06 Implement SGD

April 19, 2019

0.1 Mini-Batch SGD implementation on Boston Housing dataset

[1.1] Import Statements

```
In [868]: import warnings
          warnings.filterwarnings("ignore")
          from sklearn.datasets import load_boston
          from random import seed
          from random import randrange
          from csv import reader
          from math import sqrt
          from math import pow
          from sklearn import preprocessing
          import pandas as pd
          import numpy as np
          import matplotlib.pyplot as plt
          from prettytable import PrettyTable
          from sklearn.linear_model import SGDRegressor
          from sklearn import preprocessing
          from sklearn.metrics import mean_squared_error
          import random
          from sklearn.preprocessing import StandardScaler
          from sklearn.model_selection import train_test_split
```

[1.2] Load Dataset

```
In [869]: X = load_boston().data
          Y = load_boston().target
```

[2.1] Train Test Split

```
In [870]: X_train, X_test, y_train, y_test = train_test_split(X, Y, test_size=0.3, random_stat
          scaler = preprocessing.StandardScaler().fit(X_train)
          X_train = scaler.transform(X_train)
          X_test = scaler.transform(X_test)
```

[3.1] Applying Mini-Batch SGD

In [871]:
```python
def plott(Y_samples, y_pred, title):
    plt.scatter(Y_samples, y_pred)
    plt.xlabel('Y sample')
    plt.ylabel('Y pred')
    plt.title(title)
```

In [872]:
```python
#Source: https://stackoverflow.com/questions/15923826/random-row-selection-in-pandas
# https://stackoverflow.com/questions/17260109/sample-two-pandas-dataframes-the-same

def random_sampling(X, Y, n_samples):
    X = pd.DataFrame(X)
    X_samples = X.sample(n=n_samples)
    Y_samples = Y[X_samples.index]
    Y_samples = pd.DataFrame(Y_samples)
    return X_samples, Y_samples
```

In [873]:
```python
#Source: https://machinelearningmastery.com/implement-linear-regression-stochastic-g
# https://stackoverflow.com/questions/50328545/stochastic-gradient-descent-for-linea

def stochastic_gradient_descent(X_data, y_data, epochs, n_samples, learning_rate_give
    w_0 = np.random.normal(0,1, 13)
    eta0 = 0.01
    power_t = 0.5
    t = 1
    b_0 = np.random.rand()
    for epoch in range(epochs):
        if epoch == 0:
            w_i = w_0
            b_i = b_0
        else:
            w_i = w_latest
            b_i = b_latest

        #sampling data
        X_samples, Y_samples = random_sampling(X_data, y_data, n_samples)
        X_samples = X_samples.as_matrix()
        Y_samples = Y_samples.as_matrix()

        if learning_rate_given == 'constant':
            eta = eta0
        elif learning_rate_given == 'invscaling':
            if epoch == 0:
                eta = eta0
            else:
                t += 1
                eta = eta0 / (t**power_t)
```

```
            # Initializing derivatives to zero
            m_derivative = np.zeros(13)
            b_derivative = 0

            for X_sample, Y_sample in zip(X_samples, Y_samples):
                # m_derivative = -2*x_i(y_i - (w_T*x_i + b_i))
                # b_derivative = -2*(y_i - (w_T*x_i + b_i))
                y_hat = np.dot(w_i.T, X_sample) + b_i
                error = Y_sample - y_hat
                m_derivative += -(2/n_samples) * X_sample * error
                b_derivative += -(2/n_samples) * error

            w_latest = m_derivative * eta
            b_latest = b_derivative * eta
            w_latest = w_i - w_latest
            b_latest = b_i - b_latest
        y_pred = []
        for X_sample in X_samples:
            y_pred.append(np.dot(w_latest.T, X_sample) + b_latest)
        train_error = mean_squared_error(Y_samples, np.array(y_pred))
        return X_samples, Y_samples, y_pred, w_latest, b_latest, train_error
```

```
In [874]: def mean_sq_error(X_test, y_test, w_latest, b_latest, plot_title):

              X_test = pd.DataFrame(X_test)
              y_test = pd.DataFrame(y_test)

              X_test = X_test.as_matrix()
              y_test = y_test.as_matrix()

              y_pred = []
              for item in X_test:
                  y_pred.append(np.dot(w_latest.T, item)+b_latest)

              plott(y_test, y_pred, plot_title)

              return mean_squared_error(y_test, y_pred)
```

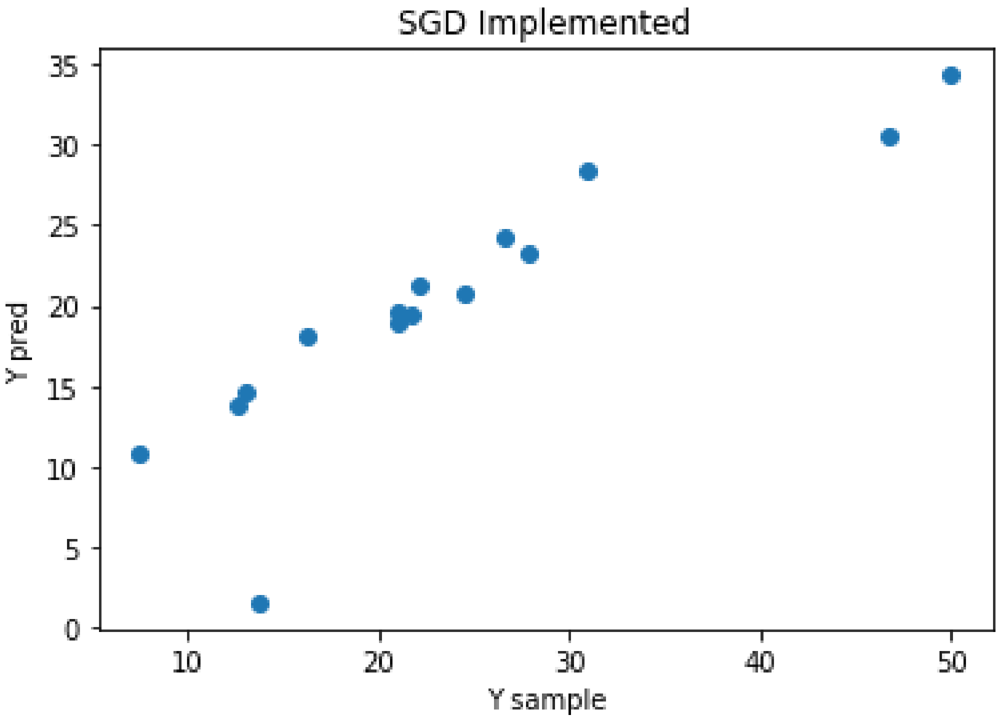
```
In [875]: def sklearn_SGD(X_train, y_train, max_iter_given, learning_rate_given, eta_given):
              if learning_rate_given == 'constant':
                  clf = SGDRegressor(max_iter = max_iter_given, learning_rate= learning_rate_gi
              elif learning_rate_given == 'invscaling':
                  clf = SGDRegressor(max_iter = max_iter_given, learning_rate= learning_rate_gi
              clf.fit(X_train, y_train)
              mse = mean_squared_error(y_train, clf.predict(X_train))
              return clf, mse
```

