DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
Series Solutions of Ordinary Differential Equations.

Course Code: MAT2002 **Experiment: 4-B**
Course Name: Application of Differential and Difference Equations **Duration: 90 Minutes**

Series Solution when $x = 0$ ia an Ordinary Point of the Equation

$$P_0 \frac{d^2y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = 0 \tag{1}$$

where P's are polynomial in x and $P_0 \neq 0$ at $x = 0$.

1. Assume its solution to be of the form

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots \tag{2}$$

2. Calculate $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, from (2) and substitute the values of y, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ in (1).

3. Equate to zero the coefficients of the various powers of x and determine a_2, a_3, a_4, $\cdots$ in terms of a_0, a_1.

4. Substituting the values of a_2, a_3, a_4, $\cdots$ in (2), we get the desired series solution having a_0, a_1 as its arbitrary constants.

1. Solve in series the equation $\frac{d^2y}{dx^2} + y = 0$.

MATLAB CODE

```
clc
clear

syms x a0 a1 a2 a3
a = [a0 a1 a2 a3];
y = sum(a.*(x).^[0:3]);

dy = diff(y);
d2y = diff(dy);
gde = collect(d2y+y,x);
cof=coeffs(gde,x);

A2=solve(cof(1),a2);
A3=solve(cof(2),a3);

y=subs(y,a2,a3,A2,A3);
y=coeffs(y,[a1 a0]);
disp('Solution is')
disp(['y=A(',char(y(1)),'+ ...)+B(',char(y(2)),'+ ...)'])
```

OUTPUT

```
Solution is
y=A(1 - x^2/2+ ...)+B(x - x^3/6+ ...)
```

Exercise

2. Solve the following:
   (a) $\frac{d^2y}{dx^2} + xy = 0$
   (b) $\frac{d^2y}{dx^2} + x^2y = 0$
   (c) $y'' + xy' + y = 0.$
   (d) $(1 - x^2)y'' + 2y = 0; y(0) = 4, y'(0) = 5$
3. The half-life of radium is 1600 years, i.e., it takes 1600 years for half of any quantity to decay. If a sample initially contains 50 g, how long will it be until it contains 45 g by power series method?

Department of Mathematics
School of Advanced Sciences
MAT 2002 – Applications of Differential and Difference Equations (MATLAB)
Experiment 3–A
Solution of a Linear differential equation by method of variation of parameters.

Method of variation of parameters:

We consider a second order linear differential equation of the form

$$F(D)y \equiv \frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = f(x) \tag{1}$$

Let the solutions of the homogeneous problem of $F(D)y = 0$ be $y_1(x)$ and $y_2(x)$.

Then the Complementary function (solution of the homogeneous problem) of (1) is

$$y_c(x) = C_1y_1 + C_2y_2 \tag{2}$$

Then by the method of variation of parameters the particular integral of (1) is of the form

$$y_p(x) = uy_1 + vy_2 \tag{3}$$

where the parameters C_1, C_2 of (2) are replaced with functions $u(x)$, $v(x)$ given by

$$u(x) = -\int \frac{y_2 f(x)}{W(x)}dx \text{ and } v(x) = \int \frac{y_1 f(x)}{W(x)}dx,$$

where the wronskian $W(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1' \neq 0$.

In this experiment we consider the coefficients p, q to be constants.

MATLAB Code:

```
clear all
close all
clc
syms c1 c2 x m
F=input('Enter the coefficients [a,b,c]: ');
f=input('Enter the RHS function f(x): ');
a=F(1);b=F(2);c=F(3);
AE=a*m^2+b*m+c; % Auxilliary Equation
m=solve(AE);
m1=m(1); m2=m(2);
D=b^2-4*a*c;
if(D>0)   % Roots are real and different
 y1=exp(m1*x);y2=exp(m2*x);
elseif (D==0)% Roots are real and equal
```

```
        y1=exp(m1*x);y2=x*exp(m1*x);
else   % Roots are complex
    alfa=real(m1);beta=imag(m1);
        y1=exp(alfa*x)*cos(beta*x);
        y2=exp(alfa*x)*sin(beta*x);
end
yc=c1*y1+c2*y2; % Complimentary Solution
%%% Particular Integral by Method of variation of parameters.
fx=f/a;
W=y1*diff(y2,x)-y2*diff(y1,x);  %%% Wronskian%%%
u=int(-y2*fx/W,x);
v=int(y1*fx/W,x);
yp=y1*u+y2*v;  %%%Particular Integral%%%
y_gen=yc+yp;   %%%General Solution%%%
check=input('If the problem has initial conditions then enter
1 else enter 2: ');
if(check==1)
cn=input('Enter the initial conditions [x0, y(x0), Dy(x0)]:');
dy_gen=diff(y_gen);
eq1=(subs(y_gen,x,cn(1))-cn(2));
eq2=(subs(dy_gen,x,cn(1))-cn(3));
[c1 c2]=solve(eq1,eq2);
y=simplify(subs(y_gen));
disp('The complete solution is');
disp(y);
ezplot(y, [cn(1),cn(1)+2]);
else
y=simplify(y_gen);
disp('The General Solution is ');
disp(y);
end
```

Example 1. Find the general solution of the differential equation $y'' - 4y = e^{2x}$.

```
Enter the coefficients [a,b,c]: [1 0 -4]
Enter the RHS function f(x): exp(2*x)
If the problem has initial conditions then enter 1 else enter 2: 2
The General Solution is
(exp(-2*x)*(16*c1 - exp(4*x) + 4*x*exp(4*x) + 16*c2*exp(4*x)))/16
```

Example 2. Find the general solution of the differential equation $y'' + y = \sec x \tan x$.

```
Enter the coefficients [a,b,c]: [1 0 1]
Enter the RHS function f(x): sec(x)*tan(x)
If the problem has initial conditions then enter 1 else enter 2: 2
The General Solution is
 (log(tan(x)^2 + 1)*sin(x))/2 - sin(x) + c1*cos(x) - c2*sin(x) + x*cos(x)
```

Example 3: Suppose that a spring with a mass of 2 kg. has natural length 0.5 m. A force of 25.6 N is required to maintain it stretched to a length of 0.2 m. The spring is immersed in a fluid with damping constant c=40. Find the position of the mass at any time t if it starts from the equilibrium position and is given a push to start with an initial velocity of 0.6 m/s.

