

## PRACTICAL QUESTIONS PRACTICE :

### BISECTION METHOD

*In[ ]:=* (\*1. Solve  $\cos[x] == 0$ \*)

```
In[ ]:= f[x_] := Cos[x];
x0 = 0;
x1 = 2.0;
Nmax = 5;
eps = 0.0001;
If[N[f[x0] * f[x1]] > 0,
  Print["\nYour values do not satisfy the IVP so use new values."],
  For[i = 1, i ≤ Nmax, i++, m =  $\frac{(x0 + x1)}{2}$ ;
    If[Abs[ $\frac{x1 - x0}{2}$ ] < eps, Return[m], Print[i, "th iteration value is : ", m];
    Print["Estimated error in ", i, "th iteration is : ",  $\frac{(x1 - x0)}{2}$ ];
    If[f[m] * f[x1] < 0, x0 = m, x1 = m]]];
Print["\nRoot is : ", m];
Print["Estimated error in ", i, "th iteration is : ",  $\frac{(x1 - x0)}{2}$ ];];
Print[Plot[f[x], {x, -1, 3}, PlotRange → {-1, 1}, PlotStyle → {Red, Thickness[0.01]},
  PlotLabel → "f[x] = " f[x], AxesLabel → {x, f[x]}]]];
```

1th iteration value is : 1.

Estimated error in 1th iteration is : 1.

2th iteration value is : 1.5

Estimated error in 2th iteration is : 0.5

3th iteration value is : 1.75

Estimated error in 3th iteration is : 0.25

4th iteration value is : 1.625

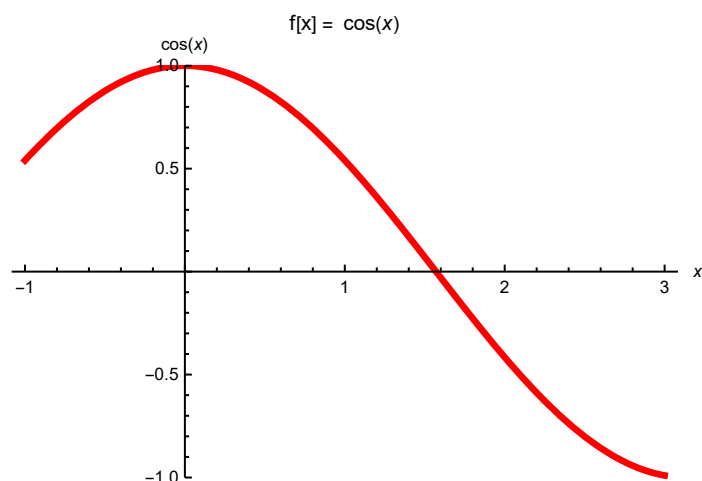
Estimated error in 4th iteration is : 0.125

5th iteration value is : 1.5625

Estimated error in 5th iteration is : 0.0625

Root is : 1.5625

Estimated error in 6th iteration is : 0.03125



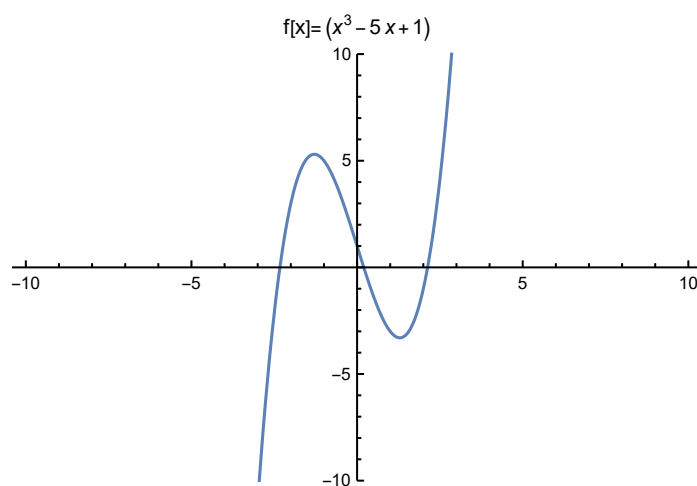
```

In[ ]:= (*Solve x3-5x+1=0*)
f[x_] := x3 - 5 x + 1;
x0 = 0;
x1 = 0.5;
Nmax = 20;
eps = 0.0001;
If[N[f[x0] * f[x1]] > 0,
  Print["\nGiven values do not satisfy the IVP, try other values."],
  For[i = 1, i ≤ Nmax, i++, m =  $\frac{x0 + x1}{2}$ ;
    If[Abs[ $\frac{x0 - x1}{2}$ ] < eps, Return[m], Print[i, "th iteration value is ", m];
    Print["Estimated error in ", i, "th iteration is : ",  $\frac{(x1 - x0)}{2}$ ];
    If[f[m] * f[x1] < 0, x0 = m, x1 = m]]];
Print["\nRoot is : ", m];
Print["Estimated error in ", i, "th iteration is : ",  $\frac{(x1 - x0)}{2}$ ];
Print[Plot[f[x], {x, -10, 10}, PlotRange → {-10, 10}, PlotLabel → "f[x] = " f[x]]];