[3.2] Applying Mini-Batch SGD with fixed learning rate itr = 100

```
In [876]: # Using implemented Mini-Batch SGD with constant learning rate
          X_samples, Y_samples, y_pred, w_latest1, b_latest1, train_error1 = stochastic_gradie
          print(train_error1)
```

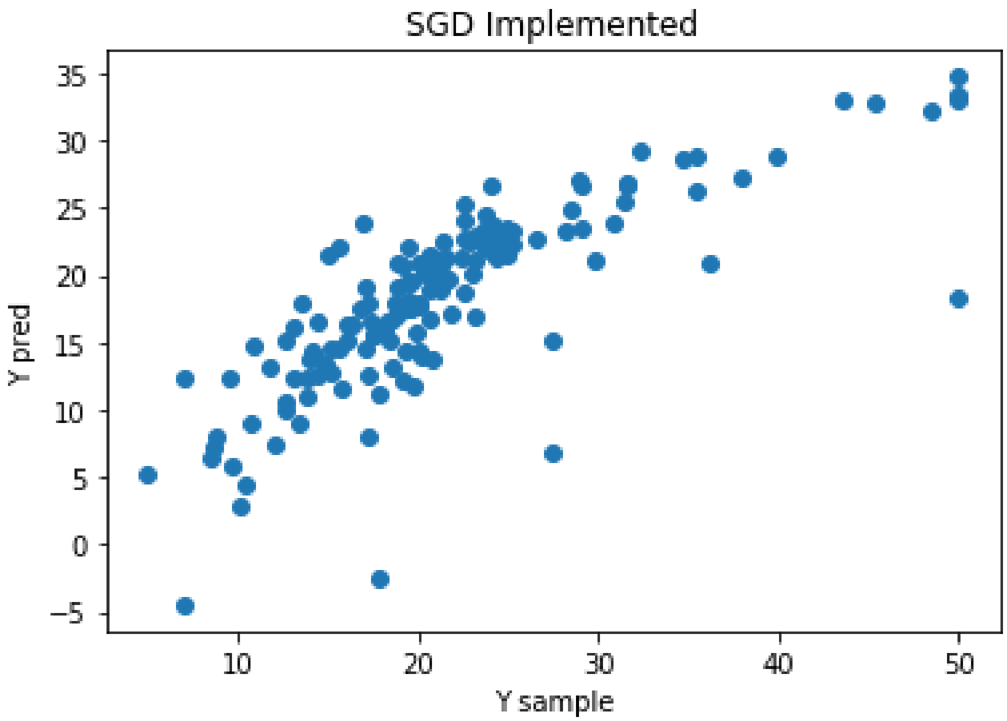
48.92878984555921

```
In [877]: plott(Y_samples, y_pred, 'SGD Implemented')
```




```
In [878]: test_error1 = mean_sq_error(X_test, y_test, w_latest1, b_latest1, 'SGD Implemented')
          print(test_error1)
```

36.732583474085736




```
In [879]: # Using Sklearn
          clf1, train_mse1 = sklearn_SGD(X_train, y_train, 100, 'constant', 0.01)
          test_mse1 = mean_squared_error(y_test, clf1.predict(X_test))
          train_mse1
```

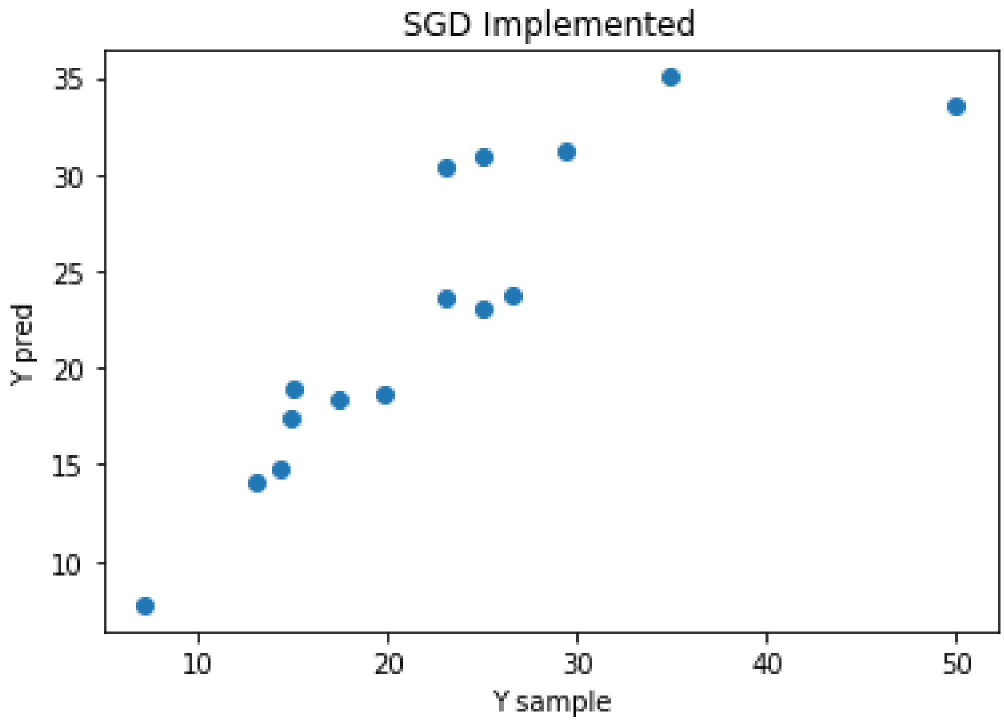
```
Out[879]: 25.44650397100788
```

itr = 1000

```
In [880]: X_samples, Y_samples, y_pred, w_latest2, b_latest2, train_error2  = stochastic_gradi
          print(train_error2)
```

