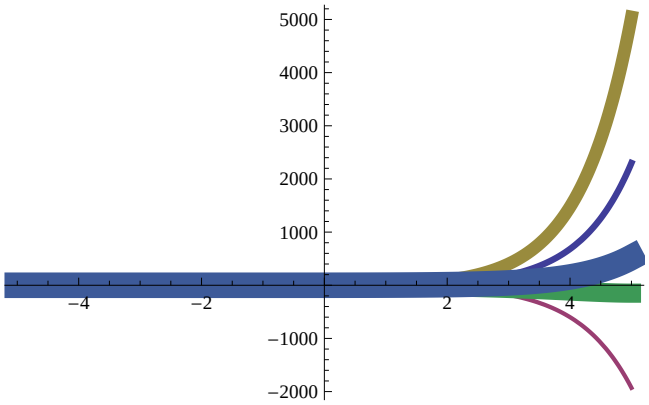
$$0.5\, e^x + 3\, e^x x$$

$$-3\, e^x - 2\, e^x x$$

$$-e^x + 7\, e^x x$$

$$-6\, e^x + e^x x$$

$$\frac{e^x}{5} + \frac{2\, e^x x}{3}$$



Question 6: Solve second order differential equation
x^2d^2y/dx^2-xdy/dx+y=2log(x)
and plot its any five solutions.

Solution:

```
Sol = DSolve[x^2 y''[x] - x y'[x] + y[x] == 2 Log[x], y[x], x]
Sol1 = Evaluate[y[x] /. [[1]] /. {C[1] -> 0.5, C[2] -> 3}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] -> -3, C[2] -> -2}
Sol3 = y[x] /. Sol[[1]] /. {C[1] -> -1, C[2] -> 7}
Sol4 = y[x] /. Sol[[1]] /. {C[1] -> -6, C[2] -> 1}
Sol5 = y[x] /. Sol[[1]] /. {C[1] -> 1 / 5, C[2] -> 2 / 3}
Plot[{Sol1, Sol2, Sol3, Sol4, Sol5}, {x, -15, 15}]
```

$$\{\{y[x] \to x\, C[1] + x\, C[2]\, \text{Log}[x] + 2\,(2 + \text{Log}[x])\}\}$$

Syntax::sntxf : "y[x] /." cannot be followed by "[[1]]".

Syntax::tsntxi : "[[1]]" is incomplete; more input is needed.

Syntax::sntxi : Incomplete expression; more input is needed .

```
Sol = DSolve[x^2 y''[x] - x y'[x] + y[x] == 2 Log[x], y[x], x]
```

$$\{\{y[x] \to x\, C[1] + x\, C[2]\, \text{Log}[x] + 2\,(2 + \text{Log}[x])\}\}$$

```
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] -> 0.5, C[2] -> 3}]
```

$$0.5\, x + 3\, x\, \text{Log}[x] + 2\,(2 + \text{Log}[x])$$

```
Sol2 = y[x] /. Sol[[1]] /. {C[1] -> -3, C[2] -> -2}
```

$$-3\, x - 2\, x\, \text{Log}[x] + 2\,(2 + \text{Log}[x])$$

```
Sol3 = y[x] /. Sol[[1]] /. {C[1] -> -1, C[2] -> 7}
```

$$-x + 7\, x\, \text{Log}[x] + 2\,(2 + \text{Log}[x])$$

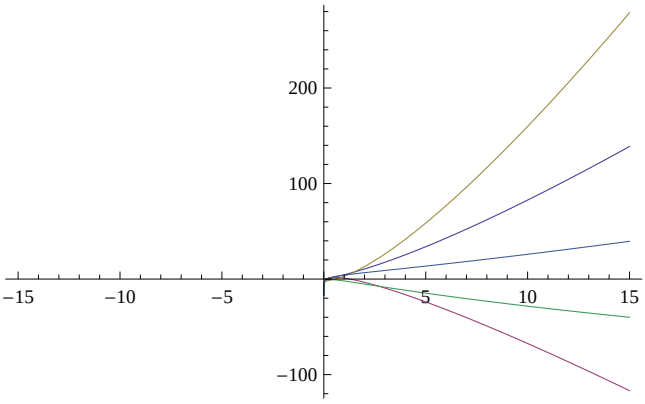
```
Sol4 = y[x] /. Sol[[1]] /. {C[1] -> -6, C[2] -> 1}
```

