Programming Exercise 6

Support Vector Machines

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In this exercise, you will be using support vector machines (SVMs) to build a spam classifier.

1. Support Vector Machines

In the first half of this exercise, we will be using support vector machines (SVMs) with various example 2D datasets. Experimenting with these datasets will help us gain an intuition of how SVMs work and how to use a Gaussian kernel with SVMs. In the next half of the exercise, we will be using support vector machines to build a spam classifier.

1.1 Example dataset 1

We will begin with a 2D example dataset which can be separated by a linear boundary. The code below will plot the training data (**Figure 1**). In this dataset, the positions of the positive examples (indicated with +) and the negative examples (indicated with o) suggest a natural separation indicated by the gap. However, notice that there is an outlier positive example (+) on the far left at about (0:1; 4:1). As part of this exercise, we will also see how this outlier affects the SVM decision boundary.

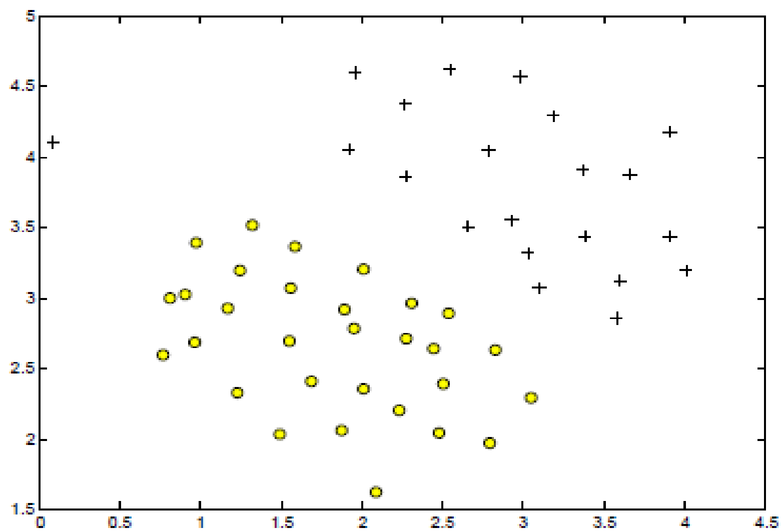
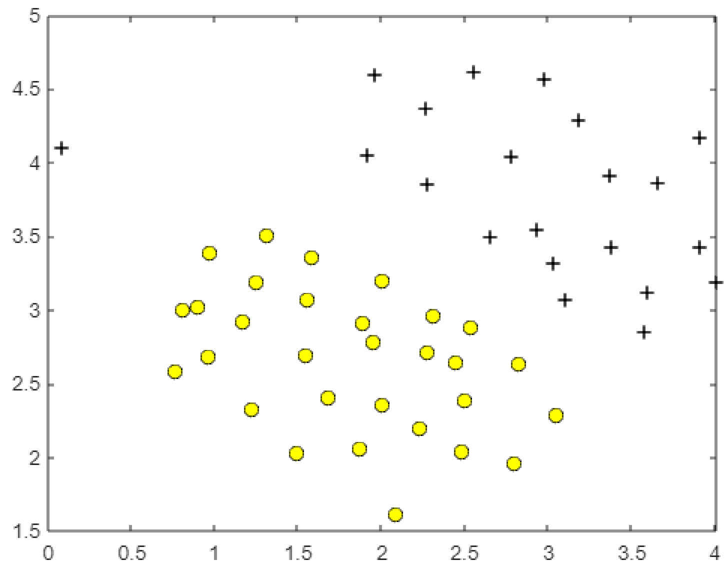


Figure 1: Example Dataset 1

```
% Load from ex6data1:
% You will have X, y in your environment
load('ex6data1.mat');

% Plot training data
plotData(X, y);
```