```

1th iteration value is 0.25  
 Estimated error in 1th iteration is : 0.25  
 2th iteration value is 0.125  
 Estimated error in 2th iteration is : 0.125  
 3th iteration value is 0.1875  
 Estimated error in 3th iteration is : 0.0625  
 4th iteration value is 0.21875  
 Estimated error in 4th iteration is : 0.03125  
 5th iteration value is 0.203125  
 Estimated error in 5th iteration is : 0.015625  
 6th iteration value is 0.195313  
 Estimated error in 6th iteration is : 0.0078125  
 7th iteration value is 0.199219  
 Estimated error in 7th iteration is : 0.00390625  
 8th iteration value is 0.201172  
 Estimated error in 8th iteration is : 0.00195313  
 9th iteration value is 0.202148  
 Estimated error in 9th iteration is : 0.000976563  
 10th iteration value is 0.20166  
 Estimated error in 10th iteration is : 0.000488281  
 11th iteration value is 0.201416  
 Estimated error in 11th iteration is : 0.000244141  
 12th iteration value is 0.201538  
 Estimated error in 12th iteration is : 0.00012207

Out[ ]:= Return[0.201599]



```

In[ ]:= (*Solve cos[x]-xe^x=0*)
f[x_] := Cos[x] - x * Exp[x];
x0 = 0;
x1 = 1;
Nmax = 20;
eps = 0.0001;
If[N[f[x0] * f[x1]] > 0,
  Print["\nGiven values do not satisfy the IVP try other values"],
  For[i = 1, i ≤ Nmax, i++, m = N[ $\frac{x0 + x1}{2}$ ];
    If[Abs[ $\frac{x0 - x1}{2}$ ] < eps, Return[m], Print[i, "th iteration value is : ", m];

    Print["Estimated error in ", i, "th iteration is ",  $\frac{x1 - x0}{2}$ ];
    If[f[m] * f[x1] < 0, x0 = m, x1 = m]]];
Print["\nRoot is ", m];
Print["Estimated error in ", i, "th iteration is ",  $\frac{x1 - x0}{2}$ ]];
Print[Plot[f[x], {x, -10, 10}, PlotRange → {-10, 10},
  PlotLabel → "f[x]=" f[x], AxesLabel → {x, f[x]}]];

```

1th iteration value is : 0.5

Estimated error in 1th iteration is  $\frac{1}{2}$

2th iteration value is : 0.75

Estimated error in 2th iteration is 0.25

3th iteration value is : 0.625

Estimated error in 3th iteration is 0.125

4th iteration value is : 0.5625

Estimated error in 4th iteration is 0.0625

5th iteration value is : 0.53125

Estimated error in 5th iteration is 0.03125

6th iteration value is : 0.515625

Estimated error in 6th iteration is 0.015625

7th iteration value is : 0.523438

Estimated error in 7th iteration is 0.0078125

8th iteration value is : 0.519531

Estimated error in 8th iteration is 0.00390625

9th iteration value is : 0.517578

Estimated error in 9th iteration is 0.00195313

10th iteration value is : 0.518555

Estimated error in 10th iteration is 0.000976563

11th iteration value is : 0.518066

Estimated error in 11th iteration is 0.000488281

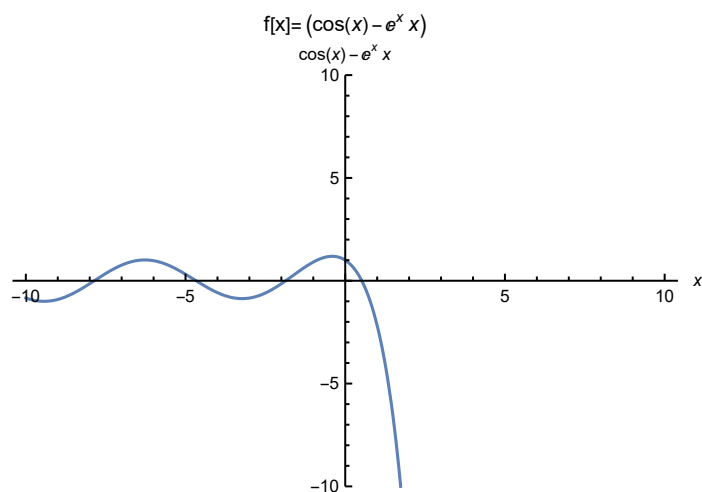
12th iteration value is : 0.517822

Estimated error in 12th iteration is 0.000244141

13th iteration value is : 0.5177

Estimated error in 13th iteration is 0.00012207

Out[ ]:= Return[0.517761]