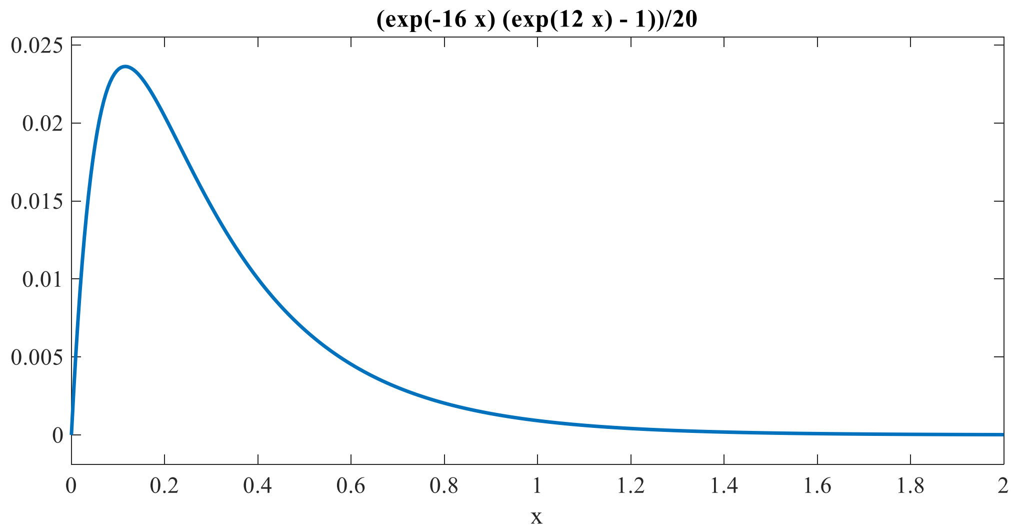
Solution: The differential equation pertaining the motion of the spring described by the differential equation $m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+ky=0$.

Given mass $m=2$, spring constant $k=\frac{25.6}{0.2}=128$, damping constant $c=40$.

Therefore, the differential equation is $\frac{d^2x}{dt^2}+20\frac{dx}{dt}+64x=0$ with $x(0)=0$; $x'(0)=0.6$.

Input/Output:

```
Enter the coefficients [a,b,c]: [1 20 64]
Enter the RHS function f(x): 0
If the problem has initial conditions then enter 1 else enter 2: 1
Enter the initial conditions [x0, y(x0), Dy(x0)]:[0 0 0.6]
The complete solution is
(exp(-16*x)*(exp(12*x) - 1))/20
```

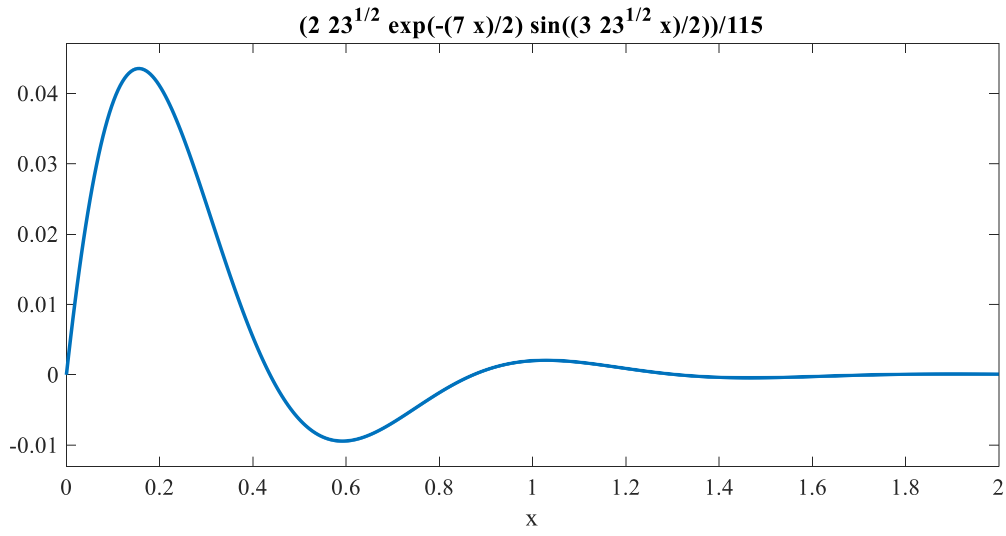


Inference: Here $c^2-4mk>0$. Hence it is a case of over damping, therefore no oscillations occur.

Example 4: Consider the above problem with the spring constant $c=14$, other parameters being the same.

The spring constant here is $k=128$. Damping constant $c=14$, the mass $m=2$.

So the differential equation becomes $\frac{d^2x}{dt^2}+7\frac{dx}{dt}+64x=0$, with $x(0)=0$; $x'(0)=0.6$.



Input/Output:

```
Enter the coefficients [a,b,c]: [1 7 64]
Enter the RHS function f(x): 0
If the problem has initial conditions then enter 1 else enter 2: 1
Enter the initial conditions [x0, y(x0), Dy(x0)]:[0 0 0.6]
The complete solution is
(2*23^(1/2)*exp(-(7*x)/2)*sin((3*23^(1/2)*x)/2))/115
```

Inference: Here $c^2-4mk<0$. It is a case of under damping. Some oscillations occur.