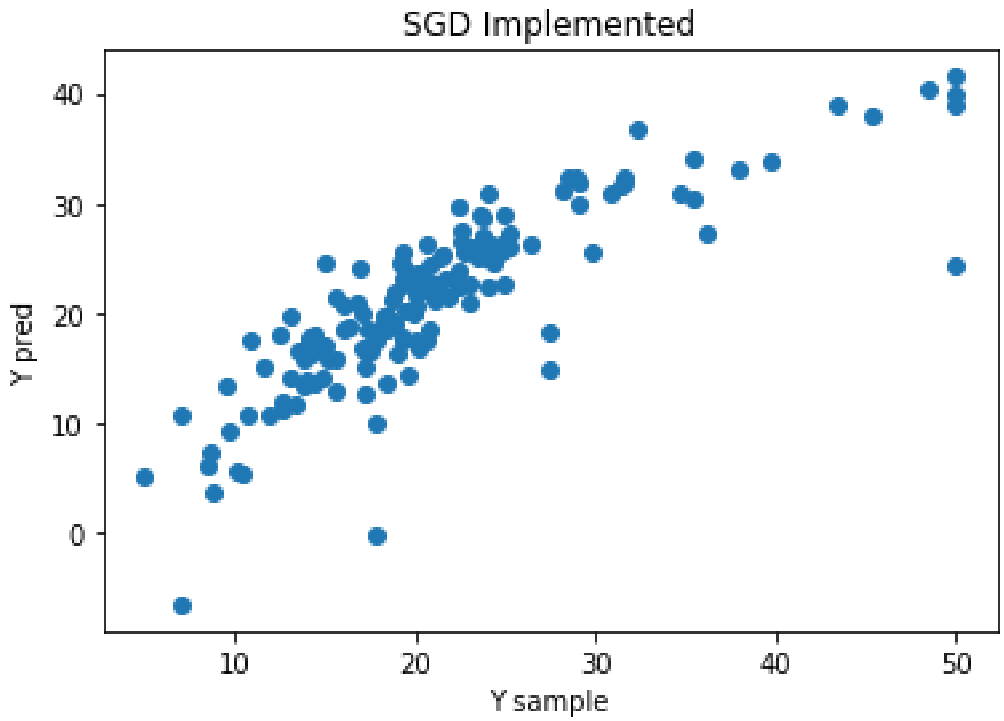
```
26.606888825077267
```

```
In [881]: plott(Y_samples, y_pred, 'SGD Implemented')
```




```
In [882]: test_error2 = mean_sq_error(X_test, y_test, w_latest2, b_latest2, 'SGD Implemented')
          print(test_error2)
```

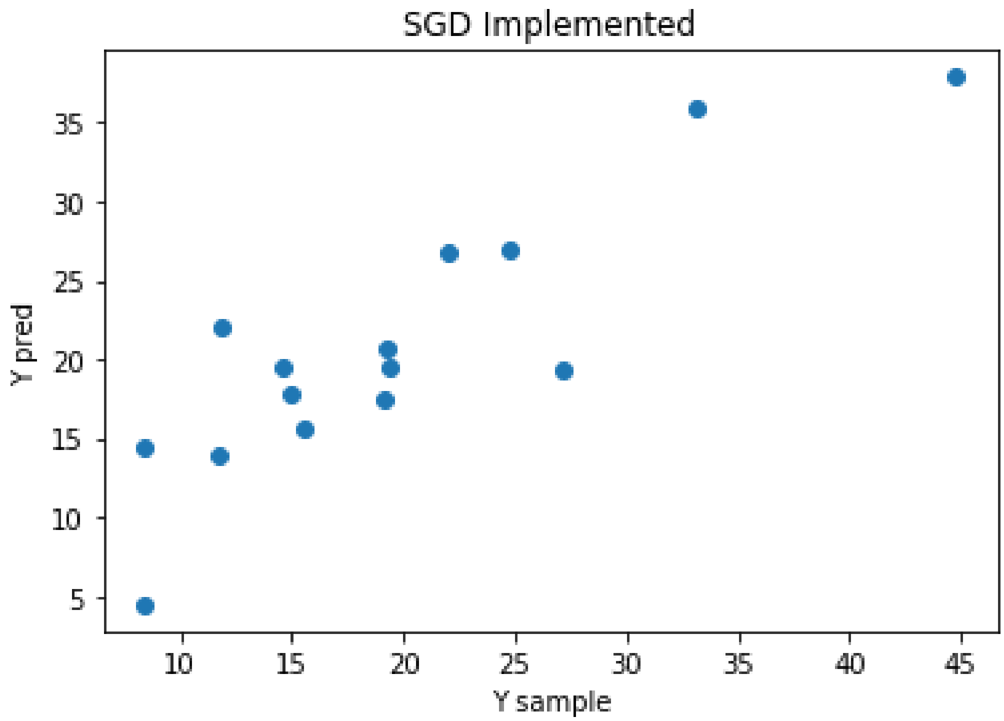
```
22.161182808502716
```




```
In [883]: clf2, train_mse2 = sklearn_SGD(X_train, y_train, 1000, 'constant', 0.01)
          test_mse2 = mean_squared_error(y_test,clf2.predict(X_test))
          train_mse2

Out[883]: 24.546898911592436
```

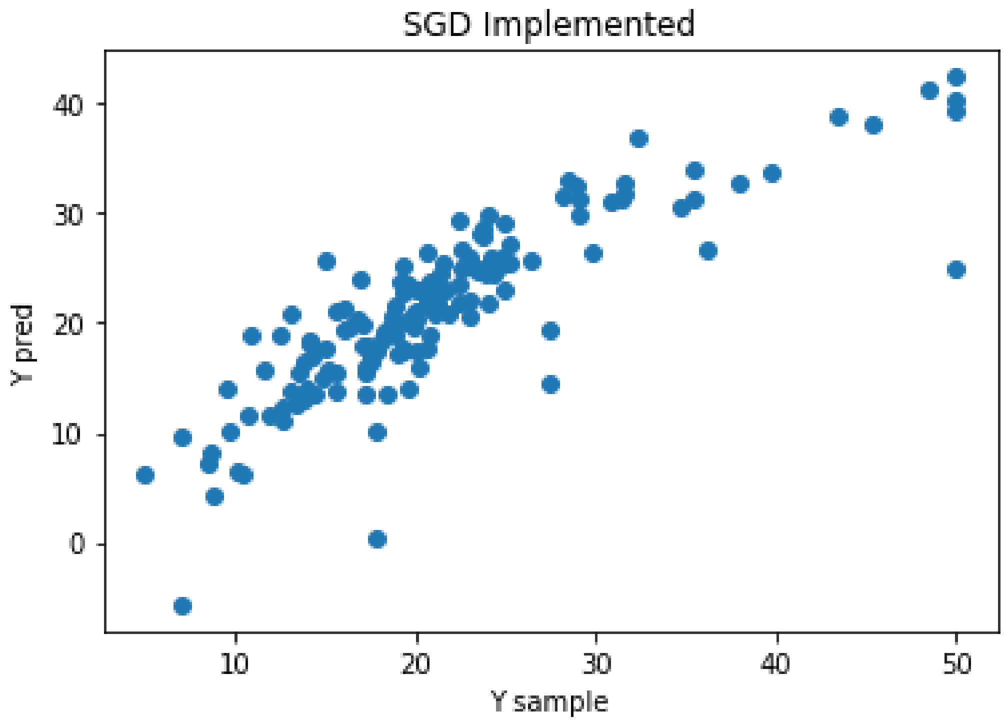
itr = 5000

```
In [884]: X_samples, Y_samples, y_pred, w_latest3, b_latest3, train_error3  = stochastic_gradi
          print(train_error3)

22.69599928570044


In [885]: plott(Y_samples, y_pred, 'SGD Implemented')
```




```
In [886]: test_error3 = mean_sq_error(X_test, y_test, w_latest3, b_latest3, 'SGD Implemented')
          print(test_error3)
```

