

Assignment – 4
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Batch – COE7

Probability and Statistics (UCS410)

Experiment 4

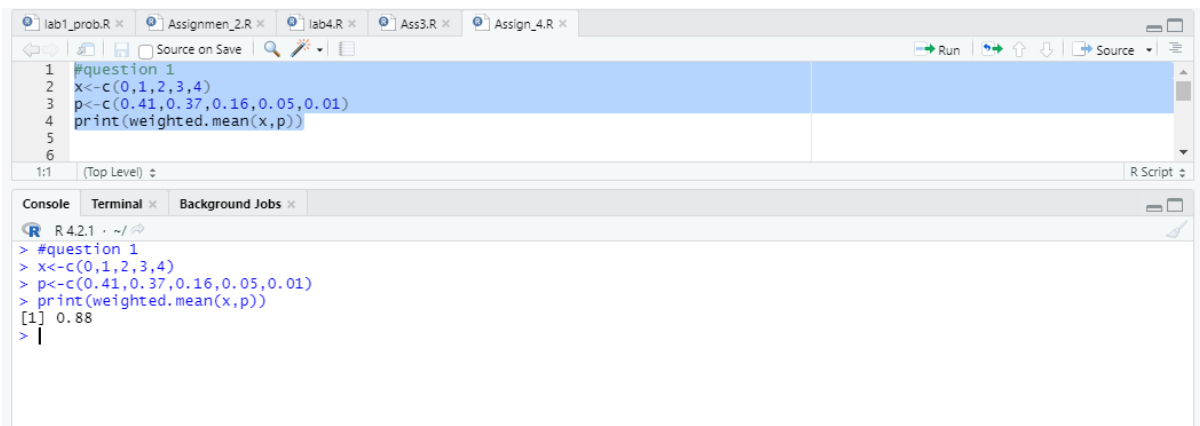
(Mathematical Expectation, Moments and Functions of Random Variables)

1. The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given as

x	0	1	2	3	4
$p(x)$	0.41	0.37	0.16	0.05	0.01

Find the average number of imperfections per 10 meters of this fabric.

(Try functions **sum()**, **weighted.mean()**, **c(a %*% b)** to find expected value/mean.



```
#question 1
x<-c(0,1,2,3,4)
p<-c(0.41,0.37,0.16,0.05,0.01)
print(weighted.mean(x,p))
```

```
R 4.2.1 ~ /
> #question 1
> x<-c(0,1,2,3,4)
> p<-c(0.41,0.37,0.16,0.05,0.01)
> print(weighted.mean(x,p))
[1] 0.88
>
```

2. The time T, in days, required for the completion of a contracted project is a random variable with probability density function $f(t) = 0.1 e^{-(0.1)t}$ for $t > 0$ and 0 otherwise. Find the expected value of T.

Use function **integrate()** to find the expected value of continuous random variable T.



The screenshot shows an R script editor with the following code:

```

5
6
7 #question 2
8 f=function(t)
9 {
10   f=(0.1*exp(-0.1*t))
11 }
12
13 print(integrate(f, lower = 0, upper = Inf))
14

```

The console output shows the result of the integration:

```

R 4.2.1 ~ /
> #question 2
> f=function(t)
+ {
+   f=(0.1*exp(-0.1*t))
+ }
>
> print(integrate(f, lower = 0, upper = Inf))
1 with absolute error < 0.00011
>

```

3. A bookstore purchases three copies of a book at \$6.00 each and sells them for \$12.00 each. Unsold copies are returned for \$2.00 each. Let $X = \{\text{number of copies sold}\}$ and $Y = \{\text{net revenue}\}$. If the probability mass function of X is

x	0	1	2	3
$p(x)$	0.1	0.2	0.2	0.5

Find the expected value of Y .