Example 5: Find the charge in the RLC circuit at time t in the circuit when a resistance of 40Ω, inductance of $1H$ and a capacitance of $16\times10^{-4}F$ are connected in series with a source of voltage $E(t)=100\cos10t$, given that initially the charge and current are both 0.

The differential equation for the RLC circuit is

$$LQ''+RQ'+\frac{1}{C}Q=E(t)$$

Here $R=40$, $L=1$, $C=16\times10^{-4}$, $E(t)=100\cos10t$.

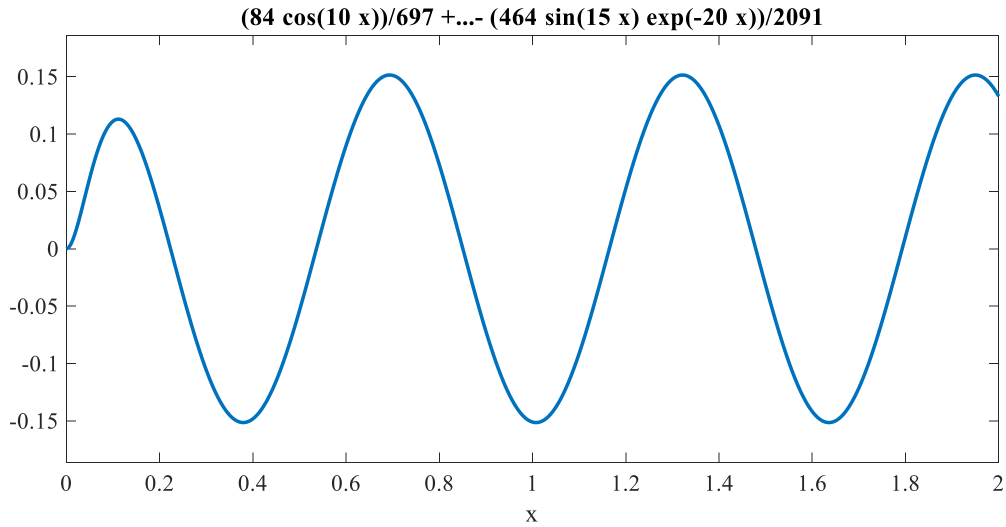
Therefore, the differential equation is

$$Q''+40Q'+625Q=100\cos10t \text{ with } Q(0)=0\text{ ; }Q'(0)=0.$$

Input/Output

```
Enter the coefficients [a,b,c]: [1,40,625]
Enter the RHS function f(x): 100*cos(10*x)
If the problem has initial conditions then enter 1 else enter 2: 1
Enter the initial conditions [x0, y(x0), Dy(x0)]:[0,0,0]
```

```
The complete solution is
(84*cos(10*x))/697 + (64*sin(10*x))/697 - (84*cos(15*x)*exp(-20*x))/697 -
(464*sin(15*x)*exp(-20*x))/2091
```



Exercise:

1. Find the general solution of the differential equation $y'' - 2y' = e^x \sin x$.

2. Solve the initial value problem

   $y'' + 4y' + 20y = 23\sin x - 15\cos x$, $y(0) = 0$, $y'(0) = -1$.

3. Find the current I(t) in an RLC circuit with R=11Ω, L=0.1 H, C=10^{-2} F , which is connected to a source of voltage E(t) = 100 sin400t. Assume that the current and the charge are zero when t=0.

4. A spring with mass of 2kg has damping constant 14, and a force of 6N is required to keep the spring stretched 0.5 m beyond its natural length. The spring is stretched 1m beyond its natural length and then released with zero velocity. Find the position of the mass at any time t.

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MAT 2002 – Applications of Differential and Difference Equations (MATLAB)
Experiment 5–A
Z-transforms and their applications for solving Difference equations

Z–Transform

If the function u_n is defined for discrete values ($n=0,1,2,...$) and $u_n=0$ for $n<0$, then its Z-transform is defined as

$$Z\{u_n\}=\overline{U}(z)=\sum_{n=0}^{\infty}\frac{u_n}{z^n}$$

whenever the infinite series converges.
The inverse Z-transform is written as $Z^{-1}\{\overline{U}(z)\}=u_n$.

MATLAB Syntax used:

`ztrans(f)`	The *z*-transform of the scalar symbol `f` with default independent variable `n`. The default return is a function of `z`.
`ztrans(f,w)`	Makes `f` a function of the symbol w instead of the default `z`.
`ztrans(f,k,w)`	Takes `f` to be a function of the symbolic variable `k`.
`iztrans(F)`	The inverse *z*-transform of the scalar symbolic object `F` with default independent variable `z`. The default return is a function of `n`.
`iztrans(F,k)`	Makes `f` a function of `k` instead of the default `n`. Here `k` is a scalar symbolic object.
`iztrans(F,w,k)`	Takes `F` to be a function of `w` instead of the default variable `z` and returns a function of `k`.
`collect(P,var)`	Rewrites `P` in terms of the powers of the variable `var`.
`stem(Y)`	Plots the data sequence `Y` as stems that extend from equally spaced and automatically generated values along the *x*-axis. When `Y` is a matrix, stem plots all elements in a row against the same *x* value.

Example 1. Find the z-transform of the function $y_n=\frac{1}{4^n}, n\geq 0.$

```
>>syms z n;
>>ztrans(1/4^n)
```

Output:
```
ans =
z/(z - 1/2)
```

Example 2. Find the inverse z-transform of the following $y(z)=\frac{2z}{2z-1}$

```
>>syms z n;
>>iztrans(2*z/(2*z-1))
```

Output:
```
ans =
(1/2)^n
```

Solution of linear difference equations with constant coefficients by Z-transforms.

