Name ____________________________
Student Number ____________________

ECE 5570 Principles of Digital Communications

Exam #1

July 5, 2016

<u>Instructions:</u>

1. Multiple-choice answers will be marked on a separate answer sheet.
2. Q-function tables and equations are provided at the end of the exam.
3. You have 1 hour and 30 minutes to complete the exam.
4. All questions are 5 points. No partial credit is given.
5. A zero mark is given for every incorrect answer.
6. Answer with the BEST of the four answers.
7. If the correct answer is "None of the above", this means the correct answer does not round to the same *first two significant digits* with the other three answers.

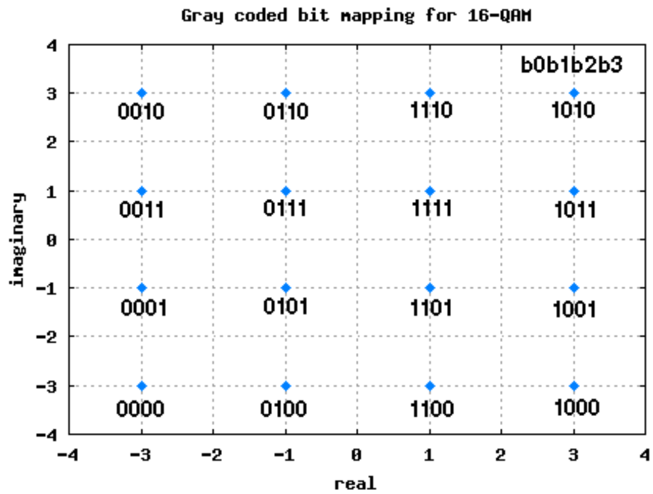
Point breakdown per problem

Problems 1-3:	15 points – Sketching signals
Problems 4-5:	10 points – Q-functions
Problems 6-7:	10 points – The Optimal Receiver
Problems 8:	5 points – Orthogonality
Problems 9-12:	20 points – Signal Energy
Problems 13-16:	20 points – General Binary Signaling
Problems 17-20:	20 points – M-PAM

Average: 64.8
High Scores: 100 (2), 95 (2), 90 (10)

Recall from the syllabus that course percentile ranges are as follows
85+ → A (top 15%)
75 – 85 → A- (next 20%)
65 – 75 → B+ (next 20%)
50 – 65 → B
(33 students below 50)

A modulation scheme is used with the following signal constellation.



This modulation scheme is used to transmit the following bit stream with the following system parameters.

Bit stream: 0101110110001011

Coordinates
Symbol 1: 0101 → (-1,-1)
Symbol 2: 1101 → (1,-1)
Symbol 3: 1000 → (3,-3)
Symbol 4: 1011 → (3,1)

Bit rate = 50.0 Mbps
Carrier frequency = 12.5 MHz

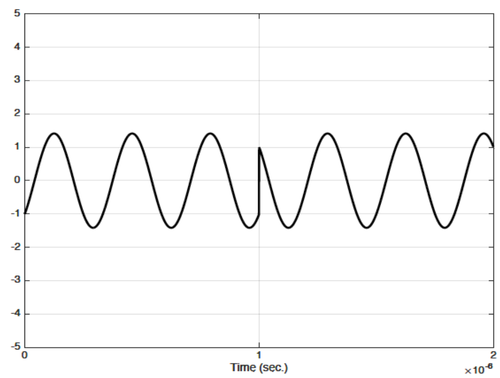
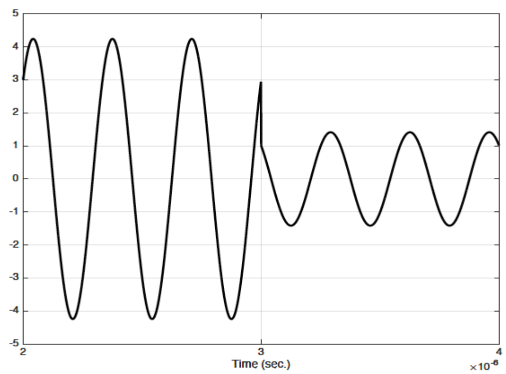
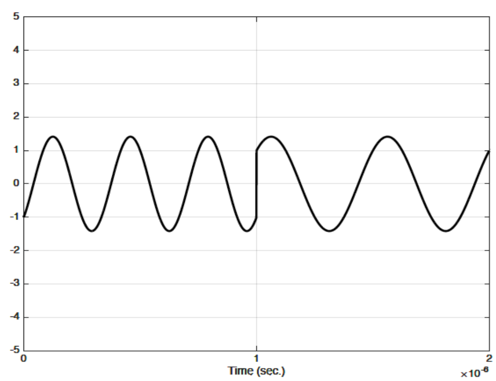
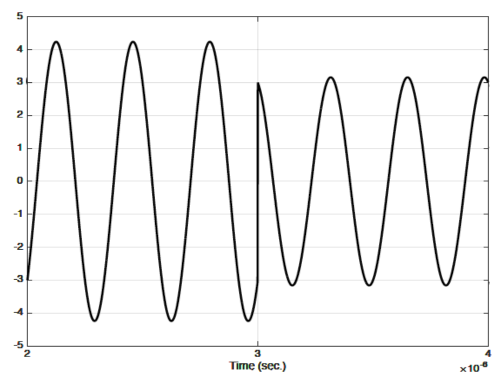
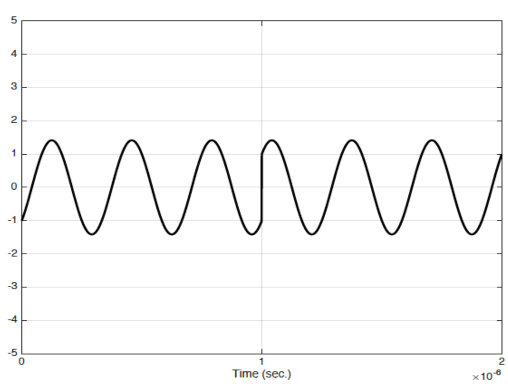
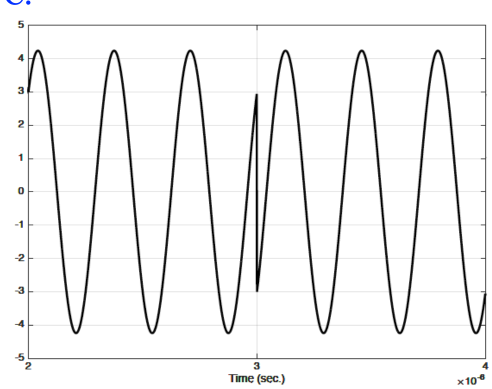
1. What is the symbol time and cycles per symbol for this system?

50 Mbps: (1/50e6 sec/bit)*(4 bits/symbol) = 0.08 microseconds
(0.08e-6 sec/symbol)*(12.5e6 cycles/sec) = 1 cycles/symbol
Carrier frequency = 12.5 MHz for 1 cycle/symbol
40 Mbps: (1/40e6 sec/bit)*(4 bits/symbol) = 0.1 microseconds
Carrier frequency = 20.0 MHz for 2 cycles/symbol
25 Mbps: (1/25e6 sec/bit)*(4 bits/symbol) = 0.16 microseconds
Carrier frequency = 25.0 MHz for 4 cycles/symbol

A. 0.1 μs, 2 cycles/symbol
B. 0.08 μs, 1 cycle/symbol
C. 0.16 μs, 4 cycles/symbol
D. None of the above

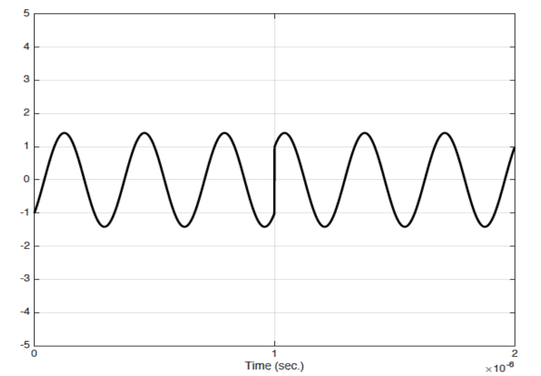
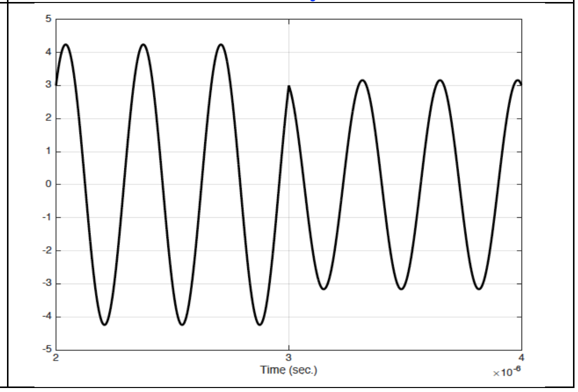
Answer __B__

<u>Now a different frequency and bit rate are used from the numbers given above.</u> Now is used a symbol time of 1 microsecond and 3 cycles per symbol. The same constellation and input data from above are used. Consider the following signals that come from plots of the cosine function.

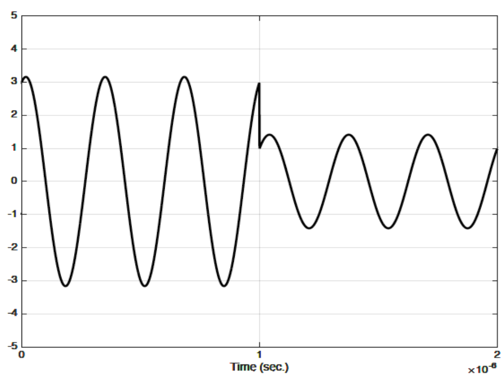
First two symbols	Last two symbols
A.  Second symbol has the wrong phase	A.  Second symbol (4^{th} overall symbol) has incorrect amplitude and phase.
B.  Second symbol has the wrong frequency	B.  First symbol has incorrect phase
C. 	C.  Second symbol has incorrect amplitude and phase.

D. None of the above	D. None of the above

Correct answer

First two symbols	Last two symbols
	

Another incorrect answer for the first two symbols



First symbol has incorrect amplitude and phase.

2. For which of the signals above are the first two symbols (0 to 2 microseconds) correct?

   Answer ___C___

3. For which of the signals above are the last two symbols (from 2 to 4 microseconds) correct?

   Answer ___D___

Q Functions

4. What is the probability that a Gaussian random variable X with mean 8 and variance 4.0 would be less than 10?

   Less than 10

   1-Q((10-8)/sqrt(4))=1-Q(1)=1-0.1587=0.8413

Less than 11

1-Q((11-8)/sqrt(4))=1-Q(1.5)=1-0.06681=0.93319

Less than 9

1-Q((9-8)/sqrt(4))=1-Q(0.5)=1-0.3085=0.6915

A. 0.1587
B. 0.6915
C. 0.8413
D. None of the above

Answer __C__

5. The bit error probability for a binary polar system is described by the following equation.

$$P_b = Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

If rectangular pulses are used, the duration of the pulse is 1 microsecond, and the value of $N/2 = 10^{-8}$, what is the minimum amplitude of the pulse that can be supported to have a bit error probability of less than 0.1255e-1?

P_b =0.1255e-1

0.1255e-1 = Q(sqrt(2*Eb/N))

2.24 = sqrt(2*Eb/N)

5.0176 = 2*Eb/N=Eb/1e-8

5.0176e-8 = Eb = A^2*Tb=A^2*10^-6

5.0176e-2=A^2

0.224 V = A = just 2.24 from above divided by 10

P_b =0.1223e-2

0.1223e-2=Q(x)

3.03=x

A = 3.03/10=0.303

P_b =0.1978e-4

0.1978e-4 =Q(x)

4.11=x

A = 4.11/10=0.411 V

A. 0.303 V
B. 0.411 V
C. 0.224 V
D. None of the above

Answer __C__

The Optimal Receiver

Consider the derivation of the optimal receiver for binary polar signaling.

6. What does the quantity $N/2$ pertain to?

   A. Duration of integration
   B. Sampling time for the Matched Filter
   C. Signal multiplied by the input
   D. White noise

Answer __D__

7. Two pulse shapes will produce the same P_b if which of the following is true?

   A. Same amplitude
   B. Same duration
   C. Both are rectangular
   D. Same energy per pulse

Answer __D__

Orthogonality

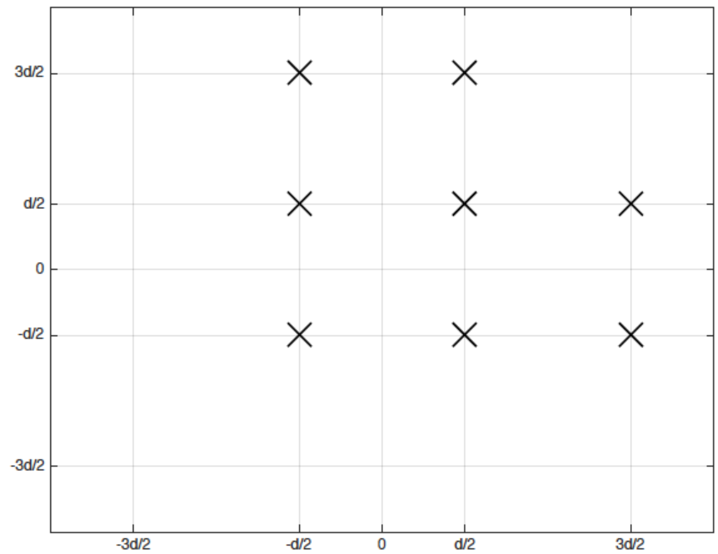
8. Which two signals are orthogonal and are the ones used by QPSK?

   A. $e^{j2\pi f_c t}, e^{-j2\pi f_c t}$
   B. $\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t), \sqrt{\frac{2}{T_b}}\sin(2\pi f_c t)$
   C. $\cos(2\pi f_1 t), \cos(2\pi f_2 t), \delta_f = \frac{n}{2T_b}$
   D. None of the above

Answer __B__

Signal Energy

9. The following form of a QAM constellation is proposed. What is the average signal energy per bit for the constellation if *d=1*?



4 points have coordinates with $\frac{d}{2}$ and $\frac{d}{2}$. 4 points have coordinates with $\frac{3d}{2}$ and $\frac{d}{2}$.

$$E_s = \frac{1}{8}\left(4\left(\left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^2\right) + 4\left(\left(\frac{3d}{2}\right)^2 + \left(\frac{d}{2}\right)^2\right)\right) = \frac{3}{2}d^2$$

$$E_b = \frac{E_s}{3} = \frac{1}{2}d^2$$

$$d = 1, E_b = \frac{1}{2}\text{ J}$$
$$d = 2, E_b = \frac{1}{2}(4) = 2\text{ J}$$
$$d = 3, E_b = \frac{1}{2}(9) = \frac{9}{2}\text{ J}$$

A. $E_b = \frac{1}{2}$ J

B. $E_b = \frac{9}{2}$ J

C. $E_b = 2$ J

D. None of the above

Answer __A__

10. Rectangular pulses are to be used to send data. 4 different pulses of different amplitudes (same durations) are used. Pulses are located at $\pm 1, \pm 3$ Volts. If a bandwidth of 100 kHz is available and 90.3% of the energy is to be transmitted, what is the data rate in bits per second that will be achieved?

100 kHz

90.3% energy in $\frac{1}{\tau} = 100$ kHz

$\frac{1}{\tau}$ =100,000 symbols/sec

2 bits/symbol, so 200 kbps

50 kHz

90.3% energy in $\frac{1}{\tau} = 50$ kHz

$\frac{1}{\tau}$ =50,000 symbols/sec

2 bits/symbol, so 100 kbps

200 kHz

90.3% energy in $\frac{1}{\tau} = 200$ kHz

$\frac{1}{\tau}$ =200,000 symbols/sec

2 bits/symbol, so 400 kbps

A. 200 kbps
B. 400 kbps
C. 100 kbps
D. None of the above

Answer __A__

11. Suppose only 6 symbols are needed of the possible 8 symbols for 8-PAM. The PAM symbols are spaced 4 volts apart, starting from ± 2 Volts on to higher voltages. When using just 6 symbols, assume those with the highest energy from the 8 symbols are not used. What is the ratio of energy per bit between 6-PAM and 8-PAM? That is to say, what is the following ratio?

$$\frac{E_{b,6-PAM}}{E_{b,8-PAM}}$$

Symbols use $\pm 2, \pm 6, \pm 10, \pm 14$ Volts.

For all eight pulses,

$$E_s = \frac{1}{8}\left(2(2^2) + 2(6^2) + 2(10^2) + 2(14^2)\right) = \frac{1}{8}(8 + 72 + 200 + 392) = 84$$

$$E_b = \frac{84}{3} = 28 \text{ J}$$

Only using 6 points with least energy:

$$E_s = \frac{1}{6}\left(2(2^2) + 2(6^2) + 2(10^2)\right) = \frac{1}{6}(8 + 72 + 200) = 46.667$$

$$E_b = \frac{46.667}{3} = 15.555 \text{ J}$$

$$\frac{E_{b,6-PAM}}{E_{b,8-PAM}} = \frac{15.555}{28} = 0.5555$$

A. 1.000
B. 0.4167

C. 0.5556 D. None of the above

Answer __C__

12. Assume a rectangular pulse is used for data transmission that has an amplitude of 0.1V and a duration of 1 μs. What pulse duration should be used to have the same energy per pulse if 0.2 V was used for the amplitude?

New amplitude = 0.2 V
$E_p = A^2 T_s = (0.1^2)10^{-6} = 10^{-8}$
New amplitude = 0.2 V
$E_p = 10^{-8} = (0.2^2)T_s$
$T_s = \frac{10^{-8}}{0.04} = 0.25\ \mu s$

New amplitude = 0.4 V
$T_s = \frac{10^{-8}}{0.4^2} = 0.0625\ \mu s$

New amplitude = 0.05 V
$T_s = \frac{10^{-8}}{0.05^2} = 4.00\ \mu s$

A. 0.5 μs B. 2 μs
C. 0.25 μs D. None of the above

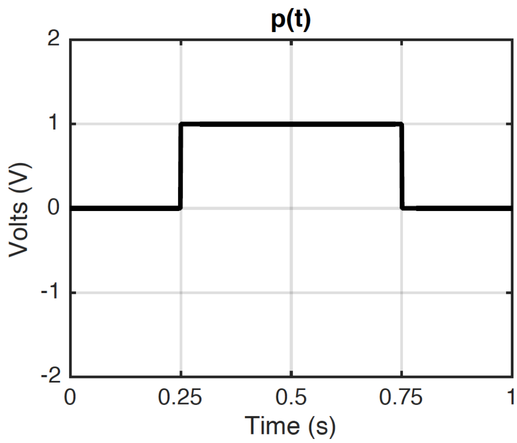
Answer __C__

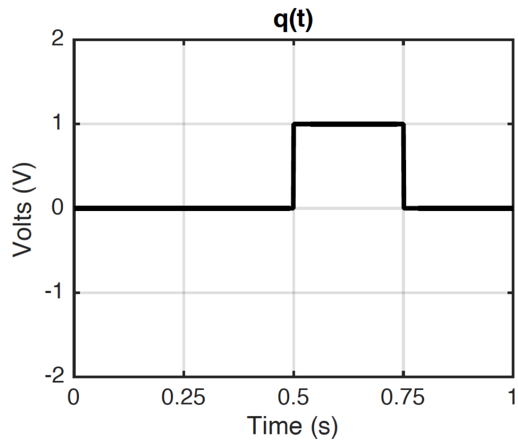
General Binary Signaling

A receiver that multiplies a received pulse by *p(t)-q(t)* before integrating uses what is known as general binary signaling. The following formula applies.

$$P_b = Q\left(\sqrt{\frac{E_p + E_q - 2E_{pq}}{2N}}\right)$$

13. Given are the following plots for a general $p(t)$ and $q(t)$, What is the expression for probability of bit error? The curves are linear.





$$E_p = \int_0^{T_b} p^2(t)dt = \int_{0.25}^{0.75} 1^2 dt = 0.5$$
$$E_q = 0.25$$
$$E_{pq} = \int_0^{T_b} p(t)q(t)dt = 0.25$$
$$P_b = Q\left(\sqrt{\frac{E_p + E_q - 2E_{pq}}{2N}}\right) = Q\left(\sqrt{\frac{0.5 + 0.25 - 2(0.25)}{2N}}\right) = Q\left(\sqrt{\frac{0.25}{2N}}\right)$$
$$= Q\left(\sqrt{\frac{0.125}{N}}\right)$$

A. $P_b = Q\left(\sqrt{\frac{0.5}{N}}\right)$

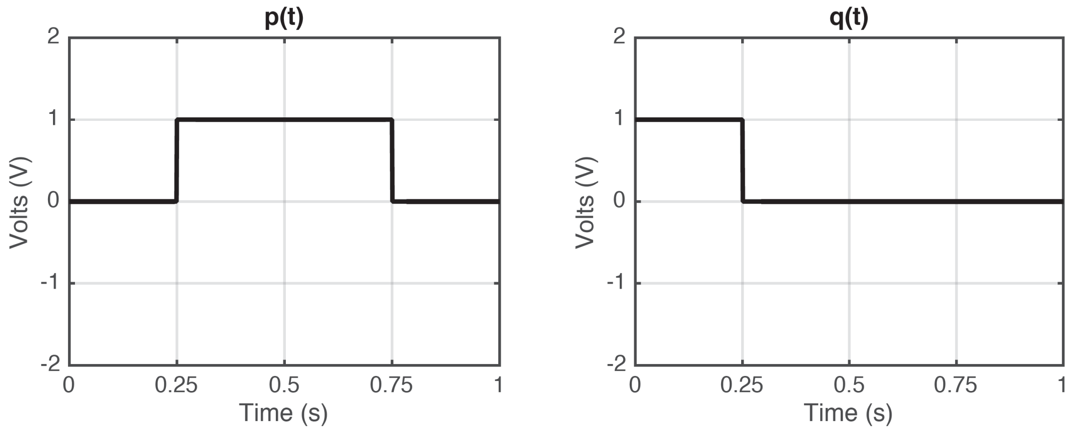
B. $P_b = Q\left(\sqrt{\frac{0.375}{N}}\right)$