The screenshot shows an R script editor with the following code:

```

14
15 #question 3
16 x<-c()
17 x1<-c(0,1,2,3)
18 p1<-c(0.1,0.2,0.2,0.5)
19 n=3
20 for(i in x1){
21   x=c(x,((i*12)+(n-i)*2)-18)
22 }
23 print(weighted.mean(x,p1))
24

```

The console output shows the result of the weighted mean calculation:

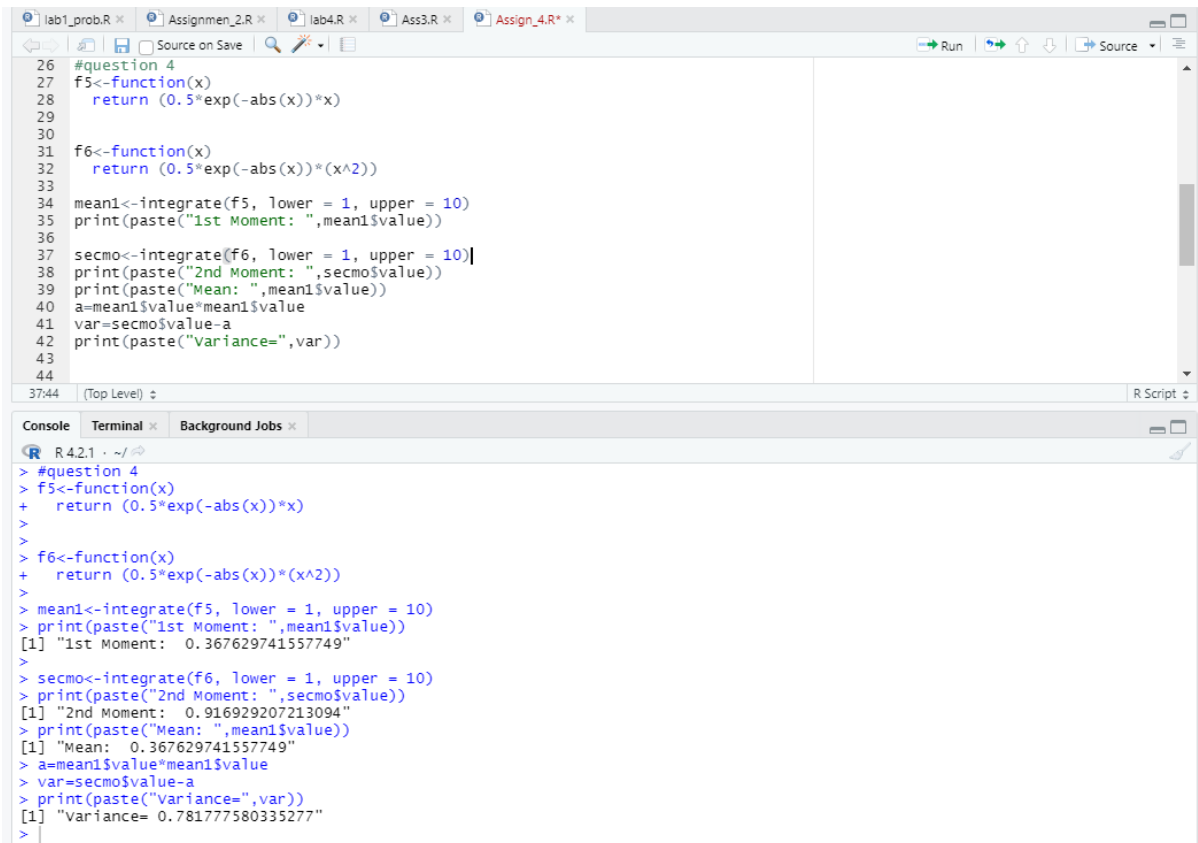
```

R 4.2.1 ~ /
> #question 3
> x<-c()
> x1<-c(0,1,2,3)
> p1<-c(0.1,0.2,0.2,0.5)
> n=3
> for(i in x1){
+   x=c(x,((i*12)+(n-i)*2)-18)
+ }
> print(weighted.mean(x,p1))
[1] 9
>

```

4. Find the first and second moments about the origin of the random variable X with probability density function $f(x) = 0.5e^{-|x|}$, $1 < x < 10$ and 0 otherwise. Further use the results to find Mean and Variance.