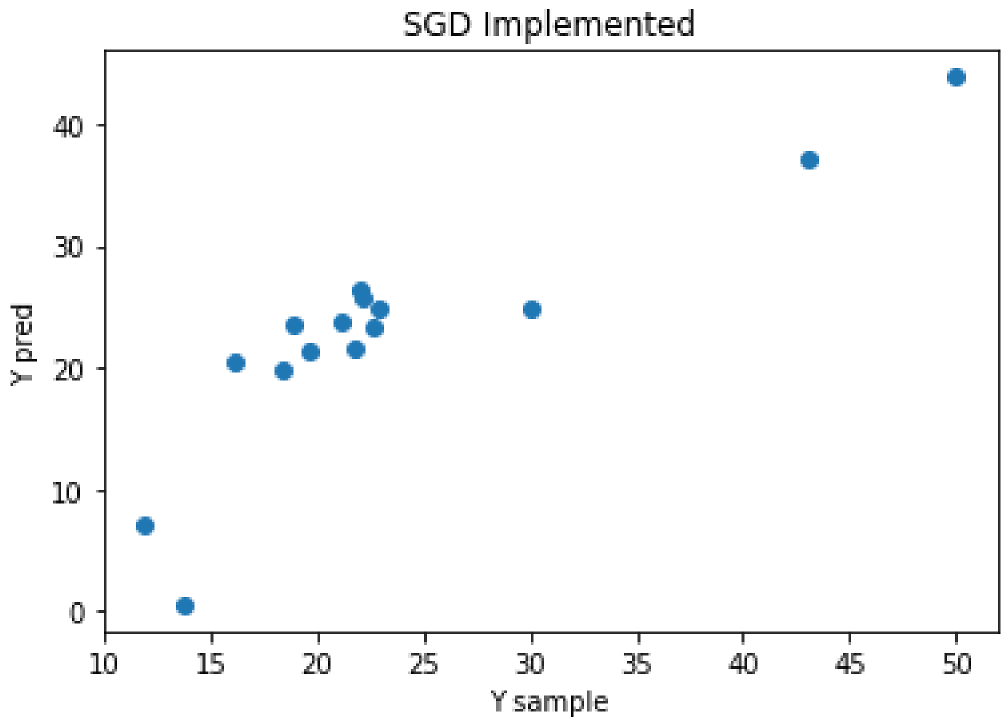
```
21.264608591302853
```




```
In [887]: clf3, train_mse3= sklearn_SGD(X_train, y_train, 5000, 'constant', 0.01)
          test_mse3 = mean_squared_error(y_test,clf1.predict(X_test))
          train_mse3

Out[887]: 22.920499619484392
```

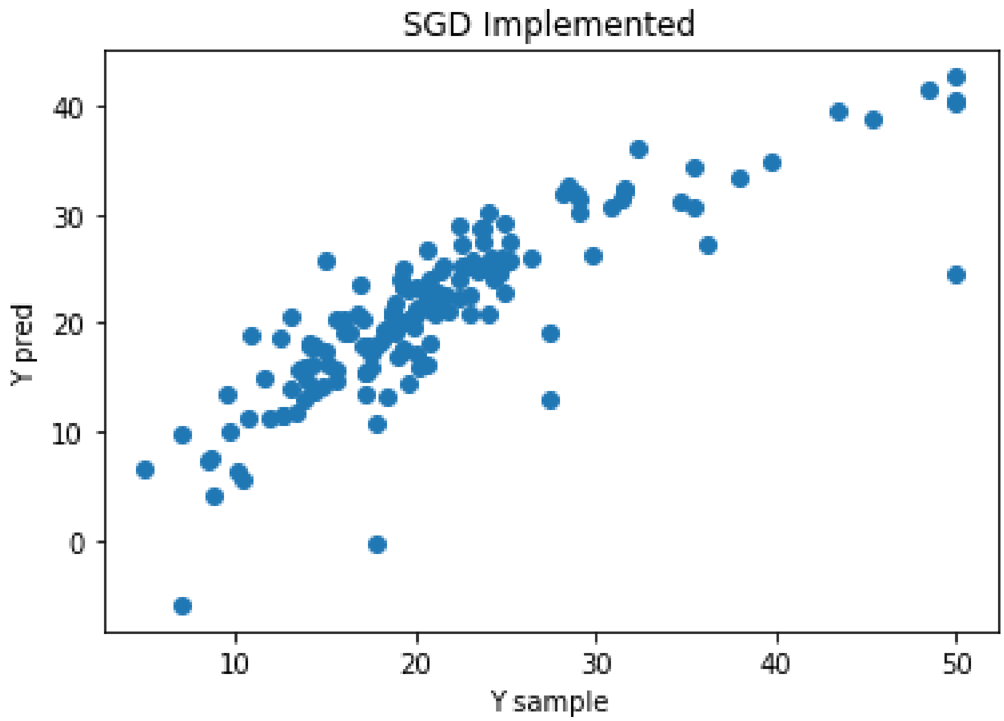
itr = 10000

```
In [888]: X_samples, Y_samples, y_pred, w_latest4, b_latest4, train_error4 = stochastic_gradie
          print(train_error4)

25.75845902947859

In [889]: plott(Y_samples, y_pred, 'SGD Implemented')
```




```
In [890]: test_error4 = mean_sq_error(X_test, y_test, w_latest4, b_latest4, 'SGD Implemented')
          print(test_error4)
```