C. $P_b = Q\left(\sqrt{\frac{0.125}{N}}\right)$

D. None of the above

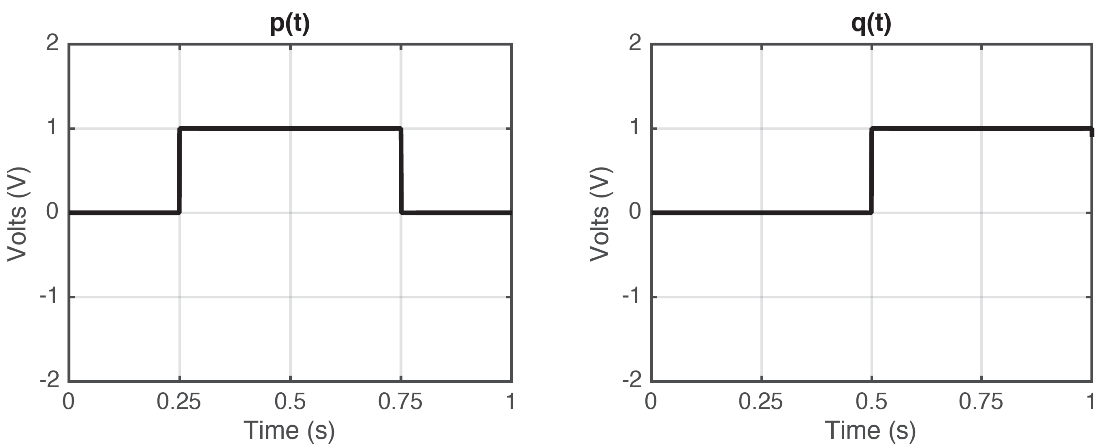
Answer ____C___

Different Pulses



$$E_p = \int_0^{T_b} p^2(t)dt = \int_{0.25}^{0.75} 1^2 dt = 0.5$$
$$E_q = 0.25$$
$$E_{pq} = \int_0^{T_b} p(t)q(t)dt = 0$$
$$P_b = Q\left(\sqrt{\frac{E_p + E_q - 2E_{pq}}{2N}}\right) = Q\left(\sqrt{\frac{0.5 + 0.25 - 0}{2N}}\right) = Q\left(\sqrt{\frac{0.75}{2N}}\right) = Q\left(\sqrt{\frac{0.375}{N}}\right)$$

Another set of pulses



$$E_p = \int_0^{T_b} p^2(t)dt = \int_{0.25}^{0.75} 1^2 dt = 0.5$$
$$E_q = 0.5$$

$$E_{pq} = \int_0^{T_b} p(t)q(t)dt = 0.25$$

$$P_b = Q\left(\sqrt{\frac{E_p + E_q - 2E_{pq}}{2N}}\right) = Q\left(\sqrt{\frac{0.5 + 0.5 - 2(0.25)}{2N}}\right) = Q\left(\sqrt{\frac{0.5}{2N}}\right)$$

$$= Q\left(\sqrt{\frac{0.25}{N}}\right)$$

14. For problem (13), what threshold will be used at the receiver to decide if a "1" or a "0" should be decoded?

First set of pulses

$$a_0 = \frac{1}{2}\left(E_p - E_q\right) = \frac{1}{2}(0.5 - 0.25) = 0.125$$

2nd set of pulses

$$a_0 = \frac{1}{2}\left(E_p - E_q\right) = \frac{1}{2}(0.5 - 0.25) = 0.125$$

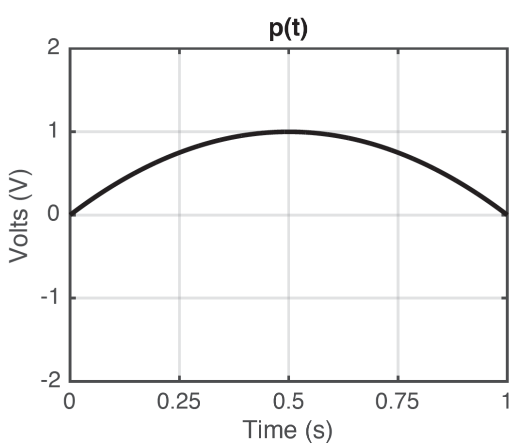
3rd set of pulses

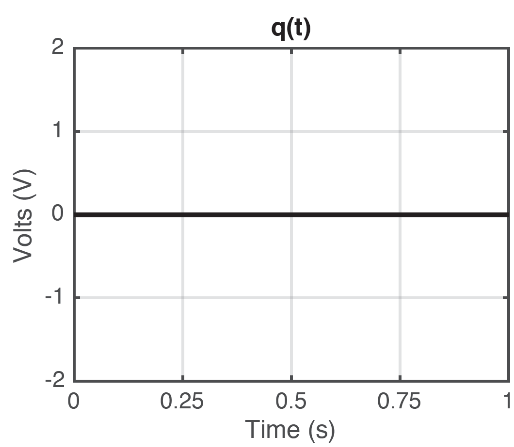
$$a_0 = \frac{1}{2}\left(E_p - E_q\right) = \frac{1}{2}(0.5 - 0.5) = 0$$

A. -0.125
B. 0.125
C. 0
D. None of the above

Answer ____B___

15. For the pulse shapes below, the energy in $p(t)$ is $\frac{2}{3}$ J, what is the expression for probability of bit error?





$$E_p = \frac{2}{3}, E_q = 0, E_{pq} = 0$$

$$P_b = Q\left(\sqrt{\frac{E_p + E_q - 2E_{pq}}{2N}}\right) = Q\left(\sqrt{\frac{\frac{2}{3} + 0 - 2(0)}{2N}}\right) = Q\left(\sqrt{\frac{1}{3N}}\right)$$

A. $P_b = Q\left(\sqrt{-\frac{1}{3N}}\right)$

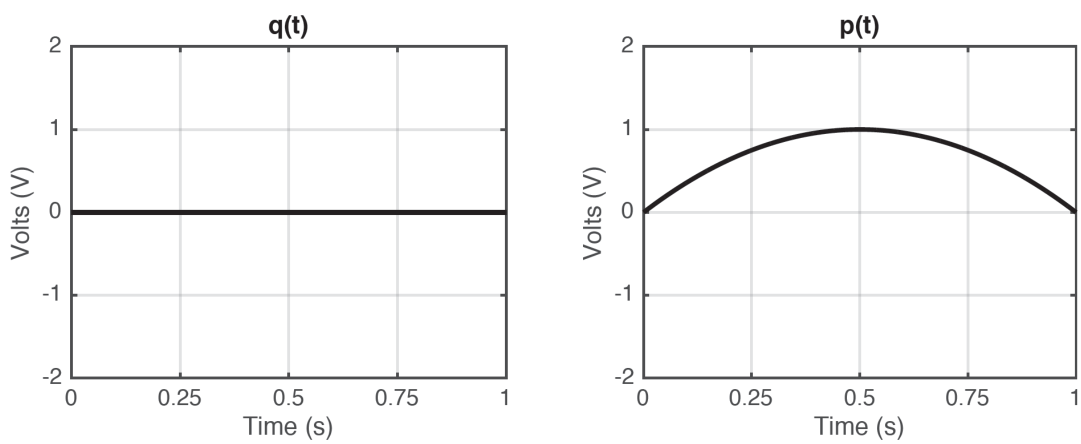
B. $P_b = Q\left(\sqrt{\frac{1}{6N}}\right)$

C. $P_b = Q\left(\sqrt{\frac{1}{3N}}\right)$

D. None of the above

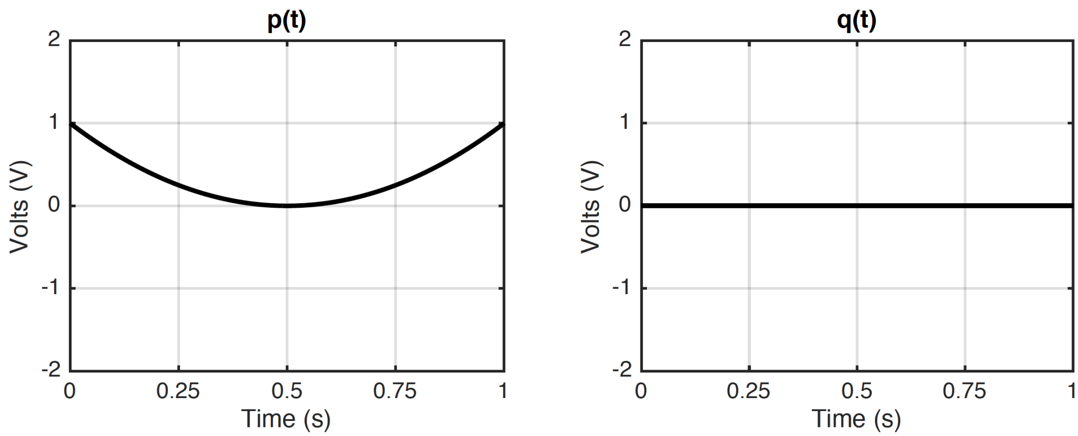
Answer ____C___

Other pulses



$$E_p = \frac{2}{3}, E_q = 0, E_{pq} = 0$$

$$P_b = Q\left(\sqrt{\frac{E_p + E_q - 2E_{pq}}{2N}}\right) = Q\left(\sqrt{\frac{\frac{2}{3} + 0 - 2(0)}{2N}}\right) = Q\left(\sqrt{\frac{1}{3N}}\right)$$



$$E_p = \frac{1}{3}, E_q = 0, E_{pq} = 0$$

$$P_b = Q\left(\sqrt{\frac{E_p + E_q - 2E_{pq}}{2N}}\right) = Q\left(\sqrt{\frac{\frac{1}{3} + 0 - 2(0)}{2N}}\right) = Q\left(\sqrt{\frac{1}{6N}}\right)$$

16. Given are the following two statements for general binary signaling.

I. $a_0 = 0$ for on-off cases.
II. $E_{pq} = 0$ only for polar functions.

A. Only I is true.
B. Only II is true.
C. Both are true.
D. Neither is true

Answer ____D___

M-ary Pulse Amplitude Modulation

M-ary pulse amplitude modulation (MPAM) has the following bit error performance equation.

$$P_{bM} = \frac{2(M-1)}{Mlog_2(M)} Q\left(\sqrt{\frac{6\log_2(M)}{M^2-1}\frac{E_{bM}}{N}}\right)$$

17. What is the approximate ratio in the amount of energy per bit needed for 16PAM divided by the energy per bit needed for 8PAM for the same P_b? Use the typical approximations used in class.

Only need to consider the $\frac{6\log_2(M)}{M^2-1}\frac{E_{bM}}{N}$ term inside the Q-function

2PAM: $\frac{6\log_2(M)}{M^2-1}\frac{E_{bM}}{N} = \frac{6\log_2(2)}{2^2-1}\frac{E_{b2}}{N} = \frac{6}{3}\frac{E_{b2}}{N} = 2\frac{E_{b2}}{N}$

4PAM: $\frac{6\log_2(M)}{M^2-1}\frac{E_{bM}}{N} = \frac{6\log_2(4)}{4^2-1}\frac{E_{b4}}{N} = \frac{6(2)}{15}\frac{E_{b4}}{N} = \frac{4}{5}\frac{E_{b4}}{N}$

8PAM: $\frac{6\log_2(8)}{8^2-1}\frac{E_{b8}}{N} = \frac{6(3)}{63}\frac{E_{b8}}{N} = \frac{2}{7}\frac{E_{b8}}{N}$

16PAM: $\frac{6\log_2(16)}{16^2-1}\frac{E_{b16}}{N} = \frac{6(4)}{255}\frac{E_{b16}}{N} = \frac{24}{255}\frac{E_{b16}}{N}$

32PAM: $\frac{6\log_2(32)}{32^2-1}\frac{E_{b32}}{N} = \frac{6(5)}{1023}\frac{E_{b32}}{N} = \frac{10}{341}\frac{E_{b32}}{N}$

$$\frac{E_{b16}}{E_{b8}}$$

$$\frac{\left(\frac{24}{255}\frac{E_{b16}}{N}\right)}{\left(\frac{2}{7}\frac{E_{b8}}{N}\right)} = 1, \frac{E_{b16}}{E_{b8}} = \frac{\left(\frac{2}{7}\right)}{\left(\frac{24}{255}\right)} = 3.0357$$

$$\frac{E_{b16}}{E_{b4}} = \frac{\left(\frac{4}{5}\right)}{\left(\frac{24}{255}\right)} = 8.500$$

$$\frac{E_{b32}}{E_{b8}} = \frac{\left(\frac{2}{7}\right)}{\left(\frac{10}{341}\right)} = 9.7428$$

A. 3.209
B. 9.743
C. 8.500
D. None of the above

Answer ____D___

18. If the average energy per pulse for a 4PAM system is 1 J, what is the average energy per *bit*?

Pulses at $p(t), -p(t), 3p(t), -3p(t)$
Average energy per symbol is $E_s = \frac{1}{4}\left(E_p + E_p + 9E_p + 9E_p\right) = 5E_p$

1 J:
$$E_b = \frac{5}{2}E_p = \frac{5}{2}(1) = \frac{5}{2}$$

2 J:
$$E_b = \frac{5}{2}E_p = \frac{5}{2}(2) = 5$$

4 J:
$$E_b = \frac{5}{2}E_p = \frac{5}{2}(4) = 10$$

A. 5 J
B. $\frac{1}{2}$ J
C. $\frac{5}{2}$ J
D. None of the above

Answer ____C___

19. For a 32PAM system, how many tails of Q functions are part of the overall average bit error probability?

30 symbols have 2 tails, 2 symbols have one tail, 30*2+2=62

A. 30
B. 64
C. 62
D. None of the above

Answer ____C___

20. If $\frac{E_b}{N} = 14$ (not in dB) for 8PAM, what is the probability of error?

$\frac{E_b}{N} = 14$, 8PAM

$$P_{bM} = \frac{2(M-1)}{Mlog_2(M)} Q\left(\sqrt{\frac{6\log_2(M)}{M^2-1}\frac{E_{bM}}{N}}\right) = \frac{2(8-1)}{8log_2(8)} Q\left(\sqrt{\frac{6\log_2(8)}{64-1}\frac{E_{bM}}{N}}\right) = \frac{2(7)}{8(3)} Q\left(\sqrt{\frac{6(3)}{63}\ 14}\right)$$
$$= \frac{14}{24} Q\left(\sqrt{4}\right) = \frac{14}{24} Q(2) = \frac{14}{24}(0.02275) = 0.01327$$

$\frac{E_b}{N} = 31.5$, 8PAM

$$P_{bM} == \frac{2(8-1)}{8log_2(8)} Q\left(\sqrt{\frac{6\log_2(8)}{64-1}\frac{E_{bM}}{N}}\right) = \frac{2(7)}{8(3)} Q\left(\sqrt{\frac{6(3)}{63}\ 31.5}\right) = \frac{14}{24} Q\left(\sqrt{9}\right) = \frac{14}{24} Q(3) =$$
$$14/24\ (0.001350) = 0.0007875$$

$\frac{E_b}{N} = 3.5$, 8PAM

$$P_{bM} == \frac{2(8-1)}{8log_2(8)} Q\left(\sqrt{\frac{6\log_2(8)}{64-1}\frac{E_{bM}}{N}}\right) = \frac{2(7)}{8(3)} Q\left(\sqrt{\frac{6(3)}{63}\ 3.5}\right) = \frac{14}{24} Q\left(\sqrt{1}\right)$$
$$= 14/24\ (0.1587) = 0.092575$$

A. 0.01327
B. 0.092575
C. 0.0007875
D. None of the above

Answer ____A___

Name ____________________________
Student Number ____________________

ECE 5570 Principles of Digital Communications

Exam #1

July 2, 2015

Instructions:

1. Before you start, put your name on the first page.
2. Multiple-choice answers will be marked on a separate answer sheet.
3. Q-function tables are provided at the end of the exam.
4. You have 1 hour and 30 minutes to complete the exam.
5. All questions are 5 points. No partial credit is given.
6. A zero mark is given for every incorrect answer.
7. Answer with the BEST of the four answers.
8. If the correct answer is "None of the above", this means the correct answer does not share the *first two significant digits* with the other three answers.

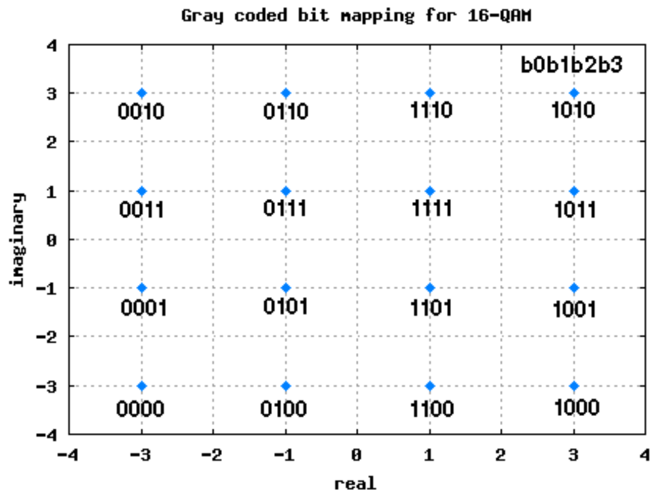
Point breakdown per problem

Problems 1-4:	20 points – Sketching signals
Problems 5-8:	20 points – Probabilities
Problems 9-12:	20 points – The Optimal Receiver
Problems 13-14:	10 points – Orthogonality
Problems 15-20:	30 points – Signal Energy

Average: 58.9
High Scores: 100, 95, 90 (9), 85 (12)

Recall from the syllabus that the grading scale will be as follows
85+ → A
75 – 85 → A-
65 – 75 → B+
50 – 65 → B

A modulation scheme is used with the following signal constellation.



This modulation scheme is used to transmit the following bit stream with the following system parameters.

Bit stream: 1111001010111000

Coordinates
Symbol 1: 1111 → (1,1)
Symbol 2: 0010 → (-3,3)
Symbol 3: 1011 → (3,1)
Symbol 4: 1000 → (3,-3)

Bit rate = 10.0 Mbps
Carrier frequency = 5.0 MHz

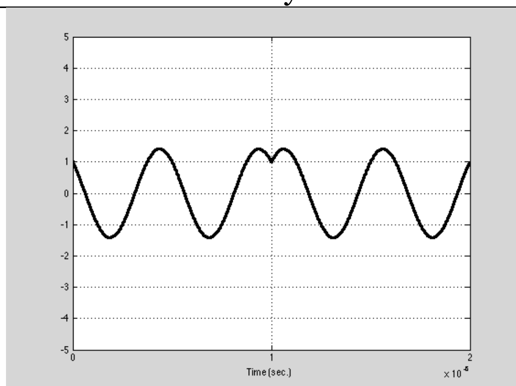
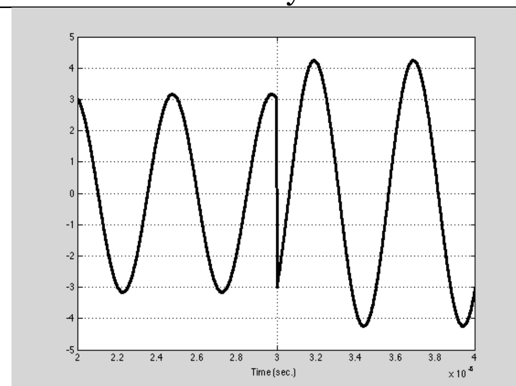
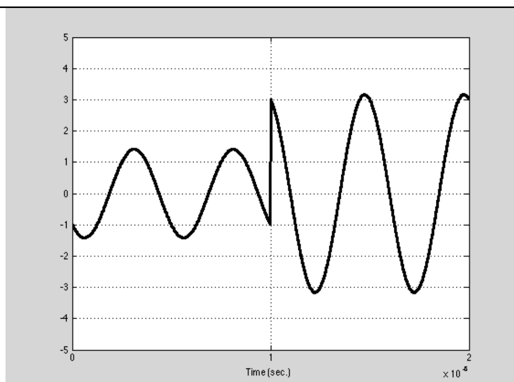
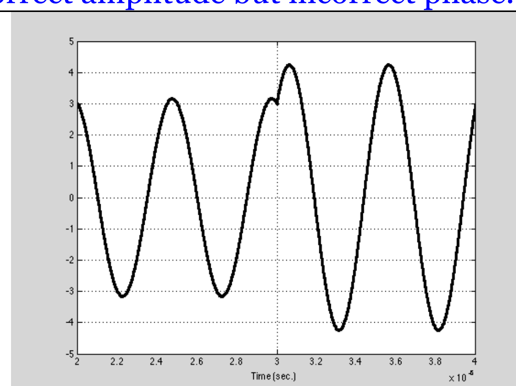
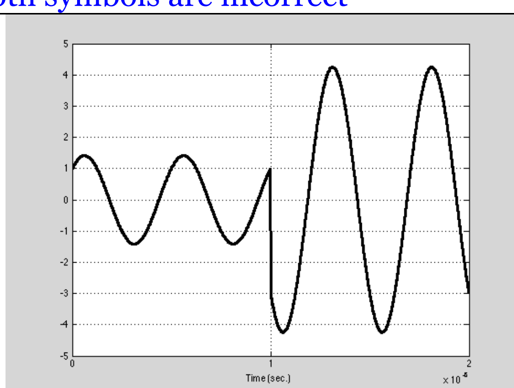
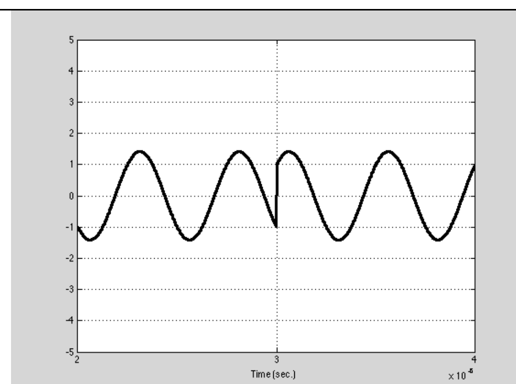
1. What is the symbol time for this system?