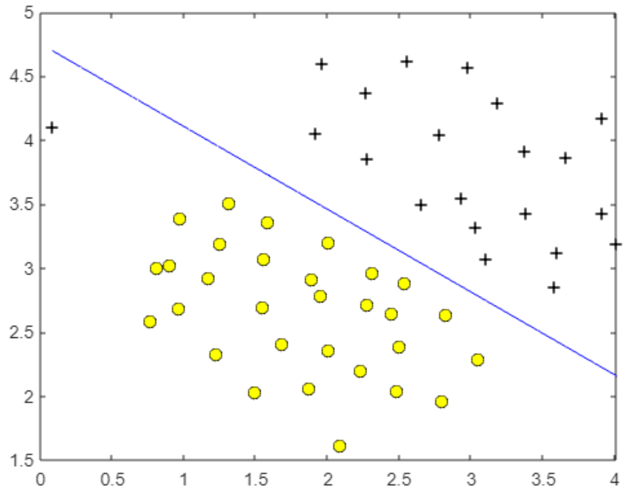


In this part of the exercise, you will try using different values of the C parameter with SVMs. Informally, the C parameter is a positive value that controls the penalty for misclassified training examples. A large C parameter tells the SVM to try to classify all the examples correctly. C plays a role similar to $1/\lambda$, where λ is the regularization parameter that we were using previously for logistic regression. The code below will run the SVM training (with $C = 1$) using SVM software that we have included with the starter code, svmTrain.m.

```
C = 1;
model = svmTrain(X, y, C, @linearKernel, 1e-3, 20);
```

```
Training ......................................................................
............................................................... Done!
```

```
visualizeBoundaryLinear(X, y, model);
```



When $C = 1$, you should find that the SVM puts the decision boundary in the gap between the two datasets and misclassifies the data point on the far left (**Figure 2**).

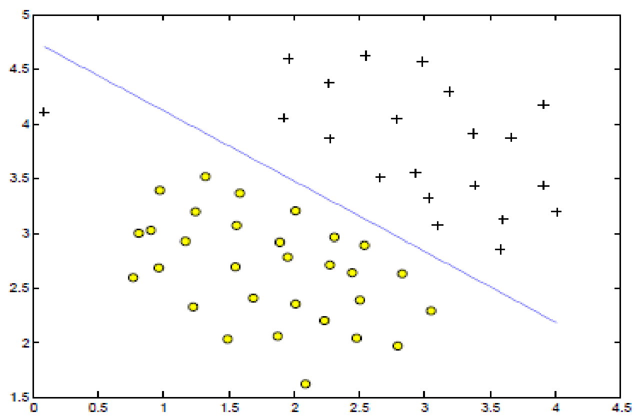


Figure 2: SVM Decision Boundary with $C = 1$ (Example Dataset 1)

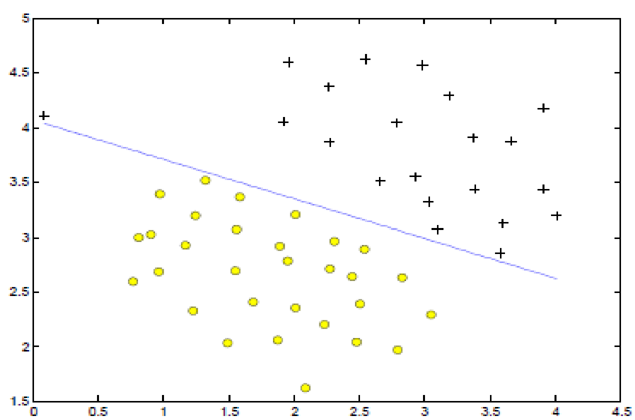


Figure 3: SVM Decision Boundary with $C = 100$ (Example Dataset 1)

Our task is to try different values of C on this dataset. Specifically, use the control to change the value of C in the script to $C = 100$ and run the SVM training code in the previous section again. When $C = 100$, we should find that the SVM now classifies every single example correctly, but has a decision boundary that does not appear to be a natural fit for the data (**Figure 3**).

1.2 SVM with gaussian kernels

In this part of the exercise, you will be using SVMs to do non-linear classification. In particular, we will be using SVMs with Gaussian kernels on datasets that are not linearly separable.

1.2.1 Gaussian kernel

To find nonlinear decision boundaries with the SVM, we need to first implement a Gaussian kernel. We can think of the Gaussian kernel as a similarity function that measures the 'distance' between a pair of examples, $(x^{(i)}, x^{(j)})$. The Gaussian kernel is also parameterized by a bandwidth parameter, σ, which determines how fast the similarity metric decreases (to 0) as the examples are further apart.