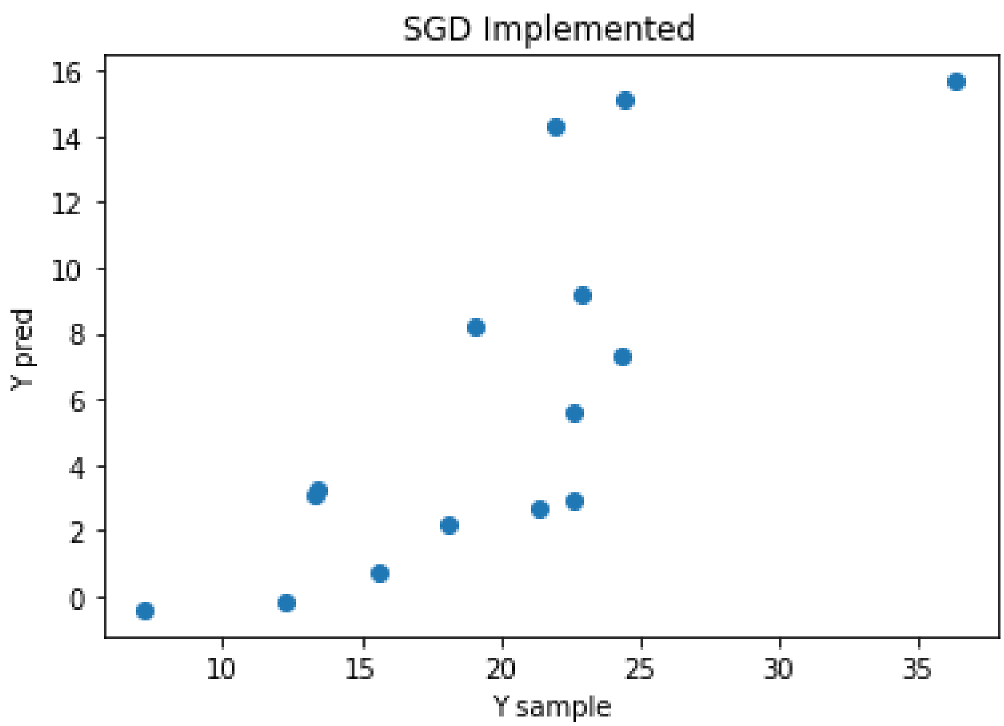
```
21.324201420092944
```



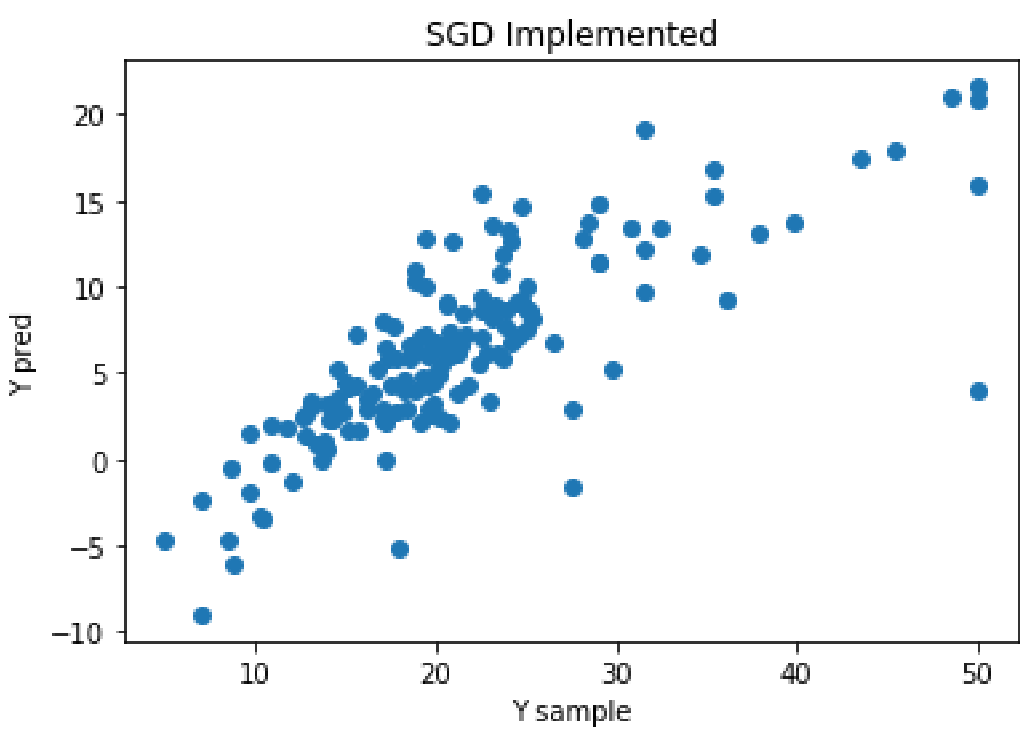

```
In [891]: clf4, train_mse4 = sklearn_SGD(X_train, y_train, 10000, 'constant', 0.01)
          test_mse4 = mean_squared_error(y_test,clf1.predict(X_test))
          train_mse4

Out[891]: 28.179224200228315
```

[3.3] Applying Mini-Batch SGD With Constant Inverse Scaling Learning Rate itr = 100

```
In [892]: X_samples, Y_samples, y_pred, w_latest5, b_latest5, train_error5 = stochastic_gradie
          train_error5

Out[892]: 206.6501596572818

In [893]: plott(Y_samples, y_pred, 'SGD Implemented')
```




```
In [894]: test_error5 = mean_sq_error(X_test, y_test, w_latest5, b_latest5, 'SGD Implemented')
```



In [895]:
```
# Using Sklearn
clf1, train_mse5 = sklearn_SGD(X_train, y_train, 100, 'invscaling', 0.01)
test_mse5 = mean_squared_error(y_test,clf1.predict(X_test))
train_mse5
```

