1. Math in Log Linear model
    a. Chain Rule
    b. Percentage change

Interpreting the Log-linear Model
log(price) = $b_0 + b_1$**lotsize*

- Increasing x by one unit will increase (natural) $log(y)$ by b_1 units
- With x = lotsize, the model $log(price) = b_0 + b_1{*}x$ is the same as $y = e^{(b_0+b_1x)}$. Hence, $dy/dx = b_1y$, or $dy/y = b_1{*}dx$
- Multiplying both sides by 100, we get $100{*}dy/y = 100{*}b_1{*}dx$
- Note that (100*dy/y) is the percentage change in Y
- If $dx = 1$, then this one unit change in x leads to a $100{*}b_1$ percentage change in Y
- Note: this interpretation works when $b_0 + b_1{*}x$ is very small
  The accurate percentage change in Y = (e^{b_1} - 1)*100 for a one unit change in X

$$y = e^{(b_0+b_1x)}$$

Let $b_0 + b_1x = u$

$$y = e^u$$

$$\frac{dy}{dx} = \frac{d(e^u)}{du} \cdot \frac{du}{dx}$$

$$= e^u \cdot \frac{du}{dx}$$

$$\left[\frac{d(e^t)}{dt} = e^t\right]$$

$$= e^u \cdot \frac{d}{dx}(b_0 + b_1x)$$

$$= e^u \cdot b_1$$

$$= y \cdot b_1$$

$$\frac{A}{B} = \frac{C}{D}$$

$$\frac{A-B}{B} = \frac{C-D}{D}.$$

Math for Log-Linear

$$\log(y_1) = b_0 + b_1 x_1$$

$$\log(y_2) = b_0 + b_1 x_2$$

$$\log(y_2) - \log(y_1) = b_1 x_2 - b_1 x_1$$

$$\log\left(\frac{y_2}{y_1}\right) = b_1(x_2 - x_1)$$

$$\frac{y_2}{y_1} = \frac{e^{b_1(x_2-x_1)}}{1}$$

By componendo dividendo,

$$\frac{y_2 - y_1}{y_1} = \frac{e^{b_1(x_2-x_1)} - 1}{1}$$

$$\frac{y_2-y_1}{y_1}$$

$$\frac{y_2-y_1}{y_1} \times 100 = \left(\frac{e^{b_1(x_2-x_1)} - 1}{1}\right) \times 100$$

1

$$\left(e^{b_1} - 1\right) \times 100$$

2. How 0.01 in Linear log model? What's the base?

Interpreting a Linear-Log Model

- Linear-log model (and most other models) needs to be interpreted carefully
- It does not make much practical sense to increase the “log(lotprice)” by one unit
- But increasing *X* by 1 percent is almost equivalent to increasing (natural) *log(X)* by 0.01 units
- Hence, a 1 percent increase in *X* increases (natural) *log(X)* by .01 and, therefore, changes the *Y* variable by $.01 * b_1$

<u>Math for Linear log</u>

$$y_1 = b_0 + b_1 \log(x)$$

$$y_2 = b_0 + b_1 \log\left(x + \frac{x}{100}\right)$$

$$= b_0 + b_1 \log\left(\frac{101x}{100}\right)$$

$$y_2 - y_1 = b_1\left[\log\left(\frac{101x}{100}\right) - \log(x)\right]$$

$$= b_1\left[\log\left(\frac{101}{100}\right)\right]$$

$$= b_1 \times 9.95 \times 10^{-3}$$

$$\approx b_1 \times 0.0095$$

$$\approx \underline{b_1 \times 0.01}$$

3. Math in Log Log model

Interpreting the Log-Log Model

- Increasing (natural) log(X) by 0.01 leads to increasing (natural) log(Y) by b_1 *0.01 units
- Increasing (natural) log(X) by 0.01 is almost equivalent to increasing X by 1 percent, which implies changing Y by b_1 percent
- In a regression setting, we'd interpret elasticity as the percent change in Y (the dependent variable), when X (the independent variable) increases by one percent
- Hence b_1 captures elasticity

$$\log(y) = b_0 + b_1 \log(x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = b_1 \cdot \frac{1}{x} \qquad \left[\frac{d}{dt}(\log(t)) = \frac{1}{t}\right]$$

$$\frac{dy}{y} = b_1 \cdot \frac{dx}{x}$$

$$\frac{dy}{y} \times 100 = b_1 \cdot \frac{dx}{x} \cdot 100$$

Increasing x by 1% $\rightarrow$

Increasing y by b_1 %

4. Interpretation of Interaction term

Interaction Term Interpretation

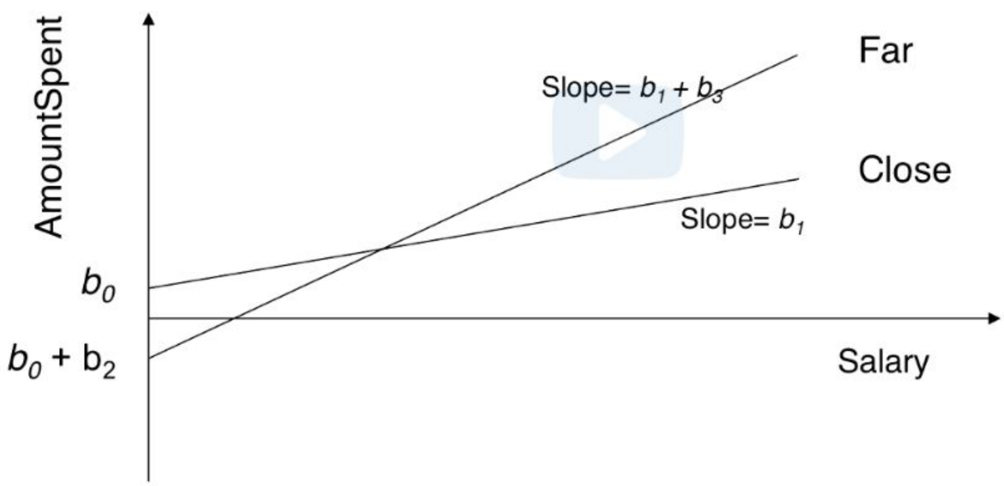
Hi-

I'm confused why we disregard certain coefficients when dealing with interaction terms. For example, from Monday nights OH and the TA instruction sheet, Video 2.4, why don't we take into account the intercept and coefficient for Far in order to calculate AmountSpent? Also, from Video 2.5, if we're calculating a private room, why don't we consider the intercept and Beta2 to calculate Price?

So when Far=1, Bo and B2 are included and will form a part of the intercept of the line, original slope will be influenced by b3. New slope = (b1 + b3), where b3 accounts for the variation in spending of the person based on salary influenced by the distance.

In the next frame, the calculation is for 'increase' in amountspent. So, the intercepts cancel out and thats why b2 is missing. Otherwise if we are just calculating amountspent, b2 will be part of it in the form of a constant.

$$AmountSpent = b_0 + b_1Salary + b_2Far + b_3SalaryFar$$



Similarly, In Video 2.5, b2 becomes part of the intercept . Its is very much a part of the calculation, just that its in the form of a constant (its variable will be 1 when the case applies). In the video b4 and b5 are stressed at because they are ones influencing the 'slope'.