We should now complete the code in **gaussianKernel.m** to compute the Gaussian kernel between two examples, $(x^{(i)}, x^{(j)})$. The Gaussian kernel function is defined as:

$$K_{gaussian}(x^{(i)}, x^{(j)}) = \exp\left(-\frac{\|x^{(i)} - x^{(j)}\|^2}{2\sigma^2}\right) = \exp\left(-\frac{\sum_{k=1}^{n}(x_k^{(i)} - x_k^{(j)})^2}{2\sigma^2}\right)$$

Once we've completed the function **gaussianKernel.m**, run the code below to test our kernel function on two provided examples, we should expect to see **a value of 0.324652** with **sigma set to 2.**

```
x1 = [1 2 1]; x2 = [0 4 -1];
sigma = 2;
sim = gaussianKernel(x1, x2, sigma);
fprintf('Gaussian Kernel between x1 = [1; 2; 1], x2 = [0; 4; -1], sigma = %f :
\n\t%g\n', sigma, sim);
```

```
Gaussian Kernel between x1 = [1; 2; 1], x2 = [0; 4; -1], sigma = 2.000000 :
0.324652
```

1.2.2 Example dataset 2

The next code section will load and plot dataset 2 (**Figure 4**). From the figure, we can observe that there is no linear decision boundary that separates the positive and negative examples for this dataset.

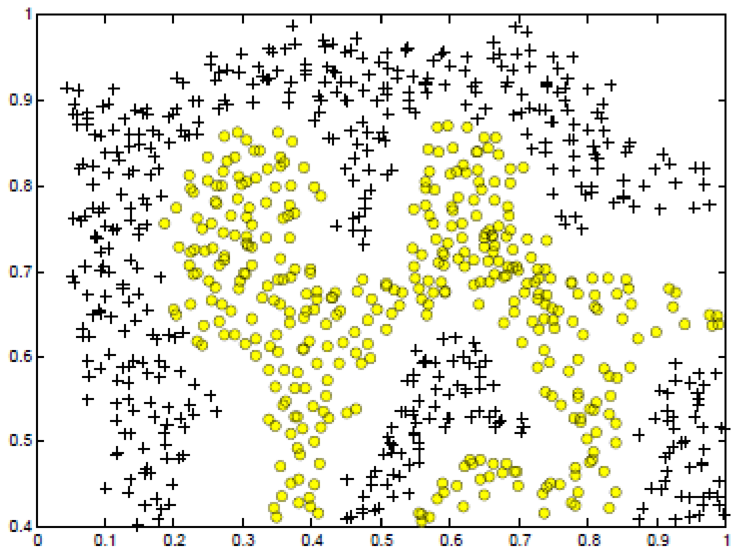
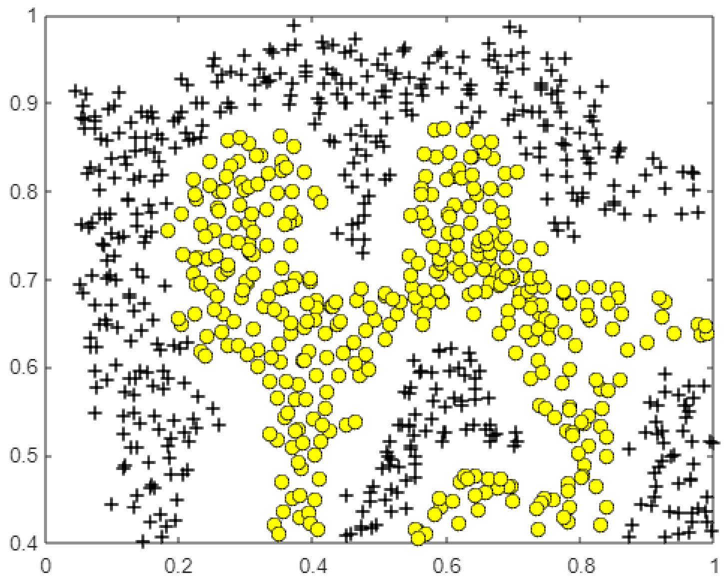