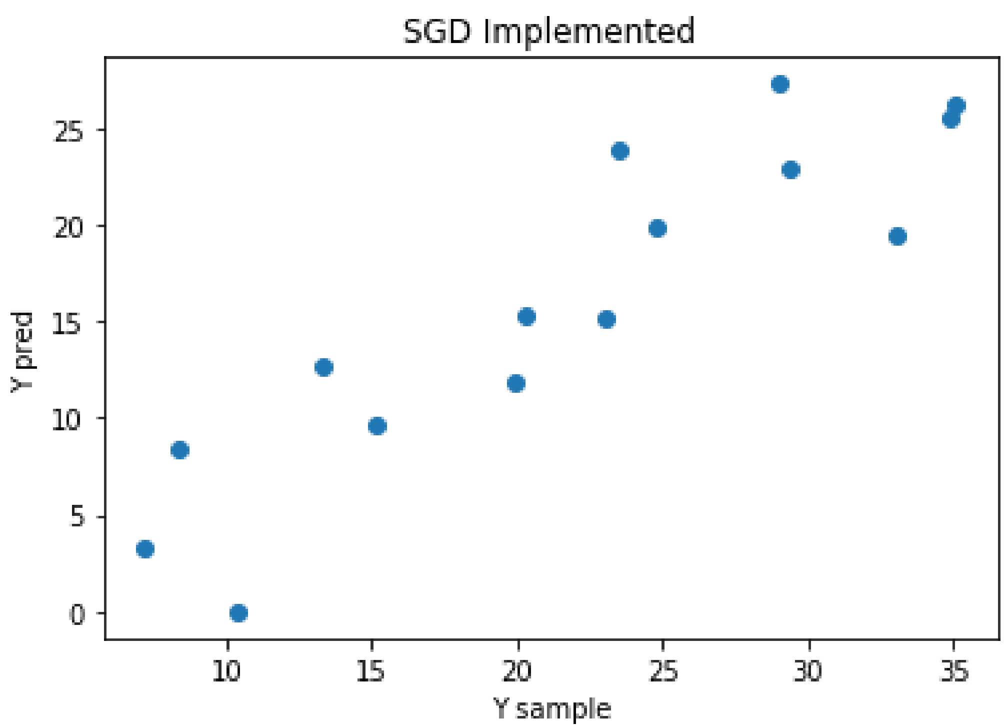
Out[895]: 22.60830921541977

itr = 1000

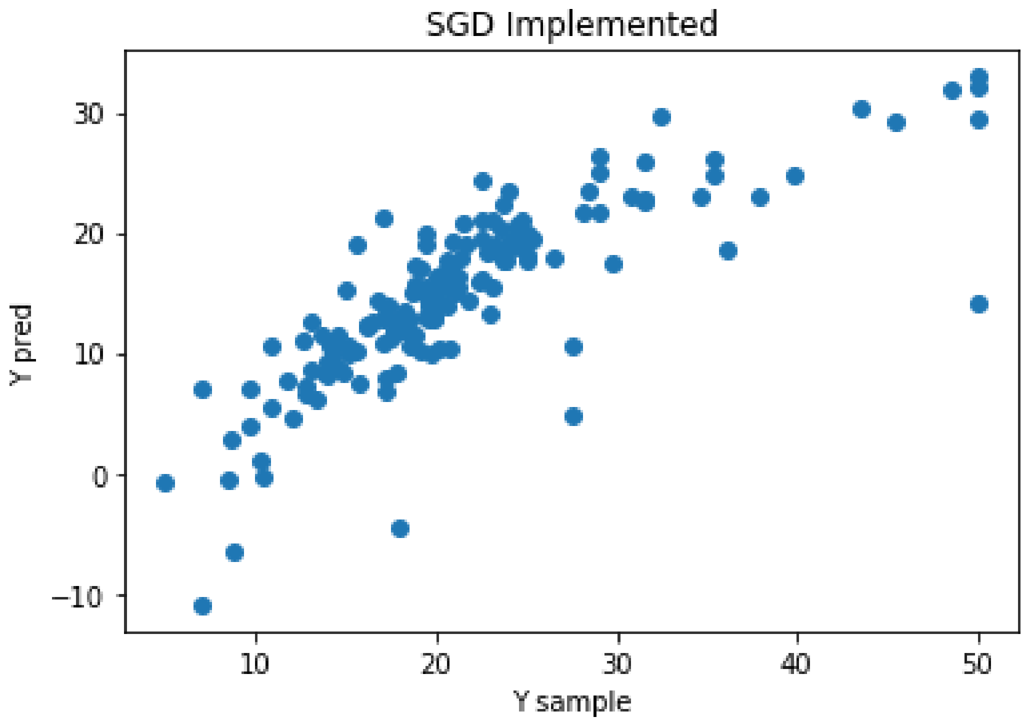
In [896]:
```
X_samples, Y_samples, y_pred, w_latest6, b_latest6, train_error6 = stochastic_gradien
train_error6
```

Out[896]: 48.2860956123789

In [897]:
```
plott(Y_samples, y_pred, 'SGD Implemented')
```



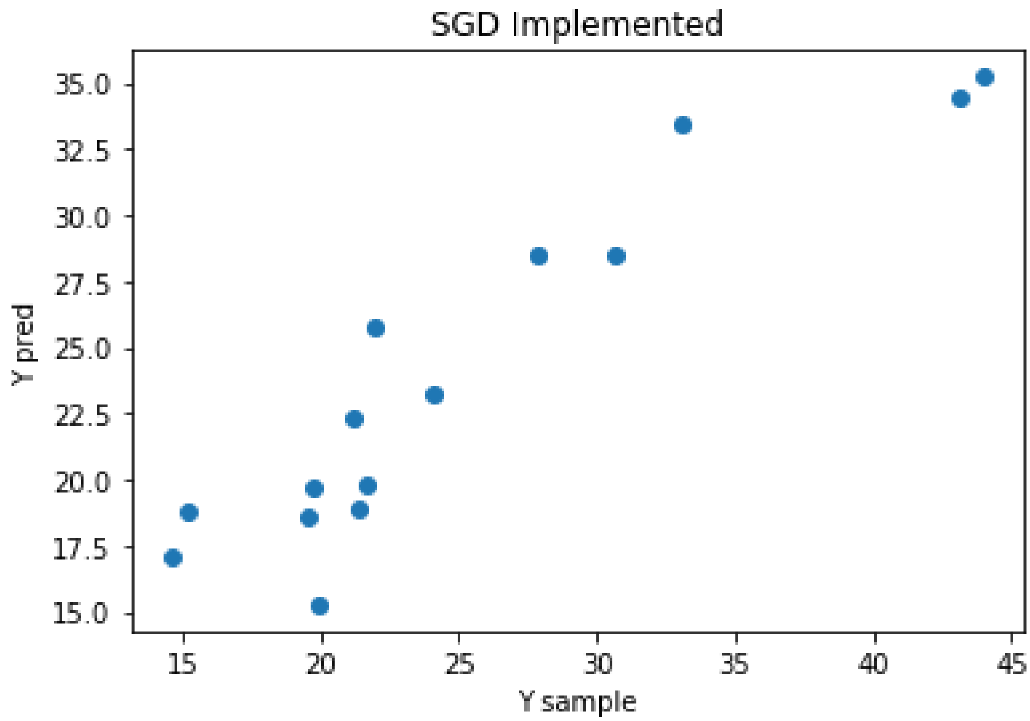
In [898]:
```
test_error6 = mean_sq_error(X_test, y_test, w_latest6, b_latest6, 'SGD Implemented')
```



