1) *You have 20 bottles of pills. 19 bottles have 1.0 gram pills, but one has pills of weight 1.1 grams. Given a scale that provides an exact measurement, how would you find the heavy bottle? You can only use the scale once.*

   The idea is to take a different number of pills from each bottle.

   - We mark each bottle with a different number from 1 to 20, and take 1 pill from bottle #1, 2 pills from bottle #2, and so till 20 pills from bottle #20.
   - Next, we put all the pills on the weighing scale. Since each bottle has a different number of pills, based on the result of the weighing scale, we can know which bottle has the heavy pills.
   - In our example, if each bottle had 1 gram pills, then the total weight given by the scale would have been: 1 + 2 + 3 + … + 20 = 20 * (20 + 1) / 2 = 210 grams.
   - If the weighing scale shows the weight as 210.1 grams, we know that there is only 1 pill with extra weight. Since we took only 1 pill from bottle #1, the heavy bottle must have been bottle #1. If the weight is 210.2, the heavy pills must have been in bottle #2.
   - In general, the bottle number can be found using the formula:

     Bottle Number = (Actual weight – Ideal Weight) * 10, where Ideal weight = 210 grams

2) *You have a basketball hoop and someone says you can play one of 2 games:*
   a. *You get one shot to make the hoop, or*
   b. *You get 3 shots and you have to make 2 of 3 shots*

   If p is the probability of making a shot, for which values of p should you pick one game or the other?

   The probability of winning game 1 is p.

   If s(n, k) is the probability of making k shots out of a total of n attempts, the probability P of winning game 2 is:

   P = s(3, 2) + s(3, 3)

   Since the probability of making one shot is p, the probability of making all 3 shots is p^3.

   s(3, 2) = P(making shots 1 and 2, missing 3)

   + P(making shots 1 and 3, missing 2)

   + P(making shots 2 and 3, missing 1)

   = 3 * p * p * (1 - p)

   = $3p^2 - 3p^3$

   Thus, P = $3p^2 - 3p^3 + p^3$

   = $3p^2 - 2p^3$

You should play game 1 rather than game 2 if

$p > 3p^2 - 2p^3$

$1 > 3p - 2p^2$
$2p^2 - 3p + 1 > 0$
$2p(p - 1) - (p - 1) > 0$
$(p - 1)(2p - 1) > 0$

For the above equation to hold true, either both terms should be positive or both terms should be negative. But we know $p < 1$, so $p - 1 < 0$.

Thus, $(2p - 1) < 0$

$p < 0.5$

Thus, if $0 < p < 0.5$, we should play game 1, and if $0.5 < p < 1$, we should play game 2. If $p = 0.5$, we can play either game.

3) *There is an 8x8 chess board in which two diagonally opposite corners have been cut off. You are given 31 dominos, and a single domino can cover exactly two squares Can you use the 31 dominos to cover the entire board? Prove your answer (by providing an example or showing why it's impossible).*

Although an 8 x 8 board has 32 back and 32 white square, by removing diagonally opposite squares (which have to be of the same color), we are left with 30 squares of one color and 32 squares of the other.

- Since differently colored squares are adjacent to each other, one domino covers 1 back and 1 white square. Thus, 31 dominos can cover 31 black and 31 white square.
- However, we have 30 squares of one color and 32 squares of the other. Thus, it is impossible to cover them with 31 dominos.

4) *There are 3 ants on different vertices of a triangle. What is the probability of (all or two of) them colliding if they start walking on the sides of the triangle? Assume that each ant randomly picks a direction with equal probability, and that they walk with the same speed.*
Similarly, find probability of collision of n ants on an n-vertex polygon.

The ants will not collide if they move in the same direction, whether it is clockwise or anti-clockwise. Since each ant may walk in one of two directions, the probability of them all walking clockwise is:

$P(\text{clockwise}) = (1/2)^3$

Similarly, $P(\text{counter-clockwise}) = (1 / 2)^3$

Thus, $P(\text{same direction}) = (1 / 2)^3 + (1 / 2)^3 = 1 / 8 + 1 / 8 = 1 / 4$

Thus, $P(\text{collision}) = 1 - P(\text{same direction}) = 1 - 1 / 4 = 3 / 4$.

Applying this concept to an n-vertex polygon with n ants,

$P(\text{same direction}) = (1 / 2)^n + (1 / 2)^n = (1 / 2)^{n - 1}$

Thus, $P(\text{collision}) = 1 - (1 / 2)^{n - 1}$

5) *You have a five-quart jug, a three-quart jug, and an unlimited supply of water (but no measuring cups). How would you come up with exactly four quarts of water? Note that the jugs are oddly shaped, such that filling up exactly "half" of the jug would be impossible.*

   We follow these steps to get 4 quarts of water.

