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SEMESTER EXAMINATION (DEC 2023)

CLASS: B. Tech SEMESTER: I

PAPER CODE: BEM-C102

PAPER TITLE: ENGINEERING MATHEMATICS-I

Time: 3 Hours Max. Marks: 70

Note: Question Paper is divided into two sections: **A and B**. Attempt both the sections as per given instructions.

SECTION-A (SHORT ANSWER TYPE QUESTIONS)

Instructions: Answer any ***five*** questions in about 150 words each. Each question carries six marks.

(5 X 6 = 30 Marks)

Question-1: (i)Expand $e^{\sin x}$ by Maclaurin's series upto the terms containing x^4.

(ii) Evaluate $\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$

Question-2: (i) Show that radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$ is $4a\cos\theta$

(ii) Find the asymptotes of the curve $x^3+3x^2y-4y^3-x+y+3=0$

Question-3: (i)If $u=log\frac{x^4+y^4}{x+y}$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3$

(ii) Find the equations of the tangent plane and normal to the surface $z^2=4(1+x^2+y^2)$ at (2,2,6).

Question-4: If $u=xyz$, $v=x^2+y^2+z^2$, $w=x+y+z$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

Question-5: Show that area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $16a^2/3$.

Question-6: Prove that $B(m,n)=\frac{\Gamma m\ \Gamma n}{\Gamma(m+n)}$

Question-7:Prove that $r^n\vec{r}$ is an irrotational vector for any value of n but is solenoidal only if n+3 =0

Question-8: Verify Green's theorem in the plane for $\oint_C (xy+y^2)dx+x^2dy$ where C is closed curve of the region bounded by $y=x$ and $y=x^2$.

Question-9: Define normal form of a matrix. Reduce the given matrix into normal form and hence find its rank.

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Question-10: Investigate the values of λ *and* μ so that the equations

$2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+\lambda z=\mu$

have (i) No solution (ii) a unique solution (iii) an infinite number of solutions

SECTION-B (LONG ANSWER TYPE QUESTIONS)

Instructions: Answer any ***four*** questions in detail. Each question carries 10 marks.

(4 X 10 = 40 Marks)

Question-1: If $x = \sin\left(\frac{1}{a} \log y\right)$ then prove that $(1-x^2)y_{n+2} - x(2n+1)y_{n+1} - (n^2+a^2)y_n = 0$ and hence using McLaurin theorem show that $e^{a\sin^{-1}x} = 1 + ax + \frac{a^2x^2}{2!} + \frac{a^2(1+a^2)x^3}{3!} + \cdots \ldots$

Question-2: (i) In a plane triangle, find the maximum value of the $\cos A \cos B \cos C$

(ii) If $u = f(r)$ and $x = r\cos\theta, y = r\sin\theta$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$

Question-3: (i)Find the area enclosed by the curve $a^2x^2 = y^3(2a-y)$.

(ii) Find the volume of the solid generated by revolving the laminscate $r^2 = a^2 \cos 2\theta$ about the line $\theta = \pi/2$.

Question-4: (i) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)i - (2xy)j$ taken around the rectangle bounded by $x = \pm a, y = 0, y = b$.

(ii) Apply Gauss's Divergence theorem to evaluate $\iint_S \left[(x^3 - yz)dydz - 2x^2 ydzdx + zdxdy\right]$ over the surface of a cube bounded by the coordinate planes and planes $x = y = z = a$.

Question-5: Verify Caley Hamilton theorem for matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find its inverse.

Question-6: Expand $f(x,y) = \tan^{-1}(y/x)$ in power of (x-1) and (y-1)upto third terms. Hence compute $f(1.1, 0.9)$ approximately.

Question-7: Find the Volume of the Greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Question-8: (i) Find the Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

(ii) Prove that $\beta(m+1,n)+\beta(m,n+1)=\beta(m,n)$.