Consider the Linear difference equation

$$a y_{n+2} + b y_{n+1} + c y_n = f(n) \qquad (1)$$

subject to the initial conditions

$$y_0 = \alpha\,,\ y_1 = \beta\,. \qquad (2)$$

The working procedure:

1. Input the difference equation coefficients and the right hand side function of (1).
2. Input the initial conditions (2).
3. Apply Z–Transform and find $Y(z)$.
4. Apply inverse Z – Transform and find y_n.

MATLAB CODE

```
clear all
clc
syms n z y(n) Y
yn=y(n);
yn1=y(n+1);
yn2=y(n+2);
F = input('Input the coefficients [a,b,c]:  ');
a=F(1);b=F(2);c=F(3);
nh = input('Enter the non-homogenous part f(n):  ');
eqn=a*yn2+b*yn1+c*yn-nh;
ZTY=ztrans(eqn);
IC=input('Enter the initial conditions in the form [y0,y1]:');
y0=IC(1);y1=IC(2);
ZTY=subs(ZTY,{'ztrans(y(n),n,z)','y(0)','y(1)'},{Y,y0,y1});
eq=collect(ZTY,Y);
Y=simplify(solve(eq,Y));
yn=simplify(iztrans(Y));
disp('The solution of the difference equation yn=')
disp(yn);
m=0:20;
y=subs(yn,n,m);
stem(y)
title('Difference equation');
xlabel('n'); ylabel('y(n)');
```

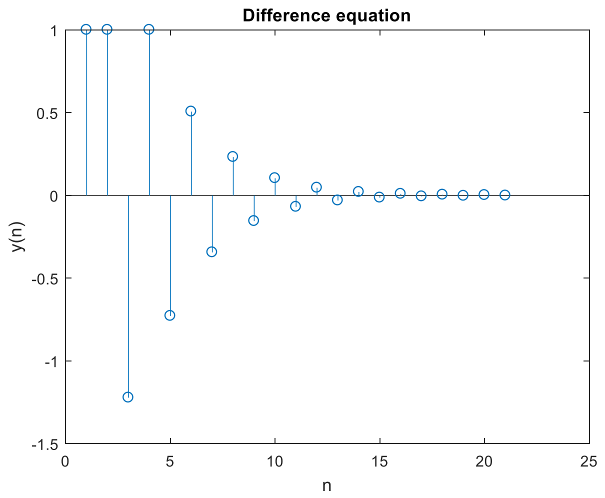
Example 3. Solve $9y_{n+2} + 9y_{n+1} + 2y_n = 0,\ n \ge 0$, with $y_0 = 1$ and $y_1 = 1$.

Input:

```
Input the coefficients [a,b,c]:  [9 9 2]
Enter the non-homogenous part f(n):  0
Enter the initial conditions in the form [y0,y1]:[1 1]
```

Output:

```
The solution of the difference equation yn=
5*(-1/3)^n - 4*(-2/3)^n
```



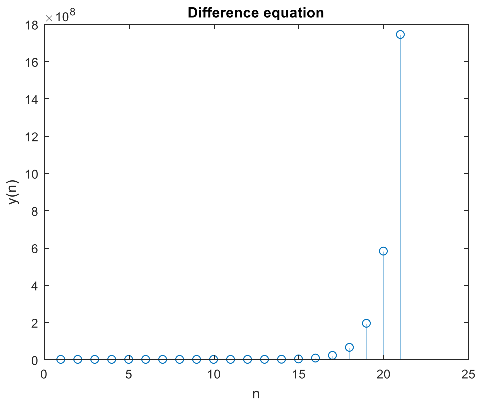
Example 4: Solve the equation $y_{n+2} - 3y_{n+1} + 2y_n = 3^n$, $y_0 = 0$, $y_1 = 1$.

Input:

```
Input the coefficients [a,b,c]:  [1 -3 2]
Enter the non-homogenous part f(n):  3^n
Enter the initial conditions in the form [y0,y1]:[0,1]
```

Output:

```
The solution of the difference equation yn=
3^n/2 - 1/2
```



Exercise

1. Solve $y_{n+2} - 5y_{n+1} + 6y_n = 5^n, n \geq 0$, $y_0 = 1$ and $y_1 = 1$.
2. Solve $y(n+2) - y(n) = 2^n, n \geq 0$, $y_0 = 0$ and $y_1 = 1$.
3. Solve $y(n+2) + 2y(n+1) + y(n) = n, n \geq 0$, $y_0 = 0$ and $y_1 = 0$.
4. Solve $y(n+2) - 4y(n+1) + 3y(n) = n.2^n, n \geq 0$, $y_0 = 0$ and $y_1 = 0$.
5. Formulate the difference equation for Fibonacci numbers and hence solve by Z-transforms.

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MAT 2002 – Applications of Differential and Difference Equations (MATLAB)
Experiment 3–B
Solution of Linear differential equations by Laplace transforms

The Laplace Transform of a function $f(t)$ is defined a

$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)dt$, provided the integral exists.