10 Mbps: (1/10e6 sec/bit)*(4 bits/symbol) = 0.4 microseconds
Carrier frequency = 5.0 MHz for 2 cycles/symbol
(0.4e-6 sec/symbol)*(5e6 cycles/sec) = 2 cycles/symbol
12.5 Mbps: (1/12.5e6 sec/bit)*(4 bits/symbol) = 0.32 microseconds
Carrier frequency = 6.25 MHz for 2 cycles/symbol
8 Mbps: (1/8e6 sec/bit)*(4 bits/symbol) = 0.5 microseconds
Carrier frequency = 4.0 MHz for 2 cycles/symbol

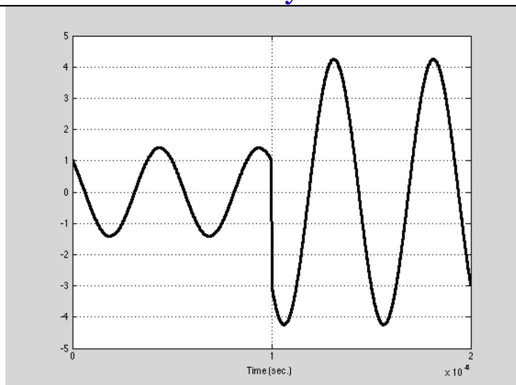
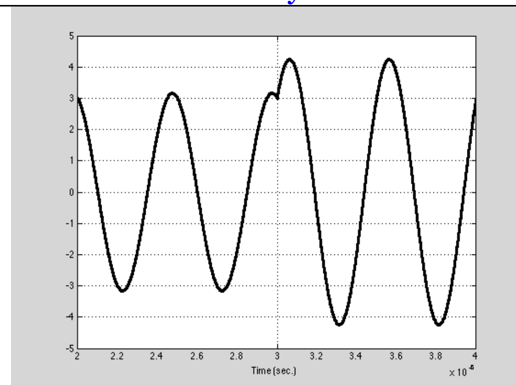
A. 0.32 microseconds
B. 0.20 microseconds
C. 0.50 microseconds
D. None of the above

Answer __D__

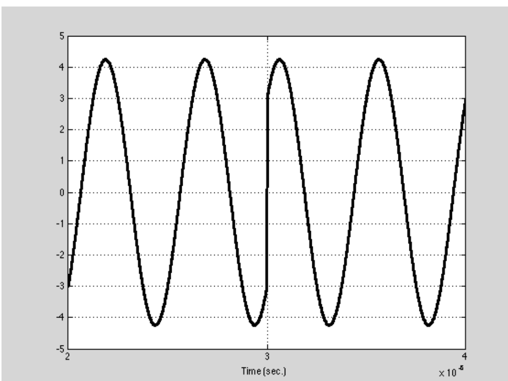
<u>Now a different frequency and bit rate are used from the numbers given above</u>. Now is used a symbol time of 1 microsecond and 2 cycles per symbol. The same constellation and input data from above are used. Consider the following signals that come from plots of the cosine function.

First two symbols	Last two symbols
 A. Second symbol is incorrect	 A. Second symbol (4th overall symbol) has correct amplitude but incorrect phase.
 B. Both symbols are incorrect	 B.
 C. The first symbol has the correct amplitude but on the correct phase.	 C. Both symbols are incorrect
D. None of the above	D. None of the above

Correct answer

First two symbols	Last two symbols
	

Another incorrect answer for the last two symbols



First symbol (3rd overall symbol) is incorrect, but the second symbol (4th overall symbol) is correct.

2. For which of the signals above are the first two symbols (0 to 2 microseconds) correct?

Answer __D__

3. For which of the signals above are the last two symbols (from 2 to 4 microseconds) correct?

Answer __B__

4. A 4-ary FSK signal has the following parameters

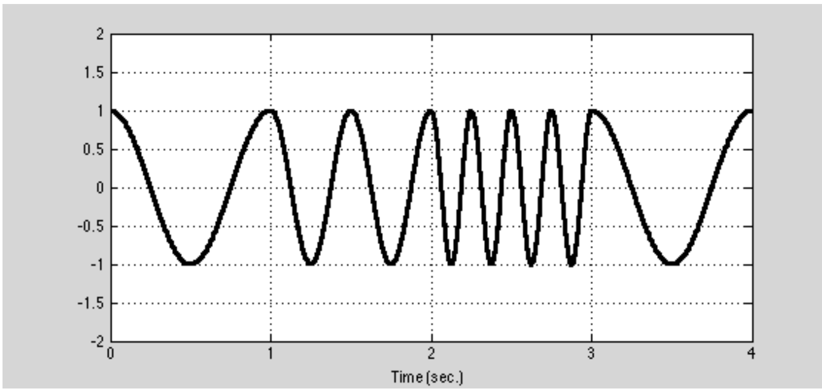
   Data stream = 01001100
   f2 f1 f4 f1

   Symbol time = 1 microsecond
   $f_i = f_1 + (i\text{-}1)\delta f$
   f_1 corresponds to one cycle per symbol and symbol 00
   f_2 corresponds to symbol 01

f_3 corresponds to symbol 10
f_4 corresponds to symbol 11
δf = one cycle per symbol
Signal amplitude = 1 volt

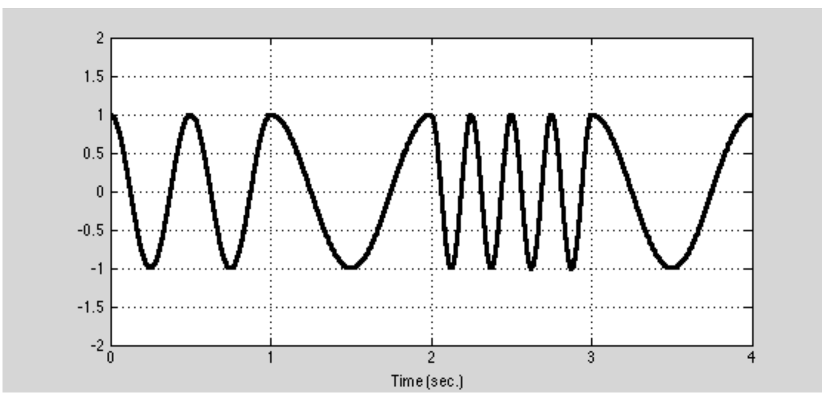
Which of the following signals is correct?

A.

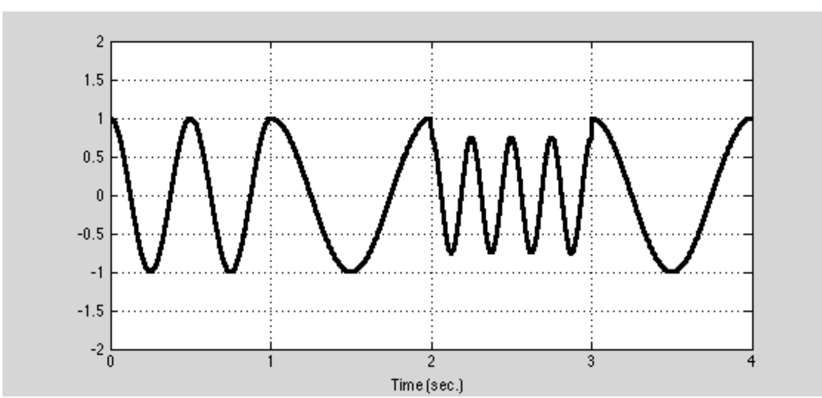


First and second frequencies inverted.

B.



C.

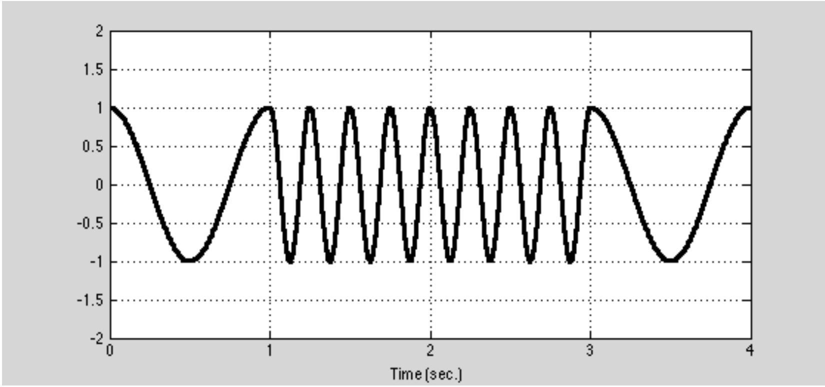


Amplitude for the third symbol is not equal to 1.0

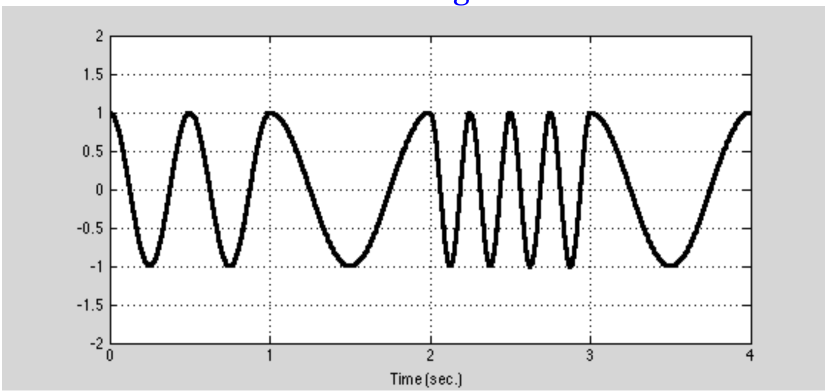
D. None of the above

Answer __B__

Another incorrect



Correct signal



Probabilities

5. What is the probability that a Gaussian random variable X with mean 8 and variance 4.0 would be less than 6 or greater than 9?

   Q((9-8)/sqrt(4))+(1-Q((6-8)/sqrt(4)) = Q(0.5)+(1-(1-Q((10-8)/2))=
   Q(0.5)+Q(1) = 0.3085+0.1587 = 0.4672

   Variance 1.0:
   Q(1/1)+Q(2/1) = 0.1587+0.2275e-1 = 0.18145

   Variance 16.0
   Q(1/4)+Q(2/4) = Q(0.25) + Q(0.5) = 0.4013+0.3446 = 0.7459

   A. 0.4672 B. 0.7459
   C. 0.1815 D. None of the above

   Answer __A__

6. A system design requires that bits be received with less than a 10^{-4} probability of bit error. What is the probability that a packet is received with 0 or 1 errors if packets are 10000 bits?

   p = 1e-4
   Pr{0 errors} = ${}_{10000}C_0\, p^0(1-p)^{10000}$=(10000!/10000!/0!)*1*(1-p) ^10000 =
   1*1*(1-p)^10000 = 0.36786
   Pr{1 error} = ${}_{10000}C_1\, p^1(1-p)^{9999}$ = (10000!/9999!/1!)*p*(1-p)^9999=
   10000*p*(1-p)^9999 = 0.367897
   Total probability = 0.36786+0.367897 = 0.735757

   A. 0.3679 B. 1.0000
   C. 0.7358 D. None of the above

   Answer __C__

7. The bit error probability for a binary polar system is described by the following equation.

$$P_b = Q\sqrt{\frac{2E_b}{N}}$$

   Note: When using the attached Q table, use the closest value. You do not need to interpolate between values.

If rectangular pulses are used, the amplitude of the pulse is 1 V, and the value of $N/2 = 10^{-8}$, what is the maximum bit rate that can be supported to have a bit error probability of less than 0.1078e-3?

0.1078e-3 = Q(sqrt(2*Eb/N))
3.70 = sqrt(2*Eb/N)
13.69 = 2*Eb/N=Eb/1e-8
13.69e-8 = Eb = 1^2*Tb
13.69e-8 = Tb
rb = 1/Tb = 7.3046e+6 = 7.3046 Mbps

If Pb = 0.3167e-4
4.00 = sqrt(2*Eb/N)
16.00 = 2*Eb/N=Eb/1e-8
16e-8 = Eb = 1^2*Tb
16e-8 = Tb
rb = 1/Tb = 6.25e+6 = 6.25 Mbps

If Pb = 0.3369e-3
3.4 = sqrt(2*Eb/N)
rb = 1/Tb = 8.6505e+6 = 8.65 Mbps

A. 7.30 Mbps
B. 8.65 Mbps
C. 6.25 Mbps
D. None of the above

Answer __A__

8. In order to achieve better bit error performance, we decrease the bit rate in problem (7) to 1/4 of what was achieved there. What is the new bit error probability? Note: It is recommended to use several digits in calculations from the answer in problem (7) to find an accurate answer here.

Eb = A^2 * Tb
To decrease the bit rate by 1/4, this increases the Tb to 4*Tb, which increases the Eb to 4*Eb

For 0.1078e-3:
Sqrt(2*Eb/N)=3.7 becomes sqrt(2*4*Eb/N)=3.7*2 = 7.4
Q(7.4) = 0.6809e-13
Using the answer from problem (7): 7.3046e6
rb = 7.3046e6/4 = 1.82615e6
Tb = 1/rb = 0.5476e-6
sqrt(2*Eb/N)=sqrt(2/N*1^2*Tb)=sqrt(1/1e-8*1*.5476e-6)= 7.399999
Q(7.40) = 0.6809e-13

For 0.3167e-4:
4 becomes 8
Q(8) = 0.6221e-15

For 0.3369e-3
3.4 becomes 6.8
Q(6.8) = 0.5231e-11

A. 0.6221e-15
B. 0.6809e-13
C. 0.5231e-11
D. None of the above

Answer __B__

The Optimal Receiver

Consider the derivation of the optimal receiver for binary polar signaling.

9. Which best describes the location in the receiver where $r(t_m)$ is used in the optimal receiver?

A. Input to multiplication by $p(t)$
B. Input to the sampling operation
C. Input to the integration
D. None of the above

Answer __D__

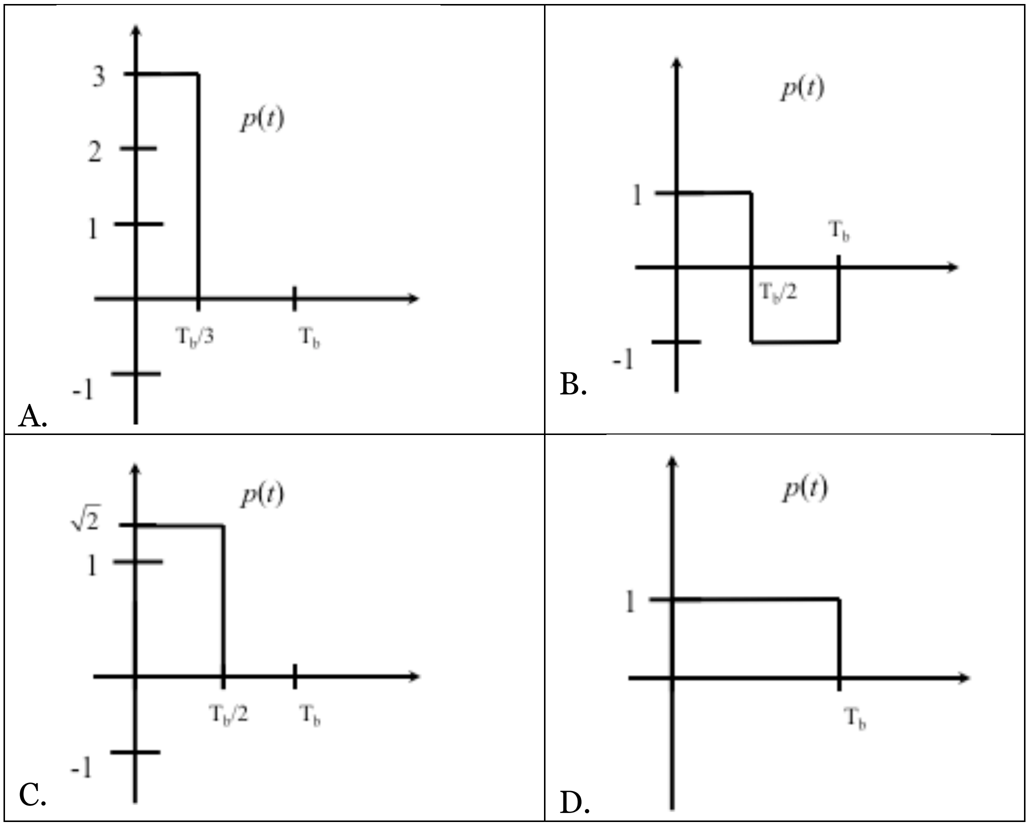
Correct answer: Input to decision of a "1" or a "0"

10. Which of the following is NOT an assumption that is made for the matched filter derivation?

A. AWGN
B. Noise with nonzero mean
C. Polar signals
D. Threshold of 0 for decisions

Answer __B__

11. Which of the following pulse shapes will NOT produce the same probability of error as the others?



Answer __A__

12. The derivation of the optimal receiver for binary polar signaling has the following equation for probability of bit error.

$$P_b = Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

The value for N in this equation (usually shown as a script N in class lectures) comes from which of the following.

A. Number of pulses
B. Power spectral density of the noise
C. Filter impulse response
D. Pulse shape

Answer __B__

Orthogonality

13. Which two signals are orthogonal and are the ones used by FSK?

A. $\cos(2\pi f_c t)\,, 3\cos(2\pi f_c t)$
B. $\cos(2\pi f_c t)\,, \sin(2\pi f_c t)$
C. $\cos(2\pi f_1 t)\,, \cos(2\pi f_2 t)\,, \delta_f = \frac{n}{2T_b}$
D. None of the above

Answer __C__

14. Which integral is the definition of orthogonality?

A. $\int_0^{T_b} \psi_1(t)\psi_2(t)dt = 0$
B. $X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}dt$
C. $E_g = \int_{-\infty}^{+\infty} g^2(t)dt = \int_{-\infty}^{+\infty} |G(f)|^2 df$
D. $\rho^2 = \frac{\int_{-\infty}^{+\infty} |X(f)Y(f)|^2 df}{\int_{-\infty}^{+\infty} |X(f)|^2 df}$

Answer __A_(any answer full credit)_

Signal Energy

15. An 8-PSK modulation scheme is proposed. If each point is located at a distance of R from the origin of a constellation diagram, what is the average energy per bit? Assume normalized basis functions are used to cancel any T_b terms.

$E_s = \frac{1}{8}(8R^2) = R^2, E_b = \frac{1}{3}E_s = \frac{R^2}{3}$

A. $E_b = R^2$ J
B. $E_b = \frac{R^2}{8}$ J
C. $E_b = \frac{R^2}{3}$ J
D. None of the above

Answer __C__

16. Rectangular pulses are to be used to send data. 8 different pulses of different amplitudes (same durations) are used. Pulses are located at $\pm1, \pm3, \pm5, \pm7$ Volts. If the duration of each pulse is 1 microsecond, what is the data rate in bits per second that is achieved?

1/1e-6 = 1e6 symbols per second
(1e6 symb/sec)*(3 bits/symb) = 3e6 = 3 Mbps

A. 1 Mbps
B. 3 Mbps

C. 8 Mbps D. None of the above

Answer __B__

17. An engineer has a 500 kHz bandwidth available to send a signal. The requirement is to send 95.0% of the signal within the bandwidth to achieve sufficient quality. Binary signals are sent using rectangular pulses. What bit rate can be achieved?

500 kHz:
$\frac{2}{\tau} = 500x10^3, \tau = \frac{2}{5}\times10^{-5}, r_b = 1/\tau = 250$ kbps

1 MHz:
$\frac{2}{\tau} = 10^6, \tau = 2\times10^{-6}, r_b = \frac{1}{\tau} = 500$ kbps

200 kHz:
200/2 = 100 kbps

A. 200 kbps B. 100 kbps
C. 500 kbps D. None of the above

Answer __D__

18. 16 QAM is now used to send data with pulses that are 4 volts apart in the x and y directions of the signal constellation. That is to say that constellation points are laid out along combinations of ± 2 and ± 6 Volts. What is the average energy per bit?