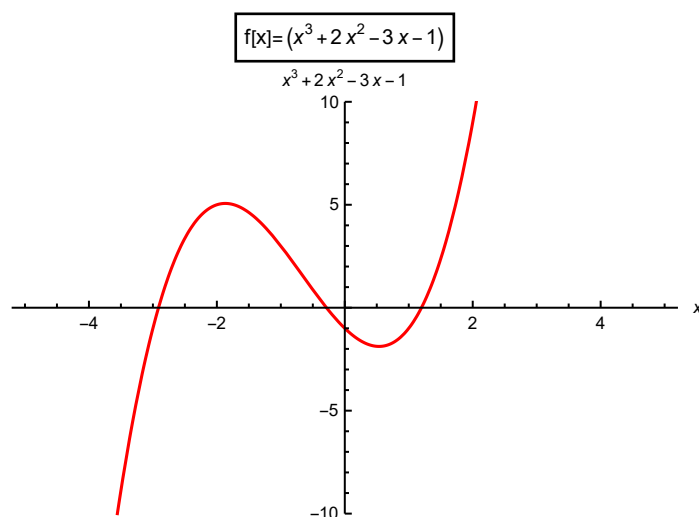


```

In[ ]:= (*Solve  $x^3+2x^2-3x-1=0$ *)
f[x_] :=  $x^3 + 2x^2 - 3x - 1$ ;
a = 1;
b = 2;
Nmax = 10;
list = {};
If[N[f[a] * f[b]] > 0, Print[
  "\nThe method won't apply in the interval [a,b] for given a,b\nTry new values."],
For[i = 1, i <= Nmax, i++, m = N[ $\frac{a+b}{2}$ ];
  list = Append[list, {i, a, m, b, f[m]}];
  If[f[m] == 0,
    Print["\nExact root of f[x] is ", m], If[f[m] * f[b] < 0, a = m, b = m]]];
Print[TableForm[list, TableHeadings -> {None, {"Iteration", "Left End Point",
  "Approximate Value", "Right End Point", "Value of func"}}]];
Print[Plot[f[x], {x, -5, 5}, PlotRange -> {-10, 10}, PlotLabel -> Framed["f[x] = " f[x]],
  AxesLabel -> {x, f[x]}, PlotStyle -> Red]];
Print["\nRoot after ", Nmax, " iterations is ", m];
Print["\nAccuracy is ", Abs[ $\frac{b-a}{2}$ ]];

```

Iteration	Left End Point	Approximate Value	Right End Point	Value of func
1	1	1.5	2	2.375
2	1	1.25	1.5	0.328125
3	1	1.125	1.25	-0.419922
4	1.125	1.1875	1.25	-0.067627
5	1.1875	1.21875	1.25	0.124725
6	1.1875	1.20313	1.21875	0.0271797
7	1.1875	1.19531	1.20313	-0.0205646
8	1.19531	1.19922	1.20313	0.00322217
9	1.19531	1.19727	1.19922	-0.00869253
10	1.19727	1.19824	1.19922	-0.00274051



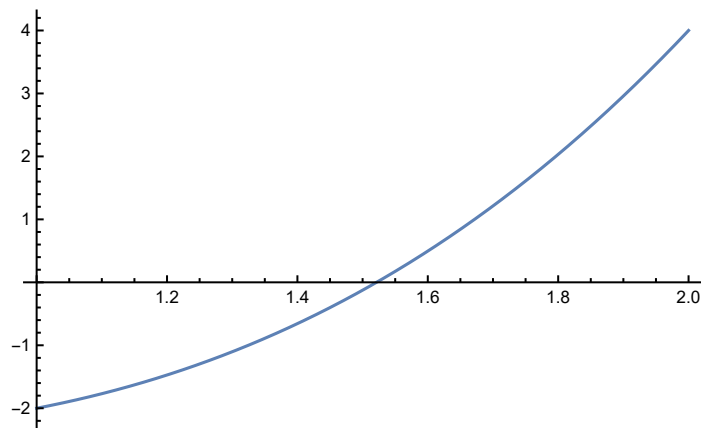
Root after 10 iterations is 1.19824

Accuracy is 0.000488281

## REGULA FALSI METHOD

*In[ ]:=* (\*Find out root of  $f[x]=x^3-x-2$  in the interval  $[1,2]$ \*)

```
In[ ]:= f[x_] := x3 - x - 2;
a = 1;
b = 2;
Nmax = 10;
Print[Plot[f[x], {x, a, b}]];
For[i = 0, i ≤ Nmax, i++, c = N[ $\frac{b * f[a] - a * f[b]}{f[a] - f[b]}$ ];
  If[f[c] * f[b] < 0, a = c, b = c];
  Print["Iteration ", i, " = ", c]]
```



Iteration 0 = 1.33333

Iteration 1 = 1.46269

Iteration 2 = 1.50402

Iteration 3 = 1.51633

Iteration 4 = 1.51992

Iteration 5 = 1.52096

Iteration 6 = 1.52126

Iteration 7 = 1.52134

Iteration 8 = 1.52137

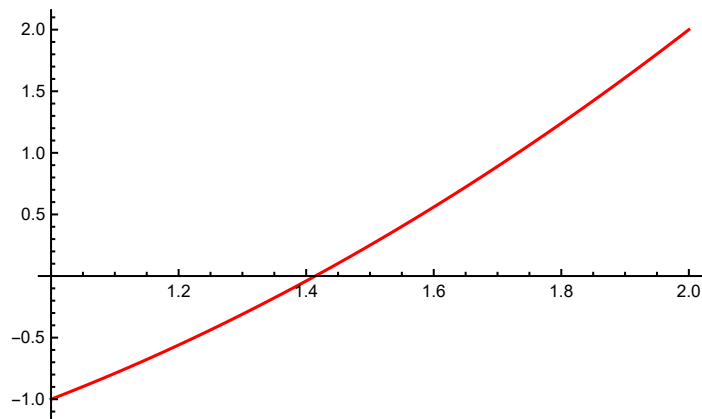
Iteration 9 = 1.52138

Iteration 10 = 1.52138

```

In[ ]:= (*Evaluate square root of 2*)
f[x_] := x2 - 2;
a = 1;
b = 2;
Nmax = 10;
Print[Plot[f[x], {x, a, b}, PlotStyle → Red]];
For[i = 1, i ≤ Nmax, i++, c = N[ $\frac{a * f[b] - b * f[a]}{f[b] - f[a]}$ ];
  If[f[c] * f[b] < 0, a = c, b = c]; Print[i, "th Iteration : ", c]];
Print["The approximate value of the square root of 2 by Regula Falsi Method after ",
  Nmax, " iterations is ", c];

