

5.2  $U \rightarrow \left( \begin{array}{c|c} 1 & u_{12} \\ \hline 0 & u_{22} \end{array} \right) \quad D \rightarrow \left( \begin{array}{c|c} \delta_1 & 0 \\ \hline 0 & D_{22} \end{array} \right)$

So,  $UD = \left( \begin{array}{c|c} 1 & u_{12} \\ \hline 0 & u_{22} \end{array} \right) \left( \begin{array}{c|c} \delta_1 & 0 \\ \hline 0 & D_{22} \end{array} \right)$

$= \left( \begin{array}{c|c} \delta_1 & \delta_1 u_{12} \\ \hline * & u_{22} D_{22} \end{array} \right)$

$UDU^T = \left( \begin{array}{c|c} \delta_1 & \delta_1 u_{12} \\ \hline * & u_{22} D_{22} \end{array} \right) \left( \begin{array}{c|c} 1 & u_{12} \\ \hline 0 & u_{22} \end{array} \right)^T = \left( \begin{array}{c|c} \delta_1 & \delta_1 u_{12}^T + u_{22} D_{22}^T u_{12} \\ \hline * & u_{22} D_{22}^T u_{22} \end{array} \right)$

$A = \left( \begin{array}{c|c|c|c} A_{00} & \alpha_{01} e_L & 0 & 0 \\ \hline * & \alpha_{11} & \alpha_{12} & 0 \\ \hline * & * & \alpha_{22} & \alpha_{23} e_F^T \\ \hline * & * & * & A_{33} \end{array} \right)$

As  $A = UDU^T$ .

By comparison:  $\alpha_{11} = \delta_1$ ,  $\delta_1 u_{12}^T (1 + u_{12}) = \alpha_{12}$

$\alpha_{22} = u_{22} D_{22} u_{22}^T$

So Algorithm for  $A = UDU^T$ .

1. Split A.
2.  $\alpha_{11} := \delta_1$
3. Compute  $\alpha_{12} := \delta_1 u_{12}^T + \delta_1 u_{12} u_{12}^T \ll \text{overwrite upper only} \gg$
4. Compute  $\alpha_{22} := u_{22} D_{22} u_{22}^T$
5. Continue Split A.