(k th moment = $E(X^k)$, Mean = first moment and Variance = second moment – Mean².)



```

26 #question 4
27 f5<-function(x)
28   return (0.5*exp(-abs(x))*x)
29
30
31 f6<-function(x)
32   return (0.5*exp(-abs(x))*(x^2))
33
34 mean1<-integrate(f5, lower = 1, upper = 10)
35 print(paste("1st Moment: ",mean1$value))
36
37 secmo<-integrate(f6, lower = 1, upper = 10)
38 print(paste("2nd Moment: ",secmo$value))
39 print(paste("Mean: ",mean1$value))
40 a=mean1$value*mean1$value
41 var=secmo$value-a
42 print(paste("Variance=",var))
43
44
37:44 (Top Level) R Script

```

```

R 4.2.1 ~ /
> #question 4
> f5<-function(x)
+   return (0.5*exp(-abs(x))*x)
>
> f6<-function(x)
+   return (0.5*exp(-abs(x))*(x^2))
>
> mean1<-integrate(f5, lower = 1, upper = 10)
> print(paste("1st Moment: ",mean1$value))
[1] "1st Moment:  0.367629741557749"
>
> secmo<-integrate(f6, lower = 1, upper = 10)
> print(paste("2nd Moment: ",secmo$value))
[1] "2nd Moment:  0.916929207213094"
> print(paste("Mean: ",mean1$value))
[1] "Mean:  0.367629741557749"
> a=mean1$value*mean1$value
> var=secmo$value-a
> print(paste("Variance=",var))
[1] "Variance= 0.781777580335277"
>

```

5. Let X be a geometric random variable with probability distribution

$$f(x) = \frac{3}{4} \left(\frac{1}{4} \right)^{x-1}, x = 1, 2, 3, \dots$$

Write a function to find the probability distribution of the random variable $Y = X^2$ and find probability of Y for X = 3. Further, use it to find the expected value and variance of Y for X = 1, 2, 3, 4, 5.

```
47
48 f<- function(x) (3/4)*(1/4^(x-1))
49 y<- function(x) (x^2)
50 x<-c(1,2,3,4,5)
51 f(x)
52
53 distr<-data.frame(x,f3(x))
54 distr
55 ans=f(1)*y(1)+f(2)*y(2)+f(3)*y(3)+f(4)*y(4)+f(5)*y(5)
56 ans
57
58 ans=f(3)*y(3)
59 ans
60 x<-c(1,2,3,4,5)
61 f(x)
62 |
63 mean<-weighted.mean(x,f(x))
64 mean
65
66 var<-ans-mean^2
67 var
68
62:1 (Top Level) ↕ R Script
```

Console Terminal Background Jobs

```
R 4.2.1 ~ /
> f<- function(x) (3/4)*(1/4^(x-1))
> y<- function(x) (x^2)
> x<-c(1,2,3,4,5)
> f(x)
[1] 0.7500000000 0.1875000000 0.0468750000 0.011718750 0.002929688
>
> distr<-data.frame(x,f3(x))
> distr
  x      f3.x.
1 1 0.7500000000
2 2 0.1875000000
3 3 0.0468750000
4 4 0.011718750
5 5 0.002929688
> ans=f(1)*y(1)+f(2)*y(2)+f(3)*y(3)+f(4)*y(4)+f(5)*y(5)
> ans
[1] 2.182617
>
> ans=f(3)*y(3)
> ans
[1] 0.421875
> x<-c(1,2,3,4,5)
> f(x)
[1] 0.7500000000 0.1875000000 0.0468750000 0.011718750 0.002929688
>
> mean<-weighted.mean(x,f(x))
> mean
[1] 1.328446
>
> var<-ans-mean^2
> var
[1] -1.342893
> |
```