MATLAB Commands used

Command	Purpose
`laplace(f)`	To find the Laplace transform of a scalar symbol f with default independent variable t. The default return is a function of s.
`ilaplace(F)`	To find the inverse Laplace transform of the scalar symbolic object F with default independent variable s. The default return is a function of t.
`heaviside(t-a)`	To input the heaviside's unit step function H(t−a).
`dirac(t-a)`	To input the dirac delta function δ(t−a).
`collect(P,`*var*`)`	Rewrites **P** in terms of the powers of the variable *var*

Example 1. The following MATLAB code finds the Laplace transform of $f(t)$.

```
clear all
clc
syms t
f=input('Enter the function of t: ');
F=laplace(f);
disp(['L{f(t)}=',char(F)]);
```

Input/Output:

```
Enter the function of t: sin(t)
L{f(t)}=1/(s^2 + 1)
```

Example 2: The following MATLAB code computes the Laplace Transform of

$$f(t) = \begin{cases} t^2, t < 2 \\ t-1, 2 < t < 3 \\ 7, t > 3 \end{cases}$$

Input/Output:

```
Enter the function of t: t^2*(heaviside(t)-heaviside(t-2))+
     (t-1)*(heaviside(t-2)-heaviside(t-3))+7*heaviside(t-3)
  L{f(t)}=(7*exp(-3*s))/s-(4*exp(-2*s))/s-(4*exp(-2*s))/s^2
         -(2*exp(-2*s))/s^3+2/s^3- (exp(-3*s)*(2*s-exp(s)-
          s*exp(s)+1))/s^2
```

Example 3. The following MATLAB code computes the inverse Laplace transform of $F(s)$

```
syms s
F=input('Enter the function of s: ');
f=ilaplace(F);
disp(['f(t)=',char(f)]);
```

Input/Output

```
Enter the function of s: 6/(s^3+2*s^2-s-2)

f(t)=2*exp(-2*t)-3*exp(-t)+exp(t)
```

To solve and visualize solutions of a second order Linear differential equation using Laplace transform.

Working Procedure:

- Input the differential equation coefficients a,b,c and the RHS function $f(x)$ of the differential equation $ay''+by'+cy=f(x)$.
- Input the initial conditions $y(0)$ and $y'(0)$.
- Apply Laplace Transform and find $Y(s)$.
- Apply inverse Transform and find $y(t)$.

MATLAB Code

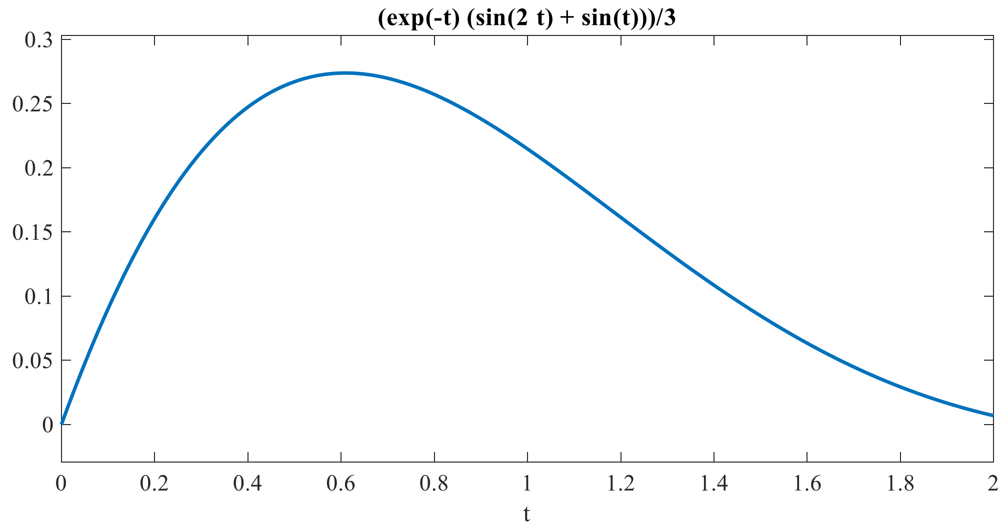
```
clear all
clc
syms t s y(t) Y
dy(t)=diff(y(t));
d2y(t)=diff(y(t),2);
 F = input('Input the coefficients [a,b,c]:  ');
 a=F(1);b=F(2);c=F(3);
nh = input('Enter the non-homogenous part f(x):  ');
eqn=a*d2y(t)+b*dy(t)+c*y(t)-nh;
LTY=laplace(eqn,t,s);
IC = input('Enter the initial conditions in the form [y0,Dy(0)]: ');
y0=IC(1);dy0=IC(2);
LTY=subs(LTY,{'laplace(y(t), t, s)','y(0)','D(y)(0)'},{Y,y0,dy0});
eq=collect(LTY,Y);
Y=simplify(solve(eq,Y));
yt=simplify(ilaplace(Y,s,t));
disp('The solution of the differential equation y(t)=')
disp(yt);
ezplot(yt,[y0,y0+2]);
```

Example 4. Solve the initial value problem $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t$, $y(0)=0$, $y'(0)=1$.

Input/Output

```
Input the coefficients [a,b,c]:  [1 2 5]
Enter the non-homogenous part f(x):  exp(-t)*sin(t)
Enter the initial conditions in the form [y0,Dy(0)]: [0,1]

The solution of the differential equation y(t)=
(exp(-t)*(sin(2*t) + sin(t)))/3
```



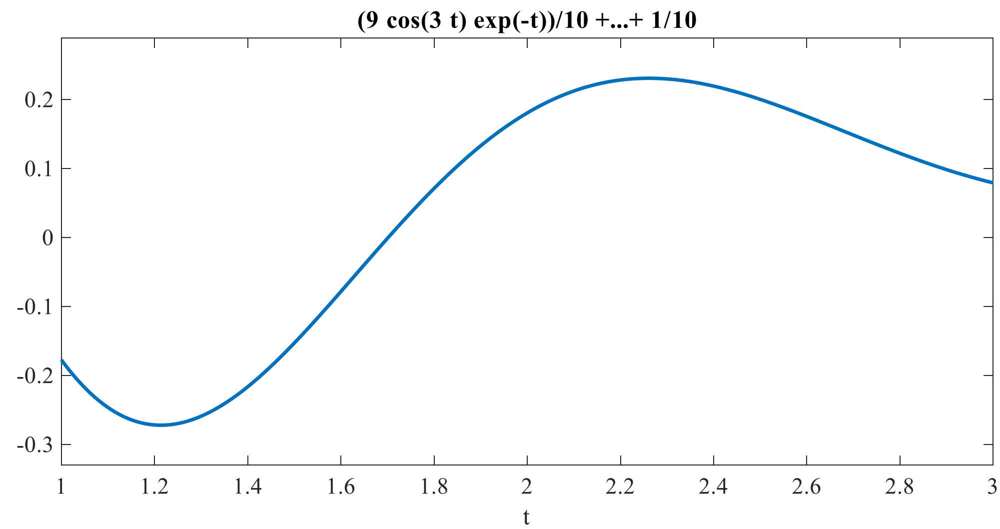
Example 5.