Figure 4: Example Dataset 2

However, by using the Gaussian kernel with the SVM, we will be able to learn a non-linear decision boundary that can perform reasonably well for the dataset.

```
% Load from ex6data2:
% You will have X, y in your environment
load('ex6data2.mat');

% Plot training data
plotData(X, y);
```



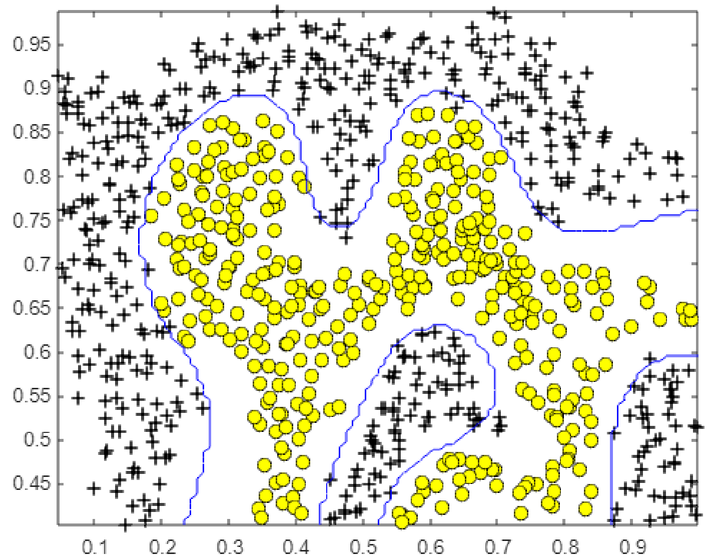
If we have correctly implemented the Gaussian kernel function, the code below will proceed to train the SVM with the Gaussian kernel on this dataset.

```
% SVM Parameters
C = 1; sigma = 0.1;

% We set the tolerance and max_passes lower here so that the code will run faster. However, in practice,
% you will want to run the training to convergence.
model= svmTrain(X, y, C, @(x1, x2) gaussianKernel(x1, x2, sigma));
```

```
Training ......................................................................
...............................................................................
...............................................................................
...............................................................................
...............................................................................
...............................................................................
...............................................................................
Done!
```

```
visualizeBoundary(X, y, model);
```



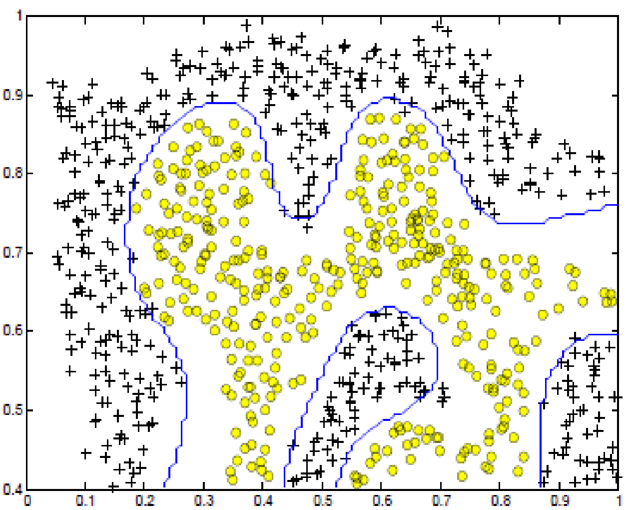


Figure 5: SVM (Gaussian Kernel) Decision Boundary (Example Dataset 2)