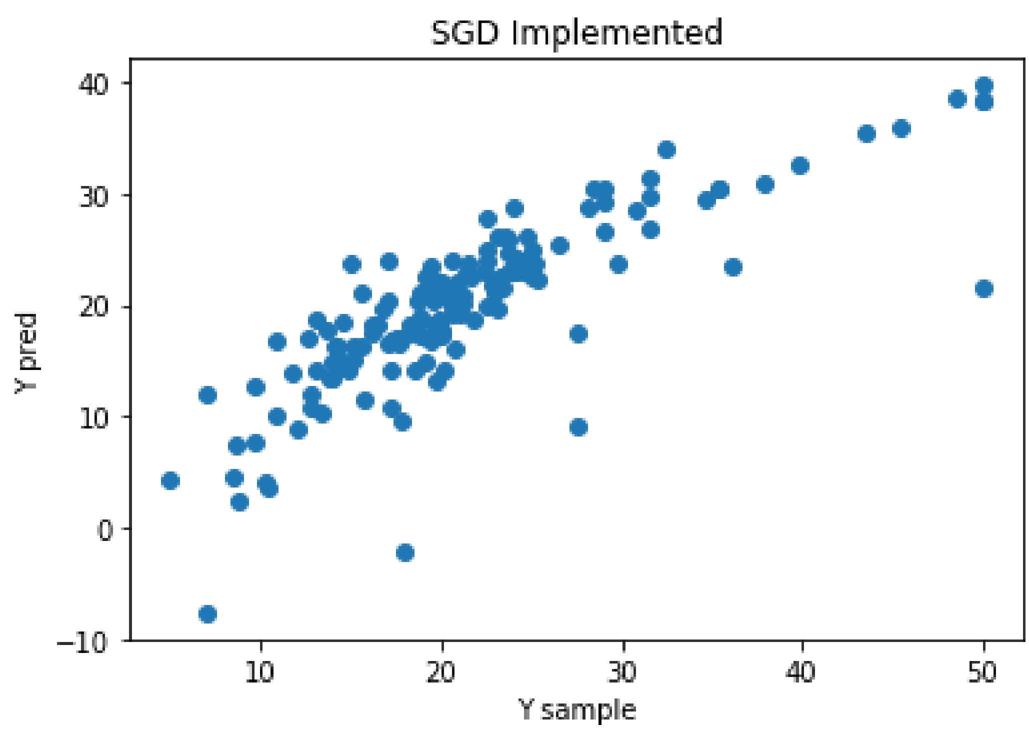

```
In [899]: # Using Sklearn
          clf1, train_mse6 = sklearn_SGD(X_train, y_train, 1000, 'invscaling', 0.01)
          test_mse6 = mean_squared_error(y_test,clf1.predict(X_test))
          train_mse6

Out[899]: 22.55418955713128
```

itr = 5000

```
In [900]: X_samples, Y_samples, y_pred, w_latest7, b_latest7, train_error7 = stochastic_gradier
          train_error7

Out[900]: 14.961183281522397

In [901]: plott(Y_samples, y_pred, 'SGD Implemented')
```



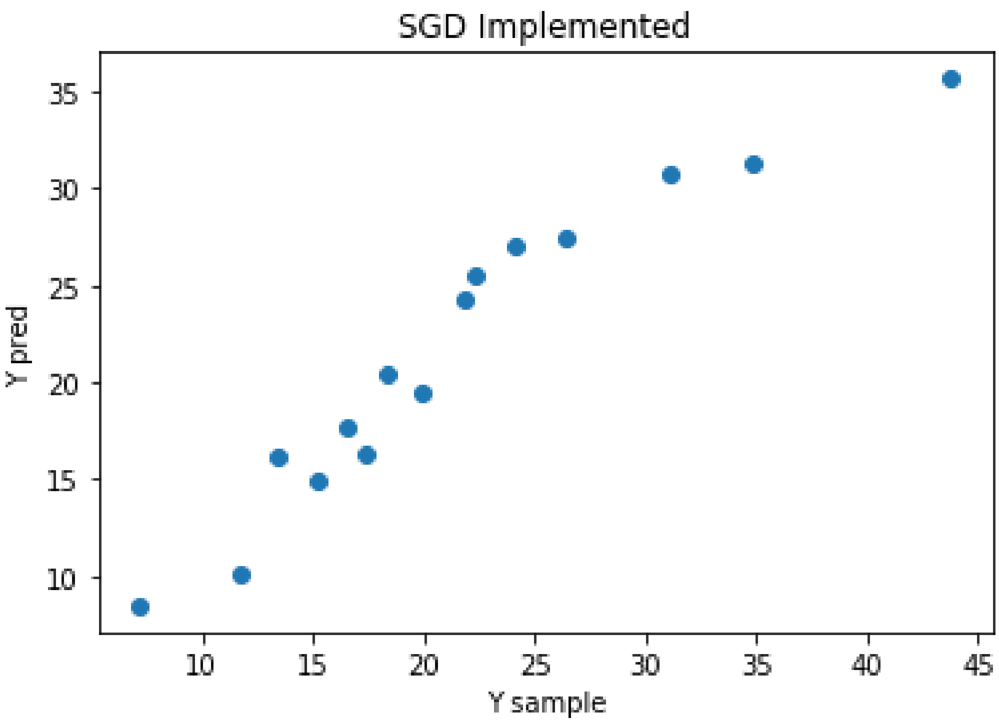

```
In [902]: test_error7 = mean_sq_error(X_test, y_test, w_latest7, b_latest7, 'SGD Implemented')
```



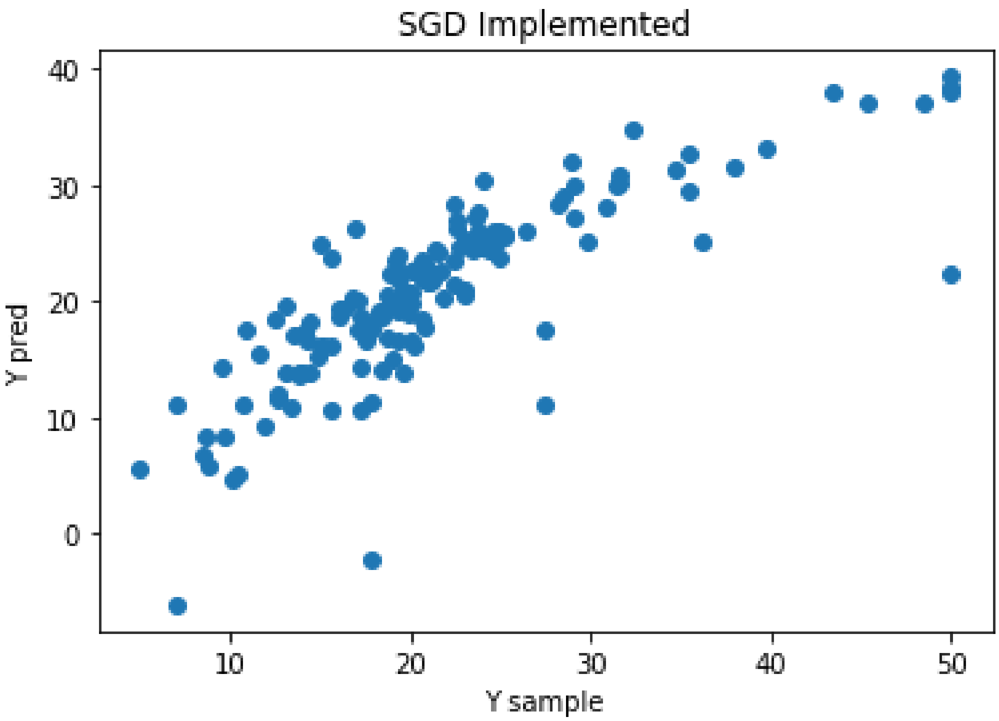
```
In [903]: # Using Sklearn
          clf1, train_mse7 = sklearn_SGD(X_train, y_train, 5000, 'invscaling', 0.01)
          test_mse7 = mean_squared_error(y_test,clf1.predict(X_test))
          train_mse7

Out[903]: 22.546122092391037
```