Since 4V spacing, constellation points are at x and y coordinates of combinations of +/-2 and +/-6 Volts
Average energy per symbol = E_s = $\frac{1}{16}(4 * (2^2 + 2^2) + 8 * (2^2 + 6^2) + 4 * (6^2 + 6^2))$=
1/16*(4* (8)+8*(40)+4*(72))=40 J

E_b = ¼*E_s = 10 J

A. 40 J B. 640 J
C. 2.25 J D. 10 J

Answer __D__

19. Suppose only 12 symbols are needed of the possible 16 symbols for 16 QAM in problem (18), and suppose the constellation points with the highest energy

are not used. What is the ratio of energy per bit between the two? That is to say, what is the following ratio?

$$\frac{E_{b,12QAM}}{E_{b,16QAM}}$$

Only using 12 points with least energy:

$E_S = \frac{1}{12}\left(4(2^2+2^2)+8(2^2+6^2)\right) = \frac{1}{12}(4\times 8+8\times 40) = 29.333$

$E_b = \frac{E_S}{4} = \frac{29.333}{4} = 7.333$

$$\frac{E_{b,12QAM}}{E_{b,16QAM}} = \frac{7.333}{10} = 0.733$$

A. 29.333 B. 0.950
C. 0.733 D. None of the above

Answer __C__

20. Assume the bit rate achieved in problem (18) was 500 kbps. If the constellation points are spread even further apart to 6 volt spacing (at ± 3 and ± 9 Volts), what would be the new data rate?

No change in data rate. Still 4 bits/symbol. Spreading constellation points changes only the energy per bit.

A. 750 kbps B. 500 kbps
C. 1.5 Mbps D. None of the above

Answer __B__

Name ____________________________
Student Number ____________________

ECE 5570 Principles of Digital Communications

Exam #1

July 3, 2014

Instructions:

1. Before you start, put your name on the first page.
2. Q-function tables are provided at the end of the exam.
3. You have 1 hour and 30 minutes to complete the exam.
4. Multiple choice answers must be clearly and unambiguously marked.
5. No partial credit is given toward multiple choice problems.
6. A zero mark is given for every incorrect answer.
7. Answer with the BEST of the four answers.
8. If the correct answer is "None of the above", this means the correct answer is not numerically close to the other three answers.

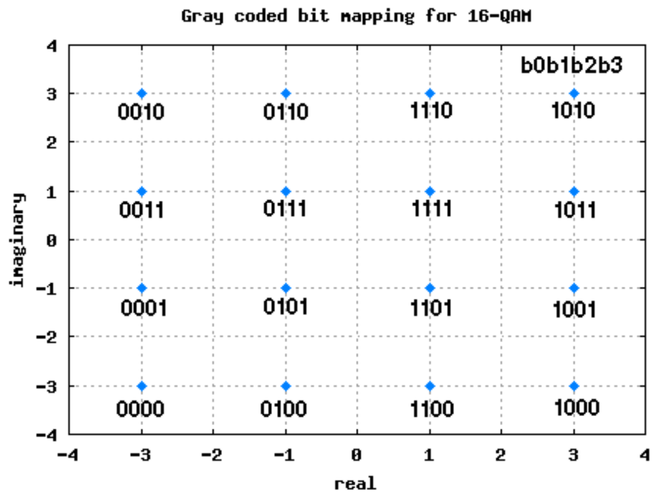
Point breakdown per problem
1. 20 Sketching signals
2. 20 Probabilities
3. 25 Proof of the Optimal Receiver
4. 10 Orthogonality
5. 25 Signal Energy

There were multiple version of the exam, so this is one set of solutions.

Average: 56.0
High Scores: 90(3), 85, 80(3), 75(5)
Rough curved grading scale (non-binding)
Rough grade ranges (non-binding):

- A → 70 – 100
- A- → 60 – 69
- B+ → 50 – 59
- B → 42 – 49
- Below B → Below 42

1. (20 points) A modulation scheme is used with the following signal constellation.



This modulation scheme is used to transmit the following bit stream with the following system parameters.

Bit stream: 0010101001000111

Bit rate = 16.0 Mbps
Carrier frequency = 12.0 MHz

a) (5 points) What is the symbol time for this system?

(1/16e6 sec/bit)*(4 bits/symbol) = 1/4e6 = 0.25 microseconds

A. 0.0625 microseconds
B. 0.0833 microseconds
C. 0.3333 microseconds
D. None of the above

Answer __D__

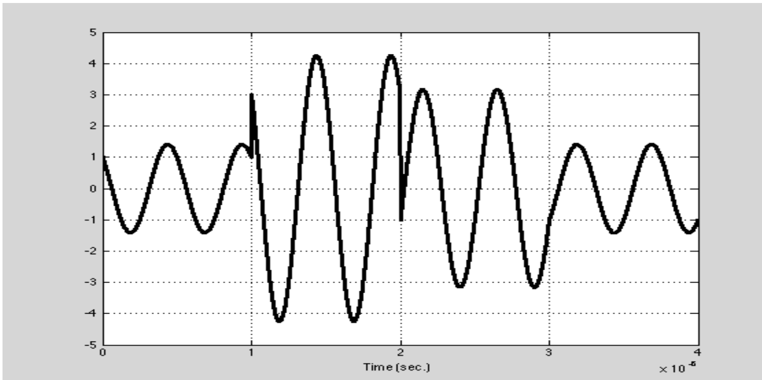
b) (5 points) The signal goes through how many cycles per each symbol?

(0.25e-6 sec/symbol)*(12e6 cycles/sec) = 3 cycles/symbol

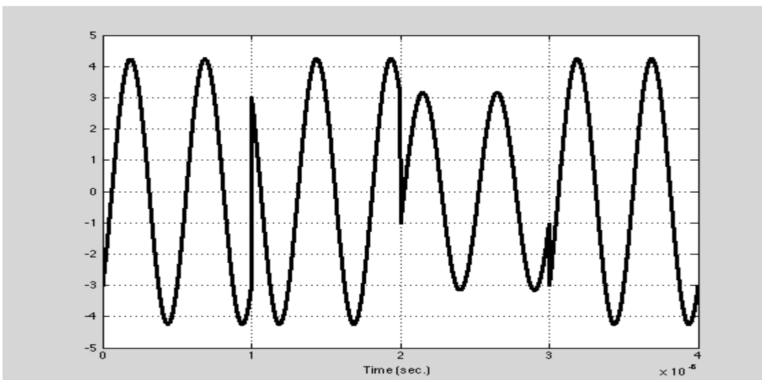
A. 3 cycles/symbol
B. 1 cycle/symbol
C. 3/4 cycles/symbol
D. None of the above

Answer __A__

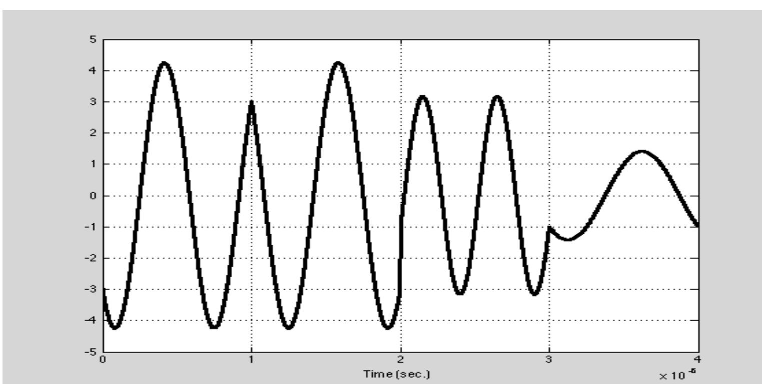
<u>Now a different frequency and bit rate are used from the numbers given above</u>. Now is used a symbol time of 1 microsecond and 2 cycles per symbol. Consider the following signals.

A.


Incorrect amplitude and phase for symbol #1, incorrect phase for symbol #4.

B.


Incorrect phase for symbol #1, incorrect amplitude and phase for symbol #4.

C.


Incorrect frequencies for symbols #1, #2, and #4.

D. None of the above

Note: in the original exam, all signals had incorrect amplitudes, so answer "D" was the correct answer for (c) and (d). But this was not the intention for the problem.

c) (5 points) For which of the signals above are the first two symbols (0 to 2 microseconds) correct?

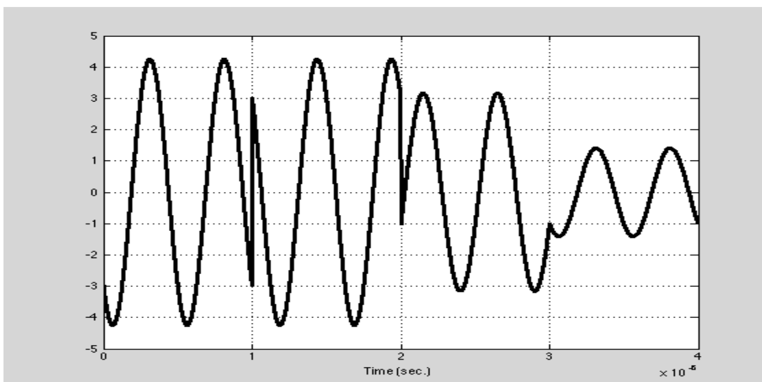
Answer __D__

d) (5 points) For which of the signals above are the last two symbols (from 2 to 4 microseconds) correct?

Answer __D__

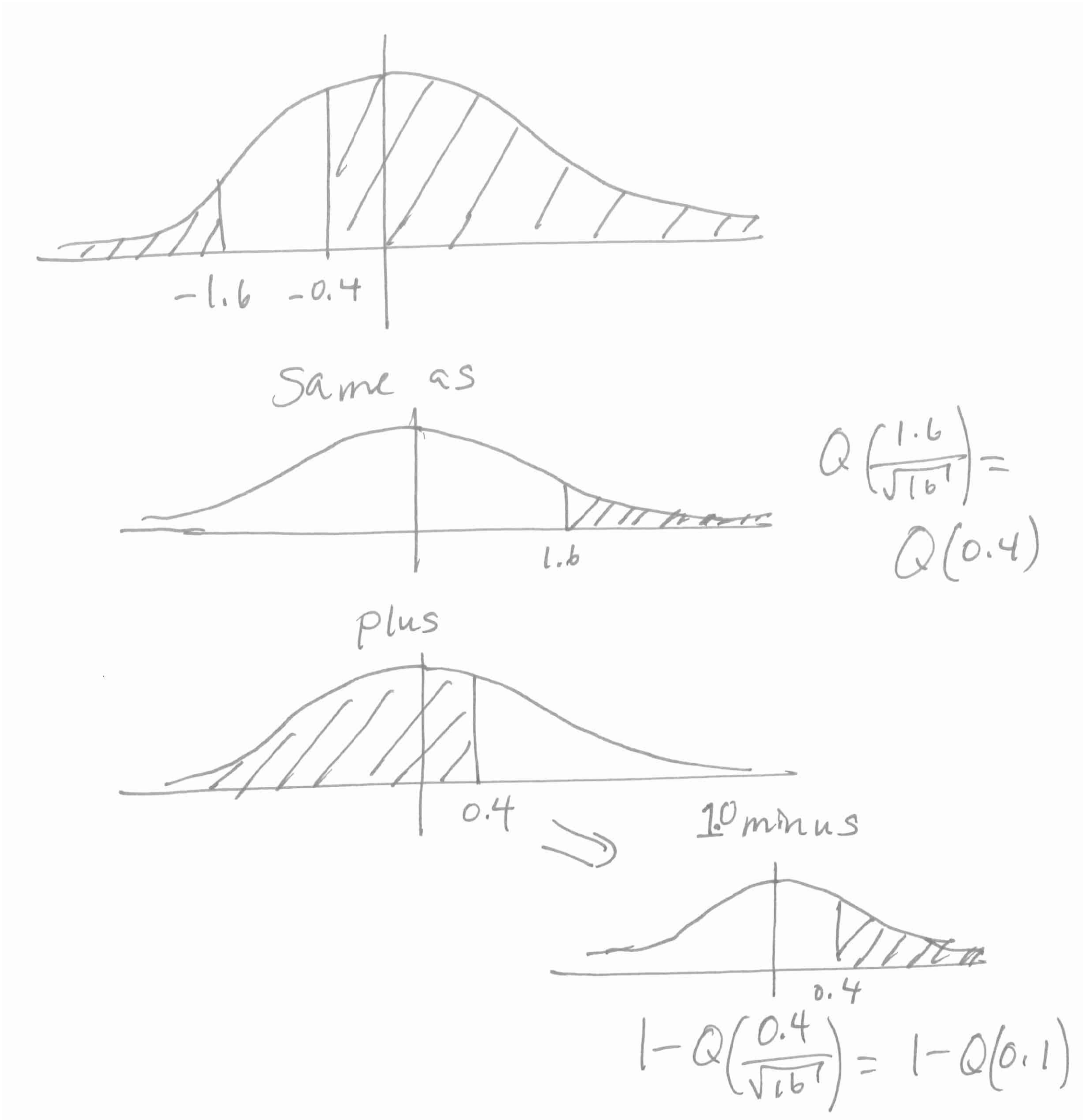
Bit stream: 0010 1010 0100 0111

Correct signal



2. (20 points) Probabilities

   (a) (5 points) What is the probability that a zero mean Gaussian random variable X with variance 16.0 would be less than -1.6 or greater than -0.4?



Q(1.6/sqrt(16))+(1-Q(-0.4/sqrt(16)) = Q(0.4)+(1-Q(0.1))
= 0.3446+1-0.4602 = 0.8844

A. 0.8844 B. 0.3446
C. 0.8048 D. None of the above

Answer __A__

(b) (5 points) An LTE system implementation requires that packets be received with less than a 10% probability of packet error. What is the corresponding requirement on the *range* of acceptable bit error probabilities if packets are 10000 bits?

1-Pr{all correct} = Pr{no errors} = 1-(1-Pb)^10000 = 0.1
(1-Pb)^10000 = 0.9
1-Pb = 0.9^(1/10000) = 0.999989464
Pb = 1.0356e-5
The probability has to be less than this value to meet the requirement.

A. $P_b < 1.0000 \times 10^{-5}$
B. $P_b < 1.0356 \times 10^{-5}$
C. $P_b > 0.00100$
D. None of the above

Answer __B__

The bit error probability for a binary system is described by the following equation.

$$P_b = Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

Note: When using the attached Q table, use the closest value. You do not need to interpolate between values.

d) (5 points) If the E_b/N is 3 dB, what is P_b?

sqrt(10^0.3*2) = 1.997629753
Q(2.000) = 0.2275e-1 = 0.02275

Incorrect answers:
Q(sqrt(3*2)) = Q(2.449) = Q(2.45) = 0.007143
Q(sqrt(10^.3)) = Q(1.41) = 0.7927e-1

A. $P_b = 0.2275$
B. $P_b = 0.007927$
C. $P_b = 0.007143$
D. None of the above

Answer __D__

e) (5 points) How much ***INCREASE (IN DB)*** is needed over part (c) to achieve a BER of 0.001350?

$Q(x) = 0.001350 = 0.1350e-2$
From table, $x = 3.0$
$\text{Sqrt}(2*E_b/N) = 3.0$
$E_b/N = 3.0^2/2 = 4.5$
$10*\log_{10}(E_b/N) = 10*\log_{10}(4.5) = 6.532$ dB
Compared to 3.0 dB, this is an increase of 3.532 dB

A. 3.532 dB	B. 1.502 dB
C. 0.1350 dB	D. None of the above

Answer __A__

Note: The original exam had 3.352 dB for (A), so this was not the exact answer and D would also have been acceptable.

3. (25 points) The Optimal Receiver

Consider the derivation of the optimal receiver for binary polar signaling

(a) (5 points) Which of the following is the expression for the noise signal after it leaves the matched filter $H(f)$ but before sampling?

A. $n_o(t_m)$
B. $n_o(t)$
C. $n(t)$
D. None of the above

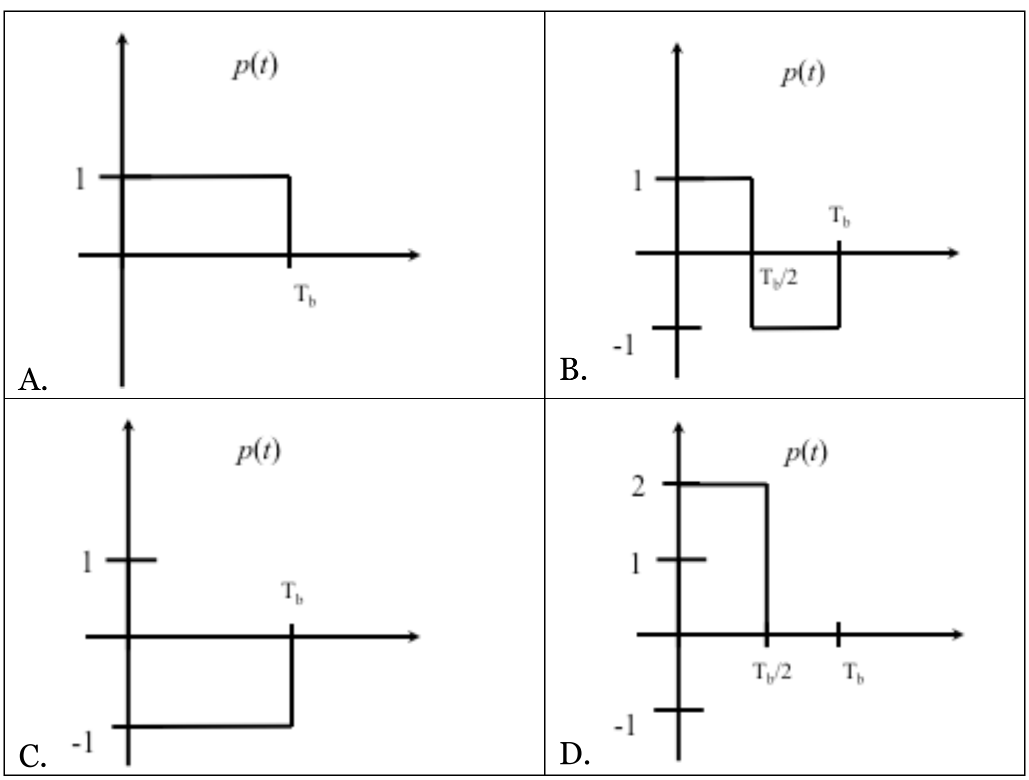
Answer __B__

(b) (5 points) Which of the following is NOT an assumption that is made about the noise for the matched filter?

A. Gaussian
B. Multiplicative
C. Constant power spectral density
D. White

Answer __B__

(c) (5 points) Which of the following pulse shapes will NOT produce the same probability of error as the others?



Answer __D__

(d) (10 points) The derivation of the optimal receiver for binary polar signaling has the following equation for probability of bit error.

$$P_b = Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

Show ONLY the part of the derivation that produces this equation when maximizing the value of ρ^2 for the special case when the power spectral density is $S_n(f)=N/2$.

4. (10 points) Orthogonality

(a) (5 points) Which two signals are orthogonal and are the ones used by QPSK?

A. $\cos(2\pi f_c t)$, $\sin(2\pi f_c t)$

B. $\cos(2\pi f_1 t)$, $\cos(2\pi f_2 t)$, $\delta_f = \frac{n}{2T_b}$

C. $\cos(2\pi f_c t)$, $-\cos(2\pi f_c t)$

D. None of the above

Answer __A__

(b) (5 points) In an MFSK system with $f_1 = 110$ kHz and a bit rate of 15 kbps, which of the following frequencies are orthogonal? (only one answer is correct)

A. 111.5 kHz

B. 220 kHz

C. 117.5 kHz

D. None of the above

Answer __C__

5. (25 points) Signal Energy

a) (5 points) A variation on the 16-PSK modulation scheme is proposed. Symbols are to be spaced around two circles instead of one. Six symbols are evenly distributed around a smaller circle of radius R, and the remaining 10 symbols are spaced around a larger circle of radius $2R$. Rectagular pulses of duration T_s are used. What is the average energy per symbol?

Energy per pulse values are R^2T_s and $(2R)^2T_s=4R^2T_s$.
Average is $1/16*(6* R^2T_s+10*4R^2T_s) = R^2T_s*1/16*(6+40) = 23/8* R^2T_s$

A. $\frac{23}{8}R^2T_s$ J
B. $46R^2T_s$ J
C. $\frac{13}{8}R^2T_s$ J
D. None of the above

Answer ___A___

a) (5 points) An engineer has a 200 kHz bandwidth available to send a signal. The requirement is to send 95.0% of the signal within the bandwidth to achieve sufficient quality. The goal is to try several transmission formats to see which is best. The first attempt is to send binary signals using rectangular pulses. What bit rate can be achieved?

$\frac{2}{\tau} = 200x10^3, \tau = 10^{-5}, r_b = 1/\tau = 100$ kbps

A. 200 kbps
B. 100 kbps
C. 2 Mbps
D. None of the above

Answer ___B___

b) (5 points) If a 3 Volt spacing is needed between the amplitudes of the pulses in part (b), what is the average signal energy per bit?

$\tau = 10^{-5}$, pulse are +1.5 V or -1.5V, so average energy is 1.5^2*1e-5 = 2.25e-5

A. 2.25×10^{-5} J
B. 10^{-5} J
C. 1.125×10^{-5} J
D. None of the above

Answer ___A___

c) Then 16QAM is considered for the system in part (b). The constellation points must be 3 V part. What bit rate can be achieved?

Same $\frac{2}{\tau} = 200x10^3, \tau = 10^{-5}$ as in part (b), but now 4 bits are transferred during each τ, so $r_b = 4 * 1/\tau = 400$ kbps

A. 400 kbps
B. 1.6 Mbps
C. 800 kbps
D. None of the above

Answer __A__

e) (5 points) For the 16QAM signal in (d), what is the average energy **per bit**?