```



1th Iteration : 1.33333

2th Iteration : 1.4

3th Iteration : 1.41176

4th Iteration : 1.41379

5th Iteration : 1.41414

6th Iteration : 1.4142

7th Iteration : 1.41421

8th Iteration : 1.41421

9th Iteration : 1.41421

10th Iteration : 1.41421

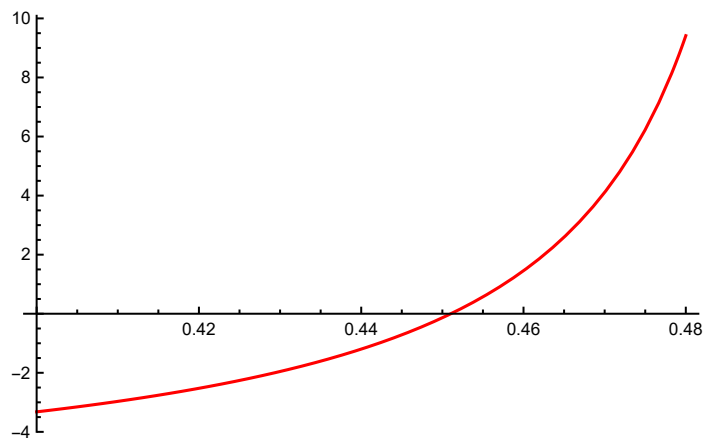
The approximate value of the square root of 2 by Regula Falsi Method after  
10 iterations is 1.41421



```

In[ ]:= (*Approximate the root of the equation f[x]= Tan[Pi*x]-x-6 in [0.40,0.48],
iterations=10 *)
f[x_] := Tan[Pi * x] - x - 6;
a = 0.40;
b = 0.48;
Nmax = 10;
Print[Plot[f[x], {x, a, b}, PlotStyle -> Red]];
For[i = 1, i ≤ Nmax, i++, c = N[ $\frac{a * f[b] - b * f[a]}{f[b] - f[a]}$ ]];
If[f[c] * f[b] < 0, a = c, b = c]; Print[i, "th iteration : ", c];
Print["\nApproximate root of f(x)=", f[x], " after ", Nmax, " iterations is ", c];

```



```

1th iteration : 0.420867
2th iteration : 0.433203
3th iteration : 0.440496
4th iteration : 0.444808
5th iteration : 0.447358
6th iteration : 0.448866
7th iteration : 0.449757
8th iteration : 0.450284
9th iteration : 0.450596
10th iteration : 0.45078

```

Approximate root of  $f(x) = -6 - x + \tan[\pi x]$  after 10 iterations is 0.45078

## SECANT METHOD

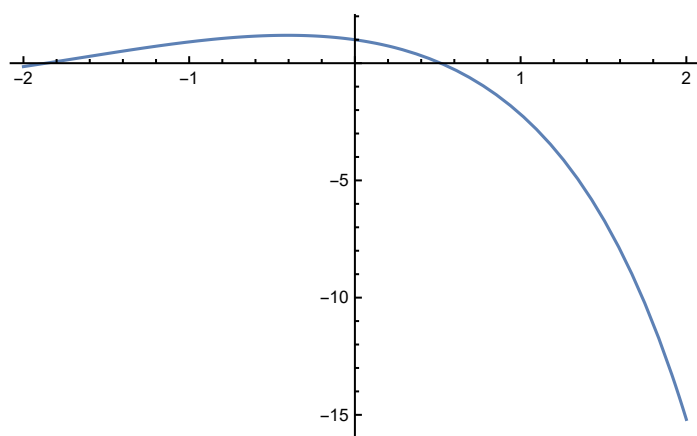
```

In[6]:= (*Solve f[x]=cos[x]-x*e^x*)
f[x_] := Cos[x] - x * Exp[x];
p0 = 0;
p1 = 1;
Nmax = 10;
Print[Plot[f[x], {x, -2, 2}]];

For[i = 1, i ≤ Nmax, i++, p2 = N[ $\frac{p0 * f[p1] - p1 * f[p0]}{f[p1] - f[p0]}$ ];

  Print["Iteration ", i, " : ", p2];
  p0 = p1;
  p1 = p2];

```



```

Iteration 1 : 0.314665
Iteration 2 : 0.446728
Iteration 3 : 0.531706
Iteration 4 : 0.516904
Iteration 5 : 0.517747
Iteration 6 : 0.517757
Iteration 7 : 0.517757
Iteration 8 : 0.517757
Iteration 9 : 0.517757
Iteration 10 : 0.517757

```

```
In[6]:= (*Find the root of f(x)=x^3-17 taking p0=3,p1=2 after 10 iterations*)
```

```
f[x_] := x^3 - 17;
```

```
p0 = 3;
```

```
p1 = 2;
```

```
Nmax = 10;
```