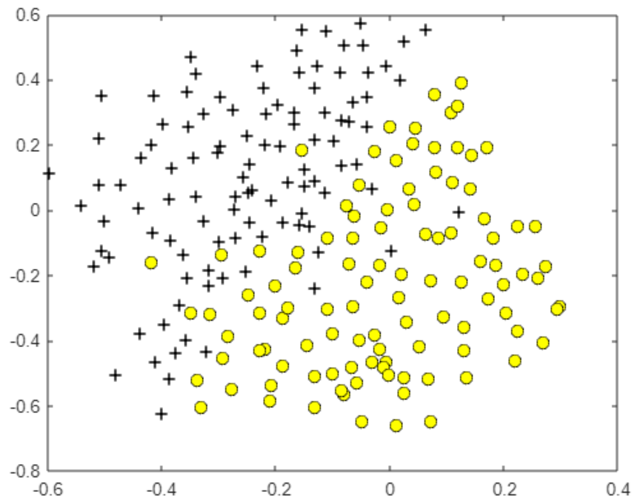
Figure 5 shows the decision boundary found by the SVM with a Gaussian kernel. The decision boundary is able to separate most of the positive and negative examples correctly and follows the contours of the dataset well.

1.2.3 Example dataset 3

In this part of the exercise, we will gain more practical skills on how to use a SVM with a Gaussian kernel. The code below will load and display a third dataset (**Figure 6**).

```
% Load from ex6data3:
% You will have X, y in your environment
load('ex6data3.mat');

% Plot training data
plotData(X, y);
```



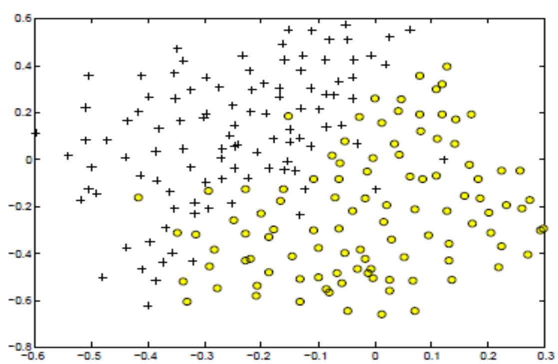


Figure 6: Example Dataset 3

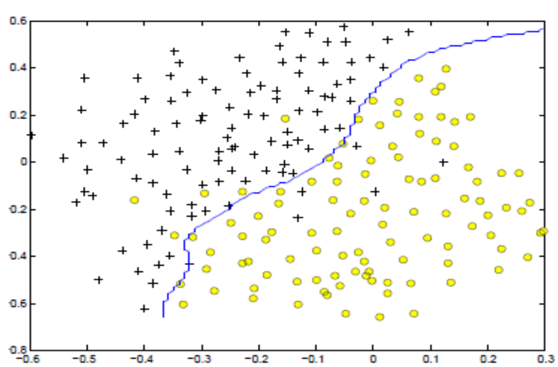


Figure 7: SVM (Gaussian Kernel) Decision Boundary (Example Dataset 3)

We will be using the SVM with the Gaussian kernel with this dataset. In the provided dataset, **ex6data3.mat**, we are given the variables X, y, Xval, yval. The provided code will train the SVM classifier using the training set (X, y) using parameters loaded from **dataset3Params.m**.

Our task is to use the cross validation set Xval, yval to determine the best C and σ parameter to use. We should write any additional code necessary to help us search over the parameters C and σ. For both C and σ we suggest trying values in multiplicative steps (e.g., 0.01, 0.03, 0.1, 0.3, 1, 3, 10, 30). Note that we should try all possible pairs of values for C and σ (e.g., $C = 0.3$ and $\sigma = 0.1$). For example, if we try each of the 8 values listed above for C and for σ^2, we would end up training and evaluating (on the cross validation set) a total of 8^2 = 64 different models. After we have determined the best C and σ parameters to use, we should modify the code in **dataset3Params.m**, filling in the best parameters.

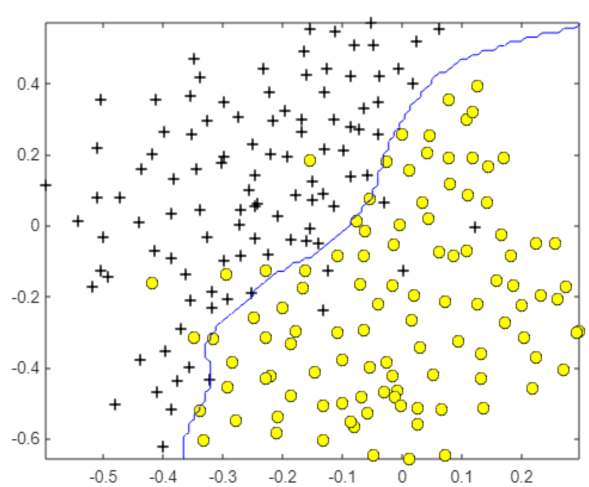
```
% Try different SVM Parameters here
[C, sigma] = dataset3Params(X, y, Xval, yval);
```

```
Training ......... Done!
Training ................................. Done!
```

```
% Train the SVM
model = svmTrain(X, y, C, @(x1, x2)gaussianKernel(x1, x2, sigma));
```

```
Training ..........................................................................
..........................................................................
..................................................................... Done!
```

```
visualizeBoundary(X, y, model);
```