$$-6\, x + x\, \text{Log}[x] + 2\,(2 + \text{Log}[x])$$

```
Sol5 = y[x] /. Sol[[1]] /. {C[1] -> 1 / 5, C[2] -> 2 / 3}
```

$$\frac{x}{5} + \frac{2}{3}\, x\, \text{Log}[x] + 2\,(2 + \text{Log}[x])$$

```
Plot[{Sol1, Sol2, Sol3, Sol4, Sol5}, {x, -15, 15}]
```



Question 7: Solve second order differential equation x^2d^2y/dx+y=3x^2 and plot its any five solutions.

Solution:

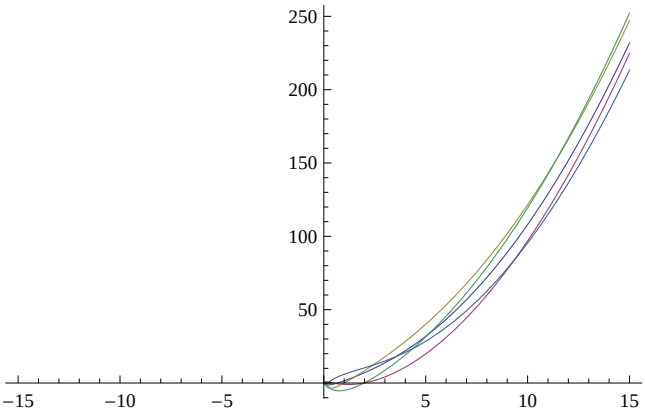
```
sol = DSolve[x^2 y''[x] + y[x] == 3 x^2, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -2, C[2] → -2}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 7}
sol4 = y[x] /. sol[[1]] /. {C[1] → -6, C[2] → 4}
sol5 = y[x] /. sol[[1]] /. {C[1] → 5, C[2] → 2 / 3}
Plot[{sol1, sol2, sol3, sol4, sol5}, {x, -15, 15}]
```

$$\left\{\left\{y[x] \to \sqrt{x}\, C[1] \cos\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right] + \sqrt{x}\, C[2] \sin\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right] + x^2 \left(\cos\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right]^2 + \sin\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right]^2\right)\right\}\right\}$$

$$0.5\sqrt{x} \cos\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right] + 3\sqrt{x} \sin\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right] + x^2 \left(\cos\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right]^2 + \sin\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right]^2\right)$$

$$-2\sqrt{x} \cos\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right] - 2\sqrt{x} \sin\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right] + x^2 \left(\cos\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right]^2 + \sin\left[\frac{1}{2}\sqrt{3}\, \text{Log}[x]\right]^2\right)$$

$$-\sqrt{x}\,\text{Cos}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]+7\sqrt{x}\,\text{Sin}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]+$$
$$x^2\left(\text{Cos}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]^2+\text{Sin}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]^2\right)$$
$$-6\sqrt{x}\,\text{Cos}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]+4\sqrt{x}\,\text{Sin}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]+$$
$$x^2\left(\text{Cos}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]^2+\text{Sin}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]^2\right)$$
$$5\sqrt{x}\,\text{Cos}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]+\frac{2}{3}\sqrt{x}\,\text{Sin}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]+$$
$$x^2\left(\text{Cos}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]^2+\text{Sin}\left[\frac{1}{2}\sqrt{3}\,\text{Log}[x]\right]^2\right)$$