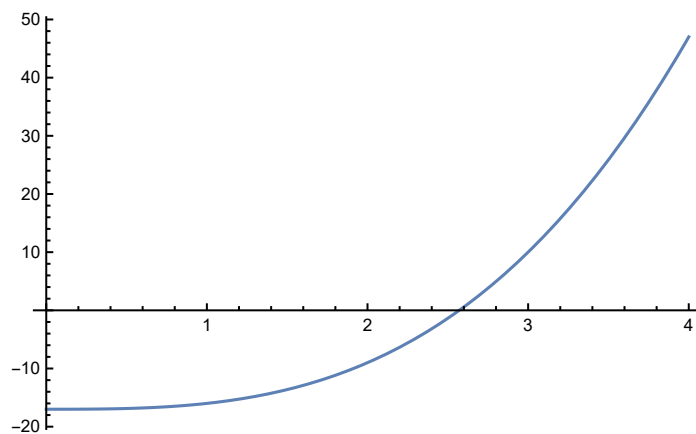
```
Print[Plot[f[x], {x, 0, 4}]];
```

```
For[i = 1, i ≤ Nmax, i++, p2 =  $\frac{p1 * f[p0] - p0 * f[p1]}{f[p0] - f[p1]}$ ;
```

```
Print["Iteration ", i, " : ", N[p2, 7]];
```

```
p0 = p1;
```

```
p1 = p2;]
```



```
Iteration 1 : 2.473684
```

```
Iteration 2 : 2.597352
```

```
Iteration 3 : 2.570274
```

```
Iteration 4 : 2.571271
```

```
Iteration 5 : 2.571282
```

```
Iteration 6 : 2.571282
```

```
Iteration 7 : 2.571282
```

```
Iteration 8 : 2.571282
```

```
Iteration 9 : 2.571282
```

```
Iteration 10 : 2.571282
```

## NEWTON RAPHSON METHOD

```
In[6]:= (*Use Newton Raphson Method to find the root of f(x)=x^3-17 taking p_0=3,  
2 after 10 iterations*)
```

```
f[x_] := x^3 - 17;
```

```
p[0] = 3;
```

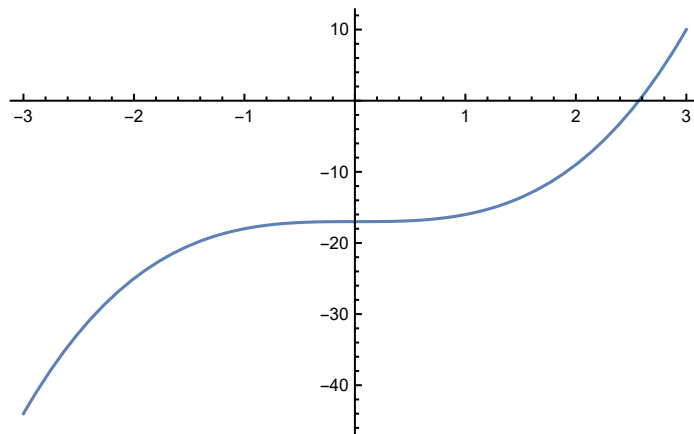
```
Print[Plot[f[x], {x, -3, 3}]];
```

```
p[n_] := p[n-1] -  $\frac{f[p[n-1]]}{f'[p[n-1]]}$ ;
```

```
err[n_] := Abs[N[p[n]] - CubeRoot[17], 7]];
```

```
Print[Grid[Prepend[Table[{n, N[p[n], 7], err[n]}, {n, 1, 10}],
```

```
{ "n", "p[n]", "Error[n]" }, Dividers → {{False, True}, {False, True}}]] // Quiet;
```



n	p[n]	Error[n]
1	2.629630	0.05834804
2	2.572567	0.001285091
3	2.571282	$6.418429 \times 10^{-7}$
4	2.571282	$1.602167 \times 10^{-13}$
5	2.571282	$9.983110 \times 10^{-27}$
6	2.571282	$3.876 \times 10^{-53}$
7	2.571282	$0. \times 10^{-57}$
8	2.571282	$0. \times 10^{-57}$
9	2.571282	$0. \times 10^{-57}$
10	2.571282	$0. \times 10^{-57}$

## GAUSS ELIMINATION METHOD

Solve the linear system of by Gauss Elimination method

$$3.15x - 1.96y + 3.85z = 12.95$$

$$2.13x + 5.12y - 2.89z = -8.61$$

$$5.92x + 3.05y + 2.155z = 6.88$$

```
m = {{3.15, -1.96, 3.85, 12.95}, {2.13, 5.12, -2.89, -8.61}, {5.92, 3.05, 2.155, 6.88}};
Print["\nAugmented Matrix [A:B]=", MatrixForm[m]];
```

$$m[[2]] = m[[2]] - \frac{m[[2, 1]]}{m[[1, 1]]} * m[[1]];$$

$$m[[3]] = m[[3]] - \frac{m[[3, 1]]}{m[[1, 1]]} * m[[1]];$$

```
Print[m // MatrixForm];
```

$$m[[3]] = m[[3]] - \frac{m[[3, 2]]}{m[[2, 2]]} * m[[2]];$$

```
Print[m // MatrixForm];
```

$$z = \frac{m[[3, 4]]}{m[[3, 3]]};$$

$$y = \frac{1}{m[[2, 2]]} * (m[[2, 4]] - m[[2, 3]] * z);$$

$$x = \frac{1}{m[[1, 1]]} * (m[[1, 4]] - m[[1, 2]] * y - m[[1, 3]] * z);$$

```
Print["\nx : ", x, "\ny : ", y, "\nz : ", z];
```