Since 3V spacing, constellation points are at x and y coordinates of combinations of +/-1.5 and +/-4.5 Volts

Average power = $\frac{1}{16}(4 * (1.5^2 + 1.5^2) + 8 * (1.5^2 + 4.5^2) + 4 * (1.5^2 + 1.5^2))$=
1/16*(4* (1.5^2+1.5^2)+8*(4.5^2+1.5^2)+4*(4.5^2+4.5^2))=22.50

E_s = 22.50*Ts = 2.25e-4
E_s = 2.25e-4/4 = 5.625e-5 (2.5 times 2-PAM)

A. 2.25 x 10^{-4} J
B. 2.5 x 10^{-5} J
C. 2.25 x 10^{-5} J
D. None of the above

Answer __D__

ECE 5570 Principles of Digital Communications

Exam #1

July 2, 2013

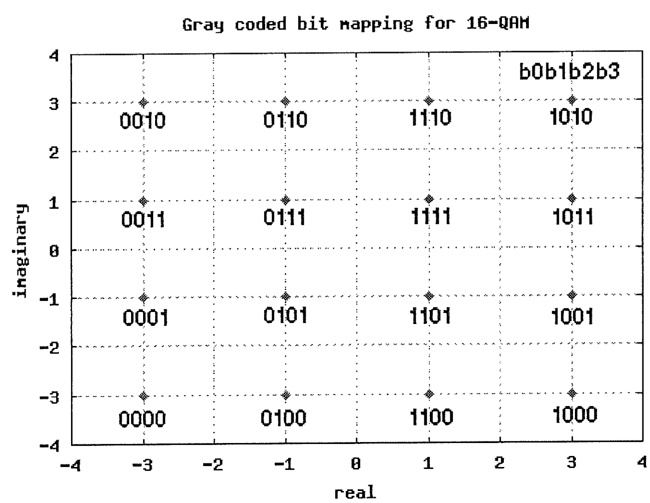
High Scores: 96, 96, 95, 93, 92, 91
Average: 76.3
Rough grade ranges (non-binding):

- A → 87 – 100
- A- → 79 – 86
- B+ → 72 – 77
- B → 60 – 71
- Below B → Below 60

Point breakdown per problem

1. 20 Sketching signals
2. 20 Probabilities
3. 25 Proof of the Optimal Receiver
4. 15 Orthogonality
5. 20 Signal Energy

1. (20 points) A modulation scheme is used with the following signal constellation.



This modulation scheme is used to transmit the following bit stream with the following system parameters.

Bit stream: 0001 0101 1111 1001

Bit rate = 20.0 Mbps
Carrier frequency = 12.5 MHz

a) (2 points) What would this modulation scheme most likely be called?

16QAM

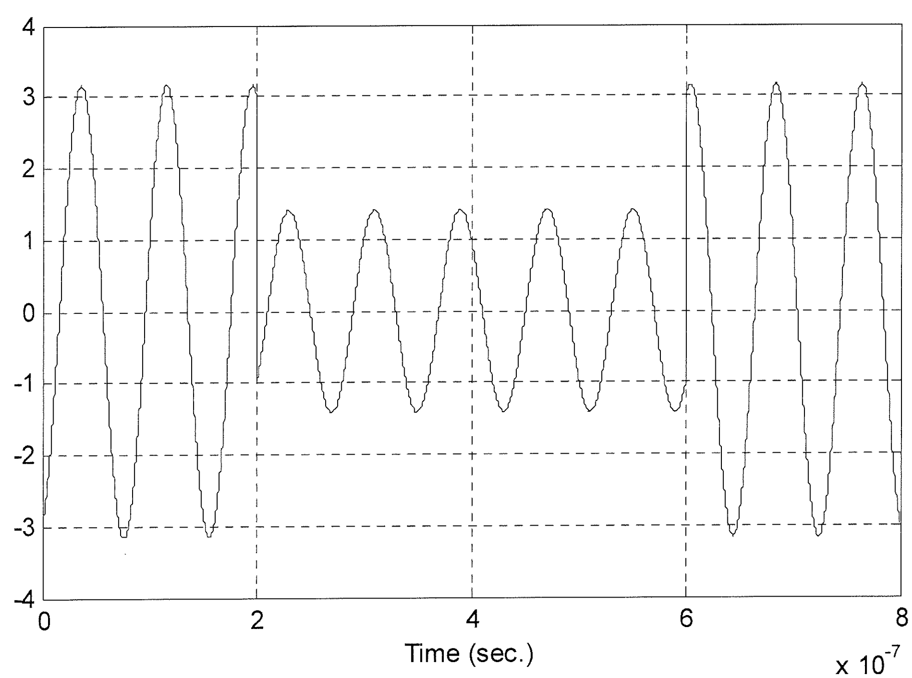
b) (3 points) What is the symbol time for this system in microseconds?

20e6/4 = 5e6 symbols/sec
1/5e6 = 0.2 microseconds/symbol

c) (3 points) The signal goes through how many cycles per each symbol?

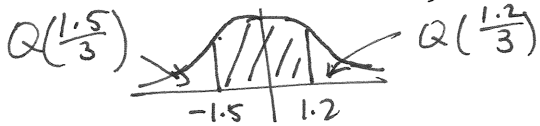
(12.5e6 cycles/sec)/(5e6 symbols/sec) = 2.5 symbols/cycle

d) (12 points) Sketch the transmitted signal. Use the **cosine** function. Be **sure** to label both axes as completely as possible.



2. (20 points) Probabilities

(a) (5 points) What is the probability that a zero mean Gaussian random variable X with variance 9.0 would be between -1.5 and +1.2?



(1-Q(1.5/3)) – Q(1.2/3) = (1-Q(0.5)) – Q(0.4) = (1-0.3085) - .3446 = 0.3469

(b) (5 points) The probability of bit error is 10^{-4}; packets are sent of size 100 bits; and 50 packets are sent. What is the probability that 2 or more packets will be received with errors? Make sure to keep enough significant digits in your calculations to have an accurate answer.

P =Pr{packet error} = 1-Pr{all bits correct} = 1-(1-1e-4)^100 = 0.00995

Pr{2 or more packets in error} = 1- Pr{0 packets in error} – Pr{1 packet in error}
= 1- $_{50}C_0$*P^0*(1-P)^50-$_{50}C_1$*P^1*(1-P)^49 = 1-1*1*(1-P)^50-50*P*(1-P)^49
= 1- 0.6065 - 0.30479
= 0.08871

(c) (10 points) The bit error probability for a binary system is described by the following equation. At first, rectangular pulses were used of duration 1 microsecond and 0.1 volt amplitude, but the bit error probability was found to be too high at a value of 0.1350 x 10^{-2}. How should the amplitude of the pulses be changed to if a bit error probability of less than 0.3908 x 10^{-4} is needed?

$$P_b = Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

First error rate:
Eb = A^2*Tb = (.1)^2*1e-6 = 1e-8
Q(sqrt(2Eb/N)) = 0.1350e-2
Sqrt(2Eb/N) = 3
2Eb/N = 9
N = 2*Eb/9 = 2*1e-8/9 = 2.22e-9

New error rate:
Q(sqrt(2Eb/N)) = 0.3908e-4
sqrt(2Eb/N) = 3.95
2*Eb/N = 3.95^2

Eb = 3.95^2/2*N = 3.95^2/2*2.22e-9 = 1.732e-8
Eb = A^2*Tb = 1.732e-8
A = sqrt(1.732e-8/1e-6) = 0.1316 volts

3. (25 points) The Optimal Receiver

Consider the derivation of the optimal receiver for binary polar signaling

(a) (5 points) Why is T_b the best time to sample the output of the integrator in the matched filter? You should only use one or two sentences to explain your answer

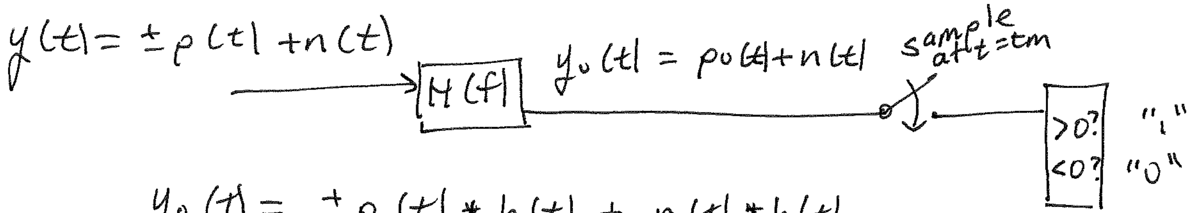
At $t=T_b$, the pulses overlap the most.
There is the best match.

(b) (20 points) Show the derivation of the optimal receiver for binary polar signaling that has the following equation for probability of bit error.

$$P_b = Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

Proof of the Matched Filter Optimum Receiver

"1" $\to +p(t)$
"0" $\to -p(t)$



$$y_o(t) = \pm p(t) * h(t) + n(t) * h(t)$$
$$= \pm p_o(t) + n_o(t)$$

$$r(t_m) = \pm p_o(t_m) + n_o(t_m)$$
$$A_p = p_o(t_m) = \int_{-\infty}^{\infty} P(f) H(f) e^{+j2\pi f t_m} df$$
$$\sigma_n^2 = \overline{n_o^2(t_m)} = \int_{-\infty}^{\infty} S_n(f) |H(f)|^2 df$$

$$P_e = Q\left(\frac{A_p}{\sigma_n}\right)$$

Maximize $\rho^2 = \left(\frac{A_p}{\sigma_n}\right)^2 = \dfrac{\left|\int_{-\infty}^{\infty} H(f) P(f) e^{+j2\pi f t_m} df\right|^2}{\int_{-\infty}^{\infty} S_n(f) |H(f)|^2 df}$

$$X(f) = H(f)\sqrt{S_n(f)}$$
$$Y(f) = \frac{P(f) e^{+j2\pi f t_m}}{\sqrt{S_n(f)}}$$

$$\rho^2 = \frac{\left|\int_{-\infty}^{\infty} X(f) Y(f) df\right|^2}{\int_{-\infty}^{\infty} |X(f)|^2 df} \le \frac{\int_{-\infty}^{\infty} |X(f)|^2 df \int_{-\infty}^{\infty} |Y(f)|^2 df}{\int_{-\infty}^{\infty} |X(f)|^2 df}$$

Schwartz Inequality

$$\le \int_{-\infty}^{\infty} |Y(f)|^2 df$$

To get maximum, set $X(f) = k\,[Y(f)]^*$

$$X(f) = H(f)\sqrt{S_n(f)} = \frac{k P(-f) e^{-j2\pi f t_m}}{\sqrt{S_n(f)}}$$

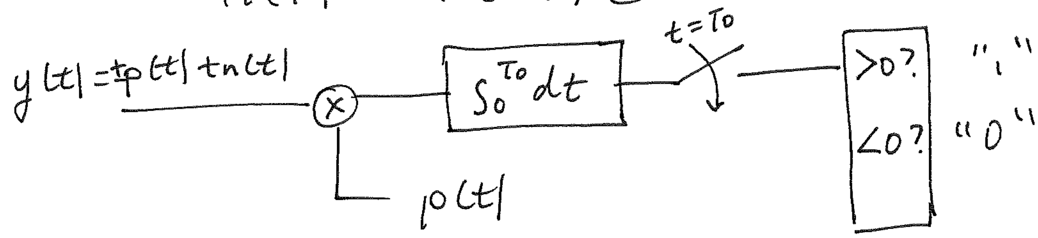
$$\boxed{H(f) = \frac{K P(-f) e^{-j2\pi f t_m}}{S_n(f)}}$$

If white noise, $S_n(f) = \frac{\eta}{2}$

Matched Filter

$$h(t) = p(T_0 - t)$$

$$H(f) = P(-f)\, e^{-j2\pi f T_0}$$



$$\text{Max } \rho^2 = \int_{-\infty}^{\infty} |Y(f)|^2 df = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{S_n(f)} df$$

$$= \frac{2}{\eta} \int_{-\infty}^{\infty} |P(f)|^2 df = \frac{2E_p}{\eta}$$

$$P_e = Q\left(\frac{A_p}{\sigma_n}\right) = Q\left(\sqrt{\rho^2}\right) = \boxed{Q\left(\sqrt{\frac{2E_p}{\eta}}\right)}$$

4. (15 points) Orthogonality

Orthogonal Frequency Division Multiplexing (OFDM) splits a high bit rate stream into many parallel lower bit rate streams (called subcarriers). The main benefit of this approach is to make the bit times very long to overcome wireless channel problems.

For this problem, a wireless designer has decided that bit times need to be at least 100 microseconds to overcome the channel problems.

(a) (3 points) If rectangular pulses are used, that what should be the bandwidth of each subcarrier so that approximately 96.6% of the energy in the pulses is transmitted?

$3/Tb = 30$ kHz

(b) (12 points) What should be the spacing between these subcarriers so that they are orthogonal? Prove this by showing through integral equations.

See class notes on orthogonality for M-FSK signals. $\delta f=1/(2^*T_b)$

5. (20 points) Signal Energy

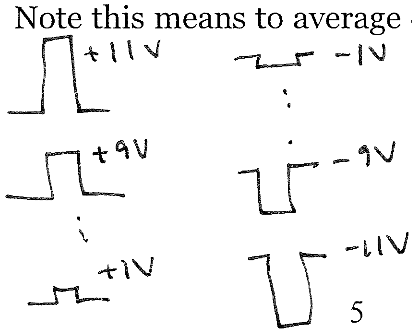
a) (5 points) Why does QPSK achieve twice the bit rate but require no increase in the average energy per bit?

It uses the orthogonal sine and cosine functions.

For the rest of the questions, consider this scenario. A system designer wishes to compare the signal energy needed for 3 different types of modulation schemes. The design needs to transmit 12 different symbols. It is clear that a 16-ary modulation scheme is needed, but 4 of the 16 symbols will never be used. The plan is to assign the 12 symbols to those of the 16 that use the least energy. In other words, the design will never use the 4 symbols in the constellation with the highest energy.

In all cases, the symbols must have a 2 volt spacing with the other closes symbols.

b) (7 points) In a pulse amplitude modulated scheme that uses various voltages for rectangular pulses, what would be the average power per symbol? Note this means to average over the 12 symbols that are actually used.



Voltages need to be +11,+9,+7,+5,+3,+1,-1,-3,-5,-7,-9,-11

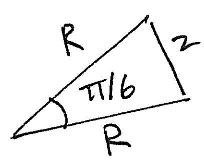
Avg. power = 1/12*(2*11^2+2*9^2+2*7^2+2*5^2+2*3^2+2*1^2)
= 47.67 W/symbol

c) (8 points) Using the constellation for Problem #1 of this exam, what would be the average power per symbol? Note that the grid is in units of volts, so the points are already spaced 2 volts apart.

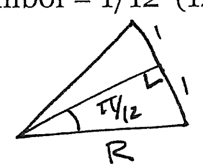
Do not use the four outside corners.

Avg. power = 1/12*(4*(1^2+1^2)+8*(3^2+1^2)) = 7.333 W/symbol

d) (5 points) Extra Credit (all correct or no points): If a 12-PSK constellation were used, what would be the average power per symbol? Note in this case that the 12 points could be evenly distributed around the circle instead of using 16 points.



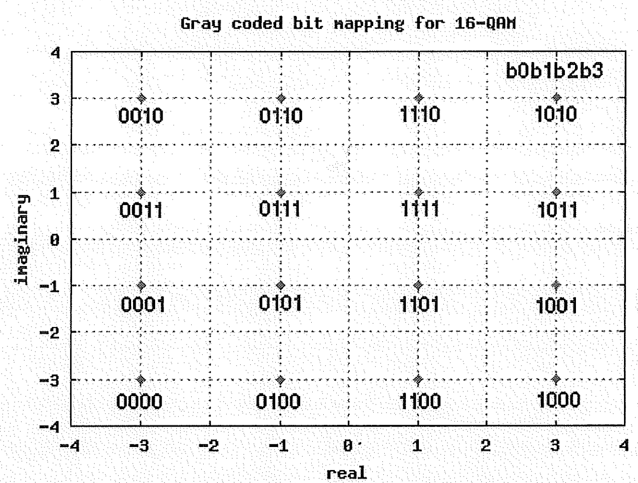
Assume one symbol is at 0 degrees.
The second symbol forms a triangle with sides R, R, and 2, with the angle 2*pi/12.
Using the law of cosines c^2=a^2+b^2-2*a*b*cos(theta)
2^2=R^2+R^2-2*R*R*cos(2*pi/12)
4=R^2*(2-2*cos(2*pi/12))
R=sqrt(4/(2-2*cos(2*pi/12))) = 3.864
All symbols at distance 3.864
Average energy per symbol = 1/12*(12*3.864^2) = 14.93 W/symbol



Alternate approach: If the law of cosines is not known:
12-PSK would use 30 degree wedges between symbol points
So, the sides of the triangle would be R, R, and 2, with 30 degrees (pi/6).
Or this could be broken into two right triangles, with sides unknown, 1, and R, with the angle 15 degrees (pi/12).
Therefore, sin(pi/12) = 1/R, R=1/sin(pi/12) = 3.864 (same result as above)

ECE 5590CT Exam#1 - Summer 2012

1. (20 points) A modulation scheme is used with the following signal constellation.



Avg = 81.0
Highs = 91 (3), 90 (3), 89 (3)
Ranges
85 + → A/A-
70 ~ 84 → B/B+
69 and below ⇒ B -

This modulation scheme is used to transmit the following bit stream with the following system parameters.

Bit stream: 1110|1101|1010|0000

Bit rate = 5.0 Mbps
Carrier frequency = 1.25 MHz

a) (2 points) What would this modulation scheme most likely be called?

16 QAM

b) (3 points) What is the symbol time for this system in microseconds?

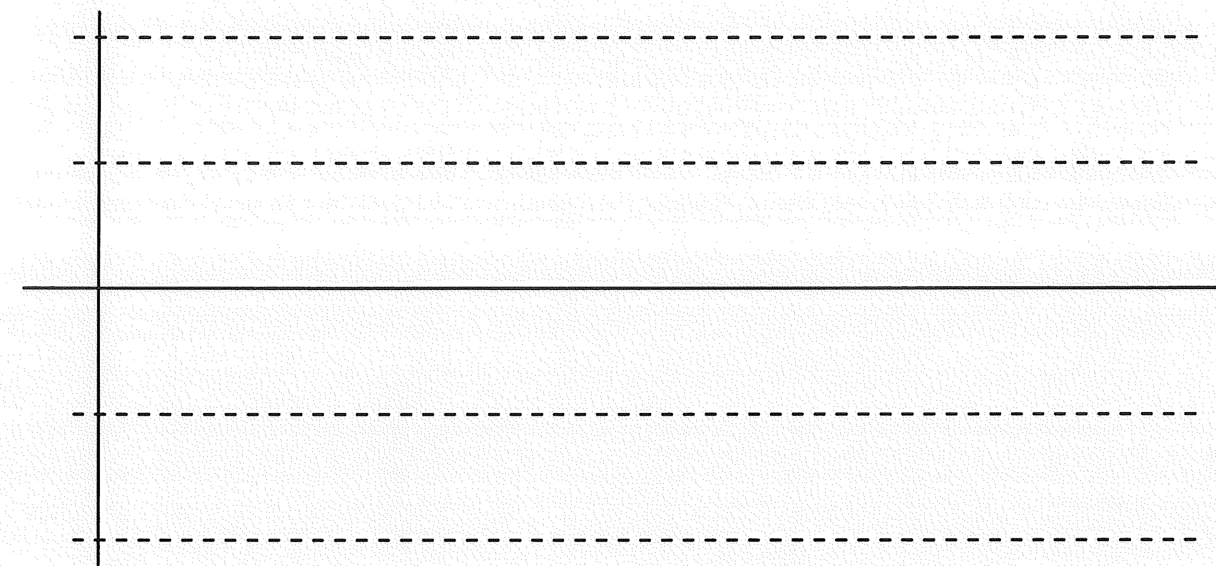
$$5.0 \times 10^6 \frac{\text{bits}}{\text{sec}} \times \frac{1 \text{ symb}}{4 \text{ bits}} = 1.25 \times 10^6 \text{ symb/sec} = r_s$$

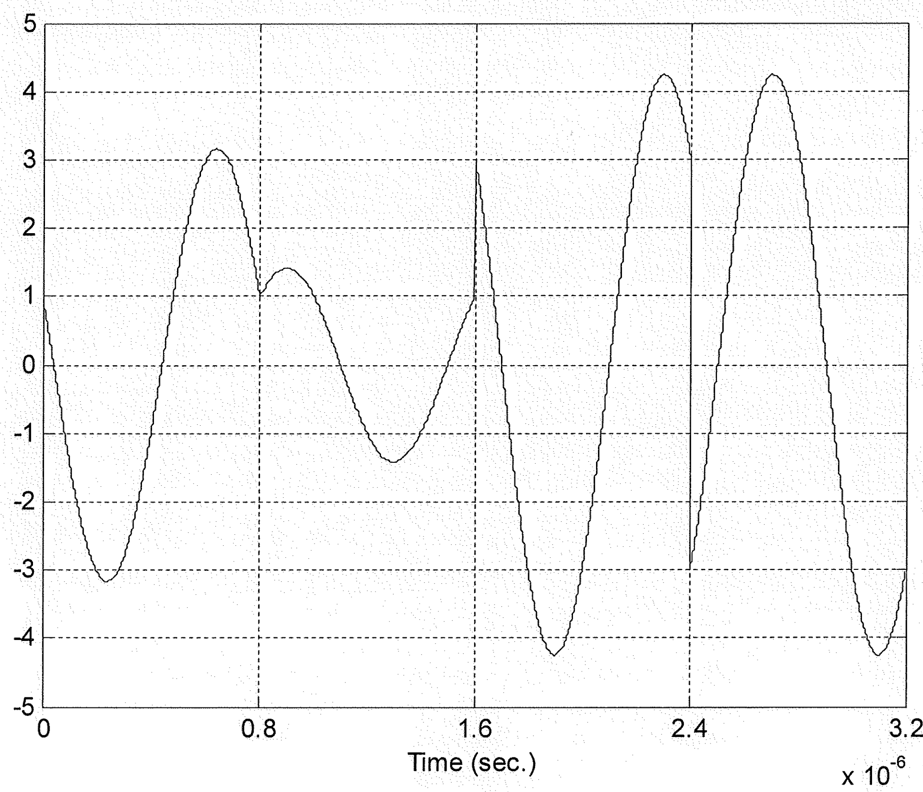
$$\frac{1}{r_s} = T_s = \frac{1}{1.25 \times 10^6} = 0.8 \times 10^{-6} = \boxed{0.8 \frac{\mu\text{sec}}{\text{symb}}}$$

c) (3 points) The signal goes through how many cycles per each symbol?

$$\left(1.25 \times 10^6 \frac{\text{cycles}}{\cancel{\text{sec}}}\right) \times \left(0.8 \times 10^{-6} \frac{\cancel{\text{sec}}}{\text{symb}}\right) = \boxed{1 \frac{\text{cycle}}{\text{symb}}}$$

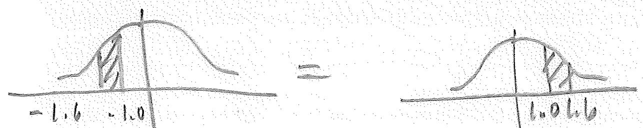
d) (12 points) Sketch the transmitted signal. Use the **cosine** function. Be **sure** to label both axes as completely as possible.