2. Spam Classification

Many email services today provide spam filters that are able to classify emails into spam and non-spam email with high accuracy. In this part of the exercise, we will use SVMs to build our own spam filter. We will be training a classifier to classify whether a given email, x, is spam ($y = 1$) or non-spam ($y = 0$). In particular, we need to convert each email into a feature vector $x \in \mathbb{R}^n$. The following parts of the exercise will walk us through how such a feature vector can be constructed from an email.

The dataset included for this exercise is based on *a subset* of the SpamAssassin Public Corpus. For the purpose of this exercise, we will only be using the body of the email (excluding the email headers).

2.1 Preprocessing emails

Before starting on a machine learning task, it is usually insightful to take a look at examples from the dataset. **Figure 8** shows a sample email that contains a URL, an email address (at the end), numbers, and dollar amounts. While many emails would contain similar types of entities (e.g., numbers, other URLs, or other email addresses), the specific entities (e.g., the specific URL or specific dollar amount) will be different in almost every email.

```
> Anyone knows how much it costs to host a web portal ?
>
Well, it depends on how many visitors youre expecting.  This can be
anywhere from less than 10 bucks a month to a couple of $100.  You
should checkout http://www.rackspace.com/ or perhaps Amazon EC2 if
youre running something big..

To unsubscribe yourself from this mailing list, send an email to:
groupname-unsubscribe@egroups.com
```

Figure 8: Sample Email

Therefore, one method often employed in processing emails is to 'normalize' these values, so that all URLs are treated the same, all numbers are treated the same, etc. For example, we could replace each URL in the email with the unique string "**httpaddr**" to indicate that a URL was present. This has the effect of letting the spam classifier make a classification decision based on whether any URL was present, rather than whether a specific URL was present. This typically improves the performance of a spam classifier, since spammers often randomize the URLs, and thus the odds of seeing any particular URL again in a new piece of spam is very small.

In **processEmail.m**, we have implemented the following email preprocessing and normalization steps:

- ✓Lower-casing: The entire email is converted into lower case, so that capitalization is ignored (e.g., IndIcaTE is treated the same as indicate).
- ✓Stripping HTML: All HTML tags are removed from the emails. Many emails often come with HTML formatting; we remove all the HTML tags, so that only the content remains.
- ✓Normalizing URLs: All URLs are replaced with the text "httpaddr".
- ✓Normalizing Email Addresses: All email addresses are replaced with the text "emailaddr".
- ✓Normalizing Numbers: All numbers are replaced with the text 'number'.
- ✓Normalizing Dollars: All dollar signs ($) are replaced with the text 'dollar'.
- ✓Word Stemming: Words are reduced to their stemmed form. For example, 'discount', 'discounts', 'discounted' and 'discounting' are all replaced with 'discount'. Sometimes, the Stemmer actually strips off additional characters from the end, so 'include', 'includes', 'included', and 'including' are all replaced with 'includ'.
- ✓Removal of non-words: Non-words and punctuation have been removed. All white spaces (tabs, newlines, and spaces) have all been trimmed to a single space character.

```
anyon know how much it cost to host a web portal well it depend on how
mani visitor your expect thi can be anywher from less than number buck
a month to a coupl of dollarnumb you should checkout httpaddr or perhap
amazon ecnumb if your run someth big to unsubscrib yourself from thi
mail list send an email to emailaddr
```