$$\text{Augmented Matrix } [A:B] = \begin{pmatrix} 3.15 & -1.96 & 3.85 & 12.95 \\ 2.13 & 5.12 & -2.89 & -8.61 \\ 5.92 & 3.05 & 2.155 & 6.88 \end{pmatrix}$$

$$\begin{pmatrix} 3.15 & -1.96 & 3.85 & 12.95 \\ 0. & 6.44533 & -5.49333 & -17.3667 \\ 8.88178 \times 10^{-16} & 6.73356 & -5.08056 & -17.4578 \end{pmatrix}$$

$$\begin{pmatrix} 3.15 & -1.96 & 3.85 & 12.95 \\ 0. & 6.44533 & -5.49333 & -17.3667 \\ 8.88178 \times 10^{-16} & 0. & 0.658428 & 0.685491 \end{pmatrix}$$

x : 1.71422  
y : -1.80713  
z : 1.0411

## GAUSS JORDAN METHOD

Solve the linear system of by Gauss JORDAN method

$$2a+b+c-2d=-10$$

$$4a+0b+2c+d=8$$

$$3a+2b+2c+0d=7$$

$$a+3b+2c-d=-5$$

```
In[ ]:= m = {{2, 1, 1, -2, -10}, {4, 0, 2, 1, 8}, {3, 2, 2, 0, 7}, {1, 3, 2, -1, -5}};
Print["\nAugmented Matrix [A:B]=", MatrixForm[m]];
n = RowReduce[m];
Print["\nRow Reduced Matrix: ", MatrixForm[n]];
{a, b, c, d} = n[[All, 5]];
Print["\na : ", a, "\nb : ", b, "\nc : ", c, "\nd : ", d];
```

$$\text{Augmented Matrix } [A:B] = \begin{pmatrix} 2 & 1 & 1 & -2 & -10 \\ 4 & 0 & 2 & 1 & 8 \\ 3 & 2 & 2 & 0 & 7 \\ 1 & 3 & 2 & -1 & -5 \end{pmatrix}$$

$$\text{Row Reduced Matrix: } \begin{pmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & -10 \\ 0 & 0 & 0 & 1 & 8 \end{pmatrix}$$

a : 5  
b : 6  
c : -10  
d : 8

Q2. Find Inverse of Matrix using gauss jordan method

```

In[6]:= A = {{2, 2, 3, 1, 0, 0}, {2, 1, 1, 0, 1, 0}, {1, 3, 5, 0, 0, 1}};
Print["\nAugmented Matrix [A:I]=", MatrixForm[A]];
n = RowReduce[A];
Print["\nRow Reduced form : ", MatrixForm[n]];
Print["\nInverse of given matrix is", MatrixForm[n[[1 ;; 3, 4 ;; 6]]];

```

$$\text{Augmented Matrix [A:I]} = \begin{pmatrix} 2 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 3 & 5 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Row Reduced form : } \begin{pmatrix} 1 & 0 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -9 & 7 & 4 \\ 0 & 0 & 1 & 5 & -4 & -2 \end{pmatrix}$$

$$\text{Inverse of given matrix is } \begin{pmatrix} 2 & -1 & -1 \\ -9 & 7 & 4 \\ 5 & -4 & -2 \end{pmatrix}$$

## GAUSS JACOBI METHOD

Q1. Solve the given system of equations using Gauss Jacobi Method by initial approximation as  $X(0)=(0,0)$ . Do 5 iterations

$$x_1 + 2x_2 = 4$$

$$5x_1 - 6x_2 = 3$$

```

In[1]:= x1[0] = 0;
x2[0] = 0;
x1[n_] := N[4 - 2 * x2[n - 1]];
x2[n_] := N[ $\frac{3 - 5 * x1[n - 1]}{-6}$ ];
Print[Grid[Prepend[Table[{n, N[x1[n], 7], N[x2[n], 7]}, {n, 1, 5}],
{"Iteration(n)", "x1[n]", "x2[n]"}], Dividers -> {AllTrue, AllTrue}]] // Quiet;

```

Iteration (n)	x1 [n]	x2 [n]
1	4.	-0.5
2	5.	2.83333
3	-1.66667	3.66667
4	-3.33333	-1.88889
5	7.77778	-3.27778

Q2. Solve the given system of equations using Gauss Jacobi Method by taking initial approximation as  $X(0) = (0, 0, 0)$ . Do 5 iterations.

$$4x_1 - x_2 - x_3 = 3$$

$$-2x_1 + 6x_2 + x_3 = 9$$

$$-x_1 + x_2 + 7x_3 = -6$$

```

In[6]:= x1[0] = 0;
x2[0] = 0;
x3[0] = 0;
x1[n_] :=  $\frac{3 + x2[n-1] + x3[n-1]}{4}$ ;
x2[n_] :=  $\frac{9 + 2 * x1[n-1] - x3[n-1]}{6}$ ;
x3[n_] :=  $\frac{-6 + x1[n-1] - x2[n-1]}{7}$ ;
Print[Grid[Prepend[Table[{n, N[x1[n], 7], N[x2[n], 7], N[x3[n], 7]}, {n, 1, 5}],
{"Iteration(n)", "x1[n]", "x2[n]", "x3[n]"}], Dividers → {AllTrue, AllTrue}]];