2. (15 points) Probabilities

(a) (5 points) What is the probability that a zero mean Gaussian random variable X with variance 4.0 would be between -1.6 and -1.0?



$$Pr\{-1.6 < X < 1.0\}$$

$$= Q\left(\frac{1.0}{\sqrt{4}}\right) - Q\left(\frac{1.6}{\sqrt{4}}\right)$$

$$= Q(0.5) - Q(0.8) = 0.3085 - 0.2119 = \boxed{0.0966}$$

(b) (5 points) The bit error performance of a modulation scheme is described by the following equation, where N = 2.0. What is the value of the signal energy per bit (regular number, not in dB) that is required to produce a bit error probability of 0.5009 x 10^{-3}?

$$P_b = Q\left(\sqrt{\frac{2E_b}{N}}\right) = 0.5009 \times 10^{-3}$$

$$\sqrt{\frac{2E_b}{N}} = 3.29$$

$$E_b = \frac{(3.29)^2}{2} N = \frac{3.29^2 \cancel{2}}{\cancel{2}} = 10.824$$

(c) (5 points) The probability of correctly receiving an 4-bit packet is found to be 0.959936377. And the bit error process is caused by noise that is known to be described by a zero-mean Gaussian random variable. An error is caused when the noise is greater than a value of 3e-6. What is the variance of this distribution? Make sure to use all of the significant digits in the values given above to get the correct answer.

$$0.959936377 = (1 - P_b)^4$$

$$0.98983 = 1 - P_b$$

$$P_b = 0.01017 = Q\left(\frac{3 \times 10^{-6}}{\sigma_n}\right)$$

$$2.32 = \frac{3 \times 10^{-6}}{\sigma_n}$$

$$\sigma_n = 1.2931 \times 10^{-6}$$

$$\boxed{\sigma_n^2 = 1.672 \times 10^{-12}}$$

3. (25 points) The Optimal Receiver

Consider the derivation of the optimal receiver for binary polar signaling

(a) (5 points) Which of the following is the BEST answer for why the result of this derivation commonly called the "matched filter"?

(A) The noise energy matches the energy in the pulse.
(B) The input signal is multiplied by a pulse shape that matches the pulse shape of the transmitted signal.
(C) The input signal is multiplied by a signal that matches the noise in the received signal.
(D) The goal is to make the bit error probability match the error probability if there were no noise in the system.

Answer B

(b) (20 points) Show the derivation of the optimal receiver for binary polar signaling that has the following equation for probability of bit error.

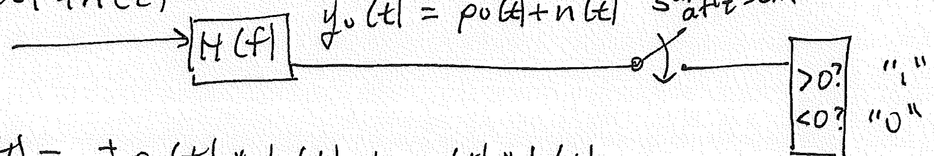
$$P_b = Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

(more space on the next page)

Proof of the Matched Filter Optimum Receiver

"1" $\rightarrow +p(t)$
"0" $\rightarrow -p(t)$

$y(t) = \pm p(t) + n(t)$



$$y_o(t) = \pm p(t) * h(t) + n(t) * h(t)$$
$$= \pm p_o(t) + n_o(t)$$

$$r(t_m) = \pm p_o(t_m) + n_o(t_m)$$

$$A_p = p_o(t_m) = \int_{-\infty}^{\infty} P(f) H(f) e^{+j2\pi f t_m} df$$

$$\sigma_n^2 = \overline{n_o^2(t_m)} = \int_{-\infty}^{\infty} S_n(f) |H(f)|^2 df$$

$$P_e = Q\left(\frac{A_p}{\sigma_n}\right)$$

Maximize $\rho^2 = \left(\frac{A_p}{\sigma_n}\right)^2 = \dfrac{\left|\int_{-\infty}^{\infty} H(f) P(f) e^{+j2\pi f t_m} df\right|^2}{\int_{-\infty}^{\infty} S_n(f) |H(f)|^2 df}$

$$X(f) = H(f)\sqrt{S_n(f)}$$
$$Y(f) = \frac{P(f) e^{+j2\pi f t_m}}{\sqrt{S_n(f)}}$$

$$\rho^2 = \frac{\left|\int_{-\infty}^{\infty} X(f) Y(f) df\right|^2}{\int_{-\infty}^{\infty} |X(f)|^2 df} \underset{\substack{\uparrow \\ \text{Schwartz} \\ \text{Inequality}}}{\leq} \frac{\int_{-\infty}^{\infty} |X(f)|^2 df \int_{-\infty}^{\infty} |Y(f)|^2 df}{\int_{-\infty}^{\infty} |X(f)|^2 df}$$

$$\leq \int_{-\infty}^{\infty} |Y(f)|^2 df$$

To get maximum, set $X(f) = k\,[Y(f)]^*$

$$X(f) = H(f)\sqrt{S_n(f)} = \frac{k P(-f) e^{-j2\pi f t_m}}{\sqrt{S_n(f)}}$$

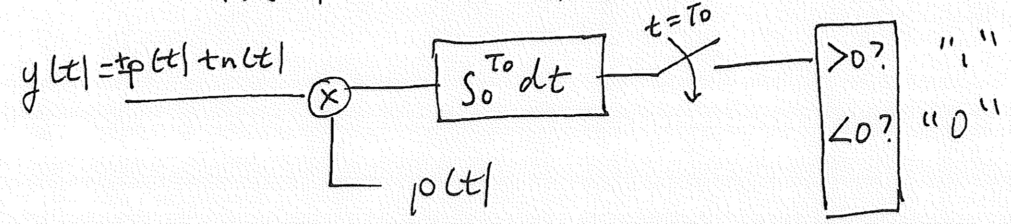
$$\boxed{H(f) = \frac{k P(-f) e^{-j2\pi f t_m}}{S_n(f)}}$$

If white noise, $S_n(f) = \frac{\eta}{2}$

Matched Filter

$$h(t) = p(T_0 - t)$$

$$H(f) = P(-f)\, e^{-j2\pi f T_0}$$

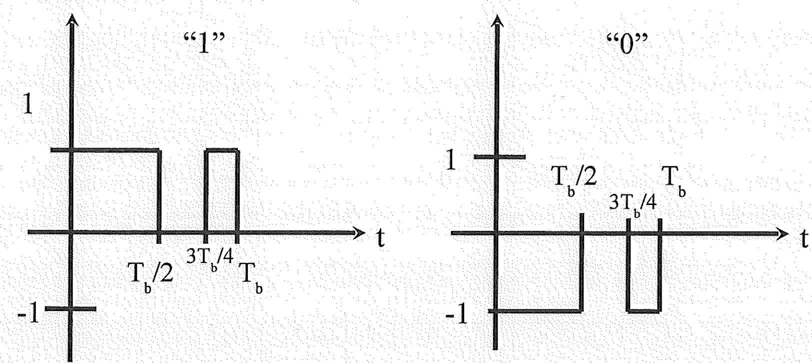


$$\text{Max } \rho^2 = \int_{-\infty}^{\infty} |Y(f)|^2 df = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{S_n(f)} df$$

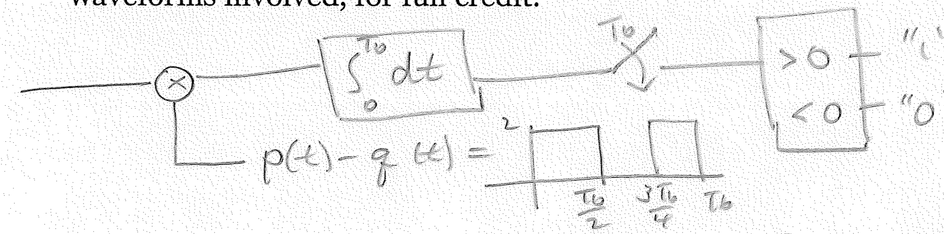
$$= \frac{2}{\eta} \int_{-\infty}^{\infty} |P(f)|^2 df = \frac{2E_p}{\eta}$$

$$P_e = Q\left(\frac{A_p}{\sigma_n}\right) = Q\left(\sqrt{\rho^2}\right) = \boxed{Q\left(\sqrt{\frac{2E_p}{\eta}}\right)}$$

4. (20 points) Given are the following two signals that are transmitted for a binary "1" and a binary "0".



a) (7 points) Show the parts of the optimal receiver for these pulses with all of the details. All details must be clearly shown, including shapes of any waveforms involved, for full credit.



$$a_0 = \frac{1}{2}\left[E_p - E_q\right] = \frac{1}{2}\left[\frac{3T_b}{4} - \frac{3T_b}{4}\right] = 0$$

$$E_p = 1^2\frac{T_b}{2} + 1^2\left(T_b - \frac{3T_b}{4}\right) = \frac{3T_b}{4}$$

$$E_q = (-1)^2\frac{T_b}{2} + (-1)^2\left(\frac{T_b}{4}\right) = \frac{3T_b}{4}$$

b) (6 points) What is the formula for the probability of bit error, with respect to T_b, when these pulses are used?

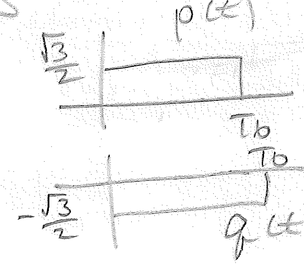
$$E_{pq} = \int_0^{T_b} p(t)q(t)\,dt = (+1)(-1)\frac{T_b}{2} + (+1)(-1)\frac{T_b}{4} = -\frac{3T_b}{4}$$

$$P_b = Q\left(\sqrt{\frac{E_p + E_q - 2E_{pq}}{2\eta}}\right) = Q\left(\sqrt{\frac{\frac{3T_b}{4} + \frac{3T_b}{4} - 2\left(-\frac{3T_b}{4}\right)}{2\eta}}\right) = Q\left(\sqrt{\frac{\frac{12T_b}{4}}{2\eta}}\right) = Q\left(\sqrt{\frac{3}{2}\frac{T_b}{\eta}}\right)$$

c) (7 points) Give an example of another set of pulse shapes for "1" and "0" that would produce the same probability of bit error.

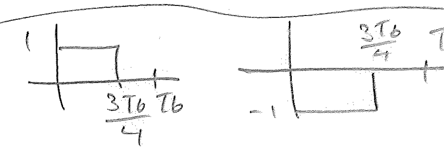
Need: Polar signals

$$E_b = \frac{3T_b}{4}$$



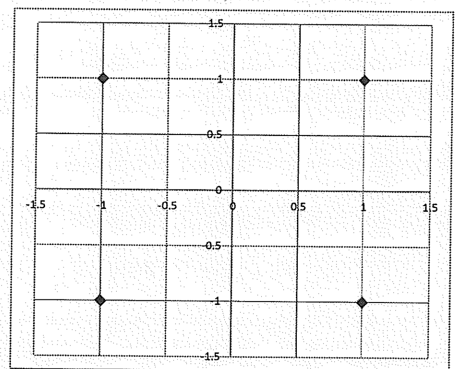
$$E_b = \left(\frac{\sqrt{3}}{2}\right)^2 T_b = \frac{3}{4}T_b$$

7 or



5. (20 points) Signal Constellations

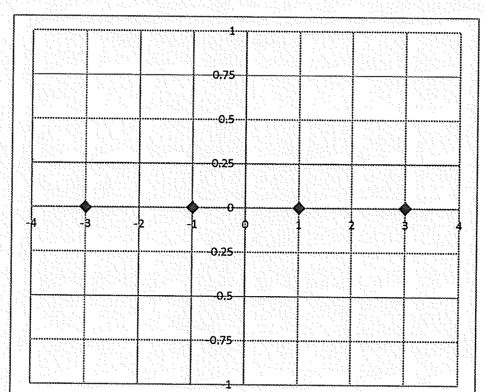
(a) (4 points) Data is originally being transmitted using QPSK with the following signal constellation. What is the signal energy per bit if rectangular pulses are used and the symbol time is 1 millisecond?



$$E_p = \frac{1}{4}\left[(\sqrt{2})^2 T_b + (\sqrt{2})^2 T_b + (\sqrt{2})^2 T_b + (\sqrt{2})^2 T_b\right] = 2T_b = 2\times10^{-3}$$

$$E_b = \frac{1}{2}E_p = \boxed{1\times10^{-3}}$$

(b) (4 points) What is the signal energy per bit for the following M-PAM constellation? Assume the same symbol time and rectangular pulse shapes.



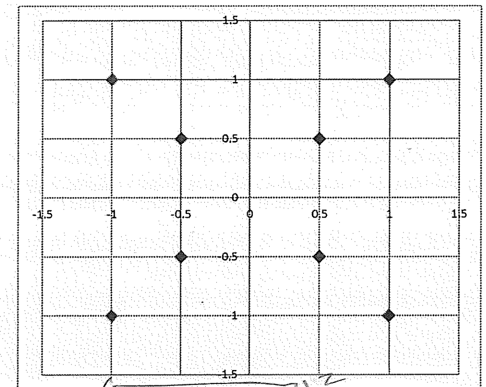
$$E_p = \frac{1}{4}\left[(1)^2 T_b + (-1)^2 T_b + 3^2 T_b + (-3)^2 T_b\right]$$

$$= \frac{1}{4}T_b[20] = 5T_b = 5\times10^{-3}$$

$$E_b = \frac{1}{2}(5\times10^{-3}) = \boxed{2.5\times10^{-3}}$$

(more on the next page)

(c) (4 points) What is the signal energy per bit for the following constellation? Assume the same symbol time and rectangular pulse shapes.



$$E_{pm} = \frac{1}{8}\left[4\times\left(\sqrt{0.5^2+0.5^2}\right)^2 + 4\left(\sqrt{1+1}\right)^2\right] T_b$$

$$= \frac{1}{8}\left[4(0.5) + 4(2)\right]T = 1.25\,T_b = 1.25 \times 10^{-3}$$

$$E_{bm} = \frac{1}{3} E_{pm} = \boxed{0.417 \times 10^{-3}}$$

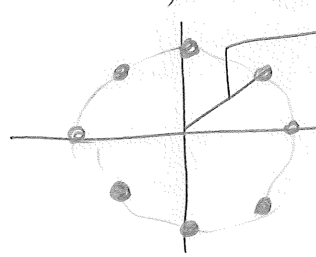
(d) (4 points) Comparing the signal constellations in (a) and (c), what is the benefit of using the constellation in (c) and the drawback to using that constellation?

Benefit: More bits per symbol

Drawback: Worse bit error probability since points are closer together

(e) (4 points) Sketch the signal constellation for 8-PSK to have the same signal energy per bit as in (c).

8 symbols, each with $E_{pm} = 1.25\,T_b$



Radius $= \sqrt{1.25} = 1.118$

TABLE 8.2[3]

$Q(x)$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0000	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
.1000	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
.2000	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
.3000	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
.4000	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
.5000	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
.6000	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
.7000	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
.8000	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
.9000	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.000	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.100	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.200	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.9853E-01
1.300	.9680E-01	.9510E-01	.9342E-01	.9176E-01	.9012E-01	.8851E-01	.8691E-01	.8534E-01	.8379E-01	.8226E-01
1.400	.8076E-01	.7927E-01	.7780E-01	.7636E-01	.7493E-01	.7353E-01	.7215E-01	.7078E-01	.6944E-01	.6811E-01
1.500	.6681E-01	.6552E-01	.6426E-01	.6301E-01	.6178E-01	.6057E-01	.5938E-01	.5821E-01	.5705E-01	.5592E-01
1.600	.5480E-01	.5370E-01	.5262E-01	.5155E-01	.5050E-01	.4947E-01	.4846E-01	.4746E-01	.4648E-01	.4551E-01
1.700	.4457E-01	.4363E-01	.4272E-01	.4182E-01	.4093E-01	.4006E-01	.3920E-01	.3836E-01	.3754E-01	.3673E-01
1.800	.3593E-01	.3515E-01	.3438E-01	.3362E-01	.3288E-01	.3216E-01	.3144E-01	.3074E-01	.3005E-01	.2938E-01
1.900	.2872E-01	.2807E-01	.2743E-01	.2680E-01	.2619E-01	.2559E-01	.2500E-01	.2442E-01	.2385E-01	.2330E-01
2.000	.2275E-01	.2222E-01	.2169E-01	.2118E-01	.2068E-01	.2018E-01	.1970E-01	.1923E-01	.1876E-01	.1831E-01
2.100	.1786E-01	.1743E-01	.1700E-01	.1659E-01	.1618E-01	.1578E-01	.1539E-01	.1500E-01	.1463E-01	.1426E-01
2.200	.1390E-01	.1355E-01	.1321E-01	.1287E-01	.1255E-01	.1222E-01	.1191E-01	.1160E-01	.1130E-01	.1101E-01
2.300	.1072E-01	.1044E-01	.1017E-01	.9903E-02	.9642E-02	.9387E-02	.9137E-02	.8894E-02	.8656E-02	.8424E-02
2.400	.8198E-02	.7976E-02	.7760E-02	.7549E-02	.7344E-02	.7143E-02	.6947E-02	.6756E-02	.6569E-02	.6387E-02
2.500	.6210E-02	.6037E-02	.5868E-02	.5703E-02	.5543E-02	.5386E-02	.5234E-02	.5085E-02	.4940E-02	.4799E-02
2.600	.4661E-02	.4527E-02	.4396E-02	.4269E-02	.4145E-02	.4025E-02	.3907E-02	.3793E-02	.3681E-02	.3573E-02
2.700	.3467E-02	.3364E-02	.3264E-02	.3167E-02	.3072E-02	.2980E-02	.2890E-02	.2803E-02	.2718E-02	.2635E-02
2.800	.2555E-02	.2477E-02	.2401E-02	.2327E-02	.2256E-02	.2186E-02	.2118E-02	.2052E-02	.1988E-02	.1926E-02
2.900	.1866E-02	.1807E-02	.1750E-02	.1695E-02	.1641E-02	.1589E-02	.1538E-02	.1489E-02	.1441E-02	.1395E-02
3.000	.1350E-02	.1306E-02	.1264E-02	.1223E-02	.1183E-02	.1144E-02	.1107E-02	.1070E-02	.1035E-02	.1001E-02
3.100	.9676E-03	.9354E-03	.9043E-03	.8740E-03	.8447E-03	.8164E-03	.7888E-03	.7622E-03	.7364E-03	.7114E-03
3.200	.6871E-03	.6637E-03	.6410E-03	.6190E-03	.5976E-03	.5770E-03	.5571E-03	.5377E-03	.5190E-03	.5009E-03
3.300	.4834E-03	.4665E-03	.4501E-03	.4342E-03	.4189E-03	.4041E-03	.3897E-03	.3758E-03	.3624E-03	.3495E-03
3.400	.3369E-03	.3248E-03	.3131E-03	.3018E-03	.2909E-03	.2802E-03	.2701E-03	.2602E-03	.2507E-03	.2415E-03
3.500	.2326E-03	.2241E-03	.2158E-03	.2078E-03	.2001E-03	.1926E-03	.1854E-03	.1785E-03	.1718E-03	.1653E-03
3.600	.1591E-03	.1531E-03	.1473E-03	.1417E-03	.1363E-03	.1311E-03	.1261E-03	.1213E-03	.1166E-03	.1121E-03
3.700	.1078E-03	.1036E-03	.9961E-04	.9574E-04	.9201E-04	.8842E-04	.8496E-04	.8162E-04	.7841E-04	.7532E-04
3.800	.7235E-04	.6948E-04	.6673E-04	.6407E-04	.6152E-04	.5906E-04	.5669E-04	.5442E-04	.5223E-04	.5012E-04
3.900	.4810E-04	.4615E-04	.4427E-04	.4247E-04	.4074E-04	.3908E-04	.3747E-04	.3594E-04	.3446E-04	.3304E-04
4.000	.3167E-04	.3036E-04	.2910E-04	.2789E-04	.2673E-04	.2561E-04	.2454E-04	.2351E-04	.2252E-04	.2157E-04
4.100	.2066E-04	.1978E-04	.1894E-04	.1814E-04	.1737E-04	.1662E-04	.1591E-04	.1523E-04	.1458E-04	.1395E-04
4.200	.1335E-04	.1277E-04	.1222E-04	.1168E-04	.1118E-04	.1069E-04	.1022E-04	.9774E-05	.9345E-05	.8934E-05
4.300	.8540E-05	.8163E-05	.7801E-05	.7455E-05	.7124E-05	.8807E-05	.6503E-05	.6212E-05	.5934E-05	.5668E-05
4.400	.5413E-05	.5169E-05	.4935E-05	.4712E-05	.4498E-05	.4294E-05	.4098E-05	.3911E-05	.3732E-05	.3561E-05
4.500	.3398E-05	.3241E-05	.3092E-05	.2949E-05	.2813E-05	.2682E-05	.2558E-05	.2439E-05	.2325E-05	.2216E-05
4.600	.2112E-05	.2013E-05	.1919E-05	.1828E-05	.1742E-05	.1660E-05	.1581E-05	.1506E-05	.1434E-05	.1366E-05
4.700	.1301E-05	.1239E-05	.1179E-05	.1123E-05	.1069E-05	.1017E-05	.9680E-06	.9211E-06	.8765E-06	.8339E-06
4.800	.7933E-06	.7547E-06	.7178E-06	.6827E-06	.6492E-06	.6173E-06	.5869E-06	.5580E-06	.5304E-06	.5042E-06
4.900	.4792E-06	.4554E-06	.4327E-06	.4111E-06	.3906E-06	.3711E-06	.3525E-06	.3448E-06	.3179E-06	.3019E-06
5.000	.2867E-06	.2722E-06	.2584E-06	.2452E-06	.2328E-06	.2209E-06	.2096E-06	.1989E-06	.1887E-06	.1790E-06
5.100	.1698E-06	.1611E-06	.1528E-06	.1449E-06	.1374E-06	.1302E-06	.1235E-06	.1170E-06	.1109E-06	.1051E-06