   - Fill up the 5 quarts jug completely with water.
   - Put water from the 5 quarts jug to the 3 quarts one till the 3 quarts jug is full.
     This leaves 5-3 = 2 quarts of water in the 5 quarts jug.
   - Drain the 3 quarts jug, and transfer the water from the 5 quarts jug to the 3 quarts one.
     Now, the 5 quarts jug is empty and the 3 quarts jug has 2 quarts.
   - Again, fill the 5 quarts jug completely. Transfer water from the 5 quarts jug to the 3 quarts one till the 3 quarts jug is full.
     Since the 3 quarts jug already had 2 quarts of water, it can take 1 more quart of water.
     This leaves 5 – 1 = 4 quarts of water in the 5 quart jug, which is what we wanted.

6) ***A bunch of people are living on an island, when a visitor comes with a strange order: all blue-eyed people must leave the island as soon as possible. There will be a flight out at 8:00pm every evening. Each person can see everyone else's eye color, but they do not know their own (nor is anyone allowed to tell them). Additionally, they do not know how many people have blue eyes, although they do know that at least one person does. How many days will it take the blue-eyed people to leave?***

   The number of days the blue eyed people will take to leave the island = the number of blue eyed people on the island.

   This can be proved in the following manner:

   - **If there is 1 blue eyed person** on the island, he will see that no other person has blue eyes. Since there he knows there is at least one blue eyed person on the island, he concludes that he is the only blue eyed person and leaves the island on the 1st day.
   - **If there are 2 blue eyed persons**, they can see that 1 person has blue eyes, but they are both unsure about the color of their own eyes. After the 1st night, if there was only one blue eyed person on the island, he would have already left (from the previous point). Thus, the only conclusion that each blue eyed man will draw is that he also has blue eyes, and both of them will leave the island on the 2nd night.
   - Extending this logic to n people, we can see that it will take them n nights to leave the island.

7) *Ratio of boy and girls.*

   Assume that whenever a family has a child we append a character representing the gender of the child onto a String. So, if a boy is born, we append a B, and if a girl is born, we append a G. We can see that at any moment, the chances of appending a G to the String is 50% since the chances of a girl being born is 50%. The fact that families do not have children after a girl is born does not impact this probability.

   Thus, gender ratio for girl : boy = 1 : 1.

Simulation Code:

```java
import java.util.Random;

public class BirthingSimulator {
    public static double simulateBirthingForNFamilies(int n) {
        int girlsCounter = 0;
        int boysCounter = 0;
        Random RNG = new Random();
        while(n != 0) {
            boysCounter += simulateBirthingFor1Family(RNG);
            girlsCounter++;
            n--;
        }
        return girlsCounter / (double) boysCounter;
    }

    private static int simulateBirthingFor1Family(Random RNG) {
        int boysCounter = 0;
        while(true) {
            if(RNG.nextBoolean()) {
                return boysCounter;
            } else {
                boysCounter++;
            }
        }
    }

    public static void main(String[] args) {
        int n = 1000000000;
        System.out.println("Simulating birthing for 1,000,000,000
families...");
        System.out.println("The ratio of girls : boys is: " +
simulateBirthingForNFamilies(n));
    }
}
```

Output: Simulating birthing for 1,000,000,000 families...
The ratio of girls : boys is: 0.9999751886156201

8) *There is a building of 100 floors. If an egg drops from the Nth floor or above, it will break. If it's dropped from any floor below, it will not break. You're given two eggs. Find N, while minimizing the number of drops for the worst case.*

We can see that once egg #1 breaks, we need to do a linear search for egg #2 from the last tested floor (where the egg didn't break) to the one where it did. We can also see that if we keep the number of floors between drops constant, then the number of drops increase with the number of floors. For example, consider the scenario in which we drop egg #1 from floor 10, then floor 20, and so on till 100.