Question 8: Solve second order differential equation

d^2y/dx^2=$\sqrt{1+(dy/dx)^2}$ and plot its any five solutions

Solution:

```
sol = DSolve[y''[x] == Sqrt[1 + (y'[x])^2], y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -2, C[2] → -2}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 7}
sol4 = y[x] /. sol[[1]] /. {C[1] → -6, C[2] → 4}
sol5 = y[x] /. sol[[1]] /. {C[1] → 5, C[2] → 2 / 3}
Plot[{sol1, sol2, sol3, sol4, sol5}, {x, -35, 35}]
```

```
{{y[x] → C[2] + Cosh[x] Cosh[C[1]] + Sinh[x] Sinh[C[1]]}}
```

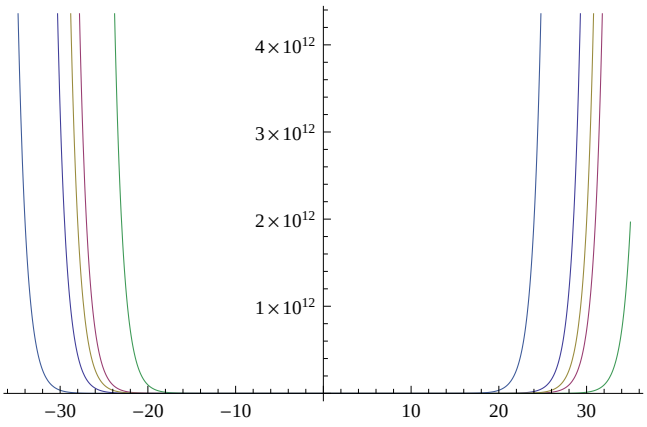
$$3 + 1.12763\,\text{Cosh}[x] + 0.521095\,\text{Sinh}[x]$$

$$-2 + \text{Cosh}[2]\,\text{Cosh}[x] - \text{Sinh}[2]\,\text{Sinh}[x]$$

$$7 + \text{Cosh}[1]\,\text{Cosh}[x] - \text{Sinh}[1]\,\text{Sinh}[x]$$

$$4 + \text{Cosh}[6]\,\text{Cosh}[x] - \text{Sinh}[6]\,\text{Sinh}[x]$$

$$\frac{2}{3} + \text{Cosh}[5]\,\text{Cosh}[x] + \text{Sinh}[5]\,\text{Sinh}[x]$$



Question 9: Solve the second order differential equation (1+x^2)d^2y/dx^2+1+(dy/dx)^2=0 and plot its any five solutionss

Solution:

```
sol = DSolve[(1 + x^2) y''[x] + 1 + (y'[x])^2 == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
sol2 = y[x] /. sol[[1]] /. {C[1] → -2, C[2] → -2}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 7}
sol4 = y[x] /. sol[[1]] /. {C[1] → -6, C[2] → 4}
sol5 = y[x] /. sol[[1]] /. {C[1] → 5, C[2] → 2 / 3}
Plot[{sol1, sol2, sol3, sol4, sol5}, {x, -15, 15}]
```

$$\left\{\left\{y[x] \to C[2] - x\,\text{Cot}[C[1]] + \text{Csc}[C[1]]^2\,\text{Log}[-\text{Cos}[C[1]] - x\,\text{Sin}[C[1]]]\right\}\right\}$$

$$3 - 1.83049\,x + 4.35069\,\text{Log}[-0.877583 - 0.479426\,x]$$

$$-2 + x\,\text{Cot}[2] + \text{Csc}[2]^2\,\text{Log}[-\text{Cos}[2] + x\,\text{Sin}[2]]$$

$$7 + x\,\text{Cot}[1] + \text{Csc}[1]^2\,\text{Log}[-\text{Cos}[1] + x\,\text{Sin}[1]]$$

$$4 + x\,\text{Cot}[6] + \text{Csc}[6]^2\,\text{Log}[-\text{Cos}[6] + x\,\text{Sin}[6]]$$

$$\frac{2}{3} - x\,\text{Cot}[5] + \text{Csc}[5]^2\,\text{Log}[-\text{Cos}[5] - x\,\text{Sin}[5]]$$