```

Iteration(n)	x1[n]	x2[n]	x3[n]
1	0.7500000	1.500000	-0.8571429
2	0.9107143	1.892857	-0.9642857
3	0.9821429	1.964286	-0.9974490
4	0.9917092	1.993622	-0.9974490
5	0.9990434	1.996811	-1.000273

## GAUSS SEIDEL METHOD

Q1 : Solve the given system of equations using Gauss Seidel Method by taking initial approximation as  $X(0) = (0, 0)$ . Do 7 iterations.

$$7x_1 + 2x_2 = 4$$

$$2x_1 + 6x_2 = 3$$

```

In[13]:= x1[0] = 0;
x2[0] = 0;
x1[n_] :=  $\frac{4 - 2 * x2[n-1]}{7}$ ;
x2[n_] :=  $\frac{3 - 2 * x1[n]}{6}$ ;
Print[Grid[Prepend[Table[{n, N[x1[n], 7], N[x2[n], 7]}, {n, 1, 7}],
{"Iteration(n)", "x1[n]", "x2[n]"}], Dividers → All]];

```

Iteration(n)	x1[n]	x2[n]
1	0.5714286	0.3095238
2	0.4829932	0.3390023
3	0.4745708	0.3418097
4	0.4737686	0.3420771
5	0.4736923	0.3421026
6	0.4736850	0.3421050
7	0.4736843	0.3421052

Q2. Solve the given system of equations using Gauss Seidel Method by taking initial approximation as  $X(0) = (0, 0, 0)$ . Do 7 iterations.

$$4x_1 - x_2 - x_3 = 3$$

$$-2x_1 + 6x_2 + x_3 = 9$$

$$-x_1 + x_2 + 7x_3 = -6$$

```

In[18]:= x1[0] = 0;
x2[0] = 0;
x3[0] = 0;
x1[n_] :=  $\frac{3 + x2[n - 1] + x3[n - 1]}{4}$ ;
x2[n_] :=  $\frac{9 + 2 * x1[n] - x3[n - 1]}{6}$ ;
x3[n_] :=  $\frac{-6 + x1[n] - x2[n]}{7}$ ;
Print[Grid[Prepend[Table[{n, N[x1[n], 7], N[x2[n], 7], N[x3[n], 7]}, {n, 1, 7}],
{"Iteration(n)", "x1[n]", "x2[n]", "x3[n]"}], Dividers → All]];

```

Iteration(n)	x1[n]	x2[n]	x3[n]
1	0.750000	1.750000	-1.000000
2	0.937500	1.979167	-1.005952
3	0.993303	1.998760	-1.000779
4	0.999495	1.999962	-1.000067
5	0.999973	2.000002	-1.000004
6	0.999999	2.000001	-1.000000
7	1.000000	2.000000	-1.000000

## LAGRANGE INTERPOLATION POLYNOMIAL

### MODULE

```

In[25]:= LagrangePolynomial[x0_, y0_] := Module[{xi = x0, yi = y0, n, m, polynomial},
  n = Length[xi];
  m = Length[yi];
  If[m ≠ n, Print["\nList of points and values are not of same size"];
  Return[]];
  For[i = 1, i ≤ n, i++, L[i, x_] =  $\left( \prod_{j=1}^{i-1} \frac{x - xi[[j]]}{xi[[i]] - xi[[j]]} \right) \left( \prod_{j=i+1}^n \frac{x - xi[[j]]}{xi[[i]] - xi[[j]]} \right)$ ;
  polynomial[x_] =  $\sum_{k=1}^n L[k, x] * yi[[k]]$ ;
  Return[polynomial[x]];

```

Q1.

```

In[26]:= nodes = {0, 1, 3};
values = {1, 3, 55};
resultingPolynomial[x_] = LagrangePolynomial[nodes, values];
Simplify[resultingPolynomial[x]]

```

Out[29]=  $1 - 6x + 8x^2$

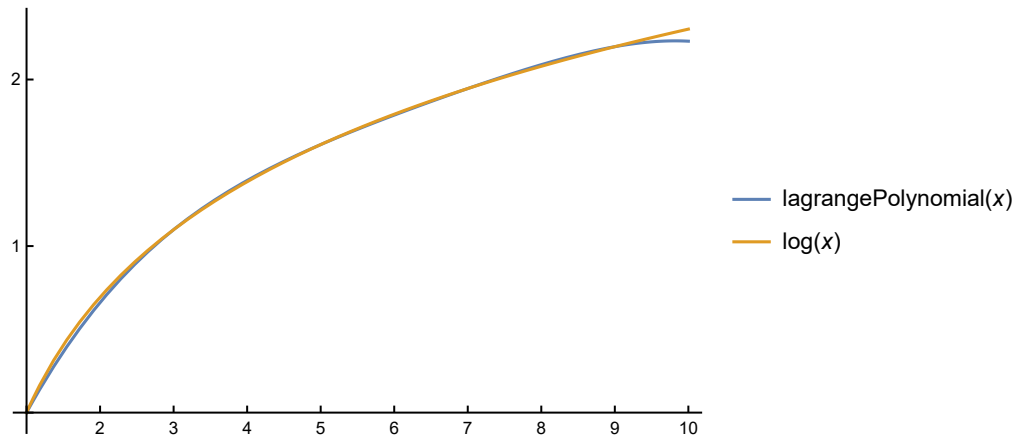
Q2.



```

In[30]:= nodes = {1, 3, 5, 7, 9};
values = N[Log[nodes]];
lagrangePolynomial[x_] = LagrangePolynomial[nodes, values];
Print["Lagrange Polynomial : ", Simplify[lagrangePolynomial[x]]];
Print[Plot[{lagrangePolynomial[x], Log[x]},
  {x, 1, 10}, Ticks -> {Range[0, 10]}, PlotLegends -> "Expressions"]];
Lagrange Polynomial :  $-0.987583 + 1.18991 x - 0.223608 x^2 + 0.0221231 x^3 - 0.000844369 x^4$ 