itr = 10000

```
In [904]: X_samples, Y_samples, y_pred, w_latest8, b_latest8, train_error8 = stochastic_gradier
          train_error8

Out[904]: 8.308924995459867

In [905]: plott(Y_samples, y_pred, 'SGD Implemented')
```




```
In [906]: test_error8 = mean_sq_error(X_test, y_test, w_latest8, b_latest8, 'SGD Implemented')
```




```
In [907]: # Using Sklearn
          clf1, train_mse8 = sklearn_SGD(X_train, y_train, 10000, 'invscaling', 0.01)
          test_mse8 = mean_squared_error(y_test,clf1.predict(X_test))
          train_mse8
```

```
Out[907]: 22.545657510733108
```

1 [4.1] Conclusions

```
In [910]: #Source: http://zetcode.com/python/prettytable/
          from prettytable import PrettyTable

          model_metric = PrettyTable()

          model_metric = PrettyTable(["Algorithm", "Learning Rate Type", "Learning Rate", 'No.

          model_metric.add_row(["SGD Implemented","constant", "0.01", "100", train_error1, tes
          model_metric.add_row(["SGD Implemented","constant", "0.01", "1000", train_error2, tes
          model_metric.add_row(["SGD Implemented","constant", "0.01", "5000", train_error3, tes
          model_metric.add_row(["SGD Implemented","constant", "0.01", "10000", train_error4, te
          model_metric.add_row(["SGD Implemented","invscaling", "0.01", "100", train_error5, te
          model_metric.add_row(["SGD Implemented","invscaling", "0.01", "1000", train_error6, t
          model_metric.add_row(["SGD Implemented","invscaling", "0.01", "5000", train_error7, t
```

```
model_metric.add_row(["SGD Implemented","invscaling", "0.01", "10000", train_error8,

model_metric.add_row(["Sklearn","constant", "0.01", "100", train_mse1, test_mse1])
model_metric.add_row(["Sklearn","constant", "0.01", "1000", train_mse2, test_mse2])
model_metric.add_row(["Sklearn","constant", "0.01", "5000", train_mse3, test_mse3])
model_metric.add_row(["Sklearn","constant", "0.01", "10000", train_mse4, test_mse4])
model_metric.add_row(["Sklearn","invscaling", "0.01", "100", train_mse5, test_mse5])
model_metric.add_row(["Sklearn","invscaling", "0.01", "1000", train_mse6, test_mse6]
model_metric.add_row(["Sklearn","invscaling", "0.01", "5000", train_mse7, test_mse7]
model_metric.add_row(["Sklearn","invscaling", "0.01", "10000", train_mse8, test_mse8

print(model_metric.get_string(start=0))
```

```
+-----------------+--------------------+---------------+-------------------+------------------
|    Algorithm    | Learning Rate Type | Learning Rate | No. of Iterations |      Train MSE
+-----------------+--------------------+---------------+-------------------+------------------
| SGD Implemented |      constant      |      0.01     |        100        | 48.92878984555921
| SGD Implemented |      constant      |      0.01     |        1000       | 26.60688882507726
| SGD Implemented |      constant      |      0.01     |        5000       | 22.69599928570044
| SGD Implemented |      constant      |      0.01     |       10000       | 25.75845902947859
| SGD Implemented |     invscaling     |      0.01     |        100        | 206.6501596572818
| SGD Implemented |     invscaling     |      0.01     |        1000       |  48.2860956123789
| SGD Implemented |     invscaling     |      0.01     |        5000       | 14.96118328152239
| SGD Implemented |     invscaling     |      0.01     |       10000       | 8.308924995459867
|     Sklearn     |      constant      |      0.01     |        100        | 25.44650397100788
|     Sklearn     |      constant      |      0.01     |        1000       | 24.54689891159243
|     Sklearn     |      constant      |      0.01     |        5000       | 22.92049961948439
|     Sklearn     |      constant      |      0.01     |       10000       | 28.17922420022831
|     Sklearn     |     invscaling     |      0.01     |        100        | 22.60830921541977
|     Sklearn     |     invscaling     |      0.01     |        1000       | 22.55418955713128
|     Sklearn     |     invscaling     |      0.01     |        5000       | 22.54612209239103
|     Sklearn     |     invscaling     |      0.01     |       10000       | 22.54565751073310
+-----------------+--------------------+---------------+-------------------+------------------
```

1.1 [4.2] Steps for implementing the Mini-Batch SGD

Equations of Mini-batch SGD for Linear regression:

$$\frac{\partial L}{\partial w} = w_0 - \sum_{k=1}^{j} -(2/j)x_i(y_i - (w^T * x_i + b_i))$$

$$\frac{\partial L}{\partial b} = b_0 - \sum_{k=1}^{j} -(2/j)(y_i - (w^T * x_i + b_i))$$

1) Using the train data to find the components of the SGD for Linear regression such as m_derivative(w_i) and b_derivative(b_i) for a batch of randomly sampled points from train data for 'n' number of epochs. For every epoch a new batch is sampled here.

2) After running for n epochs, we'll obtain the optimal m_derivative and b_derivative values for the nth batch which is sum of m_derivative and b_derivative for all the points. Now multiply both of them with the learning rate to get w_i and b_i. Now, get w_0 and b_0 from n-1 epoch of the batch and subtract with the w_i and b_i respectively to get the updated values w_updated and b_updated.

3) When a point(x_test) in test data is given we'll calculate the following for the price prediction:

```
y_pred = w_updated * x_test+ b_updated
```

4) The above step is repeated for all the test data points and all the predicted values are stored in a list.

5) If the prices of the corresponding test data is present, we can plot a scatter plot for the predicted values and the actual values for the test data. Similar plot can be done for train data as well.

6) Mean Square Error is calculated for the test data to check the performance of the model on the unseen data. If this is high then the model performance is bad.

1.2 [4.3] Observations

1. Mini-Batch SGD for linear regression is implemeted with the help of Sklearn's SGDRegressor. This algorithm uses two types of learning rates: a) Constant b) Inverse Scaling or invscaling
2. Sklearn's SGDRegressor is also utilized to generate results on the same data.
3. No. of iterations used for both type of implementations are 100, 1000, 5000, 1000
4. The Mean Square Errors(MSE) of the implemented Mini-Batch SGD both on Train and Test data are very close to that of Sklearn's SGDRegressor results except in 3 cases.
5. The results are consistent and reproducible.
6. By observation on all the graphs, the points on the graph are aligned towards a straight line.