Solve $y''+2y'+10y=1+5\delta(t-5)$, $y(0)=1,\ y'(0)=2$.

Input/Output

```
Input the coefficients [a,b,c]:  [1 2 10]
Enter the non-homogenous part f(x):  1+5*dirac(t-5)
Enter the initial conditions in the form [y0,Dy(0)]: [1,2]

The solution of the differential equation y(t)=
(9*cos(3*t)*exp(-t))/10 + (29*sin(3*t)*exp(-t))/30 + (5*heaviside(t -
5)*exp(5 - t)*sin(3*t - 15))/3 + 1/10
```



Exercise

1. Solve $y'' - 2y' + y = e^t$, subject to $y(0) = 2$, $y'(0) = -1$
2. Solve $y'' + y = f(t)$, $y(0) = 1$, $y'(0) = 0$ where $f(t) = \begin{cases} 3, & t \le 4 \\ 2t - 5, & t > 4 \end{cases}$.
3. Using Laplace transforms find the current $i(t)$ in the circuit with a resistance $R = 4\Omega$, inductance $L = 1H$, capacitance $C = 0.05F$ connected in a series with a source of voltage $v(t) = \begin{cases} 34e^{-t}, & 0 < t < 4 \\ 0, & t > 4 \end{cases}$ volts.

Department of Mathematics
School of Advanced Sciences
MAT 2002 – Applications of Differential and Difference Equations (MATLAB)
Experiment 1–B
Harmonic Analysis

Introduction:

In practice, the function is often not given by a formula, but a table of corresponding values. In such cases, the integrals in the Fourier series cannot be evaluated, instead the following forms can be used.

Since the mean value of the function $y = f(x)$ over the interval (a,b) is $\frac{1}{b-a}\int_a^b f(x)dx$, the Fourier coefficients in the interval $(\alpha, \alpha+2l)$ can be written as

$$a_0 = 2\times\frac{1}{2l}\int_{\alpha}^{\alpha+2l} f(x)dx = 2\times \text{Mean of } f(x) \text{ in } (\alpha,\alpha+2l)$$

$$a_n = 2\times\frac{1}{2l}\int_{\alpha}^{\alpha+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx = 2\times \text{Mean of } f(x)\cos\left(\frac{n\pi x}{l}\right) \text{ in } (\alpha,\alpha+2l)$$

$$b_n = 2\times\frac{1}{2l}\int_{\alpha}^{\alpha+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx = 2\times \text{Mean of } f(x)\sin\left(\frac{n\pi x}{l}\right) \text{ in } (\alpha,\alpha+2l)$$

Hence the Fourier series for the tabulated data can be written as

$$f(x) = \frac{a_0}{2} + a_1\cos\theta + b_1\sin\theta + a_2\cos 2\theta + b_2\sin 2\theta + \ldots \quad \text{where } \theta = \frac{\pi x}{l}.$$

The term $(a_1\cos x + b_1\sin x)$ is called the Fundamental Harmonic/First Harmonic, $(a_2\cos 2x + b_2\sin 2x)$ is called the Second Harmonic and so on.

Given m data points of function $y = f(x)$ of period 2π or $2l$, we find the Fourier coefficients

$$a_0 = 2\frac{\sum f(x_i)}{m},\ a_n = 2\frac{\sum_{i=1}^{m} f(x_i)\cos n\theta_i}{m},\ b_n = 2\frac{\sum_{i=1}^{m} f(x_i)\sin n\theta_i}{m},\ \theta_i = \frac{\pi x_i}{l},\ n = 1,2,\ldots$$

MATLAB Syntax Used:

`syms var1 var2`	Creates symbolic variables var1 and var2
`disp(x)`	Displays the contents of x without printing the variable name
`length(X)`	returns the length of vector X
`plot(fun)`	Plots the discrete function fun whose domain and range are given.

MATLAB Code:

```
clear all
clc
syms t
x=input('Enter the equally spaced values of x: ');
y=input('Enter the values of y=f(x): ');
m=input('Enter the number of harmonics required: ');
n=length(x);a=x(1);b=x(n);
h=x(2)-x(1);
L=(b-a+h)/2;
theta=pi*x/L;
a0=(2/n)*sum(y);
Fx=a0/2; x1=linspace(a,b,100);
for i=1:m
figure
an=(2/n)*sum(y.*cos(i*theta));
bn=(2/n)*sum(y.*sin(i*theta));
Fx=Fx+an*cos(i*pi*t/L)+bn*sin(i*pi*t/L) ;
Fx=vpa(Fx,4);
Fx1=subs(Fx,t,x1);
plot(x1,Fx1);
hold on
plot(x,y);
title(['Fourier Series with ',num2str( i ),'harmonics'])
legend('Fourier Series', 'Function Plot')
hold off;
end
disp(strcat('Fourier series with', num2str(i),'harmonics
is:',char(Fx)));
```

Example 1: Compute the first four harmonics of the Fourier series given by the following table:

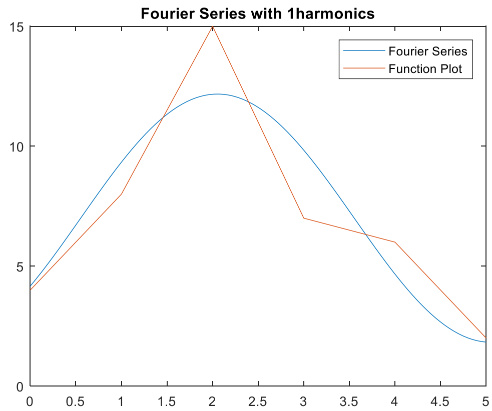
x	0	1	2	3	4	5
y	4	8	15	7	6	2

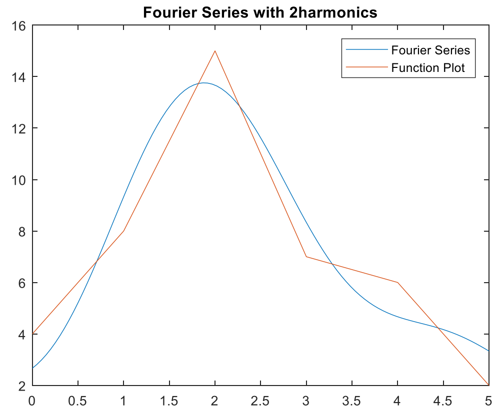
Input

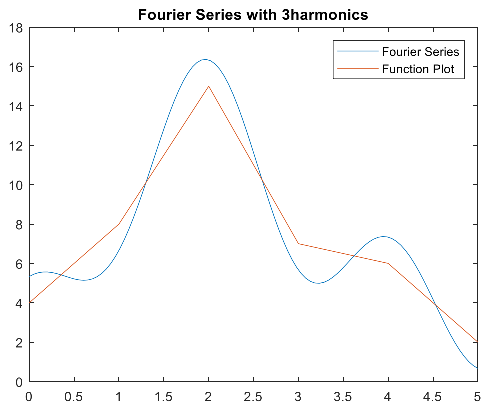
```
Enter the equally spaced values of x: 0:5
Enter the values of y=f(x): [4 8 15 7 6 2]
Enter the number of harmonics required: 4
```

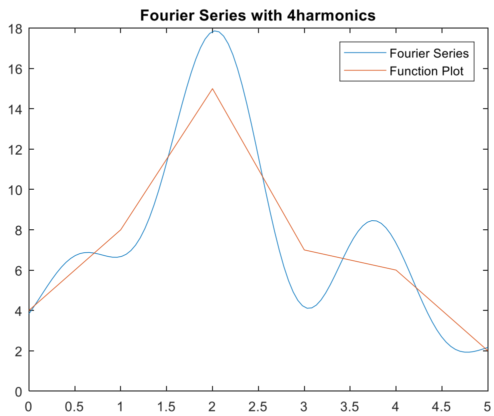
Output

```
Fourier series with4harmonics is:4.33*sin(1.047*t) -
1.5*cos(2.094*t) - 1.5*cos(4.189*t) - 2.833*cos(1.047*t) -
0.866*sin(2.094*t) + 0.866*sin(4.189*t) + 2.667*cos(3.142*t) -
6.123e-16*sin(3.142*t) + 7.0
```









Exercise:

1. The following table gives the variations of periodic current over a period T_0

T_0 sec	0	T/6	T/3	T/2	2T/3	5T/6	T
A amp	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first three harmonics (Take $T=1$).

2. Find the constant, the first sine and cosine terms in the Fourier series expansion of the function $y = f(x)$ tabulated below:

x	0	1	2	3	4	5
$y = f(x)$	6	15	18	22	17	12

DEPARTMENT OF MATHEMATICS

SCHOOL OF ADVANCED SCIENCES

Solution of homogeneous system of first order and second order differential equations by Matrix method.

Course Code: MAT2002 **Experiment: 4-A**
Course Name: Application of Differential and Difference Equations **Duration: 90 Minutes**

System of First Order Linear Differential Equations

A system of n linear first order differential equations in n unknowns (an $n \times n$ system of linear equations) has the general form:

$$\begin{cases} x_1' = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + g_1(t) \\ x_2' = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + g_2(t) \\ \vdots = \vdots + \vdots + \vdots + \vdots + \vdots \\ x_n' = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + g_n(t) \end{cases} \tag{1}$$

where the coefficients a_{ij}'s are arbitrary constants, and g_i's are arbitrary functions of t. If every term g_i is constant zero, then the system is said to be homogeneous.

The system (1) is most often given in a shorthand format as a matrix-vector equation, in the form:

$$X' = AX + G$$

where $X' = [x_i']_{n\times 1}$, $A = [a_{ij}]_{n\times n}$, $X = [x_i]_{n\times 1}$, and $G = [g_i(t)]_{n\times 1}$.

If the coefficient matrix A has two distinct real eigenvalues λ_1 and λ_2 and their respective eigenvectors are X_1 and X_2, then the 2×2 system

$$X' = AX$$

has a general solution

$$X = C_1X_1e^{\lambda_1 t} + C_2X_2e^{\lambda_2 t}$$

System of Second Order Linear Differential Equations

Consider the system of second order linear differential equations of the form

$$\begin{cases} x_1'' = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ x_2'' = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots = \vdots + \vdots + \vdots + \vdots \\ x_n'' = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n \end{cases} \tag{2}$$

where the coefficients a_{ij}'s are arbitrary constants.

Then, the solution of (2), $X'' = AX$, is

$$X = PY$$

where Y is the solution of $Y'' = DY$, P is the modal matrix of A and D is it's diagonal matrix.