```



## TRAPEZOIDAL RULE

Q1 . Intergrate  $\int_0^2 x^2 \sin[x] dx$  using Trapezoidal rule with 5 subintervals

```

In[35]:= f[x_] := x^2 * Sin[x];
a = 0;
b = 2;
n = 5; (*No of intervals*)
h = (b - a) / n;
Sol = h / 2 * (f[a] + 2 * Sum[f[i], {i, a + h, b - h, h}] + f[b]);
Print["\nSolution of the given integral is ", N[Sol]];

```

Solution of the given integral is 2.49642

Q2 . Integrate  $\int_0^1 \frac{1}{1+x} dx$  using Trapezoidal rule with 4 equal intervals

```

In[42]:= f[x_] :=  $\frac{1}{1+x}$ ;
a = 0;
b = 1;
n = 4;
h =  $\frac{b-a}{n}$ ;
Sol =  $\frac{h}{2} * (f[a] + 2 * \text{Sum}[f[i], \{i, a+h, b-h, h\}] + f[b])$ ;
Print["\nSolution of the given integral is ", N[Sol]];

```

Solution of the given integral is 0.697024

Q3. Integrate  $\int_1^5 x^3 dx$  using Trapezoidal Rule taking  $n = 10$  and calculate the absolute error in the approximation.

```

In[49]:= f[x_] = x^3;
a = 1;
b = 5;
n = 10;
h =  $\frac{b-a}{n}$ ;
Sol =  $\frac{h}{2} * (f[a] + 2 * \text{Sum}[f[i], \{i, a+h, b-h, h\}] + f[b])$ ;
Print["\nSolution of the given integral is ", N[Sol]];
actual = Integrate[x^3, {x, 1, 5}];
Print["The absolute error is ", N[Abs[actual - Sol]]];

```

Solution of the given integral is 156.96

The absolute error is 0.96

## Simpson's 1/3rd rule

Q. Calculate the integral:  $\int_0^2 e^{-x^2} dx$  using Simpson's 1/3rd rule taking  $n=10$

```

In[58]:= f[x_] := Exp[-x^2];
a = 0;
b = 2;
n = 10;
h =  $\frac{b-a}{n}$ ;
oddSum = Sum[f[i], {i, a+h, b-h, 2*h}];
evenSum = Sum[f[i], {i, a+2*h, b-2*h, 2*h}];
sol =  $\frac{h}{3} * (f[a] + 4 * \text{oddSum} + 2 * \text{evenSum} + f[b])$ ;
Print["\nValue of given integral using Simpson's method is ", N[sol]];

```

Value of given integral using Simpson's method is 0.882075

## ODE using Euler's Method

## ODE using Euler's Method

Q. Solve the initial value problem  $\frac{dy}{dx} = \frac{e^x}{y}$ ,  $y(0) = 1$  for  $y(1)$  using Euler's method

```
In[77]:= x[0] = 0;
y[0] = 1;
a = 0;
b = 1;
n = 10;
h = (b - a) / n;

f[x_, y_] := Exp[x] / y;

x[j_] := x[j - 1] + h;
y[j_] := y[j - 1] + h * f[x[j - 1], y[j - 1]];
Print[Grid[Prepend[Table[{i, N[x[i], 4], N[y[i], 7]}], {i, 1, 10}],
{"j", "x[j]", "y[j]"}], Dividers -> All]];
```

j	x[j]	y[j]
1	0.1000	1.100000
2	0.2000	1.200470
3	0.3000	1.302214
4	0.4000	1.405873
5	0.5000	1.511986
6	0.6000	1.621030
7	0.7000	1.733435
8	0.8000	1.849606
9	0.9000	1.969931
10	1.000	2.094788

## ODE using Modified Euler's Method

Q. Solve the initial value problem  $\frac{dy}{dx} = x + y$ ,  $y(0) = 1$  for  $y(0.3)$  using Euler's method

```

In[87]:= x[0] = 0;
y[0] = 1;
a = 0;
b = 0.3;
n = 10;
h =  $\frac{b - a}{n}$ ;
f[x_, y_] := x + y;
x[j_] := x[j - 1] + h;
y[j_] := y[j - 1] + h * f[x[j - 1] +  $\frac{h}{2}$ , y[j - 1] +  $\frac{h}{2}$  * f[x[j - 1], y[j - 1]]];
Print[Grid[Prepend[Table[{j, N[x[j], 4], N[y[j], 7]}], {j, 1, 10}],
{"j", "x[j]", "y[j]"}], Dividers -> All]];

```

j	x[j]	y[j]
1	0.03	1.0309
2	0.06	1.06365
3	0.09	1.09832
4	0.12	1.13495
5	0.15	1.17362
6	0.18	1.21437
7	0.21	1.25728
8	0.24	1.30241
9	0.27	1.34983
10	0.3	1.3996