- If we drop egg #1 from floor 10 and it breaks, then we have to drop egg #2 from floor 1, 2, and so on till it breaks. This results in a maximum of 10 drops.
- However, if egg #1 does not break till floor 90 and then breaks when dropped from floor 100, we have to drop egg #2 from floors 91, 92, and so on till it breaks. The total number of drops in the worst case is 19 (10 drops for egg #1 – floors 10, 20, … 100 – and 9 drops for egg #2 – floors 91, 92, .. 99).
- We should try to balance the number of drops of the 2 eggs so that as the number of drops for egg #1 increase, the number of drops for egg #2 decrease. This will result in the perfect worst-case balance between the egg drops. In other words, if we drop egg #1 from floor X on turn 1, we should drop it from floor (X - 1) on turn 2, from floor (X - 3) on turn 3 and so on. These floors will all sum to 100.

$$X + (X - 1) + (X - 2) + \dots + 1 = 100$$
$$X (X + 1) / 2 = 100$$
$$X*X + X - 200 = 0$$

Finding roots, we have X = 14. Thus, number of floors in the worst case is 14.

9) *There are 100 closed lockers in a hallway. A man begins by opening all 100 lockers. Next, he closes every second locker. Then, on his third pass, he toggles every third locker (closes it if it is open or opens it if it is closed). This process continues for 100 passes, such that on each pass i, the man toggles every ith locker. After his 100th pass in the hallway, in which he toggles only locker #100, how many lockers are open?*

A locker is toggled once for each of its factors (including 1 and itself). For example, 6 is toggled on turns 1, 2, 3 and 6.

Since on turn 1, the lockers are opened and then closed on the next appropriate turn, any locker which has an odd number of factors would remain open.

Any number would have an odd number of factors only if it is a perfect square. This is because each factor is paired with its complement, except for the root of the number. For example, in 24, factors are (1, 24), (2, 12), (4, 6). However, in a perfect square, such as 25, factors are (1, 25), (5, 5). Since (5, 5) only counts as one factor 5, 25 has an odd number of factors.

So, the number of lockers that remain open are the number of perfect squares up to and including 100: 1 x 1, 2 x 2, 3 x 3, 4 x 4, … 10 x 10.

Thus, there are 10 lockers open at the end of this process.

10) *Detect the poisoned bottle from 1000 bottles using 10 test strips.*
We use the fact that the tests take 7 days to run multiple tests at once. We divide bottles into 10 groups, with bottles 0 to 99 mapping to test strip 0, 100 to 199 mapping to strip 1, and so on. On the next day, we map bottle with 0 in their tens place to strip 0, those with 1 in the 10s place to strip 1 and so on. We carry out a similar exercise on the 3rd day as well.

	Days 0 to 7	Days 1 to 8	Days 2 to 9
Strip 0	0xx	x0x	xx0
Strip 1	1xx	X1x	xx1
Strip 2	2xx	X2x	xx2
Strip 3	3xx	X3x	xx3
Strip 4	4xx	X4x	xx4
Strip 5	5xx	X5x	xx5

Strip 6	6xx	X6x	xx6
Strip 7	7xx	X7x	xx7
Strip 8	8xx	X8x	xx8
Strip 9	9xx	X9x	xx9

For example, if on day 7, we have test strip 2 as positive; on day 8, we have strip 4 as positive; and on day 9, we have strip 3 as positive, then we know that the poisoned bottle is bottle #243.

There are edge cases that we need to consider. If on day 8, there is no new result. That means that we put the drop from the poisoned bottle on the same strip on days 0 and 1. In other words, the first and second digits are equal for the bottle.

Similarly, if there is no new result on day 9, we know that the 3rd digit of the poisoned bottle equals the 1st or the 2nd digit. To overcome this situation, we add drops from the bottle to the test strips as below:

	Days 0 to 7	Days 1 to 8	Days 2 to 9	Days 3 to 10
Strip 0	0xx	x0x	xx0	xx9
Strip 1	1xx	X1x	xx1	xx0
Strip 2	2xx	X2x	xx2	xx1
Strip 3	3xx	X3x	xx3	xx2
Strip 4	4xx	X4x	xx4	xx3
Strip 5	5xx	X5x	xx5	xx4
Strip 6	6xx	X6x	xx6	xx5
Strip 7	7xx	X7x	xx7	xx6
Strip 8	8xx	X8x	xx8	xx7
Strip 9	9xx	X9x	xx9	xx8

Now, we can overcome the ambiguous results based on the tests of day 4. For example, for a bottle such as #244, we have:

- Day 7: Strip 2's color changes
- Day 8: Strip 4's color changes
- Day 9: No color change
- Day 10: Strip 5's color changes

We could still have a case in which there is no color change even on day 10. However, that information is enough to help us detect the poisoned bottle. For example, for 2 bottles #676 and #677, we would have the following scenario:

- Day 7: Strip 6's color changes
- Day 8: Strip 7's color changes
- Day 9: No color change

  However, on day 10 if there is no color change, we know that the poisoned bottle is #676. If the poisoned bottle was #677, then the color of strip 8 would have changed.