1. Solve:

$$x_1' = x_1 + 2x_2$$
$$x_2' = 0.5x_1 + x_2$$
$$x_1(0) = 16, x_2(0) = -2$$

MATLAB CODE

```
clc
clear

syms t C1 C2

A=input('Enter A: ');

[P,D]=eig(A);
L1=D(1);L2=D(4);

y1=C1*exp(L1*t);y2=C2*exp(L2*t);
Y=[y1;y2];
X=P*Y;

Cond=input('Enter the initial conditions [t0, x10,x20]: ');
t0=Cond(1);x10=Cond(2);x20=Cond(3);

eq1=subs(X(1)-x10,t0);eq2=subs(X(2)-x20,t0);
[C1, C2] = solve(eq1,eq2);

X=subs(X);
```

INPUT

```
Enter A: [1 2;0.5 1]
Enter the initial conditions [t0, x10,x20]: [0 16 -2]
```

OUTPUT

```
X =
10*exp(t/4503599627370496) + 6*exp(2*t)
3*exp(2*t) - 5*exp(t/4503599627370496)
```

2. The governing equations of a certain vibrating system are

$$x_1'' = 2x_1 + x_2$$
$$x_2'' = 9x_1 + 2x_2$$

Solve the system of equations by matrix method.

MATLAB CODE

```
clc
clear

A=input('Enter A: ');

[P D]=eig(A);

Sol1 = dsolve(['D2y = ',num2str(D(1)),'*y']);
Sol2 = dsolve(['D2y = ',num2str(D(4)),'*y']);

X = P*[Sol1;Sol2];

disp('x1=');disp(X(1))
disp('x2=');disp(X(2))
```

INPUT

```
Enter A: [-5 2;2 -2]
```

OUTPUT

```
x1=
(10^(1/2)*(C1*exp(5^(1/2)*t) + C2*exp(-5^(1/2)*t)))/10 - (10^(1/2)*
(C3*cos(t) + C4*sin(t)))/10

x2=
(3*10^(1/2)*(C3*cos(t) + C4*sin(t)))/10 + (3*10^(1/2)*
(C1*exp(5^(1/2)*t) + C2*exp(-5^(1/2)*t)))/10
```

Exercise

3. Solve the following:
   (a) $x_1' = 3x_1 - 2x_2; x_2' = 2x_1 - 2x_2; x_1(0) = 1, x_2(0) = -1.$
   (b) $x_1' = -x_2 + x_3; x_2' = 4x_1 - x_2 - 4x_3; x_3' = -3x_1 - x_2 + 4x_3.$

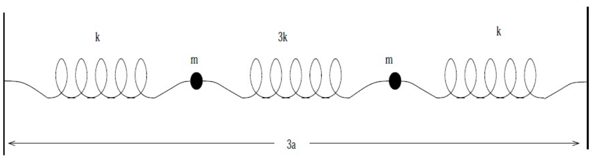
4. Solve the following:
   (a) $x_1'' = -5x_1 + 2x_2; x_2'' = 2x_1 - 2x_2.$
   (b) $x_1'' + 2x_1 - x_2 = 0; x_2'' - x_1 + 2x_2 = 0.$

5. Two particles of equal mass $m = 1$ move in one dimension at the junction of three springs. The springs each have unstretched length $a = 1$ and have spring stiffness constants, k, $3k$ and k (with $k = 1$) respectively see Figure. Applying Newton's second law and Hooke's, this mass-spring system gives rise to the differential equation system

$$x_1'' = -4x_1 + 3x_2$$

$$x_2'' = 3x_1 - 4x_2$$

   Find the displacements $x_1(t)$ and $x_2(t)$.



6. Reduce the third order equation $y''' + 2y'' - y' - 2y = 0$ to the system of first order linear equations and solve by matrix method.

7. Consider tanks T_1 and T_2 which contain initially 100 gallons of water each. In T_1 water is pure whereas 150 pounds of salt is dissolved in T_2. By circulating the liquid at the rate of 2 gallons per minute and stirring, the amount of salt $y_1(t)$ in T_1 and $y_2(t)$ in T_2 change with time t, find the amount of salt in the two tanks after a time t.

MATLAB code for solving system of first order differential equations by diagonalization process

```
clc
clear all
close all

syms x1(t) x2(t)

A=input('Enter the coefficent matrix A: ');
F=input('Enter the nonhomogenous part:');

[P D]=eig(A)

IP=inv(P);

X=[x1;x2];
FF=IP*F;

sol1=dsolve(diff(x1,1)-D(1)*x1-FF(1)==0);
sol2=dsolve(diff(x2,1)-D(4)*x2-FF(2)==0);

disp('The solution of the given system is : ')
Y=P*[sol1;sol2];
y1=simplify(Y(1))
y2=simplify(Y(2))
```

Example:
Solve the system of first order DE

$$y_1' + 3y_1 - y_2 = 3t$$

$$y_2' - 2y_1 + 4y_2 = e^{-t}$$

Solution:

$$\mathbf{Y}=c_1\begin{pmatrix}1\\1\end{pmatrix}e^{-2t}+c_2\begin{pmatrix}1\\-2\end{pmatrix}e^{-5t}+\begin{pmatrix}\frac{6}{5}\\\frac{3}{5}\end{pmatrix}t-\begin{pmatrix}\frac{27}{50}\\\frac{21}{50}\end{pmatrix}+\begin{pmatrix}\frac{1}{4}\\\frac{1}{2}\end{pmatrix}e^{-t}$$

y1 =

(6*t)/5 + exp(-t)/4 + (2^(1/2)*C1*exp(-2*t))/2 - (5^(1/2)*C2*exp(-5*t))/5 - 27/50

y2 =

(3*t)/5 + exp(-t)/2 + (2^(1/2)*C1*exp(-2*t))/2 + (2*5^(1/2)*C2*exp(-5*t))/5 - 21/50