(continued)

TABLE 8.2
Continued

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.200	.9964E-07	.9442E-07	.8946E-07	.8476E-07	.8029E-07	.7605E-07	.7203E-07	.6821E-07	.6459E-07	.6116E-07
5.300	.5790E-07	.5481E-07	.5188E-07	.4911E-07	.4647E-07	.4398E-07	.4161E-07	.3937E-07	.3724E-07	.3523E-07
5.400	.3332E-07	.3151E-07	.2980E-07	.2818E-07	.2664E-07	.2518E-07	.2381E-07	.2250E-07	.2127E-07	.2010E-07
5.500	.1899E-07	.1794E-07	.1695E-07	.1601E-07	.1512E-07	.1428E-07	.1349E-07	.1274E-07	.1203E-07	.1135E-07
5.600	.1072E-07	.1012E-07	.9548E-08	.9010E-08	.8503E-08	.8022E-08	.7569E-08	.7140E-08	.6735E-08	.6352E-08
5.700	.5990E-08	.5649E-08	.5326E-08	.5022E-08	.4734E-08	.4462E-08	.4206E-08	.3964E-08	.3735E-08	.3519E-08
5.800	.3316E-08	.3124E-08	.2942E-08	.2771E-08	.2610E-08	.2458E-08	.2314E-08	.2179E-08	.2051E-08	.1931E-08
5.900	.1818E-08	.1711E-08	.1610E-08	.1515E-08	.1425E-08	.1341E-08	.1261E-08	.1186E-08	.1116E-08	.1049E-08
6.000	.9866E-09	.9276E-09	.8721E-09	.8198E-09	.7706E-09	.7242E-09	.6806E-09	.6396E-09	.6009E-09	.5646E-09
6.100	.5303E-09	.4982E-09	.4679E-09	.4394E-09	.4126E-09	.3874E-09	.3637E-09	.3414E-09	.3205E-09	.3008E-09
6.200	.2823E-09	.2649E-09	.2486E-09	.2332E-09	.2188E-09	.2052E-09	.1925E-09	.1805E-09	.1692E-09	.1587E-09
6.300	.1488E-09	.1395E-09	.1308E-09	.1226E-09	.1149E-09	.1077E-09	.1009E-09	.9451E-10	.8854E-10	.8294E-10
6.400	.7769E-10	.7276E-10	.6814E-10	.6380E-10	.5974E-10	.5593E-10	.5235E-10	.4900E-10	.4586E-10	.4292E-10
6.500	.4016E-10	.3758E-10	.3515E-10	.3288E-10	.3077E-10	.2877E-10	.2690E-10	.2516E-10	.2352E-10	.2199E-10
6.600	.2056E-10	.1922E-10	.1796E-10	.1678E-10	.1568E-10	.1465E-10	.1369E-10	.1279E-10	.1195E-10	.1116E-10
6.700	.1042E-10	.9731E-11	.9086E-11	.8483E-11	.7919E-11	.7392E-11	.6900E-11	.6439E-11	.6009E-11	.5607E-11
6.800	.5231E-11	.4880E-11	.4552E-11	.4246E-11	.3960E-11	.3692E-11	.3443E-11	.3210E-11	.2993E-11	.2790E-11
6.900	.2600E-11	.2423E-11	.2258E-11	.2104E-11	.1960E-11	.1826E-11	.1701E-11	.1585E-11	.1476E-11	.1374E-11
7.000	.1280E-11	.1192E-11	.1109E-11	.1033E-11	.9612E-12	.8946E-12	.8325E-12	.7747E-12	.7208E-12	.6706E-12
7.100	.6238E-12	.5802E-12	.5396E-12	.5018E-12	.4667E-12	.4339E-12	.4034E-12	.3750E-12	.3486E-12	.3240E-12
7.200	.3011E-12	.2798E-12	.2599E-12	.2415E-12	.2243E-12	.2084E-12	.1935E-12	.1797E-12	.1669E-12	.1550E-12
7.300	.1439E-12	.1336E-12	.1240E-12	.1151E-12	.1068E-12	.9910E-13	.9196E-13	.8531E-13	.7914E-13	.7341E-13
7.400	.6809E-13	.6315E-13	.5856E-13	.5430E-13	.5034E-13	.4667E-13	.4326E-13	.4010E-13	.3716E-13	.3444E-13
7.500	.3191E-13	.2956E-13	.2739E-13	.2537E-13	.2350E-13	.2176E-13	.2015E-13	.1866E-13	.1728E-13	.1600E-13
7.600	.1481E-13	.1370E-13	.1268E-13	.1174E-13	.1086E-13	.1005E-13	.9297E-14	.8600E-14	.7954E-14	.7357E-14
7.700	.6803E-14	.6291E-14	.5816E-14	.5377E-14	.4971E-14	.4595E-14	.4246E-14	.3924E-14	.3626E-14	.3350E-14
7.800	.3095E-14	.2859E-14	.2641E-14	.2439E-14	.2253E-14	.2080E-14	.1921E-14	.1773E-14	.1637E-14	.1511E-14
7.900	.1395E-14	.1287E-14	.1188E-14	.1096E-14	.1011E-14	.9326E-15	.8602E-15	.7934E-15	.7317E-15	.6747E-15
8.000	.6221E-15	.5735E-15	.5287E-15	.4874E-15	.4492E-15	.4140E-15	.3815E-15	.3515E-15	.3238E-15	.2983E-15
8.100	.2748E-15	.2531E-15	.2331E-15	.2146E-15	.1976E-15	.1820E-15	.1675E-15	.1542E-15	.1419E-15	.1306E-15
8.200	.1202E-15	.1106E-15	.1018E-15	.9361E-16	.8611E-16	.7920E-16	.7284E-16	.6698E-16	.6159E-16	.5662E-16
8.300	.5206E-16	.4785E-16	.4398E-16	.4042E-16	.3715E-16	.3413E-16	.3136E-16	.2881E-16	.2646E-16	.2431E-16
8.400	.2232E-16	.2050E-16	.1882E-16	.1728E-16	.1587E-16	.1457E-16	.1337E-16	.1227E-16	.1126E-16	.1033E-16
8.500	.9480E-17	.8697E-17	.7978E-17	.7317E-17	.6711E-17	.6154E-17	.5643E-17	.5174E-17	.4744E-17	.4348E-17
8.600	.3986E-17	.3653E-17	.3348E-17	.3068E-17	.2811E-17	.2575E-17	.2359E-17	.2161E-17	.1979E-17	.1812E-17
8.700	.1659E-17	.1519E-17	.1391E-17	.1273E-17	.1166E-17	.1067E-17	.9763E-18	.8933E-18	.8174E-18	.7478E-18
8.800	.6841E-18	.6257E-18	.5723E-18	.5234E-18	.4786E-18	.4376E-18	.4001E-18	.3657E-18	.3343E-18	.3055E-18
8.900	.2792E-18	.2552E-18	.2331E-18	.2130E-18	.1946E-18	.1777E-18	.1623E-18	.1483E-18	.1354E-18	.1236E-18
9.000	.1129E-18	.1030E-18	.9404E-19	.8584E-19	.7834E-19	.7148E-19	.6523E-19	.5951E-19	.5429E-19	.4952E-19
9.100	.4517E-19	.4119E-19	.3756E-19	.3425E-19	.3123E-19	.2847E-19	.2595E-19	.2365E-19	.2155E-19	.1964E-19
9.200	.1790E-19	.1631E-19	.1486E-19	.1353E-19	.1232E-19	.1122E-19	.1022E-19	.9307E-20	.8474E-20	.7714E-20
9.300	.7022E-20	.6392E-20	.5817E-20	.5294E-20	.4817E-20	.4382E-20	.3987E-20	.3627E-20	.3299E-20	.3000E-20
9.400	.2728E-20	.2481E-20	.2255E-20	.2050E-20	.1864E-20	.1694E-20	.1540E-20	.1399E-20	.1271E-20	.1155E-20
9.500	.1049E-20	.9533E-21	.8659E-21	.7864E-21	.7142E-21	.6485E-21	.5888E-21	.5345E-21	.4852E-21	.4404E-21
9.600	.3997E-21	.3627E-21	.3292E-21	.2986E-21	.2709E-21	.2458E-21	.2229E-21	.2022E-21	.1834E-21	.1663E-21
9.700	.1507E-21	.1367E-21	.1239E-21	.1123E-21	.1018E-21	.9223E-22	.8358E-22	.7573E-22	.6861E-22	.6215E-22
9.800	.5629E-22	.5098E-22	.4617E-22	.4181E-22	.3786E-22	.3427E-22	.3102E-22	.2808E-22	.2542E-22	.2300E-22
9.900	.2081E-22	.1883E-22	.1704E-22	.1541E-22	.1394E-22	.1261E-22	.1140E-22	.1031E-22	.9323E-23	.8429E-23
10.00	.7620E-23	.6888E-23	.6225E-23	.5626E-23	.5084E-23	.4593E-23	.4150E-23	.3749E-23	.3386E-23	.3058E-23

Notes: (1) E-01 should be read as $\times 10^{-1}$; E-02 should be read as $\times 10^{-2}$, and so on.
(2) This table lists $Q(x)$ for x in the range of 0 to 10 in the increments of 0.01. To find $Q(5.36)$, for example, look up the row starting with $x = 5.3$. The sixth entry in this row (under 0.06) is the desired value 0.4161×10^{-7}.

ECE 5590CT Communication Theory
Exam #1
July 12, 2011

High scores: 97, 97, 96, 96, 95, 94

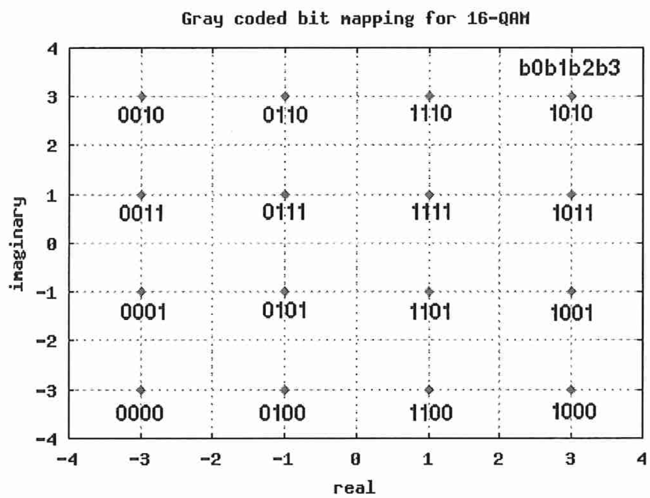
Rough grade ranges

90+ → A

80-89 → A-/B+

Under 80 → B

1. (25 points) A modulation scheme is used with the following signal constellation.



This modulation scheme is used to transmit the following bit stream with the following system parameters.

Bit stream: 0010011110001011

Bit rate = 6.4 Mbps

Carrier frequency = 4.8 MHz

a) (4 points) What would this modulation scheme most likely be called?

16-QAM

b) (3 points) What is the symbol time for this system in microseconds?

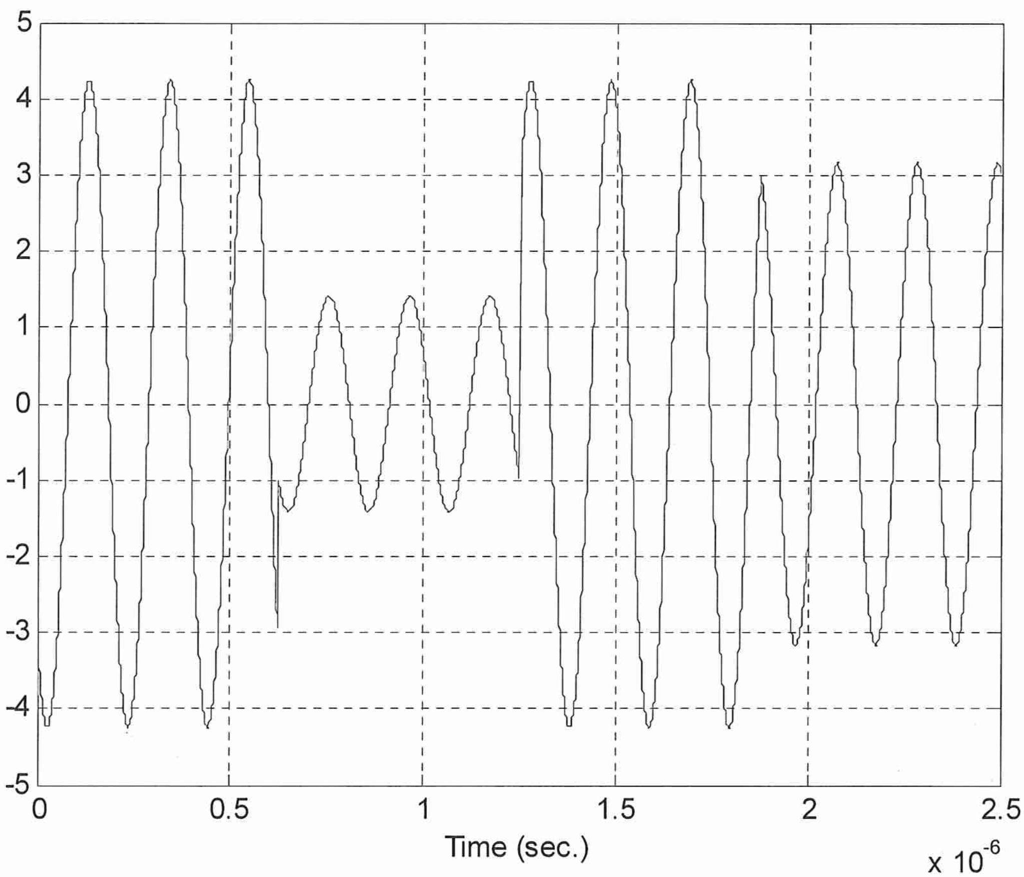
(6.4 Mbps)*(1 symb/4 bits) = 1.6 million symb/sec.
Symbol time = 1/1.6e6 = 0.625 microseconds

c) (4 points) The signal goes through how many cycles per each symbol?

0.625e-6 sec/symb*(4.8e6 cycles/sec) = 3 cycles/symb

d) (14 points) Sketch the transmitted signal. Use the **cosine** function. Be **sure** to label both axes as completely as possible.

0010 0111 1000 1011



2. (30 points) Gaussian Distribution

(a) (5 points) The probability of a bit error is found from a zero mean random variable X when it is greater than 3.0 and distributed according to a Gaussian distribution with a variance of 1.0. What is the probability of bit error?

$$P_b = Q\left(\frac{3-0}{1}\right) = Q(3) = 0.1350\times 10^{-2}$$
$$= 1.350\times 10^{-3}$$

(b) (~~5~~ 7 points) For the answer from part (a), what is the probability that a packet of size 5 bits will be received with one, two, three, four, or five errors?

Easy way:

$$1 - \Pr\{0 \text{ errors}\} = 1 - \Pr\{\text{all correct}\}$$
$$= 1 - (1 - 1.35\times 10^{-3})^5$$
$$= 1 - 0.993268 = 6.732\times 10^{-3}$$

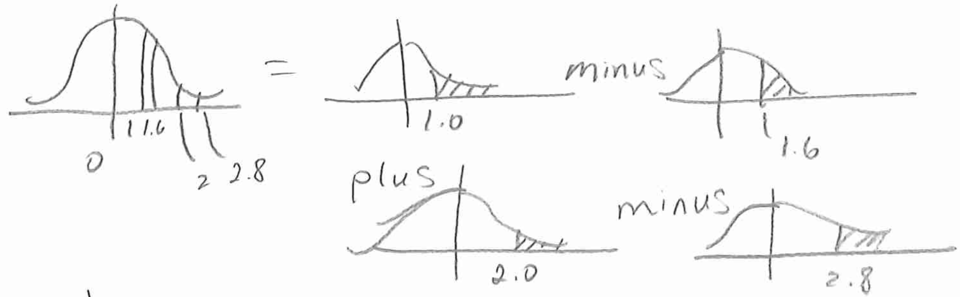
Long way

$$\Pr\{1 \text{ error}\} = \binom{5}{1}(1.35)^1(1-1.35)^4 = 6.67136\times 10^{-3}$$
$$\Pr\{2 \text{ errors}\} = \binom{5}{2}(1.35)^2(1-1.35)^3 = 1.815\times 10^{-5}$$
$$= 0.001815\times 10^{-3}$$

$$\Pr\{3 \text{ errors}\} = 0.00002437\times 10^{-3}$$
$$\Pr\{4 \text{ errors}\} = 0.000000016585\times 10^{-3}$$
$$\Pr\{5 \text{ errors}\} = P_b^5 = 4.84\times 10^{-15}$$

$$\text{Sum} = 6.732\times 10^{-3}$$

(c) (~~5~~ 8 points) What is the probability that a zero mean random variable X with variance 4.0 would be between 1.0 and 1.6 or between 2.0 and 2.8?



$$= Q\left(\frac{1}{\sqrt{4}}\right) - Q\left(\frac{1.6}{2}\right) + Q\left(\frac{2}{2}\right) - Q\left(\frac{2.8}{2}\right) = 0.3085 - 0.2119 + 0.1587 - 0.08076$$

$\boxed{0.1245}$

(d) (5 points) What is the Central Limit Theorem and how is it relevant to communications systems?

Noise processes add together to form approximately Gaussian random processes because the Central Limit Theorem says many processes add to create Gaussian

(e) (5 points) The bit error performance of a modulation scheme is described by the following equation, where $N = 2.0$. What is the value of E_b (regular number, not in dB) that is required to produce a bit error probability of 0.1662×10^{-4}?

$$P_b = Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

$$Q\left(\sqrt{\frac{2(E_b)}{2}}\right) = 0.1662\times10^{-4}$$

$$\sqrt{\frac{2E_b}{2}} = 4.15$$

$$E_b = 4.15^2 = \boxed{17.22}$$

3. (25 points) The Optimal Receiver

Show the derivation for the optimal receiver for binary polar signaling that has the following equation for probability of bit error.

