6.1

$$LD = \begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & v_{12} e_F^T \\ 0 & 0 & U_{22} \end{pmatrix} \begin{pmatrix} D_{00} & 0 & 0 \\ 0 & \phi_1 & 0 \\ 0 & 0 & E_{22} \end{pmatrix}$$

$$= \begin{pmatrix} L_{00} D_{00} & 0 & 0 \\ \lambda_{10} e_L^T D_{00} & \phi_1 & v_{12} e_F^T E_{22} \\ 0 & 0 & U_{22} E_{22} \end{pmatrix}$$

$$LDL^T = \begin{pmatrix} L_{00} D_{00} & 0 & 0 \\ \lambda_{10} e_L^T D_{00} & \phi_1 & v_{12} e_F^T E_{22} \\ 0 & 0 & U_{22} E_{22} \end{pmatrix} \begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & v_{12} e_F^T \\ 0 & 0 & U_{22} \end{pmatrix}^T$$

$$= \begin{pmatrix} L_{00} D_{00} L_{00}^T & 0 & 0 \\ \lambda_{10} e_L^T D_{00} L_{00}^T + \phi_1 \lambda_{10} e_L^T & \phi_1 & v_{12} e_F^T E_{22} U_{22} + \phi_1 v_{12} e_F^T \\ 0 & 0 & U_{22} E_{22} U_{22}^T \end{pmatrix}$$

$$A = LDL^T = \begin{pmatrix} A_{00} & \alpha_{01} e_L & 0 \\ \alpha_{01} e_L^T & \alpha_{11} & \alpha_{21} e_F^T \\ 0 & \alpha_{21} e_F & A_{22} \end{pmatrix} = UEU^T$$

$$UEU^T = \begin{pmatrix} U_{00} E_{00} & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & E_{22} U_{22} \end{pmatrix} \begin{pmatrix} U_{00} & v_{01} e_L & 0 \\ 0 & 1 & v_{12} e_F^T \\ 0 & 0 & U_{22} \end{pmatrix}^T$$

Where $U = \begin{pmatrix} U_{00} & v_{01} e_L & 0 \\ 0 & 1 & v_{12} e_F^T \\ 0 & 0 & U_{22} \end{pmatrix}$ and $E = \begin{pmatrix} E_{00} & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & E_{22} \end{pmatrix}$

$$UEU^T = \begin{pmatrix} U_{00} E_{00} U_{00}^T & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & E_{22} U_{22} U_{22}^T \end{pmatrix}$$

By comparing
we see ϕ, ε_1, α_{11} relate.

1. Cost of 1 twisted factorization $\Rightarrow UEU^T$ 1 iteration is vector – vector 2 operations. cost $\simeq mn$. m, n size of U and E
2. Cost of all twisted factorization $= mn *$ number of partitions $\simeq n^2$ range.