Figure 9: Preprocessed Sample Email

```
1 aa
2 ab
3 abil
...
86 anyon
...
916 know
...
1898 zero
1899 zip
```

Figure 10: Vocabulary List

```
86 916 794 1077 883
370 1699 790 1822
1831 883 431 1171
794 1002 1893 1364
592 1676 238 162 89
688 945 1663 1120
1062 1699 375 1162
479 1893 1510 799
1182 1237 810 1895
1440 1547 181 1699
1758 1896 688 1676
992 961 1477 71 530
1699 531
```

Figure 11: Word Indices for Sample Email

The result of these preprocessing steps is shown in **Figure 9**. While preprocessing has left word fragments and non-words, this form turns out to be much easier to work with for performing feature extraction.

2.1.1 Vocabulary list

After preprocessing the emails, we have a list of words (e.g., **Figure 9**) for each email. The next step is to choose which words we would like to use in our classifier and which we would want to leave out. For this exercise, we have chosen only the most frequently occurring words as our set of words considered (the vocabulary list). Since words that occur rarely in the training set are only in a few emails, they might cause the model to overfit our training set. The complete vocabulary list is in the file **vocab.txt** and also shown in **Figure 10**. Our vocabulary list was selected by choosing all words which occur at least a 100 times in the spam corpus, resulting in a list of 1899 words. In practice, a vocabulary list with about 10,000 to 50,000 words is often used.

Given the vocabulary list, we can now map each word in the preprocessed emails (e.g., **Figure 9**) into a list of word indices that contains the index of the word in the vocabulary list. **Figure 11** shows the mapping for the sample email. Specially, in the sample email, the word 'anyone' was first normalized to '**anyon**' and then mapped onto the index 86 in the vocabulary list.

Our task now is to complete the code in **processEmail.m** to perform this mapping. In the code, we are given a string **str** which is a single word from the processed email. We should look up the word in the vocabulary list vocabList and find if the word exists in the vocabulary list. If the word exists, we should add the index of the word into the word indices variable. If the word does not exist, and is therefore not in the vocabulary, we can skip the word.

Once we have implemented **processEmail.m**, the code below will run our code on the email sample and we should see an output similar to **Figures 9 & 11**.

```
%% Initialization
clear;

% Extract Features
file_contents = readFile('emailSample1.txt');
word_indices  = processEmail(file_contents);
```

```
==== Processed Email ====

anyon know how much it cost to host a web portal well it depend on how mani
visitor you re expect thi can be anywher from less than number buck a month
to a coupl of dollarnumb you should checkout httpaddr or perhap amazon ecnumb
if your run someth big to unsubscrib yourself from thi mail list send an
email to emailaddr

=========================
```

```
% Print Stats
disp(word_indices)
```

```
          86
         916
         794
        1077
         883
         370
        1699
         790
        1822
        1831
         883
         431
        1171
         794
        1002
        1893
        1364
         592
        1676
         238
         162
          89
         688
         945
        1663
        1120
        1062
        1699
         375
        1162
         479
        1893
        1510
         799
        1182
        1237
         810
        1895
        1440
        1547
         181
        1699
        1758
        1896
         688
        1676
         992
         961
        1477
          71
         530
        1699
         531
```

2.2 Extracting features from emails

We will now implement the feature extraction that converts each email into a vector in $\mathbb{R}^n$. For this exercise, we will be using words in vocabulary list. Specially, the feature $x_i \in \{0, 1\}$ for an email corresponds to whether the i-th word in the dictionary occurs in the email. That is, $x_i = 1$ if the i-th word is in the email and $x_i = 0$ if the i-th word is not present in the email.