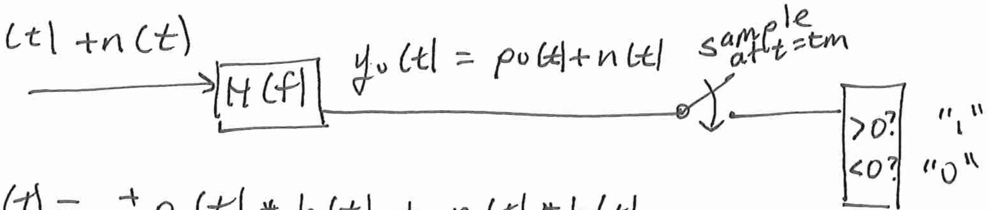
$$P_b = Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

(more space on the next page)

Proof of the Matched Filter Optimum Receiver

"1" $\rightarrow +p(t)$
"0" $\rightarrow -p(t)$

$y(t) = \pm p(t) + n(t)$



$$y_o(t) = \pm p(t) * h(t) + n(t) * h(t)$$
$$= \pm p_o(t) + n_o(t)$$

$$r(t_m) = \pm p_o(t_m) + n_o(t_m)$$

$$A_p = p_o(t_m) = \int_{-\infty}^{\infty} P(f) H(f) e^{+j2\pi f t_m} df$$

$$\sigma_n^2 = \overline{n_o^2(t_m)} = \int_{-\infty}^{\infty} S_n(f) |H(f)|^2 df$$

$$P_e = Q\left(\frac{A_p}{\sigma_n}\right)$$

Maximize $\rho^2 = \left(\frac{A_p}{\sigma_n}\right)^2 = \dfrac{\left|\int_{-\infty}^{\infty} H(f) P(f) e^{+j2\pi f t_m} df\right|^2}{\int_{-\infty}^{\infty} S_n(f) |H(f)|^2 df}$

$$X(f) = H(f)\sqrt{S_n(f)}$$
$$Y(f) = \frac{P(f) e^{+j2\pi f t_m}}{\sqrt{S_n(f)}}$$

$$\rho^2 = \frac{\left|\int_{-\infty}^{\infty} X(f) Y(f) df\right|^2}{\int_{-\infty}^{\infty} |X(f)|^2 df} \le \frac{\int_{-\infty}^{\infty} |X(f)|^2 df \int_{-\infty}^{\infty} |Y(f)|^2 df}{\int_{-\infty}^{\infty} |X(f)|^2 df}$$

Schwartz Inequality

$$\le \int_{-\infty}^{\infty} |Y(f)|^2 df$$

To get maximum, set $X(f) = k[Y(f)]^*$

$$X(f) = H(f)\sqrt{S_n(f)} = \frac{k P(-f) e^{-j2\pi f t_m}}{\sqrt{S_n(f)}}$$

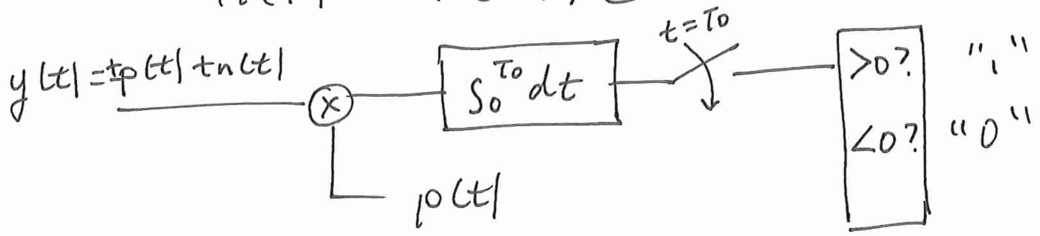
$$\boxed{H(f) = \frac{k P(-f) e^{-j2\pi f t_m}}{S_n(f)}}$$

If white noise, $S_n(f) = \frac{\eta}{2}$

Matched Filter

$$h(t) = p(T_0 - t)$$

$$H(f) = P(-f)\, e^{-j2\pi f T_0}$$



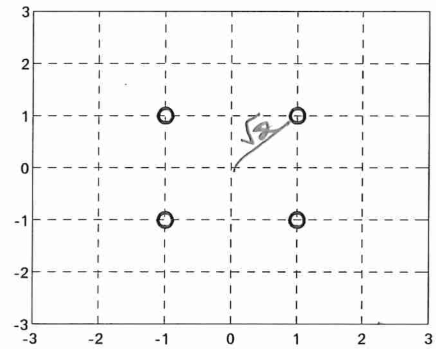
$$\text{Max } \rho^2 = \int_{-\infty}^{\infty} |Y(f)|^2 df = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{S_n(f)} df$$

$$= \frac{2}{\eta} \int_{-\infty}^{\infty} |P(f)|^2 df = \frac{2E_p}{\eta}$$

$$P_e = Q\left(\frac{A_p}{\sigma_n}\right) = Q\left(\sqrt{\rho^2}\right) = \boxed{Q\left(\sqrt{\frac{2E_p}{\eta}}\right)}$$

4. (20 points) Signal Constellations

Data is originally being transmitted using QPSK with the following signal constellation.

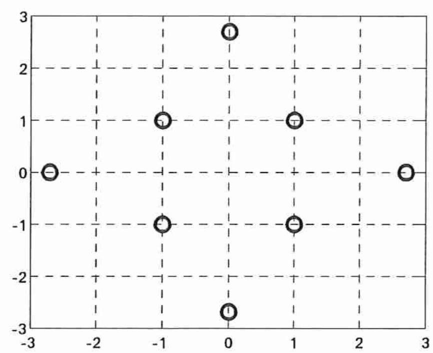


$$Avg = \frac{1}{4}\left(4(\sqrt{2})^2\right) = 2$$

$$PSD = \frac{2}{T_s} = \frac{1}{T_b}$$

$$T_s = 2T_b$$

a. (5 points) By what factor has the power per bit changed compared to QPSK for the following constellation? The point on the far right is at (2.7,0) and other similar points are the same distance from the origin.



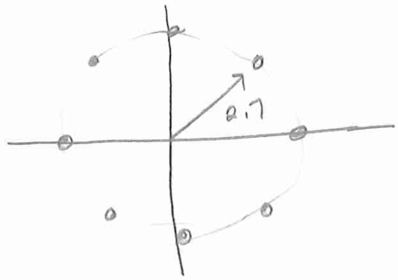
$$Avg = \frac{1}{8}\left[4(2.7)^2 + 4(\sqrt{2})^2\right] = 4.645$$

$$PSD = \frac{4.645}{T_s} = \frac{4.645}{3T_b} = \frac{1.543}{T_b}$$

$$\boxed{1.543 \text{ factor increase}}$$

(more on the next page)

b. (5 points) Sketch the signal constellation for 8-PSK if all of the constellation points were to be 2.7 from the origin as were some of the points in part (a).



c. (5 points) By what factor has the power per bit changed for the 8-PSK constellation compared to the QPSK constellation?

$$\text{Avg} = \frac{1}{8}\left(8(2.7)^2\right) = 7.29$$

$$\text{PSD} = \frac{7.29}{T_S} = \frac{7.29}{3T_b} = \frac{2.43}{T_b}$$

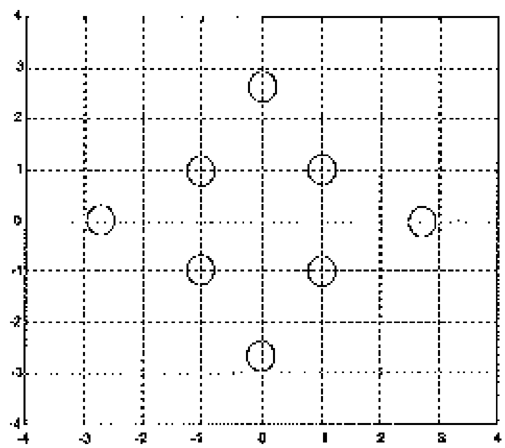
$$\boxed{\text{Factor of } 2.43}$$

d. (5 points) Which constellation had the greater power increase compared to QPSK? For full credit also explain why.

8-PSK needed more power.
The 4 constellation points at $\pm\frac{\pi}{4}$ and $\pm\frac{3\pi}{4}$ are farther from the origin and need more power.

Comm. Theory Summer 2010
ECE 5590CT Exam #1

1. (25 points) A modulation scheme is used with the following signal constellation. Given in the table are the coordinates of each point and the corresponding symbols for each point.



Symbol	In-phase value (x-axis)	Quadrature value (y-axis)
000	+1	+1
001	-1	+1
010	+1	-1
011	-1	-1
100	$\sqrt{3}+1=+2.7$	0
101	0	+2.7
110	0	-2.7
111	-2.7	0

This modulation scheme is used to transmit the following bit stream with the following system parameters.

Bit stream: 100010110101

Bit rate = 2.4 Mbps
Carrier frequency = 1.6 MHz

a) (4 points) What would this modulation scheme most likely be called?

QAM

b) (3 points) What is the symbol time for this system?

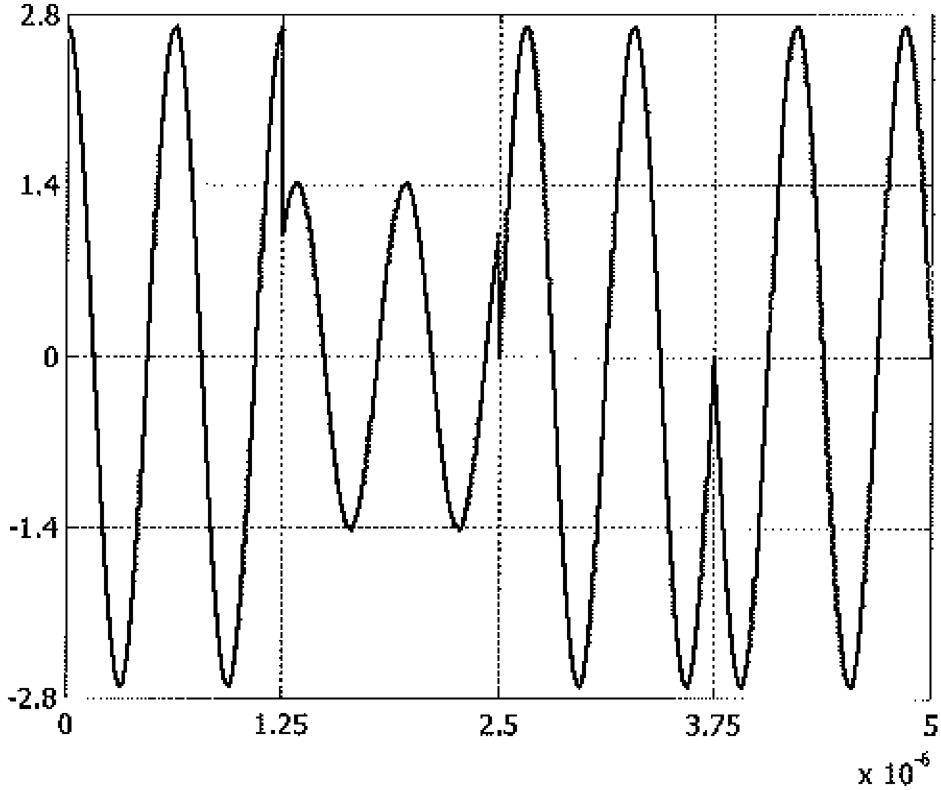
(2.4 Mbps)*(1 symb/3 bits) = 0.8 million symb/sec.
Symbol time = 1/0.8e6 = 1.25 microseconds

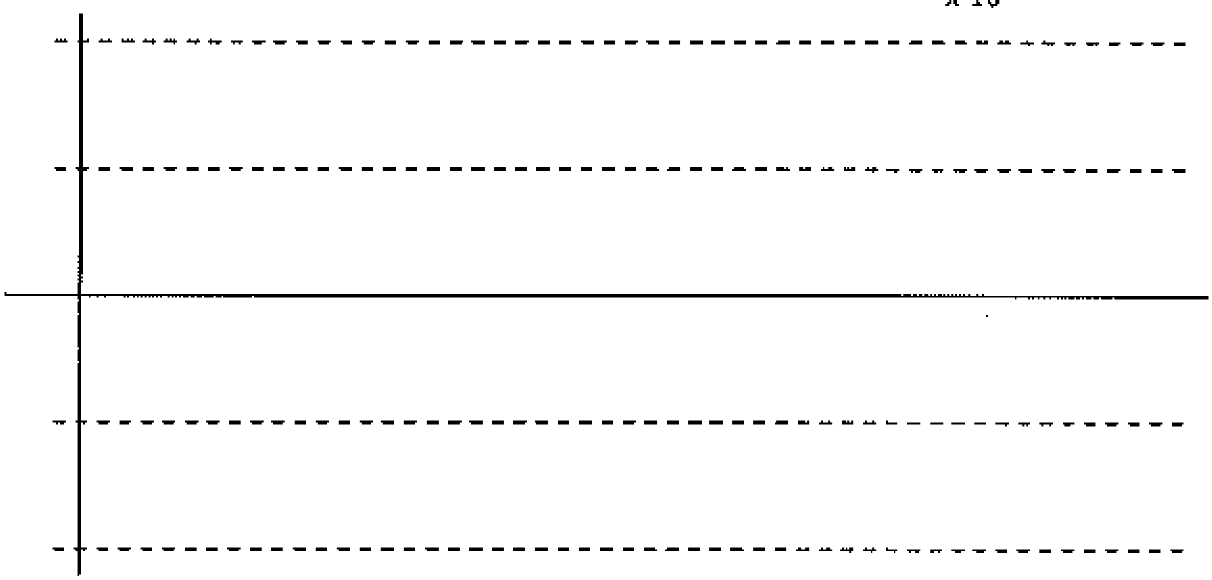
c) (4 points) The signal goes through how many cycles per each symbol?

1.25e-6 sec/symb*(1.6e6 cycles/sec) = 2 cycles/symb

High scores: 99, 92, 89, 87
Avg: 74.2
Grade ranges A/A- : 80+
B/B+ : 50-79

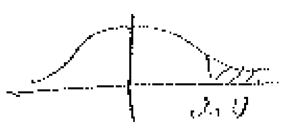
d) (14 points) Sketch the transmitted signal. Use the **cosine** function. Be **sure** to label both axes.





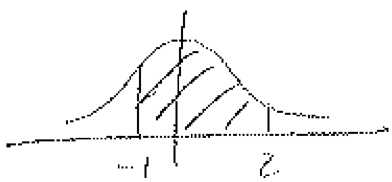
2. (25 points) Gaussian Distribution

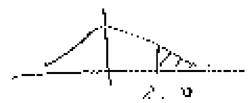
(a) (5 points) What is the probability that a random variable X is greater than 2.0 if it is distributed according to a Gaussian distribution with a variance of 1.0? zero mean

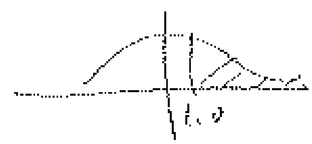


$$Q\left(\frac{2.0}{\sigma}\right) = Q(2.0) = 0.2275\times10^{-1} = \boxed{0.02275}$$

(b) (5 points) What is the probability that a random variable X is greater than -1.0 and less than 2.0 if it is distributed according to a Gaussian distribution with a standard deviation of 2.0? zero mean



$= 1 -$ 

$-$ 

$$= 1 - Q\left(\frac{2}{2}\right) - Q\left(\frac{1}{2}\right)$$

$$= 1 - 0.1587 - 0.3085 = \boxed{0.5328}$$

(c) (5 points) What property of random variables allows noise processes to commonly be assumed to be Gaussian?

Central Limit Theorem

(d) (5 points) The bit error performance of a modulation scheme is described by the following equation, where N = 2.0. What is the value of E_b that is required to produce a bit error probability of 0.1698 x 10[-6]?

$$P_e = Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

$Q(x) = 0.1698 \times 10^{-6}$; $x = 5.1$

$\sqrt{\frac{2Eb}{N}} = 5.1$

$Eb = \frac{(5.1)^2 N}{2} = \boxed{26.01}$

(e) (5 points) If a new modulation scheme is used that is described according to the new formula below, what would be the new required E_b' to produce the same error probability as part (c) with the same assumptions as part (c)?

$$P_e = Q\left(\sqrt{\frac{E_b'}{N}}\right)$$

$\sqrt{\frac{Eb'}{N}} = 5.1$

$Eb' = (5.1)^2 N = \boxed{52.02}$

(twice as much energy)

3. (10 points) Orthogonality

Over a bit period of 0.1 seconds, determine if the following two signals are orthogonal. Show your work to justify your answer.

$$x_1 = e^{-j2\pi 100t}$$

$$x_2 = e^{-j2\pi 110t}$$

Two signal are orthogonal if

$$\int_0^{T_b} x_1 x_2 \, dt = 0$$

$$= \int_0^{0.1} e^{-j2\pi 100t} e^{-j2\pi 110t} \, dt$$

$$= \int_0^{0.1} e^{-j2\pi(220)t} \, dt$$

$$= \frac{1}{-j2\pi 220} \left(e^{-j2\pi 220t} \right) \Big|_0^{0.1}$$

$$= \frac{1}{-j2\pi 220} \left(\cancel{e^{-j2\pi(220)0.1}} - \cancel{e^{0}} \right) = 0$$

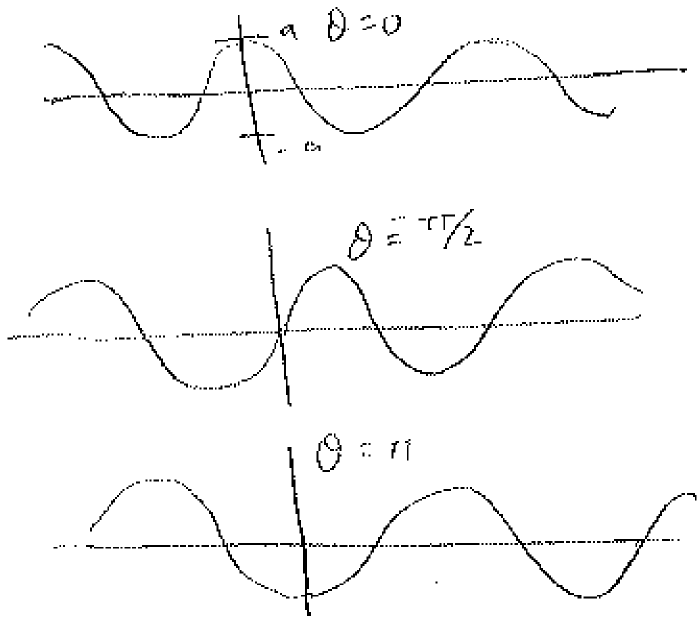
$$e^{-j2\pi(22)} = 1$$

4. (20 points) Autocorrelation

Given is the following random process where f and a are constants and θ is distributed uniformly between $-\pi$ and π.

$$x(t) = a\cos(2\pi ft + \theta)$$

a. (5 points) Sketch the ensemble of the random process.



b. (7 points) Find the mean of the random process.

$$\overline{x(t)} = \int_{-\pi}^{\pi} a\cos(2\pi ft+\theta)\, p_\theta(\theta)\, d\theta$$

$$= \frac{a}{2\pi}\int_{-\pi}^{\pi}\cos(2\pi ft+\theta)\, d\theta$$

$$= \frac{a}{2\pi}\sin(2\pi ft+\theta)\Big|_{-\pi}^{\pi}$$

$$= \frac{a}{2\pi}\Big[\underbrace{\sin(2\pi ft+\pi) - \sin(2\pi ft-\pi)}_{\text{opposites}}\Big] = 0$$

c. (8 points) Find the autocorrelation function.

$$R_X(t_1,t_2) = \overline{X(t_1)\,X(t_2)}$$

$$= a^2\int_{-\pi}^{\pi} \cos(2\pi f t_1+\theta)\cos(2\pi f t_2+\theta)\,p_\theta(\theta)\,d\theta$$

$$\cos A\cos B = \tfrac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$= \frac{a^2}{2(2\pi)}\int_{-\pi}^{\pi}\left[\cancel{\cos\left(2\pi f(t_1+t_2)+2\theta\right)}^{\,0}+\cos\left(2\pi f(t_1-t_2)\right)\right]d\theta$$

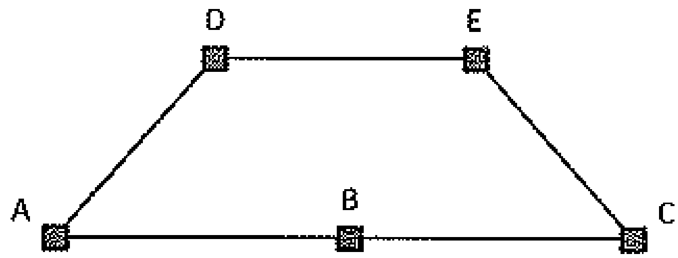
$$= \frac{a^2}{2}\cos\left(2\pi f(t_1-t_2)\right)\frac{\cancel{2\pi}}{\cancel{2\pi}}$$

$$= \boxed{\frac{a^2}{2}\cos\left(2\pi f(t_1-t_2)\right)}$$

d. (5 points) Is the random process wide sense stationary? Explain your answer. To get full credit, you justify your answer using a mathematical explanation.

WSS because $R_X(t_1,t_2)$ above only depends on $(t_1-t_2)=\tau$.

5. (20 points) Given is the following network. Each link has a probability of failure of 0.05.



a. (10 points) If node A were trying to send a packet to node C just over the path A-B-C, what is the probability that it would not be successful?

$1 - \Pr\{\text{both links are good}\}$

$= 1 - (0.95)^2 = \boxed{0.0975}$

Alternatively

$\Pr\{\text{one link fails}\} + \Pr\{\text{both fail}\}$

$= \binom{2}{1}(0.05)^1(1-0.05)^1 + \binom{2}{2}(0.05)^2 = 0.095 + 0.0025$

$= \boxed{0.0975}$

b. (10 points) Now node A is trying to send a packet to node C, but it sends the packet over both possible paths A-D-E-C and A-B-C. What is the probability that even when using these two paths it would not be successful?

$\Pr\{\text{both are bad}\} = \Pr\{\text{A-B-C is bad}\}\Pr\{\text{A-D-E-C bad}\}$

$= (0.0975)(1 - (0.95)^3)$

$= (0.0975)(0.142625)$

$= \boxed{0.01391}$