Thus, for a typical email, this feature would look like:

$$x = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in R^n$$

We should now complete the code in **emailFeatures.m** to generate a feature vector for an email, given the word indices. Once we have implemented **emailFeatures.m**, the code below will run on the email sample. We should see that the feature vector had **length 1899 and 45 non-zero** entries.

```
% Extract Features
features = emailFeatures(word_indices);

% Print Stats
fprintf('Length of feature vector: %d\n', length(features));
```

```
Length of feature vector: 1899
```

```
fprintf('Number of non-zero entries: %d\n', sum(features > 0));
```

```
Number of non-zero entries: 45
```

2.3 Training SVM for spam classification

After you have completed the feature extraction functions, the code in this section will load a preprocessed training dataset that will be used to train an SVM classifier. spamTrain.mat contains 4000 training examples of spam and non-spam email, while spamTest.mat contains 1000 test examples. Each original email was processed using the processEmail and emailFeatures functions and converted into a vector $x^{(i)} \in \mathbb{R}^{1899|}$. After loading the dataset, the code will proceed to train a SVM to classify between spam ($y = 1$) and non-spam ($y = 0$) emails. Once the training completes, you should see that the classifier gets a training accuracy of about **99.8%** and a test accuracy of about **98.5%**.

```
% Load the Spam Email dataset
% You will have X, y in your environment
load('spamTrain.mat');
C = 0.1;
model = svmTrain(X, y, C, @linearKernel);
```

```
Training ............................................................
.....................................................................
.....................................................................
............................................... Done!
```

```
p = svmPredict(model, X);
fprintf('Training Accuracy: %f\n', mean(double(p == y)) * 100);
```

```
Training Accuracy: 99.850000
```

```
% Load the test dataset
% You will have Xtest, ytest in your environment
load('spamTest.mat');

p = svmPredict(model, Xtest);
fprintf('Test Accuracy: %f\n', mean(double(p == ytest)) * 100);
```

```
Test Accuracy: 98.700000
```

2.4 Top predictors for spam

To better understand how the spam classifier works, we can inspect the parameters to see which words the classifier thinks are the most predictive of spam. The code below finds the parameters with the largest positive values in the classier and displays the corresponding words (**Figure 12**). Thus, if an email contains words such as 'guarantee', 'remove', 'dollar', and 'price' (the top predictors shown in **Figure 12**), it is likely to be classified as spam.

```
our click remov guarante visit basenumb dollar will price pleas nbsp
most lo ga dollarnumb
```

Figure 12: Top predictors for spam email

```
% Sort the weights and obtin the vocabulary list
[weight, idx] = sort(model.w, 'descend');
vocabList = getVocabList();
for i = 1:15
    if i == 1
        fprintf('Top predictors of spam: \n');
    end
    fprintf('%-15s (%f) \n', vocabList{idx(i)}, weight(i));
end
```

```
 Top predictors of spam:
our             (0.496981)
click           (0.466770)
remov           (0.422633)
guarante        (0.386021)
visit           (0.366995)
basenumb        (0.346722)
dollar          (0.325156)
will            (0.262955)
pleas           (0.261635)
price           (0.256674)
most            (0.255992)
nbsp            (0.255622)
lo              (0.252053)
al              (0.238782)
ga              (0.238554)
```

2.5 Optional (ungraded) exercise: Try our own emails

Now that we have trained a spam classifier, we can start trying it out on our own emails. In the starter code, we have included two email examples (**emailSample1.txt and emailSample2.txt**) and two spam examples (**spamSample1.txt and spamSample2.txt**). The code below runs the spam classifier over the first spam example and classifies it using the learned SVM. We should now try the other examples we have provided and see if the classifier gets them right. We can also try our own emails by replacing the examples (plain text files) with our own emails.

2.6 Optional (ungraded) exercise: Build our own dataset

In this exercise, we provided a preprocessed training set and test set. These datasets were created using the same functions (**processEmail.m and emailFeatures.m**) that we now have completed. For this optional (ungraded) exercise, we will build our own dataset using the original emails from the SpamAssassin Public Corpus. Our task in this optional (ungraded) exercise is to download the original files from the public corpus and extract them. After extracting them, we should run the processEmail* and emailFeatures functions on each email to extract a feature vector from each email. This will allow us to build a dataset X, y of examples. We should then randomly divide up the dataset into a training set, a cross validation set and a test set.

While we are building our own dataset, we also encourage to try building our own vocabulary list (by selecting the high frequency words that occur in the dataset) and adding any additional features that we think might be useful. Finally, we also suggest trying to use highly optimized SVM toolboxes such as LIBSVM or the SVM functionality contained within MATLAB's Statistics and Machine Learning Toolbox.